THE WATT BALANCE ROUTE TOWARDS A NEW DEFINITION OF THE KILOGRAM

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ABSTRACT

The kilogram is the last unit of the international system (SI) still based on a material artefact, the international prototype of the kilogram (IPK). The comparisons made in the last hundred years have clearly revealed a long term relative drift between the IPK and a set of copies kept under similar conditions. Since the long term stability is one of the major conditions set on the SI base units, this situation is no longer satisfactory and a new definition of the mass unit becomes a priority for the metrology community. A promising route towards a new definition based on fundamental constants is given by the watt balance experiment which links the mass unit to the Plank constant.

***Index Terms*—** Kilogram, Planck constant, System of units, watt balance.

1. INTRODUCTION

Today, the kilogram is the last unit of the International System of Units (SI) still based on an artifact. Its present definition in the SI is: "The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram (IPK)". The international prototype $\mathcal{K}$, kept at the Bureau International des Poids et Mesures (BIPM), is a cylinder of platinum-iridium alloy (Pt 90% - Ir 10% in mass) whose height ($39\,\mathrm{mm}$) is equal to its diameter (see Figure 1). It has been machined in 1878, together with several copies some years later, in an alloy devised by Johnson Matthey. Six copies were designated as official copies and are kept in the same conditions as the international prototype. Seventeen others were given (at that time) to the member states of the meter convention to materialize their national prototype. Since then, other countries have joined the meter convention and new national prototypes have been machined and added to the existing set of international prototype copies.

Since 1880, only three comparisons of the international prototype with the official copies and the seventeen original national prototypes have been performed

Fig. 1. The Swiss official kilogram (prototype No 38).

[1]. Figure 2 summarizes these comparisons. Despite the cleaning-washing procedures used before each comparison, the successive measurements have shown a drift between the international prototype and its copies, whose relative mean value is of the order of $3 \cdot 10^{-8}$. The scatter between individual values is $1 \cdot 10^{-7}$ in one century.With the present definition of the mass unit, it is impossible to assign this drift to the IPK or to the copies (or eventually to both). Moreover, variations of the mass unit directly reflect on the ampere definition and therefore on the whole set of electrical units.

Since it is now possible to compare two mass standards made out of the same material with an uncertainty of about $1\,\mu$g, the instability among the international mass prototypes - including IPK - is a major contribution to the final uncertainty. Clearly, such a situation is no longer satisfactory for one of the base units of the SI. There is now a general consensus in the metrology community that the time for a redefinition of the kilogram has come. The new definition should link

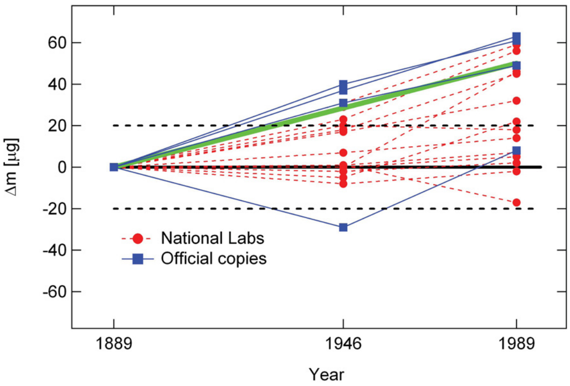

Fig. 2. The relative change in mass of four out of the six official copies (circles) and fourteen national prototypes (squares) with respect to the mass of the international prototype kilogram. The horizontal solid line is the assumed constant value of $\mathcal{K}$ according to the definition. The slope of the other solid line corresponds to the average mass drift of the official copies. The horizontal dotted lines represent the uncertainty to be reached before considering a new definition of the kilogram.

the kilogram to fundamental constants with a relative uncertainty of a few parts in 10^8. Several experiments have been attempted to realize the new definition [2, 3]. Up to now, the most successful electrical approach has been the watt balance which was proposed by B. Kibble in 1975 [4]. Its principle consists in linking the mass unit to the Planck constant h using the equivalence between the electrical and the mechanical power. Such a definition would not only allow realizing the unit of mass at different places at the same time but also improve the consistency of the SI and drastically reduce the uncertainties on a large number of other constants. All units depending on the kilogram such as the ampere, the mole or the candela will no longer depend on the behavior of a material artefact.

Several watt balances are now in operation around the world (see [5] for the latest review). At the National Physical Laboratory (NPL, UK) and at the National Institute of Standards and Technology (NIST, USA), moving coil apparatus have been in operation for a long time and have already produced several results for the Planck constant. At the Laboratoire National de Métrologie et d'Essais (LNE, France), a watt balance is under development and should soon provide a value for h. Other experiments are in progress at the BIPM and at the National Institute of Metrology (NIM, China).

Recently, the NPL instrument was transferred to the National Research Council (NRC, Canada) where the experimental work will be pursued. In 1997, the development of a watt balance started at the Federal Office of Metrology (METAS). After more than ten years of continuous improvements, systematic characterization and thorough investigations, a final result for the Planck constant has been published [6].

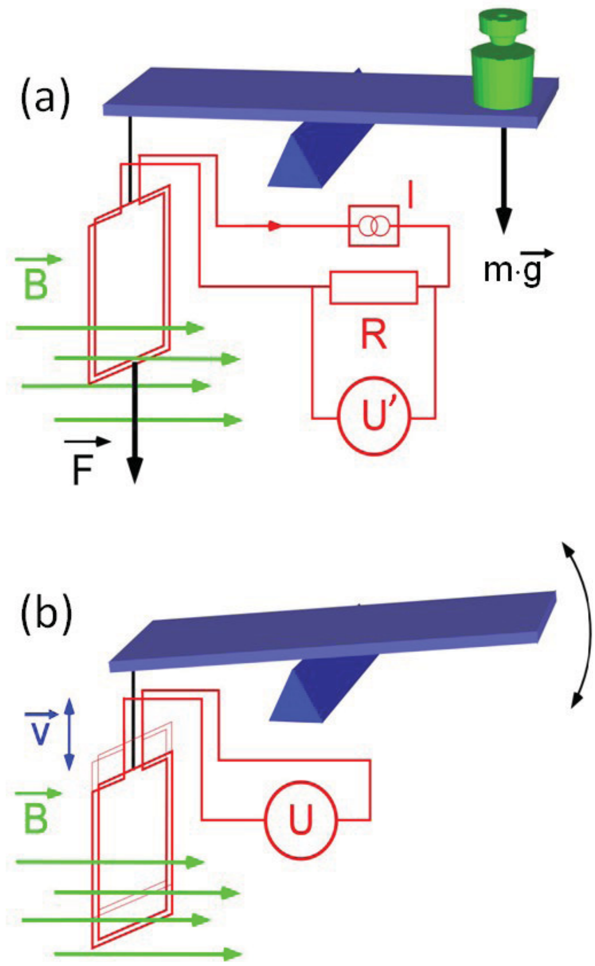

Fig. 3. (a) Static mode: The electromagnetic force acting on the current carrying coil is balanced against the weight of the test mass m.(b) Dynamic mode: the coil is moved in the vertical direction through the magnetic field and the induced voltage U is measured.

2. THE WATT BALANCE APPROACH

The principle of the watt balance experiment is based on the comparison of mechanical and electrical virtual powers. The experiment is performed in two steps with the same experimental setup: the static or weighing mode and the dynamic or induction mode (see Figure 3).

In the static phase, the force acting on a mass m placed in the local gravity field $\vec{g}$, is balanced by the vertical component of the electromagnetic force $\vec{F}$ produced by a current I flowing in a coil immersed in a magnetic field $\vec{B}$. The electromagnetic force can be expressed by

$$\vec{F} = I \cdot \oint d\vec{\ell} \times \vec{B}, \quad (1)$$

where ℓ is the conductor length of the coil. In the dynamic phase, the coil is moved vertically at a velocity $\vec{v}$ in the magnetic field. This motion induces a voltage U across the coil that can be expressed by

$$U = -\oint (d\vec{\ell} \times \vec{B}) \cdot \vec{v}. \quad (2)$$

If the mechanical dimensions of the coil and the magnetic field are strictly identical in both modes, and under the hypothesis that the coil passes through its weighing position during the velocity mode with a strictly vertical motion, the combination of Equations (1) and (2) leads to the expression

$$U \cdot I = m \cdot g \cdot v. \quad (3)$$

The watt balance thus allows a comparison between the electrical and the mechanical virtual powers by combining the static phase, where the velocities and voltages are not relevant, and the dynamic phase, where the forces and currents are not important. This means that real energy dissipation does not enter into the basic equation of the experiment. Using the expressions of the Josephson and quantum Hall effects, Equation (3) can be rewritten as

$$m = C \cdot \frac{f_J \cdot f'_J}{gv} \cdot h, \quad (4)$$

where C is a calibration constant, f_J and f'_J are the Josephson frequencies used during the static and the dynamic phases and h the Planck constant. The watt balance experiment relates therefore the unit of mass to the meter, the second and the Planck constant. A possible new definition for the mass unit could then be based on a fixed value of h [7].

3. THE METAS DESIGN

The original idea of the METAS design is to use a $100\,\mathrm{g}$ test mass (instead of the traditional $1\,\mathrm{kg}$) with a commercial mass comparator to realize the static phase. The velocity mode is then performed with a separated mechanical system that translates the coil in the magnetic field generated by a permanent magnet. The small test mass enables an important reduction of the overall size of the apparatus whereas the use of two separate measurement systems, for the moving and the weighing mode, makes it possible to optimize each setup separately but forces the transfer of the measuring coil between both systems during the measurement sequence.

This separation has been realized by means of a parallelogram structure that moves the coil through the magnetic field, and by two mechanical lifters which position and transfer this coil to the mechanical suspension hanging under the mass comparator used for the weighing mode. The coil position on the suspension can be kept within a range of the order of $1\,\mu\mathrm{m}$ in each direction during more than 500 transfers.

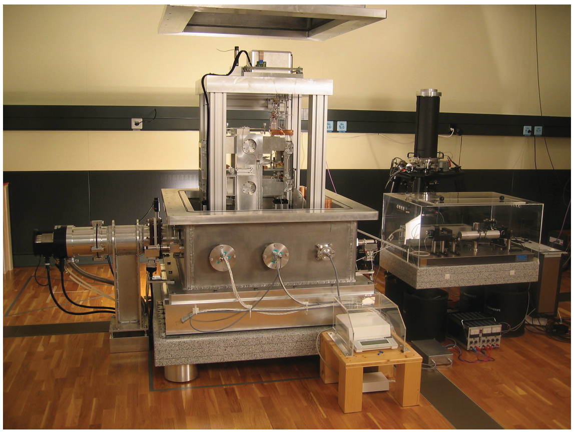

Fig. 4. A picture showing the METAS watt balance with the constant pressure chamber open. The mass comparator is placed on an aluminum table and the mechanical translation system (seesaw) is located under this table. On the right, the optical table for the velocity determination and, on the back, the absolute gravimeter are also visible. The base of the vacuum chamber is approximately $1\,\mathrm{m} \times 1\,\mathrm{m}$.

In the static phase (Figure 5a), the weighings are performed with a customized commercial mass comparator from Mettler-Toledo. This phase is composed of two steps: the conventionally called positive step where the test mass is placed on the frame supporting the coil, and a stabilized current produces an electrical force to compensate half of the weight of the test mass, and a negative step where the test mass is lifted and the electrical current running in the coil is reversed to generate a force in the opposite direction. By combining these two steps the ratio mg/I, corresponding to the so-called "mechanical geometric factor G_m" can be evaluated. In the dynamic phase (Figure 5b), the coil is removed from the comparator frame and placed on the translation table. This table is clamped to the vertical side of the parallelogram structure and is rolling at the end of the two horizontal arms. The vertical movement is generated by a voice coil motor regulated at the desired velocity v with the signal of a laser interferometer associated with a feedback loop. The induced voltage U is measured and the ratio U/v represents the "electrical geometric factor G_e" that is compared to G_m.

Several improvements have been implemented to the initial project during the last years, mainly related to alignment capabilities and control of the coil position during the measurement. A redesigned magnetic circuit was implemented to reduce hysteresis behavior (see Figure 6) [8]. A new suspension coupled to a mass handler allowed the release of several degrees of

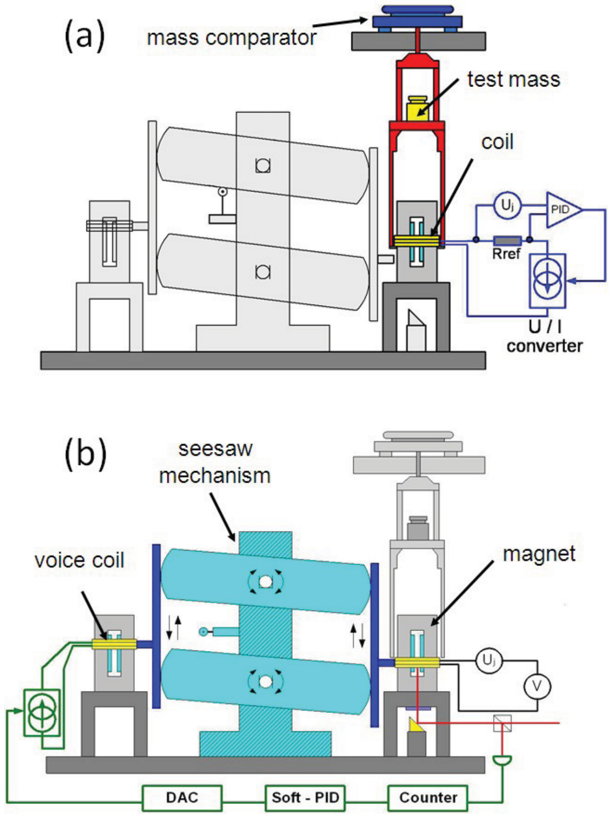

Fig. 5. (a) Static phase: the coil and the test mass are suspended under the comparator. A stabilized current is injected into the coil to produce the required force. (b) Dynamic phase: The coil is attached to the parallelogram and moved up and down in the magnetic field produced by a permanent magnet. The signal of a laser interferometer associated to a feedback loop is used to stabilize the coil velocity.

freedom to facilitate the alignment procedure [9] and optical sensors were added to monitor the coil position in both modes [10]. A picture of the system is shown in Figure 4.

4. RESULTS FROM THE METAS EXPERIMENT

Six sets of data representing a total of more than 3400 hours of operation have been analyzed to determine a value of the Planck constant. The different sets are at least composed of 500 data points. To minimize the influence of atmospheric pressure variations, the whole experiment is built in a hermetically sealed chamber whose pressure, temperature and relative humidity are monitored during the measurements. A single determination of h can be achieved after a 60 minutes cycle. Each measurement cycle is composed of three sets of weighings and two sets of induced voltage measurements that are separated by a coil transfer; every operation lasts roughly one third of this total time. Different

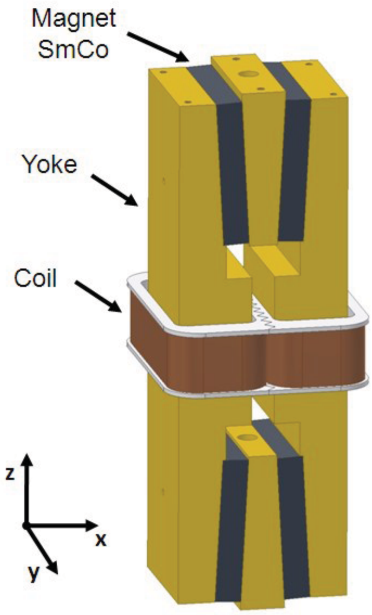

Fig. 6. View of the magnetic circuit.

corrections of systematic effects have been taken into account. Residual misalignments of the coil position between the two modes have been corrected using an analytical model where the parameters are determined with a least square fit procedure. A detailed description of the measurement sequence and the data analysis is given in [6].

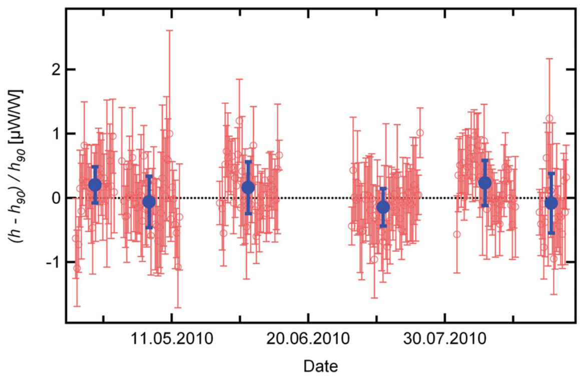

Fig. 7. Set of data used for the determination of the Planck constant with the METAS experiment. The open dots represent the mean value over 10 individual determinations of h (representing a period of 10 hours) and the plain dots are the mean values for each bloc, both with their associated standard deviation. Note that $h_{90} = {}^{4}/_{R_{K\text{-}90} K_{J\text{-}90}^{2}}$.

The value of $h = 6.626\,069\,1(20) \cdot 10^{-34}$ Js extracted from this data set differs by $0.024\,\mu\mathrm{W/W}$ from the CODATA 2006 value [11]. The results are presented in Figure 7 where each point (open dot) is the mean value over 10 hours and the mean value (black plain dot) of each set is shown with its standard deviation.

The time between each data set was used for consistency checks and secondary measurements (like the verticality of the laser beam, transfer and horizontal motion of the coil, etc...). The standard deviation of the mean of these six values ($0.07\,\mu\mathrm{W/W}$) can therefore be considered as the reproducibility of the apparatus. A summary of the Planck constant determined with watt balances is presented in Figure 8.

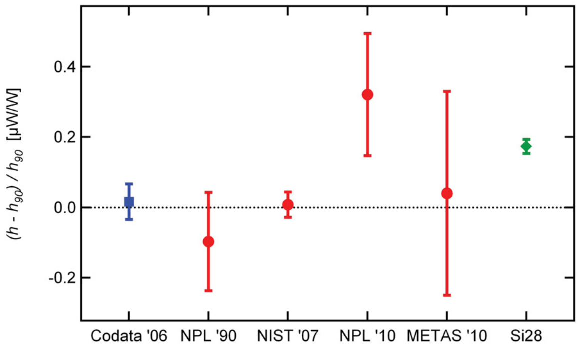

Fig. 8. Summary of the Planck constant determination with the three operational watt balances in the world (circles) and the Avogadro project on ^{28}Si (diamond) compared to the CODATA 2006 value (square). Details about the Avogadro project can be found in the related contribution in this colloquium or in reference [12].

The global uncertainty associated to the Planck constant determination is $0.29\,\mu\mathrm{W/W}$. Note that the dominant part of this budget is related to alignment issues and their combined contribution to the total uncertainty adds to $0.20\,\mu\mathrm{W/W}$. Due to intrinsic limitations in the mechanical setup, it is not possible to significantly reduce this uncertainty contribution.

Based on the experience gathered over the last ten years, it was decided to start a new watt balance project at METAS. This new project is already in progress in close collaboration with external partners like the Ecole Polytechnique Fédérale de Lausanne EPFL (Lausanne, Switzerland), Mettler-Toledo (Greiffensee, Switzerland) and the European Organization for Nuclear Research (CERN, Geneva, Switzerland). This new experimental setup is meant to reach a relative uncertainty close to $10^{-8}\,\mathrm{W/W}$.

5. CONCLUSIONS

A major requirement for changing the mass unit is to have several coherent results provided by independent experiments. The most probable value of the constant chosen to be the basis of the new definition could then be deduced from these independent results.

Today, The situation of the determination of the Planck constant may arise from unresolved systematic errors in the different experiment. The largest diversity in the design of the different watt balances around the world must then be encouraged. This is the only way to check in every detail each possible source of systematic errors. An agreement between the values of the Planck constant obtained from these various experiments would definitely provide a convincing argument to metrologists working on the new definition of the mass unit based on fundamental constants.

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