URN (Paper): urn:nbn:de:gbv:ilm1-2011iwk-022:1

56TH INTERNATIONAL SCIENTIFIC COLLOQUIUM
Ilmenau University of Technology, 12 – 16 September 2011
URN: urn:nbn:gbv:ilm1-2011iwk:5

VARIOUS ADAPTIVE CONTROL STRATEGIES APPLIED TO A BIO-INSPIRED RECEPTOR MODEL

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ABSTRACT

The motivation of this work is formed by the biologically inspired vibrissa sensory system. It is modeled as a spring-mass-damper oscillator with a spacial disturbance signal acting on the frame and an inner active element that generates a force acting on the mass. Both the system parameters and the excitation signal are supposed to be unknown. The goal is to achieve a predefined movement of the mass, such as tracking a set point trajectory or stabilization. Thus, a controller is needed to act on the system using the control force as input in such a way that the desired behavior is generated. This is done by means of high-gain-stabilization. Like its biological paradigm, the receptor is in a permanent state of adaption. This means that recurring disturbances, such as wind acting on the vibrissa, are damped in order to achieve λ-stabilization. To achieve this control goal and at the same time deal with unknown systems, adaptive controllers are introduced.

Index Terms— Adaptive control, PID-feedback, fuzzy control, uncertain system, bio-inspired system, high-gain-stabilization.

1. INTRODUCTION

In nature there are various senses that allow animals to perceive their environment, such as the senses of touch. The sense of vibration is a special case of perceiving touch. Vibrations are an important piece of environmental information that insects rely on, especially arachnids, such as spiders and scorpions. To perceive vibrations, they have different types of sensilla (see Fig. 1).

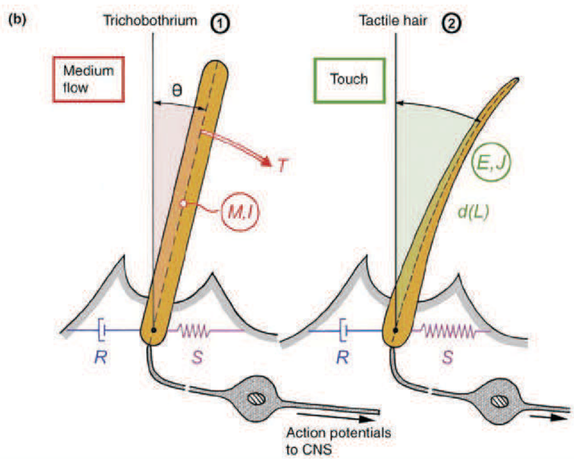

Fig. 1. Spider sensilla, [1]

Vertebrates may also possess the sense of vibration, such as cats, rats and sea lions. They can perceive vibrations with the help of their vibrissae.
Although these biological vibration sensors have a different physiology, they share common properties. When in touch with an oscillating object, they are stimulated by mechanical oscillation energy, which is then transmitted to the receptor cells (see [11]). The sensibility of these cells is continuously adjusted so that the receptor system converges to the rest position, despite the continued excitation (see [5]). This means that the receptors are in a permanent state of adaptation and the continuous excitation is cancelled out. Therefore, the information about the continuous excitation is considered irrelevant, once it has been perceived. If however a different excitation, such as a sudden deviation of the vibrissa sensor, occurs, this information is relevant and the receptor has to be sensitive to perceive it. Therefore, the adaption process has to ensure sensitivity.

The receptor is modeled as a spring-mass-damper system as shown in Fig. 2. It consists of a mass m under the action of an internal force $u(t)$ within a rigid frame that is forced by an unknown external time dependent excitation $f(t)$.

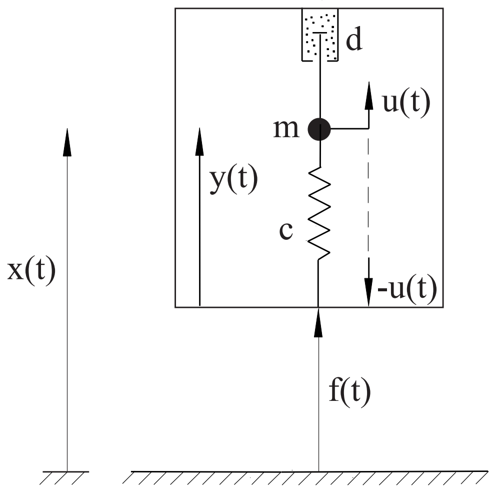

Fig. 2. Receptor Model: Spring-mass-damper oscillator, [3]

The goal is to act on the system in such a way that the system output $y(t)$ is λ-tracked, i.e., the system output is forced into an error neighborhood λ around a set point trajectory $y_{\text{ref}}(t)$. The principle of λ-tracking (see [6]) is shown in Fig. 3. The tolerated tracking error λ can be chosen arbitrarily with $\lambda > 0$. However it is set to a fixed value.

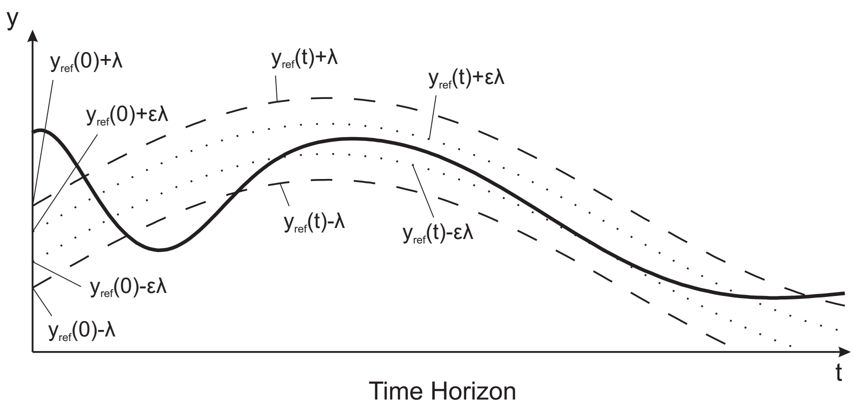

Fig. 3. Visualization of λ-tracking of the system output $y(t)$, [10]

If $y_{\text{ref}}(t) \equiv 0$, the problem is known as λ-stabilization. In this case, the receptor is supposed to remain in its equilibrium state (rest position).
It is important to point out that all system parameters are supposed to be unknown because of the sophisticated nature of the biological system. This is why traditional control methods fail, as they rely on the knowledge of those

parameters. Adaptive controllers are a way of dealing with this control problem.
To achieve λ-tracking, an adaptive output feedback controller in the form of a PD-controller is implemented. This way, the control gain is not pre-specified by the designer, but is a function of time. The gain is determined by an adaption law that uses information only from the system output to generate an adaptive control gain. The following adaptive controller is taken from [3], modified from [6]:

$$\begin{array}{rcl} e(t) & := & y(t) - y_{\text{ref}}(t) \\ u(t) & = & -k(t)\, e(t) - k(t)\, \dot{e}(t) \\ \dot{k}(t) & = & \max\left\{0, \|e(t)\| - \lambda\right\}^2 \end{array} \tag{1}$$

The controller works as follows: If the absolute error value $\|e\|$ is greater than or equal to λ, k is being increased by the square of the difference between $\|e\|$ and λ. If $\|e\|$ is smaller than λ, the increase of k is set to zero, so there is no further increase. If the system output leaves the λ-neighborhood again, k is increased further. This constitutes a dead-zone behavior. In this paper, various approaches in designing such an adaptor are investigated.

Goal of the paper
There are a number of goals to be achieved by the adaptive controllers. First of all, λ-tracking has to be achieved. It is also imperative to keep the sensitivity of the system high. If, for example, a recurring excitation signal $f(t)$ acts on the system, the adaptor is supposed to increase the control gain as high as necessary so that the system output may reach the λ-tube. If this recurring excitation subsides or if it is replaced by one with a much lower amplitude, the system is supposed to be sensitive and, as a result, the control gain is supposed to decrease to a level that is absolutely necessary to ensure λ-tracking. Controller (1) is incapable of this behavior.
Another desired quality of adaptive controllers is finite time behavior. Since all system parameters are unknown and the level of the necessary control gain cannot be anticipated, it is also unknown at which point in time λ-tracking will be achieved. Controllers with finite time behavior enable the user to specify a time at which the control goal will be achieved at the latest.

2. SYSTEM PROPERTIES & ADAPTIVE CONTROL

The equations of motion of the system modeled in Fig. 2 are:

$$\begin{array}{rcl} m(\ddot{y} + \ddot{f}) + d\dot{y} + cy & = & u(t) \\ \ddot{y} + \dfrac{d}{m}\dot{y} + \dfrac{c}{m}y & = & \dfrac{1}{m}u(t) - \ddot{f} \end{array}$$

with output $y(t)$ being the relative coordinate of the mass m. This system can be expressed in state-space notation, with $x_1 := y$ and $x_2 := \dot{y}$ (see [2]):

$$\begin{array}{rcl} \begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} & = & \underbrace{\begin{bmatrix} 0 & 1 \\ -\frac{c}{m} & -\frac{d}{m} \end{bmatrix}}_{=:\mathbf{A}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}}_{=:\mathbf{b}} u - \begin{bmatrix} 0 \\ \ddot{f} \end{bmatrix} \\ y & = & \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{=:\mathbf{c}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{array} \tag{2}$$

To investigate the stability of the closed-loop system and thus the structure of the required feedback law, the strict relative degree of system (2) is determined. It indicates which derivative order of y the input u directly acts on: begin with $r = 1$ and increase r by 1 at each iteration until $\det\left(\mathbf{CA}^{r-1}\mathbf{B}\right) \neq 0$, while $\mathbf{CA}^{r-2}\mathbf{B} = 0$. Then, r equals the strict relative degree of the system. The computation for system (2) yields:

$$r = 1 : \mathbf{cb} = 0$$

$$r = 2 : \mathbf{CA}^{r-1}\mathbf{B} = \mathbf{cAb} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -\frac{c}{m} & -\frac{d}{m} \end{bmatrix} \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} = \frac{1}{m} \neq 0$$

Therefore, the strict relative degree of the system is 2. To ensure stability, the feedback structure must contain feedback terms up to the order of $r - 1 = 1$ (see [12]). Therefore, a derivative term is needed. Thus, all adaptive controllers discussed here have a PD-feedback structure. The general controller structure is given by:

$$\left.\begin{array}{rcl} e(t) & := & y(t) - y_{\text{ref}}(t) \\ u(t) & = & -k(t)\, e(t) - \kappa\, k(t)\, \dot{e}(t) \\ \dot{k}(t) & = & \text{Adaptor,} \end{array}\right\} \tag{3}$$

with κ being a factor to set the ratio between P- and D-gain of the controller. Otherwise, the global control gain $k(t)$ would serve as both P- and D-gain. It also ensures equality of dimensions of the terms. Independent gains with independent adaption laws are not discussed.
For simulation purposes, the following parameters are fixed to these constants:

Table 1. Global simulation parameters

m	$1\,\mathrm{kg}$
d	$5\,\frac{\mathrm{Ns}}{\mathrm{m}}$
c	$10\,\frac{\mathrm{N}}{\mathrm{m}}$
λ	$0.2\,\mathrm{m}$

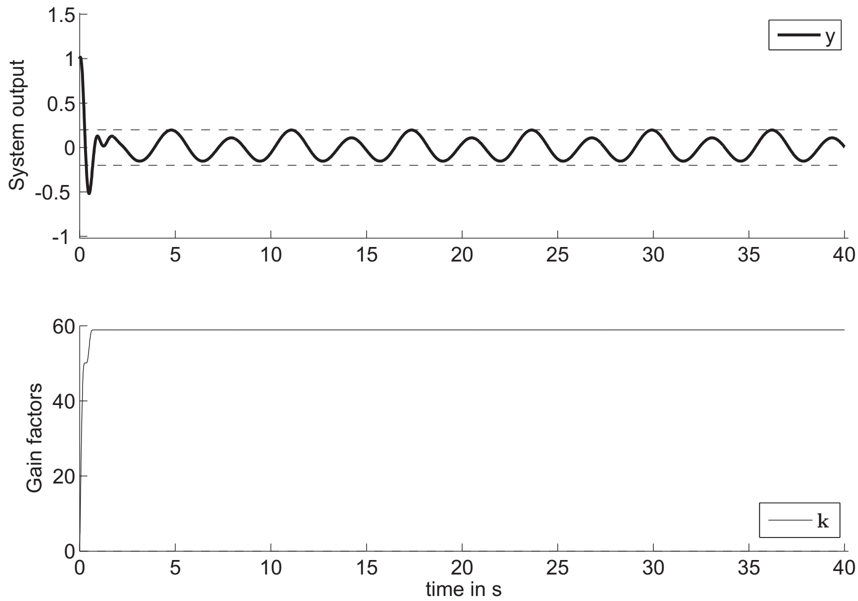

Fig. 4. Simulation of adaptive controller (1) with $\gamma = 500$

Fig. 4 shows a simulation of controller (1). λ-tracking is achieved. However, the controller is incapable of decreasing k, which stays constant after the λ-tube has been reached.
In order to ensure the aforementioned ability to both increase and decrease k if necessary, the adaptive controller is extended (see [3]). Also, the exponent for the increase term is varied according to the proximity of the system output the λ-tube. Thus, the growth of k is not halted when values smaller than 1 are squared. Now, a square root function is in effect:

$$\left.\begin{array}{rcl} e(t) & := & y(t) - y_{\text{ref}}(t) \\ u(t) & = & -k(t)\,e(t) - \kappa\,k(t)\,\dot{e}(t) \\ \dot{k}(t) & = & \left\{\begin{array}{cl} \gamma\,(\|e(t)\| - \lambda)^2 & ,\lambda + 1 \leq \|e(t)\| \\ \gamma\,(\|e(t)\| - \lambda)^{\frac{1}{2}} & ,\lambda \leq \|e(t)\| < \lambda + 1 \\ 0 & ,\|e(t)\| < \lambda \quad \wedge \quad t - t_e < t_d \\ -\sigma k(t) & ,\|e(t)\| < \lambda \quad \wedge \quad t - t_e \geq t_d \end{array}\right. \end{array}\right\} \tag{4}$$

with $\lambda > 0, \gamma \gg 1, \sigma > 0, t_d > 0, \kappa > 0$.
The term $-\sigma k(t)$ enables the controller to decrease k exponentially as soon as a pre-specified time t_d has passed since the system output entered the λ-tube. Fig. 5 shows two simulations of controller (4).

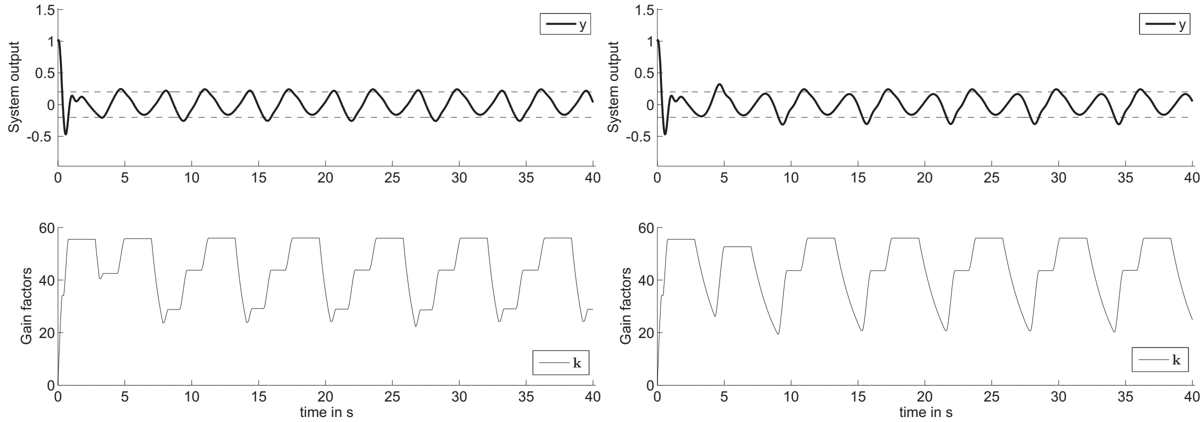

Fig. 5. Simulation of adaptive controller (4) with $\sigma = 0.5$ (left) and $\sigma = 1$ (right)

The exponential decrease coefficient is chosen to $\sigma = 0.5$, which shows smoother behavior than $\sigma = 1$.

Now, the effect of the coefficient κ in the feedback law is investigated. It is used to set a relation between the D- and P-gains of the controller:

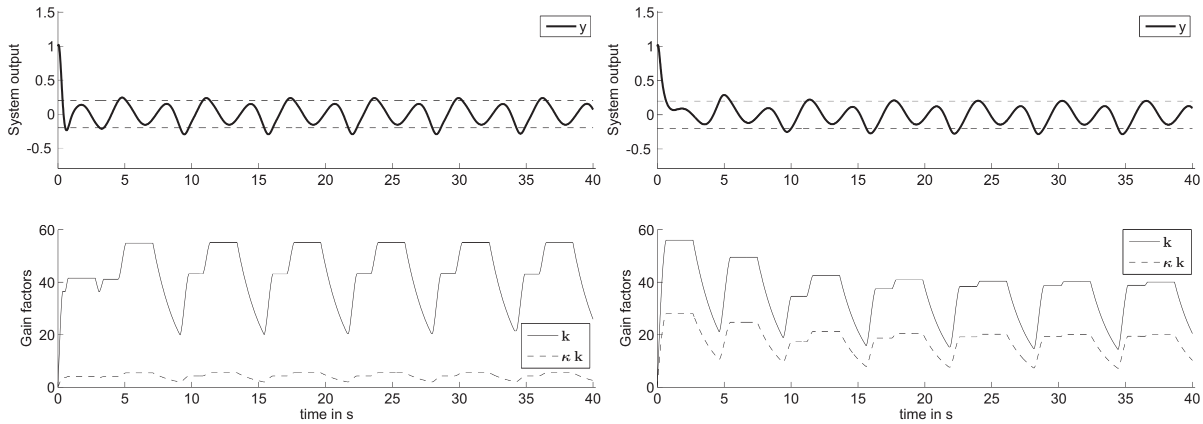

Fig. 6. System performance with $\kappa = 0.1\,\mathrm{s}$ (left) and $\kappa = 0.5\,\mathrm{s}$ (right)

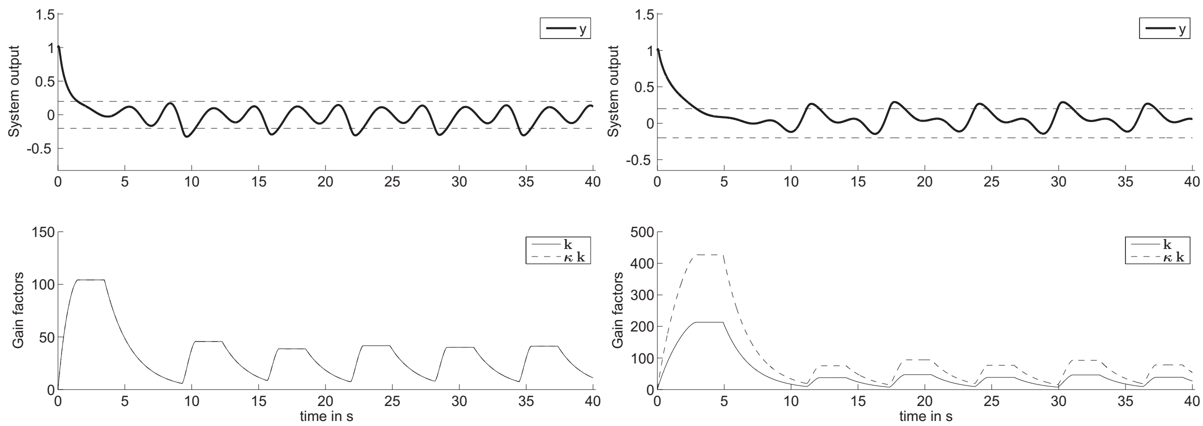

Fig. 7. System performance with $\kappa = 1\,\mathrm{s}$ (left) and $\kappa = 2\,\mathrm{s}$ (right)

As shown in Figs. 6 and 7, the settling time of the system increases as κ increases. On order to keep the settling time low and at the same time incorporate a D-feedback term into the controller, it is set to $\kappa = 0.1\,\mathrm{s}$ (see [10]).

3. ADAPTIVE λ-TRACKING: FIRST-ORDER CONTROLLER

In order to better keep the system output inside the λ-tube once it has been entered, a smaller $\varepsilon\lambda$-neighborhood is introduced in the adaptor. Now, k is increased if $y(t)$ is outside the $\varepsilon\lambda$-tube. If ε is set to a value smaller than 1, this helps keeping the system output inside the λ-tube, even if the $\varepsilon\lambda$-tube is left due to disturbances. Such an adaptive controller that achieves λ-tracking of unknown systems is taken from [10]:

$$\left.\begin{array}{rcl} e(t) & := & y(t) - y_{\text{ref}}(t) \\ u(t) & = & -k(t)\, e(t) - \kappa\, k(t)\, \dot{e}(t) \\ \dot{k}(t) & = & \begin{cases} \gamma\, (\|e(t)\| - \varepsilon\lambda)^2 & , \varepsilon\lambda + 1 \leq \|e(t)\| \\ \gamma\, (\|e(t)\| - \varepsilon\lambda)^{\frac{1}{2}} & , \varepsilon\lambda \leq \|e(t)\| < \varepsilon\lambda + 1 \\ 0 & , \|e(t)\| < \varepsilon\lambda \quad \wedge \quad t - t_e < t_d \\ -\sigma \left(1 - \frac{\|e(t)\|}{\varepsilon\lambda}\right) k(t) & , \|e(t)\| < \varepsilon\lambda \quad \wedge \quad t - t_e \geq t_d \end{cases} \\ k(0) & = & 0 \end{array}\right\} \tag{5}$$

with $\lambda > 0, \gamma \gg 1, \sigma > 0, t_d > 0, 0 < \varepsilon \leq 1, \kappa > 0$.

The adaptor works as follows: If y is far away from the $\varepsilon\lambda$-tube, k is increased with the square of the distance between y and the $\varepsilon\lambda$-tube. If it is close to the $\varepsilon\lambda$-tube, k is increased with the square root function of that distance. This ensures that the growth of k is not halted by the proximity to the $\varepsilon\lambda$-tube. As soon as the distance becomes smaller than 1, the square function would lead to an even smaller number and k would be increased very slowly. If the $\varepsilon\lambda$-tube has been entered, k is kept constant for a pre-defined time t_d. After, k is decreased exponentially while y is still inside $\varepsilon\lambda$-tube. During this decrease period, the factor $\left(1 - \frac{\|e(t)\|}{\varepsilon\lambda}\right)$ becomes smaller, as y deviates from $y = 0$. This results in a smaller exponential decrease of k. If $y = \lambda$, the term becomes 0. Therefore, the decrease of k is halted as y approaches the boarder of the λ-tube. If it is left, the growth of k starts again.

Figures 8, left and 8, right show the simulation of the system with the standard reduction term from [3] and the new reduction term in (5), respectively. The parameters used here are given in Table 2.

Table 2. Simulation parameters for adaptive reduction term

κ	0.1 s
γ	150
t_d	2 s
σ	0.5
$y_{\text{ref}}(t)$	1 m
$\ddot{f}(t)$	0

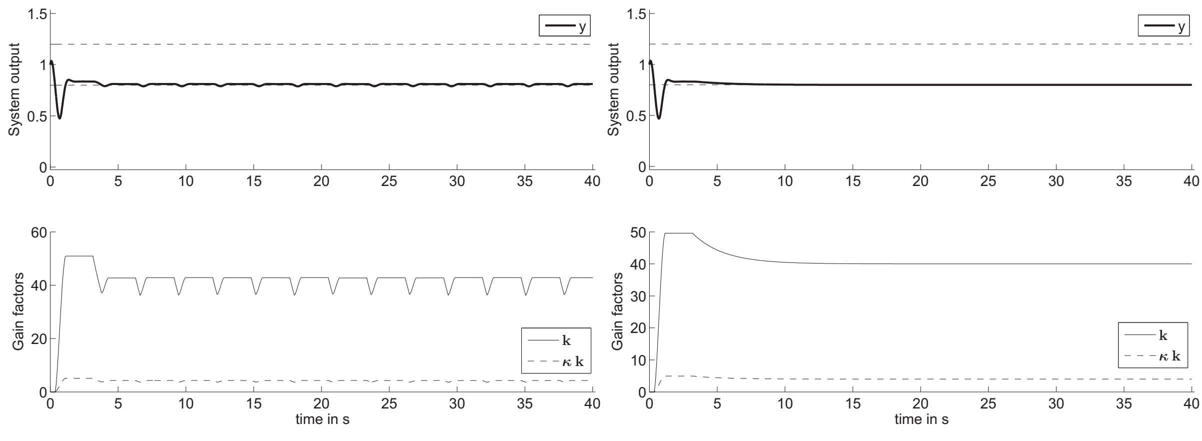

Fig. 8. System performance with regular reduction term from [3] (left) and with adapted reduction term from (5) (right)

To show the functionality of the function $\delta\left(\|e(t)\|, \lambda\right)$, the simulations are conducted without any excitation signal and with a set point trajectory of $y_{\text{ref}}(t) \equiv 1\,\text{m}$. This requires the controller to keep a certain level of k,

while it can be reduced as long as y stays inside the λ-tube. The absence of any disturbance shows the abilities of the new reduction term much more clearly: The decrease of k progresses much smoother as shown in Fig. 8, right. It is no longer a strictly exponential decrease, but rather one restrained by the proximity to the outer borders of the λ-tube. The reduction is halted, when y reaches the edge of the λ-tube, which means that $\|e\| = \lambda$.
Note that there no longer exists an alternating behavior of decrease and increase of k, while y exceeds λ repeatedly as shown in Fig. 8, left.

As mentioned above, the adaptors are supposed to have finite time capabilities. This is not yet present in adaptor (5) and will be provided in the following section.

4. FINITE TIME CONTROL

One finite time controller can be found in [9]:

$$\left.\begin{array}{rcl} e(t) & := & y(t) - y_{\text{ref}}(t) \\ u(t) & = & -k(t)\, e(t) \\ k(t) & = & k_0 + \gamma_1 \int\limits_0^t d_\lambda(e(s))\|e(s)\| ds + \gamma_2 \begin{cases} \frac{\|e(t)\|^2}{T-t} & , t \in [0, t^*) \\ k^* & , t \geq t^* \end{cases} \\ d_\lambda(e) & := & \begin{cases} (\|e(t)\| - \lambda) & , \|e\| \geq \lambda \\ 0 & , \|e\| < \lambda \end{cases} \\ k^* & := & \frac{\|e(t^*)\|^2}{T-t^*} \\ t^* & := & \min\left\{t \in [0,T); \|e(t)\| = \frac{3}{4}\lambda\right\} \end{array}\right\} \tag{6}$$

with $\lambda > 0, \gamma_1 > 1, \gamma_2 > 1, T > 0$.
The controller (6) was used to control pH-values in a biogas tower reactor, which constitutes a different system class. It serves only as a proportional feedback controller. For the purpose of applying it to systems of strict relative degree two, the feedback structure has to be changed into a PD-type, i.e., adding the term $-\kappa\, k(t)\, \dot{e}(t)$:

$$u(t) \quad = \quad -k(t)\, e(t) - \kappa\, k(t)\, \dot{e}(t)$$

The adaption law for determining $k(t)$ in (6) gives an explicit equation for the gain parameter k, not for $\dot{k}$. It is a zero-order controller, since there exists no ODE inside the controller.
Controller (6) shows several limitations:

- In order to reach the finite time goal at $t = T$, the gain k may be increased arbitrarily. This is the result of a pole that exists in the adaptor equation. However, such a sudden and strong increase of k causes the ODE system to become very stiff, which is problematic in numerical simulations. Also, a large value for k implies that the physical controller, such as a force-generating actuator as given in the system shown in Fig. 2, has to be able to generate an arbitrarily large control input, which is not feasible.
- There exist two control parameters in controller (6): γ_1 and γ_2. They have to be set by the user in advance and are therefore not part of the adaptive nature of the controller. The parameter γ_1 contributes to the regular increase of k outside the λ-tube, whereas γ_2 contributes to the increase of k caused by the finite time feature of the controller. It would be advantageous to modify the controller in such a way that the setting of γ_1 and γ_2 can be done independently of the system parameters.
- There exists no term for decreasing k in controller (6). However, this feature is absolutely necessary.

In order to overcome these deficiencies, various modifications are made to the adaption law (see [10]).

First, various terms with finite time behavior are introduced into the controller equations. They also have a pole at a specified time T, so that k may be increased infinitely, if necessary. The goal is to find a function $g_f(t,T)$ that supports the increase of k well before T, so that no sudden increase resulting in a stiff ODE may occur. Some new finite time terms are:

- the addition of a linear term to the existing one: $g_{f1}(t,T) := \left(\frac{\overline{T}}{T-t} + \frac{T}{\overline{T}^2} \cdot t\right)$
  with $\overline{T} = 1\,s$ (ensuring correct dimensions),

- the inverse hyperbolic tangent: $g_{f2}(t,T) := \operatorname{atanh}\left(\frac{t}{T}\right)$,

- a term using the natural logarithm: $g_{f3}(t,T) := \ln\left(\frac{T}{T-t}\right)$.

These terms are put in the controller instead of the standard finite time term:

$$k(t) = k_0 + \gamma_1 \int\limits_0^t d_\lambda(e(s))\|e(s)\|ds + \gamma_2 \begin{cases} \|e(t)\|^2 \cdot g_f(t,T) & ,t \in [0,t^*) \\ k^* & ,t \geq t^* \end{cases}$$

with $\gamma_1 > 1, \gamma_2 > 1, T > 0$.
To evaluate the performance of each modification, numerical simulations have been conducted with a set of fixed system parameters given in Table 1 to determine the minimum values for γ_1 and γ_2, at which the simulation is carried out without any numerical errors. If the parameters are lowered too far, the system becomes too stiff and the numerical simulation halts with an error message. This indicates that the setting of γ_1 and γ_2 would generate an unfeasible adaptive controller. Although this method can only provide an exemplary investigation, it enables a comparison between the finite time term modifications. Tables 3 to 5 show the results.

Table 3. Minimum values of γ_1 and γ_2 for linear term modification

γ_1	0	20	50	70	90	100	110
γ_2	30	30	40	25	15	5	0

Table 4. Minimum values of γ_1 and γ_2 for inverse hyperbolic tangent modification

γ_1	0	20	50	60	65	70	100	105	110
γ_2	1700	1400	850	710	650	320	120	30	0

Table 5. Minimum values of γ_1 and γ_2 for natural logarithm modification

γ_1	0	20	50	70	100	105	110
γ_2	510	430	260	190	110	20	0

The simulations show that if the finite time terms are turned off, i.e., $\gamma_2 = 0$, the value for γ_1 is the same for each modification. This is to be expected, as the adaption laws are equal without their finite time terms.
The linear term modification shows a nearly constant relation between γ_2 and γ_1 (see Fig. 9).

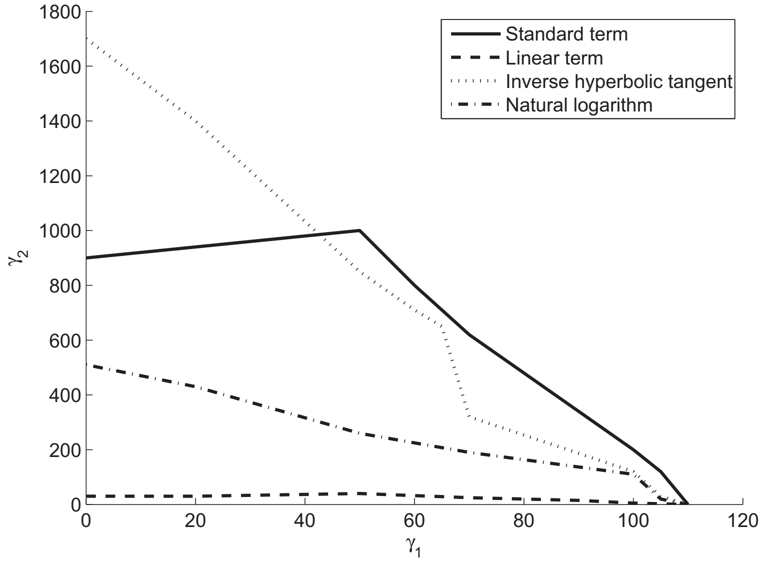

Fig. 9. Plot of γ_1 versus γ_2 for each finite time term

This means that the setting of γ_1 is almost irrelevant, as long as γ_2 is set to a certain value. In this example, it is never greater than 40, which is also the lowest overall value for all terms investigated here. Therefore, the linear term is superior to the other modifications concerning the setting of γ_2 and γ_1.

When applying a quadratic exponent to the error value $\|e(t)\|$, poor controller performance is to be expected when the error is smaller than 1 (see controller (5) ff.). Small values are diminished even further, so that there is only a small effect of the respective part of the equation. This affects the finite time terms, which are weighted with the factor $\|e(t)\|^2$. It can be expected that the performance of the controller improves, if the exponent is changed. To investigate this, the linear term modification is simulated again, with a generalized exponent α:

$$g_{f4}(\|e\|, t, T) \quad = \quad \|e(t)\|^{\alpha} \left(\frac{\overline{T}}{T - t} + \frac{T}{\overline{T}^2} \cdot t \right)$$

with $\alpha > 0, T > 0, \overline{T} = 1\,s$.
The results are shown in Table 6.

Table 6. Minimum values of γ_1 and γ_2 with linear term modification $g_{f4}(\|e\|, t, T)$

$\alpha = 2$	γ_1	0	20	50	70	90	100	110
	γ_2	30	30	40	25	15	5	0
$\alpha = 1$	γ_1	0	10	50	80	100	110	
	γ_2	10	5	4	3	2	0	
$\alpha = \frac{1}{2}$	γ_1	0	20	50	70	100	110	
	γ_2	4	2	6	3	1	0	

The overall values for γ_1 and γ_2 were further reduced by changing α from 2 to 0.5. It is therefore a useful modification.
In conclusion, the controller performance improves if the error value exponent α is lowered to 0.5, from an original value of 2. Additionally, the finite time term with the additional linear term modification outperforms all other

modifications, including the standard term. Therefore, the improved finite time controller is:

$$\left.\begin{array}{rcl} e(t) & := & y(t) - y_{\text{ref}}(t) \\ u(t) & = & -k(t)\, e(t) - \kappa\, k(t)\, \dot{e}(t) \\ k(t) & = & k_0 + \gamma_1 \int\limits_0^t d_\lambda(e(s)) \|e(s)\| ds + \gamma_2 \begin{cases} \|e(t)\|^{\frac{1}{2}} \left(\frac{\overline{T}}{T-t} + \frac{T}{\overline{T}^2} \cdot t \right) & , t \in [0, t^*) \\ k^* & , t \geq t^* \end{cases} \\ d_\lambda(e) & := & \begin{cases} (\|e(t)\| - \lambda) & , \|e\| \geq \lambda \\ 0 & , \|e\| < \lambda \end{cases} \\ t^* & := & \min \left\{ t \in [0, T); \|e(t)\| = \frac{3}{4}\lambda \right\} \\ k^* & := & \|e(t^*)\|^{\frac{1}{2}} \left(\frac{\overline{T}}{T-t^*} + \frac{T}{\overline{T}^2} \cdot t^* \right) \end{array}\right\} \tag{7}$$

with $\lambda > 0, \gamma_1 > 1, \gamma_2 > 1, T > 0, \overline{T} = 1\, s, \kappa > 0$.
It is still not possible to decrease k, once λ-tracking has been achieved. In controller (5), the reduction of k is achieved by an exponential decrease term after a predefined time t_d has passed. It would be advantageous to include this ability into the finite time controller. However, this cannot be achieved easily, as the controller is of zero order. An exponential decrease term is much easier to implement into a first-order controller. Thus, the finite time capability is transferred from controller (7) into controller (5).
The adaption law from controller (7) is:

$$k(t) = k_0 + \gamma_1 \int\limits_0^t d_\lambda(e(s)) \|e(s)\| ds + \gamma_2 \begin{cases} \|e(t)\|^{\frac{1}{2}} \left(\frac{\overline{T}}{T-t} + \frac{T}{\overline{T}^2} \cdot t \right) & , t \in [0, t^*) \\ k^* & , t \geq t^* \end{cases}$$

To incorporate this into the first-order-controller, the derivative of the term is computed:

$$\dot{k}(t) = \gamma_2 \begin{cases} \frac{1}{2} \|e(t)\|^{-\frac{1}{2}} \operatorname{sign}(\|e(t)\|)\, \dot{e}(t) \left(\frac{\overline{T}}{T-t} + \frac{T}{\overline{T}^2} \cdot t \right) + \|e(t)\|^{\frac{1}{2}} \left(\frac{\overline{T}}{(T-t)^2} + \frac{T}{\overline{T}} \right) & , t \in [0, t^*) \\ 0 & , t \geq t^* \end{cases}$$

The term is added to the increase terms for $\dot{k}$ in controller (5), and the new controller now reads:

$$\left.\begin{array}{rcl} e(t) & := & y(t) - y_{\text{ref}}(t) \\ u(t) & = & -k(t)\, e(t) - \kappa\, k(t)\, \dot{e}(t) \\ \dot{k}(t) & = & \begin{cases} \gamma \left((\|e(t)\| - \varepsilon\lambda)^2 + f_T(t) \right) & , \varepsilon\lambda + 1 \leq \|e(t)\| \quad \wedge \quad t < T \\ \gamma \left(\|e(t)\| - \varepsilon\lambda \right)^2 & , \varepsilon\lambda + 1 \leq \|e(t)\| \quad \wedge \quad t \geq T \\ \gamma \left((\|e(t)\| - \varepsilon\lambda)^{\frac{1}{2}} + f_T(t) \right) & , \varepsilon\lambda \leq \|e(t)\| < \varepsilon\lambda + 1 \quad \wedge \quad t < T \\ \gamma \left(\|e(t)\| - \varepsilon\lambda \right)^{\frac{1}{2}} & , \varepsilon\lambda \leq \|e(t)\| < \varepsilon\lambda + 1 \quad \wedge \quad t \geq T \\ 0 & , \|e(t)\| < \varepsilon\lambda \quad \wedge \quad t - t_e < t_d \\ -\sigma \left(1 - \frac{\|e(t)\|}{\varepsilon\lambda} \right) k(t) & , \|e(t)\| < \varepsilon\lambda \quad \wedge \quad t - t_e \geq t_d \end{cases} \\ f_T(t) & := & \frac{1}{2} \|e(t)\|^{-\frac{1}{2}} \operatorname{sign}\|e(t)\| \dot{e}(t) \left(\frac{\overline{T}}{T-t} + \frac{T}{\overline{T}^2} \cdot t \right) + \|e(t)\|^{\frac{1}{2}} \left(\frac{\overline{T}}{(T-t)^2} + \frac{T}{\overline{T}} \right) \end{array}\right\} \tag{8}$$

with $\lambda > 0, \sigma > 0, \gamma \gg 1, t_d > 0, 0 < \varepsilon \leq 1, T > 0, \overline{T} = 1\, s, \kappa > 0$.
Controller (8) is simulated using the following parameters:

Table 7. Simulation parameters for controller (8)

ε	0.7
κ	0.1 s
γ	10
σ	0.2
t_d	2 s
T	5 s
$y_{\text{ref}}(t)$	0
$\ddot{f}(t)$	$3\,\sin(t) + 10\,\cos(2t)\,\frac{\text{m}}{\text{s}^2}$

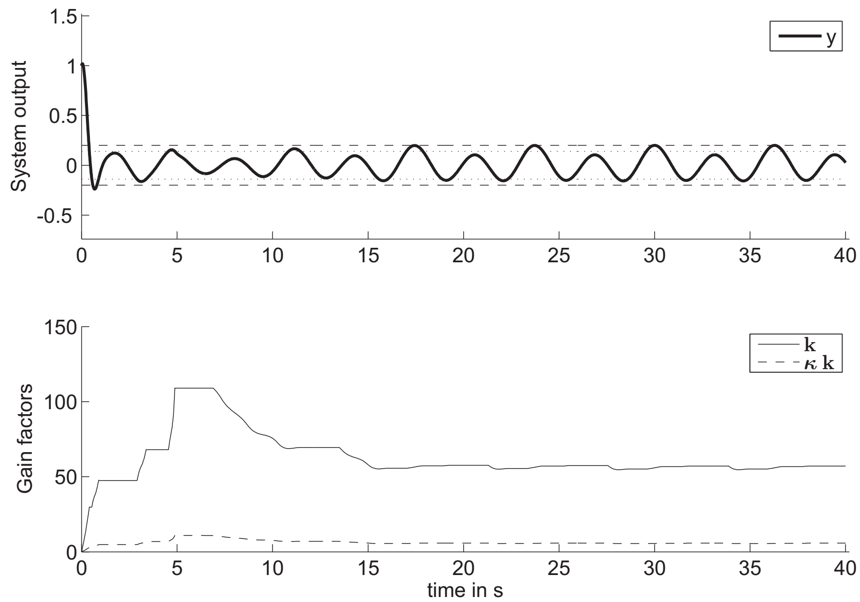

Fig. 10. Simulation of controller (8)

As shown in Fig. 10, k increases just before $T = 5\,s$ and keeps constant for $t_d = 2\,s$. After, k decreases exponentially while y is inside the $\varepsilon\lambda$-tube and increases, as soon as $\|e\| > \varepsilon\lambda$. Since γ can be chosen to be small ($\gamma = 10$) in this example, it is conceivable that γ does not depend much on the system or the disturbance signal. This improves the adaptive nature of controller (8), since γ might be chosen independently of the system. A further investigation of the influence of γ is not carried out at this point.
Thus, an adaptive controller that achieves λ-tracking with finite time capabilities is found.

5. FUZZY CONTROL

Fuzzy control systems are another way of designing controllers for unknown systems. The design process does not require any knowledge of the system parameters and is therefore suitable for the task of designing an adaptive controller for the vibrissa sensory system. There are a number of different approaches in designing a fuzzy controller:

- It is conceivable to directly implement a fuzzy inference system to act as a controller. The output of the fuzzy system would act directly on the system. This strategy is unsuitable for the control objectives at hand. The output of a fuzzy system is bounded, which makes it difficult to design a controller that can be applied on a vast number of different systems. The maximum control value u cannot be anticipated.

- Another possibility is to design a fuzzy controller that changes the parameters of an underlying classical controller, such as a PD-controller. This is similar to the adaptive controllers discussed above, where the gain parameter k is changed according to an adaption law. This strategy is implemented here as well. A fuzzy control system is designed to generate a global gain factor k, which is used to drive a PD-controller.

Again, it is important to point out that the output of the fuzzy controller is still bounded by the choice of the output membership functions. Therefore, when generating the gain parameter k, there exists always an upper limit of k. Since it cannot be anticipated how high k needs to be in a specific case, the direct computation of k is unfeasible. It would compromise the adaptive nature of the controller. Therefore, a different approach is introduced. It consists of a very simple adaption law for k, whose parameter is changed using a fuzzy controller. The basic structure of the controller is:

$$\left.\begin{array}{rcl} e(t) & := & y(t) - y_{\text{ref}}(t) \\ u(t) & = & -k(t)\, e(t) - \kappa\, k(t)\, \dot{e}(t) \\ \dot{k}(t) & = & \sigma_f(t)\, k(t) \\ \sigma_f(t) & = & \text{Fuzzy Controller}\left(\|e(t)\|, t, T, t_d, \varepsilon\lambda\right) \\ k(0) & = & 1 \end{array}\right\} \tag{9}$$

with $\lambda > 0, 0 < \varepsilon \leq 1, t_d > 0, T > 0, \kappa > 0$.

Note that the basic adaptive structure of the controller consists of a local exponential increase (or decrease) of k. The rate σ_f of this increase is determined by the fuzzy controller, and therefore bounded. However, the value of k is not bounded.

The rule set of a fuzzy controller can be obtained in two different ways: based on expert knowledge or based on measurement data. Since the goal is to design an adaptive controller that is applicable to a large number of systems, the design based on measurement data is unfeasible. It would only be useful for one specific system. Therefore, expert knowledge is used to design a rule set (see [13]).

A total of six rules is sufficient to describe the desired behavior of the controller:

- $(\mathbf{e}_{\textbf{high}} \wedge \mathbf{T}_{\textbf{low}}) \rightarrow \sigma_{\textbf{medium}}$
  If the error value is outside the $\varepsilon\lambda$-tube, and the finite time T is not close, the increase of k is medium.
- $(\mathbf{e}_{\textbf{high}} \wedge \mathbf{T}_{\textbf{medium}}) \rightarrow \sigma_{\textbf{high}}$
  If the error value is outside the $\varepsilon\lambda$-tube, and the finite time T is somewhat close, the increase of k is high.
- $(\mathbf{e}_{\textbf{high}} \wedge \mathbf{T}_{\textbf{high}}) \rightarrow \sigma_{\textbf{very high}}$
  If the error value is outside the $\varepsilon\lambda$-tube, and the finite time T is very close, the increase of k is very high.
- $(\mathbf{e}_{\textbf{high}} \wedge \mathbf{T}_{\textbf{passed}}) \rightarrow \sigma_{\textbf{medium}}$
  If the error value is outside the $\varepsilon\lambda$-tube, and the finite time T has passed, the increase of k is medium.
- $(\mathbf{e}_{\textbf{low}} \wedge \mathbf{t}_{\textbf{e, low}}) \rightarrow \sigma_{\textbf{zero}}$
  If the error value is inside the $\varepsilon\lambda$-tube, and the time t_d has not passed, keep k constant.
- $(\mathbf{e}_{\textbf{low}} \wedge \mathbf{t}_{\textbf{e, high}}) \rightarrow \sigma_{\textbf{neg}}$
  If the error value is inside the $\varepsilon\lambda$-tube, and the time t_d has passed, decrease k.

Note that these rules incorporate all aspects of the desired control behavior into the controller: a dead-zone behavior with an $\varepsilon\lambda$-tube, finite time behavior and the ability to decrease k in order to ensure sensitivity. The knowledge is taken from controller (8) to mimic its behavior.

Note that the membership functions for input and output are chosen only by experience from experimenting with other adaptive controllers. To ensure finite time behavior, it is necessary to enable k to grow infinitely, if the situation requires it. This is not possible, since the output of the fuzzy controller is always bounded, as discussed above. Therefore, an adequately large value for the maximum of σ_f has been chosen. However, there may be cases where the growth of k is not sufficient to achieve finite time behavior.

The fuzzy controller (9) is tested in simulations. Table 8 shows the parameters used.

Table 8. Simulation parameters for controller (9)

ε	0.7
κ	0.1 s
T	5 s | 3 s | 1 s
t_d	2 s
$y_{\text{ref}}(t)$	0
$\ddot{f}(t)$	$3\,\sin(t) + 10\,\cos(2t)\,\frac{\text{m}}{\text{s}^2}$

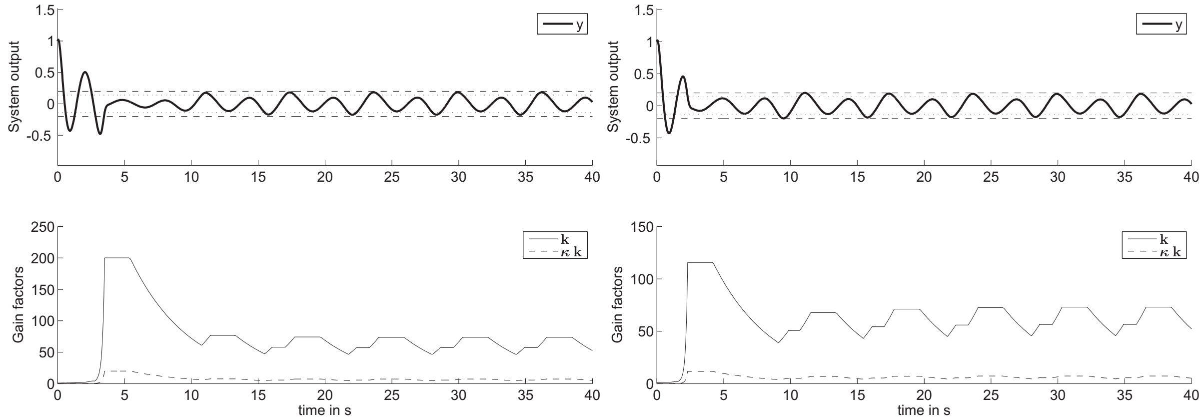

Fig. 11. Simulation of controller (9) with $T = 5\,\mathrm{s}$ (left) and $T = 3\,\mathrm{s}$ (right)

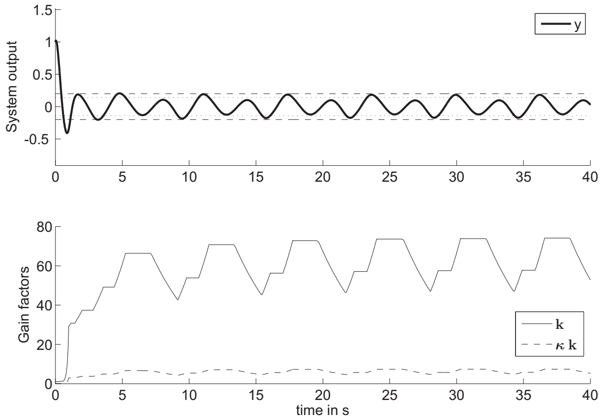

Fig. 12. Simulation of controller (9), $T = 1\,\mathrm{s}$

Figures 11 and 12 show the results of the simulation. The fuzzy controller (9) shows good performance and finite time behavior. Therefore, an adaptive fuzzy controller suitable to achieve λ-tracking with finite time behavior is found.

6. COMPARATIVE SIMULATIONS

6.1. Adaptive λ-stabilization of the receptor model

The controllers (i.e., controllers (8) and (9)) are now tested and compared with each other in order to determine if one adaptive controller surpasses the other regarding performance. The spring-mass-damper system is investigated under the influence of the excitation signal as given in Fig. 13.

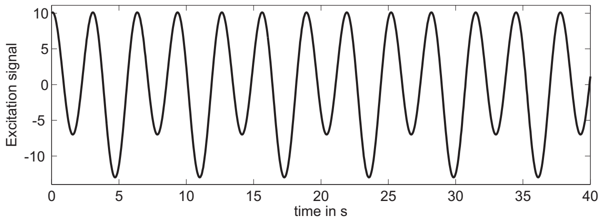

Fig. 13. Plot of excitation signal $\ddot{f}(t)$

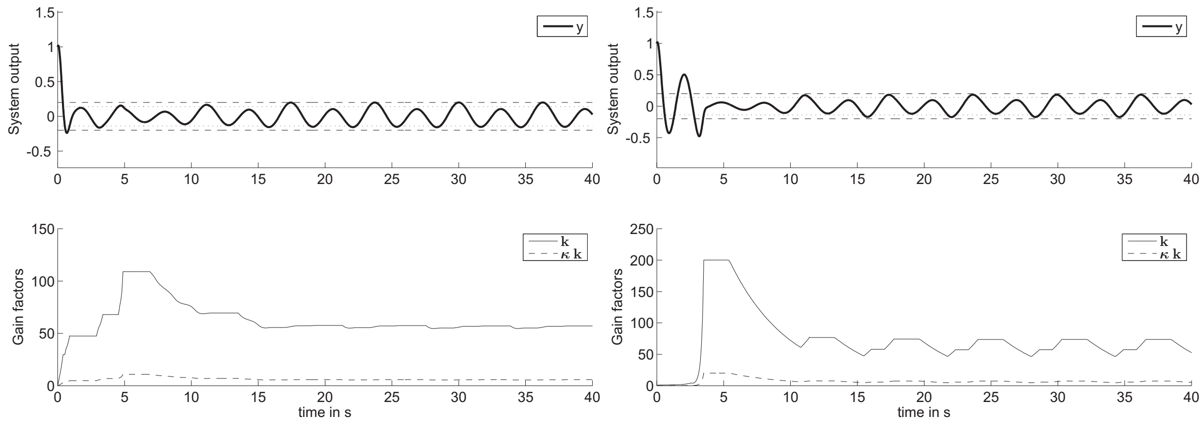

Fig. 14. Simulation of controllers (8) (left) and (9) (right) with excitation signal $\ddot{f}(t)$

Controller (8) shows better overall performance. The $\varepsilon\lambda$-tube is reached much faster than with the fuzzy controller (9). Furthermore, k does not oscillate as much as with controller (9), once the $\varepsilon\lambda$-tube has been entered and k is being decreased and increased periodically. The fuzzy controller shows a strong increase of k at the beginning of the simulation in order to reach the finite time goal. Afterwards, k is reduced to a much lower value. However, this behavior is unfavorable, since it increases the system's stiffness unnecessarily at the beginning of the simulation.
In conclusion, for the example with the parameters chosen in this simulation, controller (8) outperforms the fuzzy controller (9).
In order to show the ability to keep the system's sensitivity high, both controllers are simulated with another disturbance signal $\ddot{f}_1(t)$. The corresponding function is:

$$\ddot{f}_1(t) = 2\,\cos(t)\,\left(1 + 3\,e^{-0.5(t-20.5)^2}\right)\,\frac{\mathrm{m}}{\mathrm{s}^2}$$

Figure 15 shows the disturbance signal $\ddot{f}_1(t)$ used in the following simulations.

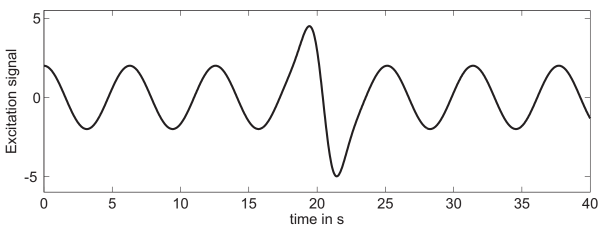

Fig. 15. Plot of excitation signal $\ddot{f}_1(t)$

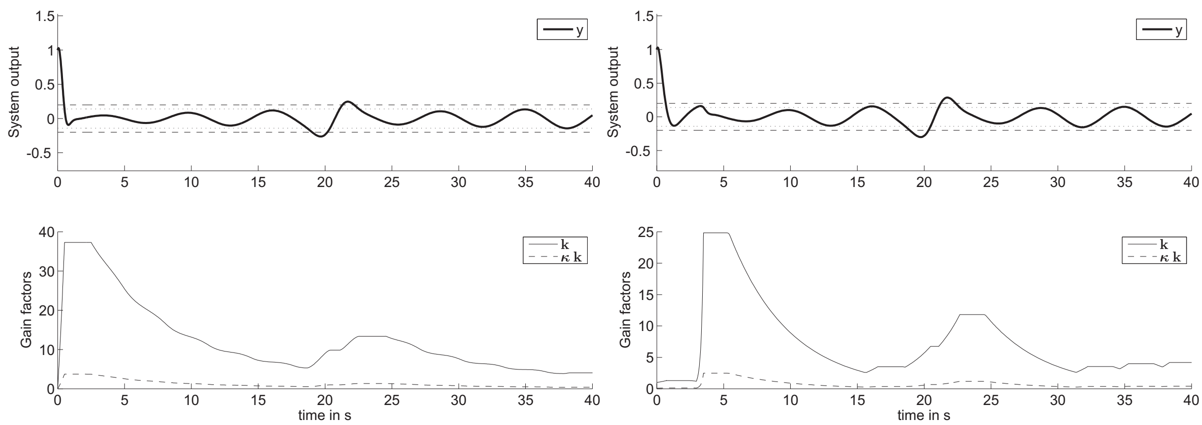

Fig. 16. Simulation of controllers (8) (left) and (9) (right) with excitation signal $\ddot{f}_1(t)$

Controller (8) enables the system to be sensitive to a sudden peak in the excitation signal, as shown in Fig. 16. The gain k is increased to compensate for the peak in $\ddot{f}_1(t)$ at $t = 20.5\,s$. Afterwards, k is decreased again to the previous low level that is absolutely necessary to damp the recurring excitation. Controller (9) also achieves the desired sensitivity in the system. The response of controller (9) to the excitation is comparable to the one of controller (8).

In conclusion, controller (8) outperforms controller (9). It shows

- better transient behavior of the system output, although the peak of the initial increase of k is much higher than with controller (9),
- shorter settling time and
- overall faster adaption of k.

6.2. PID-feedback extension to adaptive λ-stabilization

So far, there were only P- or PD-controllers discussed. When designing classic controllers for linear time-invariant (LTI) systems, the usual approach is to implement a PID-controller. It consists of a proportional, a derivative and an integral term, each with its specific gain parameter.

The idea behind incorporating an integral part into the control feedback is the reduction of steady-state errors. The drawback of this modification is that an integral term may lead to an overshoot of the system output.

To implement the PID-feedback structure into the adaptive controllers, the feedback law is extended:

$$u(t) \quad = \quad -\underbrace{k(t)\,e(t)}_{P-term} - \underbrace{\kappa\,k(t)\,\dot{e}(t)}_{D-term} - \underbrace{\eta\,k(t)\int_0^t e(\tau)\mathrm{d}\tau}_{I-term}$$

with $\kappa, \eta > 0$.

Remark 6.1. *κ and η have to be greater than 1, since the spectrum of the gain matrix is positive. Therefore, all feedback terms need to have a negative sign.* ◇

There now exists the additional tuning parameter η in the feedback law with $[\eta] = \frac{1}{s}$. It can be chosen by the designer to implement a relation between the P- and I-gain.

Controller (8) is investigated using a PID-feedback. The controller is now:

$$\left.\begin{array}{rcl}
e(t) & := & y(t) - y_{\text{ref}}(t) \\
u(t) & = & -k(t)\,e(t) - \kappa\,k(t)\,\dot{e}(t) - \eta\,k(t)\int_0^t e(\tau)\mathrm{d}\tau \\
\dot{k}(t) & = & \begin{cases}
\gamma\left((\|e(t)\| - \varepsilon\lambda)^2 + f_T(t)\right) & ,\varepsilon\lambda + 1 \le \|e(t)\| \quad \wedge \quad t < T \\
\gamma\left(\|e(t)\| - \varepsilon\lambda\right)^2 & ,\varepsilon\lambda + 1 \le \|e(t)\| \quad \wedge \quad t \ge T \\
\gamma\left((\|e(t)\| - \varepsilon\lambda)^{\frac{1}{2}} + f_T(t)\right) & ,\varepsilon\lambda \le \|e(t)\| < \varepsilon\lambda + 1 \quad \wedge \quad t < T \\
\gamma\left(\|e(t)\| - \varepsilon\lambda\right)^{\frac{1}{2}} & ,\varepsilon\lambda \le \|e(t)\| < \varepsilon\lambda + 1 \quad \wedge \quad t \ge T \\
0 & ,\|e(t)\| < \varepsilon\lambda \quad \wedge \quad t - t_e < t_d \\
-\sigma\left(1 - \frac{\|e(t)\|}{\varepsilon\lambda}\right)k(t) & ,\|e(t)\| < \varepsilon\lambda \quad \wedge \quad t - t_e \ge t_d
\end{cases} \\
f_T(t) & := & \frac{1}{2}\|e(t)\|^{-\frac{1}{2}}\text{sign}\|e(t)\|\dot{e}(t)\left(\frac{\overline{T}}{T-t} + \frac{T}{\overline{T}^2}\cdot t\right) + \|e(t)\|^{\frac{1}{2}}\left(\frac{\overline{T}}{(T-t)^2} + \frac{T}{\overline{T}}\right)
\end{array}\right\} \qquad (10)$$

with $\lambda > 0, 0 < \varepsilon \le 1, \gamma > 1, \sigma > 0, t_d > 0, T > 0, \overline{T} = 1\,s, \kappa > 0, \eta > 0$.

Controller (10) is simulated using the parameters given in Table 9. The set point of $y_{\text{ref}}(t) = 1\,\text{m}$ is chosen since it does not constitute an equilibrium of the system. Thus, steady-state errors will occur and be dealt with using the I-term of the feedback structure.

Table 9. Simulation parameters for PID-controller (10)

ε	0.7
κ	$0.1\,\mathrm{s}$
η	$0\,\mathrm{Hz} \mid 0.1\,\mathrm{Hz} \mid 0.5\,\mathrm{Hz}$
γ	10
T	$5\,\mathrm{s}$
t_d	$2\,\mathrm{s}$
σ	0.2
$y_{\mathrm{ref}}(t)$	$1\,\mathrm{m}$
$\ddot{f}(t)$	$3\,\sin(t) + 10\,\cos(2t)\,\frac{\mathrm{m}}{\mathrm{s}^2}$

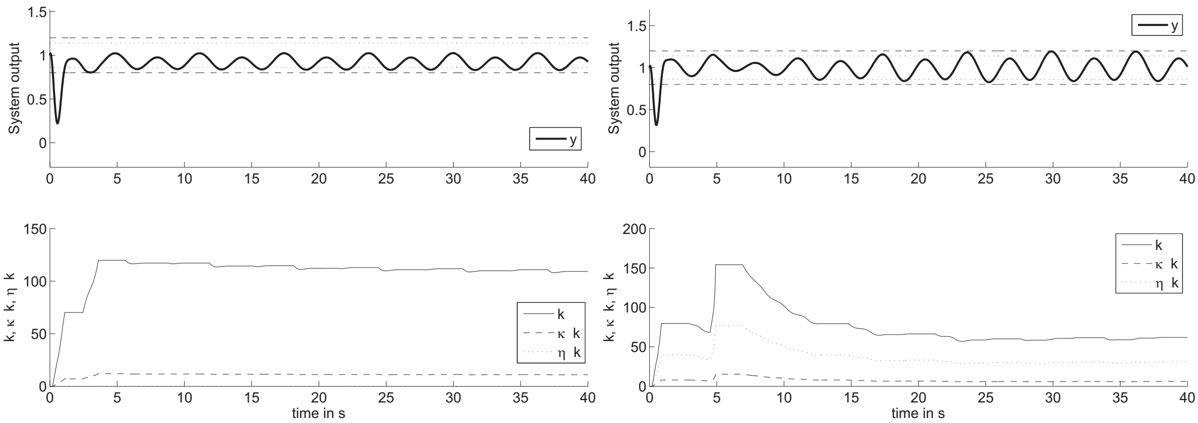

Fig. 17. Simulation of PID-controller (10) with $y_{\mathrm{ref}} \equiv 1\,\mathrm{m}$: $\eta = 0\,\mathrm{Hz}$ (left) and $\eta = 0.5\,\mathrm{Hz}$ (right)

Figure 17 shows that controller (10) performs better in eliminating steady-state errors when the I-gain is introduced. The system output y oscillates around the center of the $\varepsilon\lambda$-tube (Fig. 17, right), rather than its border (Fig. 17, left). Also, the final value for k is much smaller ($k \approx 70$) with the I-gain than without it ($k \approx 120$).
In conclusion, the addition of an integral term to the feedback structure is useful. It leads to better system performance, as it eliminates steady-state errors.

6.3. Adaptive λ-tracking of an inverted pendulum

Now, the stabilization of an inverted mathematical pendulum is investigated. Consider such a pendulum:

$$\begin{aligned} m\,L^2\,\ddot{\varphi}_p(t) + m\,g\,L\sin\left(\varphi_p(t)\right) &= u(t) \\ \text{with} & \\ m = 1, g = 1, L &= 1 \qquad \text{(dimensionless chosen)} \\ \ddot{\varphi}_p(t) + \sin\left(\varphi_p(t)\right) &= u(t) \end{aligned}$$

where φ_p is the angular displacement, $u(t)$ an external control torque. Mass, gravitational constant and length of the pendulum are set to 1 for simplification. The goal is to act on the pendulum with $u(t)$, so that it stays at the instable equilibrium $(\varphi_p, \dot{\varphi}_p) = (\pi, 0)$. This constitutes a nonlinear system with additional nonlinearity caused by the disturbance $\ddot{f}(t)$ and the adaptive controller with the time-varying gain $k(t)$. In state-space notation with $\varphi_1 = \varphi_p$ and $\varphi_2 = \dot{\varphi}_p$, the pendulum system is:

$$\begin{aligned} \begin{bmatrix} \dot{\varphi}_1 \\ \dot{\varphi}_2 \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 0 \\ -\sin(\varphi_1) \end{bmatrix} + \begin{bmatrix} 0 \\ -\ddot{f} \end{bmatrix} \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} \end{aligned} \tag{11}$$

With the computation

$$\mathbf{cb} = 0$$
$$\mathbf{cAb} = 1,$$

the strict relative degree of system (11) is 2. Controller (10) is simulated acting on the pendulum system with $y_{\text{ref}}(t) = \pi$. The corresponding simulations are shown in Fig. 18.

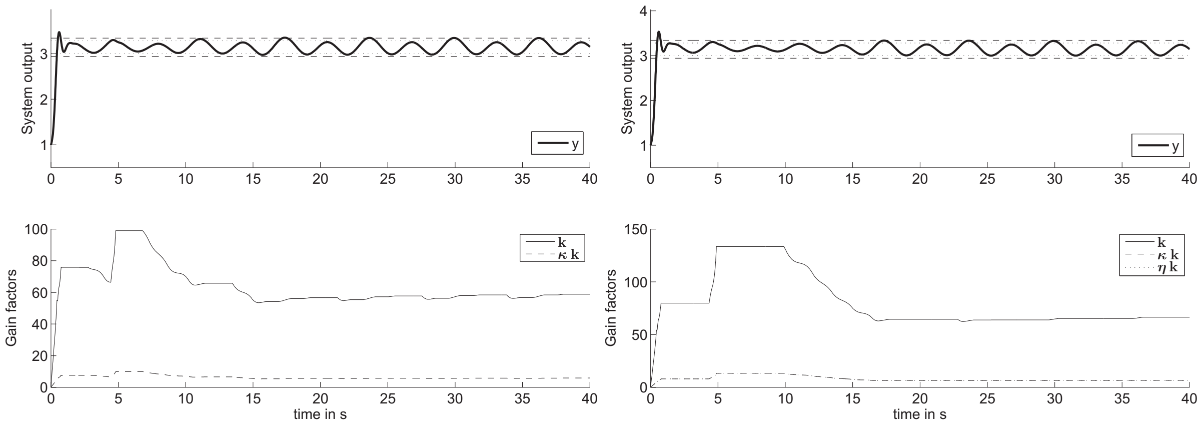

Fig. 18. Simulation of controller (10) with $\eta = 0\,\text{Hz}$ (left) and $\eta = 0.1\,\text{Hz}$ (right) acting on the inverted pendulum (11)

Controller (10) is able to achieve λ-stabilization of the nonlinear pendulum system (11) as well. However, the incorporation of the I-term as shown in the simulation in Fig. 18 (left) has almost no effect on the system performance. This is to be expected, as an equilibrium position is to be reached by the controller.

7. CONCLUSION AND OUTLOOK

Controller (10) emerges as the recommendation for an adaptive controller which incorporates the findings of this work and shows best overall performance. It shows

- best transient behavior,
- proper stabilization inside the λ-tube,
- shortest settling time and
- overall fast adaption of k.

However, the fuzzy controller (9) can be considered as an alternative. It provides a much simpler adaptive architecture and the ability to include more expert knowledge via the use of additional linguistic rules. If the short-term transient behavior is of little concern, the fuzzy controller constitutes an efficient alternative to other adaptive controllers.

There are various aspects of adaptive controllers that can be improved and modified in future work on the subject.

- **Avoidance of error derivative**
  Since the derivative term is difficult to measure, it might be possible to implement an observer in order to estimate the system state (see [4]). If this is also not possible, it would be best to omit the derivative term. This would avoid the occurrence of noise in the feedback loop. However, such a term is imperative to achieve stability. Thus, methods of not having to differentiate the system output can be investigated.
  One possibility to do so is the control with output delay feedback (see [7]). With delay feedback, the output derivative is approximated by computing a difference quotient with a fixed time span:

$$\dot{y} \approx \frac{y(t) - y(t-h)}{h}$$

with $h > 0$.
This method computes a value for $\dot{y}$, which is not exact, but might be sufficiently approximated if h is chosen sufficiently small. However, there remains an error in the derivative feedback term.

- **Constrained control input**
In technical realizations of controllers, there usually exists a limit for the control value that cannot be exceeded. This is quite obvious, as there are no actuators that can generate an infinite force, for example. Therefore, some adaptive controllers may not be implemented in certain applications, as they rely on the possibility to increase the control value as high as necessary. In order to cope with this issue, controllers with constrained input values might be investigated (see [8]).

- **Shortening of finite time term**
The term responsible for finite time behavior in controller (10) might be shortened and still produce the desired performance. If further investigations can show that a shortened term provides a comparable performance, it would help in simplifying the adaption law and therefore help with the compliance of the corresponding requirement. This is another possibility for future investigations.

- **Intelligent control**
The fuzzy controller is built upon expert knowledge that is used to form the rule set of the controller. This knowledge is not given a priori and is obtained from experimenting with previous controllers. However, it is possible to generate expert knowledge by using intelligent control methods, such as artificial neural nets. The expert knowledge – or "intelligence" – in a neural net is obtained by training the controller with data generated by the system. However, this training process takes time and therefore diminishes the adaptive capabilities of the controller.

8. ACKNOWLEDGEMENT

The authors thank Prof. Dr. rer. nat. habil. (em.) Joachim Steigenberger (Ilmenau) for fruitful discussions, hints and continuous interest.

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