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RELIABILITY PREDICTION USING THE COX PROPORTIONAL HAZARDS MODEL

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ABSTRACT

Currently, for a variety of mechatronic systems and components, sufficient failure behaviour data are not available. Endurance tests at customer-specific operating conditions provide manufacturers with specific failure time data. However, they are time-consuming and expensive. Findings gained through experiments are valid only for the applied test conditions and loads. On the other hand, developers require, as early as possible, meaningful key figures characterizing the applied components to determine the overall reliability. Often, modified components using the same technology basis are applied with other load profiles, so that available test data can not be used without further steps. Alternatively, one can try to derive sufficiently precise predictions for newly developed components or new application environments from a variety of existing data sets from endurance tests of similar components and other load cases. To this end, well-known regression models of survival analysis have been developed further. To illustrate the transferability to applications for reliability prediction, test data of DC motors from in-house experiments and simulated data sets are adapted to a Cox proportional hazards model.

Index Terms **–** Reliability prediction, Cox proportional hazards model, DC motors, mechatronic systems, regression

1. INTRODUCTION

Manufacturers of mechatronic drive systems investigate the reliability of their products performing endurance tests. The aim of these experiments is to gain findings about the failure behaviour and key figures characterizing it. Tests are expensive, which is the reason, why not all possible load profiles and combinations of impact parameters can be investigated. To provide significant statistical information, the sample parts have to be tested with respect to a specific test goal. Using the test results, the failure probability under the tested load and surrounding conditions can be estimated statistically, which is described in [3]. The statistical statement resulting from the experiment is valid only for the investigated load and test conditions, however.

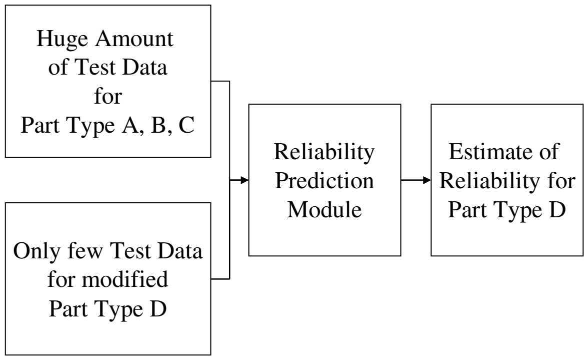

Fig. 1. Vision of a reliability prediction

Suppliers of mechatronic systems are careful when they provide reliability data about their products. If customers want to acquire meaningful information about the reliability of their products, they or the supplier himself need to perform tests for the specific application. In order to be able to estimate the reliability in early product development stages, there exists the alternative to calculate estimates from existing data of similar parts for other load cases and other part sizes. On this basis, a prediction of failure behaviour becomes possible.

For this purpose, stochastic models based on well-known regression models of survival analysis are developed further, adopted and tested for engineering applications. We investigate approaches to be able to analyze test data and apply it for prediction purposes. By a reliability prediction of this manner, it shall become possible to estimate the failure behaviour for untested load cases or new components.

Fig. 1 illustrates the vision presented in this publication. A future aim is to provide a software tool for the prediction of reliability (Reliability Prediction Module) using the Cox model.

2. SURVIVAL ANALYSIS

Lifetimes of mechatronic systems cannot be predicted exactly, they usually depend on many uncontrollable (random) influencing factors. Therefore it is important to set up mathematical models of probability distributions of the lifetime. Suppose that the distribution of the lifetime T of an object is given by the probability distribution function $F(t) = P(T \le t)$, the density $f(t) = dF(t)/dt$ and the failure (hazard) rate λ, where the failure rate at time t is defined as

$$\lambda(t) = \frac{f(t)}{1 - F(t)}.$$

The failure rate, the density and the distribution function provide alternative but equivalent characterizations of the distribution of T. The probability distribution function may be calculated from the failure rate by the well known equation

$$F(t) = 1 - \exp\left\{-\int_0^t \lambda(s)ds\right\}.$$

Comparatively simple lifetime models are given by parametric families of distributions as the Weibull distribution or the lognormal distribution which depend on a finite number of parameters. In many applications the populations of systems are considerably more heterogeneous, such that more complex models should be considered. Models of the so called *Survival Analysis* are particularly useful.

Survival analysis or failure time analysis means the statistical analysis of data, where the response of interest is the time from a well-defined time origin (birth, start of treatment, start of operating a machine) to the occurrence of a given event (death, relapse, failure). In biomedicine lifetimes or survival times are the key examples. In an obvious way these models can be transferred to engineering applications where lifetimes of technical systems are considered.

These more complex models of survival analysis take the following two main features into account:

● Censoring. The most common form of which is right-censoring: Here, the period of observation expires, or an object is removed from the study, before the event of interest (failure) occurs. For example, some of the technical systems under consideration can still be functioning at the end of a lifetime experiment.

● Covariates. The objective may be to incorporate different types of technical systems and different (environmental) conditions during the lifetime experiment like load, temperature or air pressure. This leaves us with a statistical regression problem. Additional variables, so called covariates, are introduced, which characterize the objects under consideration in more detail. The link between these covariates and the lifetime distribution is given in regression models by the intensities of corresponding lifetime models. The intensity is closely related to the failure rate or hazard rate. Roughly, the intensity function is the probability that an object which is at risk now, will fail in the next small unit of time.

3. THE COX PROPORTIONAL-HAZARDS REGRESSION MODEL

One of the most popular regression models is the Cox model (or proportional hazards model). For each object i, i = 1, …, n, there are, in addition to the possibly censored lifetime, k covariates $Y_{i,1},\dots,Y_{i,k}$ observed, which describe the object and the environmental conditions in more detail. The conditional hazard of the lifetime T given the vector of covariates Y at time t is

$$\lambda(t;Y) = \lim_{h \to 0+} \frac{P(T \le t + h \,|\, T > t, Y)}{h}.$$

Cox suggested that this so called intensity could be modeled as the product of an arbitrary unknown deterministic baseline hazard λ_0 and an exponential function with an argument linear in the covariates. This leads to the following model of the intensity for item i:

$$\lambda_i(t) = \lambda_0(t)\ R_i(t)\ \exp\left(\beta_1 Y_{i,1} + \dots + \beta_k Y_{i,k}\right),$$

where R_i is the risk indicator, equal to one as long as object i is observed (at risk). $\beta^T = (\beta_1, \beta_2, \dots, \beta_k)$ is the vector of the unknown regression parameters. Based on the lifetime data of an experiment the baseline hazard function λ_0 and the regression parameters have to be estimated statistically. For each item the data consist of

- the possibly censored failure time T;
- an indicator equal to one if T is a true failure time, zero if it is censored;
- and the vector of explanatory variables Y.

The Cox model itself makes three assumptions: first, that the ratio of the hazards of two objects is the same at all times; secondly, that the explanatory variables act multiplicatively on the hazard; and thirdly, that conditionally on the explanatory variables, the failure times of two individuals are independent. As with all regression models, one also assumes that the explanatory variables have been transformed so that they may be entered without further transformation and that all interactions have been included explicitly.

The regression coefficients β are estimated by maximizing the so-called partial likelihood. Having computed $\hat{\beta}$, the estimated vector of regression coefficients, one can calculate the estimate of the cumulative baseline hazard

$$\Lambda_0(t) = \int_0^t \lambda_0(s)ds$$

explicitly. Estimation of the intensity function itself can be done by taking a smooth derivative of the cumulative hazard. The standard Cox model and the estimation methods are implemented in the most statistic software tools [4, 7].

4. SURVIVAL ANALYSIS APPROACH TO RELIABILITY

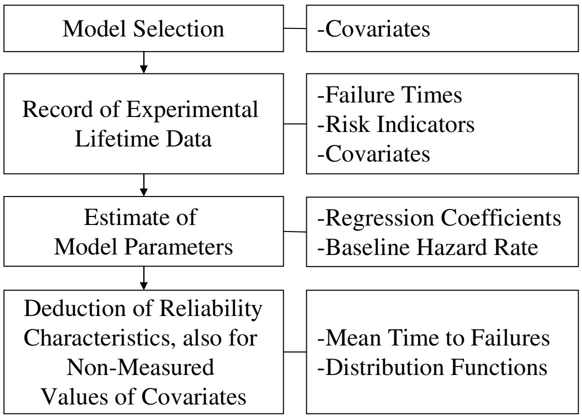

Fig. 2. Steps to predict reliability by means of a Cox model

In mechanical engineering models of survival analysis can be used if in lifetime experiments censoring effects arise. Concurrently, during the tests additional information is available, described by covariates, which characterize the objects under consideration in more detail. Such covariates can, for example, be different loads or a variety of construction forms. [2]

At the beginning of a reliability analysis by means of a Cox model, one has to fix the explanatory variables (the covariates). It is quite challenging to find out a good choice of variables influencing the reliability. Indicator variables also can be used as covariates. They describe for an object of investigation whether a property is present (value = 1) or not (value = 0). During the test phase one observes for each object the failure time, the risk indicator and the covariates. From these data the vector of the regression parameters and the baseline hazard can be estimated. These estimates can be used for the deduction of reliability characteristics like the mean time to failure, quantiles or the whole distribution function. Since a regression model is used, one can predict such characteristics also for values of covariates, which were not actually measured. This means in particular, that one can interpolate (in some sense made precise below) the lifetime distributions between different lines of objects under considerations. To verify the fit of the model to the data, so called Goodness-of-fit tests can be applied.

Fig. 2 summarizes the procedure to generate a failure probability prediction as we performed it. The model approach is chosen, and life time data including the covariates is recorded. Then, algorithms are used to calculate estimates for the model parameters. For arbitrary values of covariates, the result can be illustrated by a plot of the failure probability function. During development, these steps are part of an iteration loop to improve the prediction results. Especially the choice and the implementation of the covariates need to be modified. The fact that the performance of experiments is time-consuming and expensive must be taken into account at iteration. For the end-user, no iteration should be necessary.

5. MODELING OF DC-MOTORS AS AN EXAMPLE

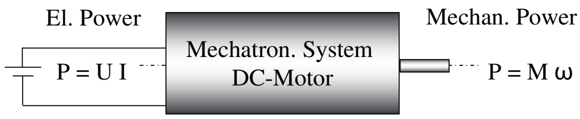

Fig. 3. Abstraction of a DC motor

In the previous chapter, it was shown, how one can proceed to estimate the regression coefficients for a Cox model from a given data set of failure times, risk indicators and explaining covariates. First of all, one must decide which of the possible impact parameters can be used as covariates in the model. Regarding a complex mechatronic system, it might not be obvious, which influence parameters on the system reliability are useful. There are influence parameters which are difficult or impossible to monitor. Besides data gained performing endurance tests, further information might be available using existing data which had been generated for other purposes. If one uses such existing data, the choice of covariates is restricted to the monitored sets of parameters.

In order to obtain reliability information, endurance tests are performed. For the investigations presented in this publication, failure data of 232 parts of three different DC-motor types (12V, 18V, 24V) were available. This data had been recorded until 2006 in a DFG research project [5, 6]. Besides real endurance test data, simulated data were applied to verify the statistical methods.

If one regards DC-Motors with brushes, significant impact parameters on lifetime are operating point, mode of operation and environmental influences. For this publication analysed experiments were run with constant load and rotation speed and at similar surrounding conditions (climate in endurance test room).

The operating point for the considered DC-Motor means the electric current, the associated load torque and the resulting rotation speed [2]. In this example, the operating point is the central covariate that is to be used in the model. Fig. 3 shows a diagram of a DC-motor.

The speed-torque characteristic specifies the mechanical operating point: when a specific torque load is applied, the rotation speed reaches the corresponding speed value and applying the operating voltage, the system responds with the motor current corresponding with the mechanical operating point. If the motor heats up during service, its characteristic changes. The characteristic curve might change from a straight to a curved line.

In order to represent the operating point in the model, one of the parameters motor current, rotation speed or torque can be used as a covariate. Assuming a linear motor characteristic, one can calculate the corresponding parameters respectively. Difficulties occur especially when several motor types with different characteristics and hence different operating points are to be compared. For example, a motor type at a given load torque, experiences an operating point at *M = 300 mNm*, *n = 3000 rpm*, *I = 200 mA*, whereas at an other motor type, the same load torque of *M = 300 mNm* might result in a rotation speed of *n = 3000 rpm* and a current of *I = 400 mA*.

For our experiments, a motor series was chosen, where different motor types posses a nearly identical speed-torque curve. So, the mechanical operating point of different motor types is nearly the same, when driven with corresponding nominal voltage for the motor type. However, the electrical operating point (voltage and current), as well as the current-torque characteristic differs from motor type to motor type. When comparing different motor types of such a motor series, one can use the torque as a reference. When the three different motor types considered at their nominal voltage are driven with the same load torque, the motor types reach about the same rotation speed according to their speed-torque characteristic. But each motor type will be operated at a different voltage and current. To compare different motor types, the motor current is not suitable to be used as a covariate without further steps. Voltage and current are comparable only within one motor type. The current is usable only when comparing data related to just one motor type.

6. EXPERIMENTAL CALCULATIONS

For the investigations, Monte Carlo simulated data were generated. Assuming, that with increasing load, the DC-Motors fail earlier, loads and failure times were assigned to 5 different specimens consisting of 16 parts each. It was set value on the fact that the empirical distribution functions of the 5 different specimens did not intersect and that these failure probabilities did not follow a parametric model like the Weibull or the exponential distribution.

With the simulated failure data of the 80 test samples, the failure probabilities were calculated using the open source Statistics software R. Fig. 4 shows simulated data for a motor type A. The dashed lines form the empirical failure probability of the individual load levels (2.5 mNm, 3.75 mNm, 5 mNm, 6.25 mNm and 7.5 mNm) for the 5 specimen with 16 parts.

The loads and the failure times of all 80 parts build the data base for the calculation using a Cox model, which was set up with one covariate for the load torque. With the help of the Statistics software R, at first the regression parameters and the baseline hazard of the Cox model were determined. Secondly, any

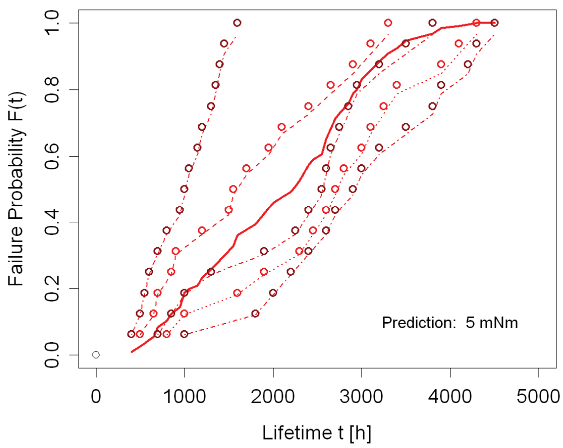

Fig. 4. Example of simulated data sets and prediction curve

desired load case can be calculated specifying the covariate torque. The bold solid line shows the prediction curve of the failure probability distribution function for the 5 mNm load level.

The estimated curve (bold solid line) does not meet the dash-dotted curve (empirical failure probability curve for the 5 mNm load) entirely. This is no contradiction, as the data of all 80 parts were used for the calculation of the bold solid prediction curve by a regression model. In this case, the Cox model has the advantage of the availability of estimated failure probabilities for non-measured values of covariates (here: loads).

Now, let us look at real test data. Through experiments observed failure times of five load levels (2.5 mNm, 3.75 mNm, 5 mNm, 6.25 mNm and 7.5 mNm) of different motor types were available as data base.

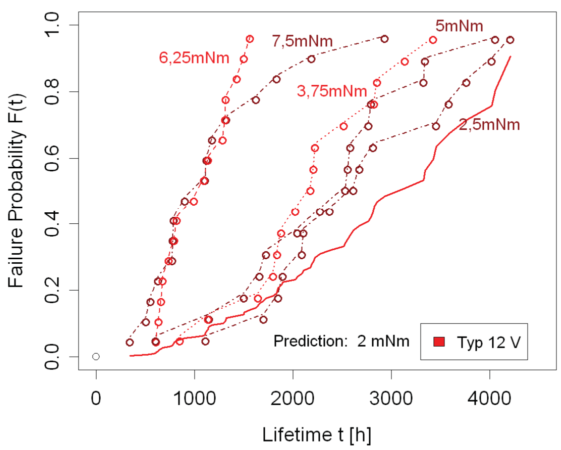

Fig. 5. Real test data for one motor type at different load levels

Fig. 5 shows the failure times of the motor type 12V. For 5 load levels, the empirical failure distribution is displayed (fine dashed lines). Here, a Cox model with the covariate load was applied. The

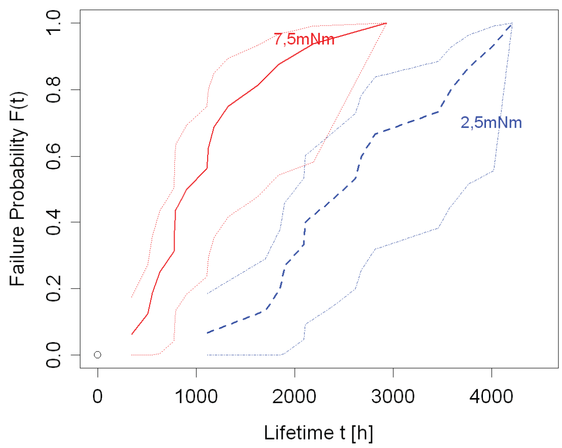

Fig. 6. Two load levels for 12V-motors and associated 95%-confidence intervals

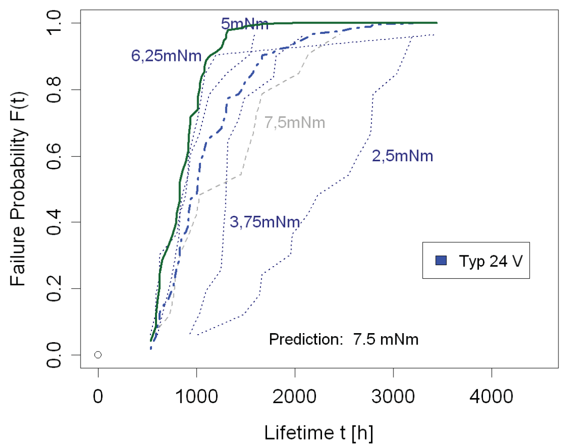

Fig. 7. Different prediction curves for real test data in comparison

prediction curve for the load case 2 mNm (bold solid line) based on the Cox model is situated approximately in the expected area. At the real test data (Fig. 5) the emerging pattern of failure probability curves becomes complicated because of existing deviations of measurement values and partly unknown influences.

The existing uncertainty can be illustrated by confidence intervals. Fig. 6 shows 95% point-wise confidence intervals of the empirical failure probabilities for the load levels 2.5 mNm and 7.5 mNm. A reason for the wide range of these intervals is the small number of measurements. If one increases the number of parts, we expect that the fluctuations decrease. Hence one achieves better predictions.

In Fig. 7, the failure probabilities of five load levels are displayed as an example for the 24V-motor type using fine dashed lines. These load level lines do not follow a monotonous load order. The bold lines represent two different prediction curves for a load case of 7.5 mNm.

For the bold dash-dotted (blue) line, all five load levels for the 24V-motor were entered as data base (2.5 mNm, 3.75 mNm, 5 mNm, 6.25 mNm and 7.5 mNm). The data base for the bold solid (green) line consists only of four load cases for the 24V-motor (2.5 mNm, 3.75 mNm, 5 mNm and 6.25 mNm). The load level of 7.5 mNm was suppressed. For both bold lines, the solid and also the dash-dotted prediction, a Cox model with the load torque as covariate was used.

As different basis data are used, between the two prediction lines shown in Fig. 7 considerable deviations emerge. In the used Cox model, strict monotony of the hazard rate with respect to the load is assumed. In the real experiment, this was not true for the load case of 7.5 mNm, however. The fine gray dashed line in Fig. 7 for the 7.5 mNm load level lays considerably to the right of the empirical failure probability curve for the 6.25 mNm and also for the 5 mNm load level.

This contradicts the natural imagination that, with increasing load, the parts fail earlier. The solid (green) prediction curve lays to a large extend left of the empirical curve for the load level of 6.25 mNm, as expected because of the monotony assumption in the model. The bold dash-dotted (blue) prediction line on basis of all five load levels of the 24V-motor type is also fitted with respect to the monotony assumption in the model. The dash-dotted (blue) prediction curve predicts the load level of 7.5 mNm too far left, to fit the data with least possible deviations to the real data sets, and still meeting the monotony assumption. When we assume the data to precisely project reality, then the model needs to be modified. In this case, we should admit non-linear influences of the load as argument in an exponential function at modeling the hazard rate, for example quadratic or piecewise linear approaches. The routines need to be investigated and modified in further research.

In another simulation experiment, data of four different motor types was artificially generated (Fig. 8). Type 1 was set up in a similar way as described before. This means that to 80 parts, failure times were assigned, consisting of specimens to 16 parts for each of the five different load levels (2.5 mNm, 3.75 mNm, 5 mNm, 6.25 mNm and 7.5 mNm). The data was adjusted so that the specimen sets for each motor type, show the same failure pattern, but the failure time is shifted 4000 h for each type.

The idea of this approach is that from system type to type, similar failure patterns exist, when the types are of similar technology basis. For example, failure mechanisms should be approximately similar, when geometries are just scaled in different sizes. So, it is assumed at this approach, that there is an interrelation between different types of similar technology (stretching or shifting of patterns in the failure probability functions at different load levels).

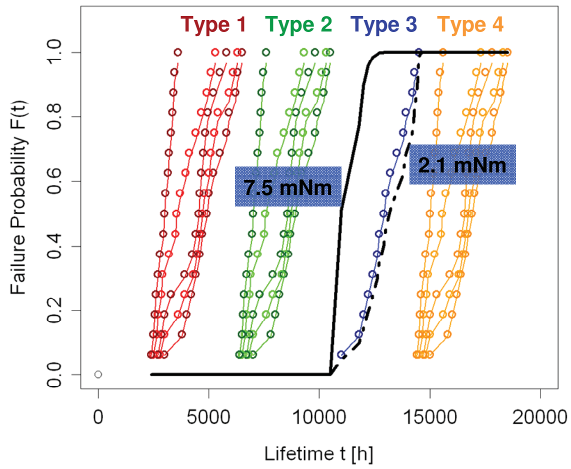

Fig. 8. Predictions for two different load cases on basis of different system types with simulated data as an example

In Fig. 8, failure probability functions for four different system types of similar technology are displayed. The 16 fine solid lines represent the failure times and associated empirical probabilities of the specimens with 16 parts each. The failure times of all 256 data points and associated load levels build the data base for a Cox model with 5 covariates. One covariate is the load torque, the others are indicator variables describing the motor type (1 to 4). For example, we want to indicate a part of type 2. In this case, the value of the covariate for type 2 is 1 and the value of the other types is zero.

The two bold lines are predictions of the failure probabilities for a system of type 3 with load level 2.1 mNm (dash-dotted) and with load level 7.5 mNm (solid).

Although one load level for type 3 is known only, the prediction curves are situated with slight deviations in the expected area, when we assume type 3 to have a shifted failure probability function pattern similar to the other three types. This shows, that the data of the other system types 1, 2 and 4 provide additional input information over the degree of spreading of the curves when the load is increased.

7. RESULTS

The Cox model can be employed as regression model for the reliability prediction. As can be expected from a regression model, it is possible to predict reliability characteristics for loads between and beyond the given boundary areas of the load levels used as data base.

However, one must be careful at modelling. The choice of covariates is of particular importance. Covariates, that are valid only for one type, should not be used without type-specific indicators in order to compare different system types. If data of different motor types are investigated, the implementation of indicator variables is useful.

Investigations at simulated data show, that, using a Cox model, the regression of existing data can be turned into praxis. Real data sometimes does not follow strict monotony of the hazard rate with respect to some covariates as is assumed in the standard Cox model used in the calculation experiments. Using the Cox model at this kind of data, prediction curves can be derived as well, but modifications might become necessary. To illustrate the prediction results in this publication, the Cox model was compared to the empirical failure probability estimates. The estimated failure probability distributions by a Cox model meet the empirical with certain deviations.

In future, the Cox model shall also be investigated with other test data, for example of mechatronic gears [1], concerning the applicability for reliability prediction.

In this publication, the goal was to illustrate the general procedure along simple examples. The statistical analysis has much potential for the future of reliability prediction.

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