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HOMODYNE INTERFERENCE SIGNAL DEMODULATION FOR NANOPOSITIONING AND NANOMEASURING MACHINES

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ABSTRACT

The Nanopositioning and Nanomeasuring Machine NMM-1 was designed for measurements within a measuring volume of 25 mm by 25 mm by 5 mm. The interferometric length measuring and drive systems make it possible to move the stage and corner mirror with a resolution of 0.1 nm in all three axes. The object being measured is placed on the corner mirror and can be measured with different probe systems. The very high precision of the machine can be attributed to several factors, the accuracy of the interferometric measuring systems, the three-dimensional realization of the Abbe comparator principle, the precise reference coordinate system defined by the corner mirror and the additional compensation of angular deviations. This article describes a small part of the measurement uncertainty analysis for a displacement measurement using two positions of the measuring mirror. In particular this article discusses the influence of offset, amplitude and phase deviations in the interference signals.

Index Terms— nanomeasuring, nanopositioning, homodyne interferometer, uncertainty analysis, demodulation, Heydemann correction

1. INTRODUCTION

Over the last several years the demands have risen on the measurement of micro- and nanostructures over larger measurement ranges with increasing accuracy and precision. Specimens with micro- and nanostructures are becoming larger and larger on the one hand, yet the structures themselves are becoming smaller and more complex. Measuring devices must provide multiple orders of magnitude for positioning and measurement, from sub-nanometres up to hundreds of millimetres. Surface scans must be realised in $2^1/_2$ dimensions over very large regions with nanometre precision. The Nanomeasuring Machine is equipped with three single-beam homodyne plane-mirror miniature interferometers for the measurement of the displacement of a movable corner mirror [1, 2, 3]. The object being measured is placed on the corner mirror, which is positioned by a three-axis drive system. The plane-mirror miniature interferometers and the probing system are fixed on a metrology frame made of Zerodur®.

The machine is capable of carrying out both $2^1/_2$-D surface scans and 3-D coordinate measurements [4, 5, 6]. Depending on the installed probing system, surface scans and three-dimensional point measurements and scans, including freeform scans, are possible within a range of 25 mm by 25 mm by 5 mm with this machine. The overall 3-D uncertainty for measurements done with the machine depends on the machine itself and the probe system in use as well as the specific measuring task.

2. DEMODULATION PRINCIPLE

Interferometers are often-used length measuring systems. In a Michelson-type interferometer, light from a light source is split into a reference beam and a measuring beam. The beams are reflected back from the reference and measuring mirrors, respectively. Recombination of the beams at the output of the beam splitter allows the two waves to interfere. The different optical paths result in two different phases of the two waves. Only the phase difference can be determined from the interference signal by the demodulation system. It is not possible to specify which mirror has moved and

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therefore, one of the mirrors is fixed. Then the phase changes can definitely be attributed to the movement of the other mirror. The wavelength of light in the arm with the moveable mirror serves as the measurement scale for the interferometric measurement. For measurements in air the actual wavelength depends on the vacuum wavelength $\lambda_{\text{vac}}(t)$ and the refractive index of air $n(t)$. The unequal paths in the measuring and reference arms along with changes in the wavelength lead to variations in the measurement value, although the mirrors have not actually changed position. The conversion of the discrete demodulation value $N(t)$ from the demodulation electronics into an actual position value $l_{\text{m}}(t)$ must take into account the dead path length l_{t}, which is the length difference between the measuring and reference paths at the moment the fringe counter was reset [7]. The discrete demodulation value $N(0)$ are the corresponding values shortly after the counter system was set to zero.

$$l_{\text{m}}(t) = \frac{\lambda(t)}{2k_{\text{TF}}}\left(N(t) - N(0)\right) + \left(\frac{\lambda(t)}{\lambda(0)} - 1\right) l_{\text{t}} \quad (1)$$

This equation for a λ/2-interferometer includes the wavelength $\lambda(0) = \lambda_{\text{vac}}(0)/n(0)$ when the counter was set to zero as well as the current wavelength $\lambda(t) = \lambda_{\text{vac}}(t)/n(t)$. According to the second term in equation, a non-zero dead path length leads to an additional uncertainty contribution for the measurement because of possible changes in the refractive index and the vacuum wavelength as well as the determination of the refractive index from the environment sensor values. The equation can be reduced to a linear function with two coefficients through substitution.

$$l_{\text{m}}(t) = k_1(t)\left(N(t) - N(0)\right) + k_0(t) \quad (2)$$

Coefficients $k0(t)$ and $k1(t)$ only need to be recalculated when measured environment values change. This equation enables the correction of systematic deviations caused by the refractive index of air in the dead path and reduces the measuring uncertainty for the measuring length. The evaluation of the measured signals based on the registration of the number and the fraction of the fringes passed through. The amplitudes of the two 90°-phase-shifted sinusoidal and offset-free signals U_{A} and U_{B} are sampled and discretised with two very fast ADCs. The most significant bits in the digital signals (quadrature signals) are fed to an up-down counter. This counter value $N_{\text{cnt}}(t)$ is incremented or decremented at a selected transition in accordance with the motion direction. The counter and the two analogue-to-digital converter values are used to determine the length measurement value. The fractional part of the demodulation phase is derived using the arctan function on the quotient of the two ADC values D_{A} and D_{B} using equation (4).

$$\varphi = \arctan\left(\frac{U_{\text{A}}}{U_{\text{B}}}\right) \quad (3)$$

ADC width in bits	arctan width in bits	maximum deviation in pm	standard deviation in pm	mean distance in pm
6	8	1760.8	1016.6	1678.8
8	12	353.31	203.99	367.81
10	14	89.88	51.89	107.04
12	16	22.03	12.72	24.26
14	16	7.36	4.25	7.68
16	16	3.62	2.09	5.72

Table 1. Demodulation deviations caused by the quantisation in the ADC and round-off in the arctan function (calculation of length for a $\lambda/2$-interferometer and a wavelength $\lambda = 632.82\,nm$)

$$N_{\text{arctan}}(t) = \frac{k_{\text{TF}}}{2\pi}\arctan\left(\frac{D_{\text{A}} + 0,5}{D_{\text{B}} + 0,5}\right) \quad (4)$$

The factor k_{TF} is the number of steps per fringe (for the NMM-1 $k_{\text{TF}} = 16384$). The analogue-to-digital converters round down the values; therefore, a 0.5 could be added to two ADC values. The demodulation value $N(t)$ can be determined from the demodulation value part $N_{\text{arctan}}(t)$ and counter value N_{cnt}.

$$N(t) = k_{\text{TF}} N_{\text{cnt}}(t) + N_{\text{arctan}}(t) \quad (5)$$

A component of the demodulation value $N_{\text{arctan}}(t)$ remains unchanged at the moment the counter is set to zero. In order to bring the length value $l_{\text{m}}(t)$ fully to zero, the demodulation value $N(0)$ is captured shortly after the counter is reset and is thereafter always subtracted from the current values as an offset value.

3. QUANTISATION AND ROUNDING DEVIATIONS

Some demodulation deviations arise due to the quantisation associated with the analogue-to-digital converters and the need for additional round-off for the arctan function. The maximum deviations for different analogue-to-digital converters and arctan register widths were calculated through a simulation for a $\lambda/2$-interferometer. For the demodulation with a 10-bit ADC and a 14-bit arctan register, the uncertainty contribution due to quantisation and round-off is $u = 52\,\text{pm}$. The analogue-to-digital converter values D_{A} and D_{B} can be read and used for interference signal monitoring and the reduction of the uncertainty through real-time ellipse regression and correction. Preferred values were able to be identified from the histograms of the length values. These are caused by the quantisation of the ADC and the round-off in the arctan function. The average distances between these preferred values was obtained from a simulation (see table 1). These distances correspond to the nominal resolution.

4. INTERFERENCE SIGNAL NOISE

At higher relative resolutions the quantisation of the analogue-to-digital conversion is less important than the signal-to-noise ratio. The achievable measurement resolution without "useless magnification" depends on the noise of the interference signals. The signal-to-noise ratio of the interference signals should be greater than or equal to the signal-to-noise ratio of the analogue-to-digital converter. The noise of the interference signals depends on the power stability of the laser and the various noise sources in the photo detectors and signal amplifiers. In the Nanomeasuring Machine, the diodes operate in quasi-short-circuit mode ($R_{\mathrm{L}} = 0$, $U = 0$, $I < 0$), in which no dark current occurs and only thermal noise (or Johnson noise) arises in the photodiode due to the shunt resistor [8]. The photoamplifier circuits with operational amplifiers also possess several noise sources. Subsequently the signals of the interferometers are further amplified and deviations in offset and amplitude corrected. Each amplifier stage transmits the noise of the previous stages and is itself a source of noise. Electromagnetic interference (EMI) and radio frequency interference (RFI) cause additional noise in the interference signals. A theoretical determination of the noise for the estimation of measurement uncertainty does not make sense due to the complexity of signal processing. A better measurement of length noise can be done by disabling the NMM-1's drive system. Here, the influence on the ADC values and thus on the length measurements almost exclusively stems from noise and quantisation effects. Standard deviations of $0.079\,\mathrm{nm}$ (x-axis), $0.094\,\mathrm{nm}$ (y-axis) and $0.065\,\mathrm{nm}$ (z-axis) were determined using this type of measurement. The larger values of the x- and y-axes are caused by the larger lateral mechanical vibrations.

5. OFFSET, AMPLITUDE AND PHASE DEVIATIONS

The demodulation of the interference signals requires two offset-free signals ($\bar{U}_{\mathrm{A}} = \bar{U}_{\mathrm{B}} = 0$) of equal amplitude ($\hat{U}_{\mathrm{A}} = \hat{U}_{\mathrm{B}}$) and a phase angle of 90° between the two signals (phase difference $\alpha = 0$). The output signals from the electronics most likely exhibit low amplitude, offset deviations and deviations of the phase angle, which cause periodic nonlinearities of the demodulated length measurement signals. Maximum relative offset and amplitude deviations of $\leq 3\%$ were determined for various measurements with the NMM-1 [9]. The deviations arise from electronic component tolerances and movement-dependent dynamic deviations which can only be corrected by the automatic control with a delay. An adjustment of the phase angle to 90° based on the Lissajous figure on a oscilloscope can reduce the maximum phase difference α to 1.5°. To describe the resulting demodulation deviations, the two voltage values in equation (3) can be replaced by sinusoidal voltage characteristics depending on the ideal demodulation phase γ (see equation (6)).

$$\varphi = \arctan\left(\frac{\hat{U}_{\mathrm{A}}\sin(\gamma+\alpha)+\bar{U}_{\mathrm{A}}}{\hat{U}_{\mathrm{B}}\cos(\gamma)+\bar{U}_{\mathrm{B}}}\right) \quad (6)$$

The offset deviations $\Delta\bar{U}_{\mathrm{A}}$ and $\Delta\bar{U}_{\mathrm{B}}$ cause offset-free and sinusoidal periodic demodulation deviations. The effects of amplitude deviations $\Delta\hat{U}_{\mathrm{A}}$ and $\Delta\hat{U}_{\mathrm{B}}$ also have a sinusoidal curve with half of the period length. The phase difference α also causes a sinusoidal periodic demodulation deviation.

The additionally read ADC values allow the compensation of static offset, amplitude and phase deviations. The offset, amplitude and phase deviations of the digitally converted interferometer signals can be determined using with an ellipse-shaped regression. The determined phase difference value can be used for fine adjustment of the phase difference α to $< 0.1°$ by slanting a beam splitter in the detection unit. The phase difference remains unchanged in subsequent measurements and leads to a maximum length measurement deviation of $< 0.088\,\mathrm{nm}$. The static offset and amplitude deviations can be reduced by altering the signal control set points to approximately $\leq 1\%$. The amplitude and offset deviations vary during the subsequent measurements and could be only avoided using continuous monitoring and correction. The ellipse regression and computational correction of the two sinusoidal signals to an offset-free circle was originally proposed by Heydemann [10] and is based on equation (7).

$$AD_{\mathrm{A}}^2 + BD_{\mathrm{B}}^2 + CD_{\mathrm{A}}D_{\mathrm{B}} + DD_{\mathrm{A}} + ED_{\mathrm{B}} = 1 \quad (7)$$

The ellipse equation coefficients A, B, C, D, E can be determined by direct or recursive estimation using the least squares method on the ADC values D_{A} und D_{B}. The quality of the estimation depends on the number of data points and the noise of the two input signals [11]. The greater the noise of the input signals, the greater amount of observation amount must used in the estimation. These parameters can then be used to correct the demodulation signals [10].

6. UNCERTAINTY CONTRIBUTION FROM THE DEMODULATION

For a simple differential measurement between two measured corner mirror positions, the difference length l_{d} corresponding to equation (8) are calculated by:

$$\begin{aligned} l_{\mathrm{d}} &= l_{\mathrm{m}}(t_2) - l_{\mathrm{m}}(t_1) \qquad (8)\\ &= \frac{\lambda(t_2)(N(t_2)-N(0)) - \lambda(t_1)(N(t_1)-N(0))}{2k_{\mathrm{TF}}}\\ &+ \frac{\lambda(t_2)-\lambda(t_1)}{\lambda(0)} l_{\mathrm{t}} \end{aligned}$$

Calculation of the uncertainty must take into account any correlation of the wavelengths. However, the wavelengths can be decoupled for the uncertainty analysis by replacing the wavelengths with $\lambda(t_1) = \lambda(0) + \Delta\lambda_1$ and $\lambda(t_2) = \lambda(t_1) + \Delta\lambda_2$. After inserting these factors into equation (8), multiplying out and combining yields the following equation:

$$l_{\mathrm{d}} = \frac{(\lambda(0) + \Delta\lambda_1)(N(t_2) - N(t_1))}{2k_{\mathrm{TF}}} + \Delta\lambda_2 \left(\frac{N(t_2) - N(0)}{2k_{\mathrm{TF}}} + \frac{l_{\mathrm{t}}}{\lambda(0)} \right) \tag{9}$$

Starting from equation (9), the uncertainty contribution of demodulation values $N(t)$ are determined for the differential measurement (see equation (10)).

$$u_{1b}(l_{\mathrm{d}}) = \sqrt{\left(\frac{\lambda(t_2)}{2k_{\mathrm{TF}}}\right)^2 u^2(N(t_2)) + \left(\frac{\lambda(t_1)}{2k_{\mathrm{TF}}}\right)^2 u^2(N(t_1)) + \left(\frac{\Delta\lambda_2}{2k_{\mathrm{TF}}}\right)^2 u^2(N(0))} \tag{10}$$

The sensitivity coefficients for demodulation values $N(t_1)$ and $N(t_2)$ are determined with the corresponding wavelengths $\lambda(t_1)$ and $\lambda(t_2)$. In contrast, the sensitivity coefficient for demodulation value $N(0)$ only depends on the wavelength difference $\Delta\lambda_2$ between the two measured points. The uncertainties of demodulation values $N(0)$, $N(t_1)$ and $N(t_2)$ are influenced by the signal-to-noise ratio of analogue interference signals U_{A} and U_{B} as well as rounding and quantisation deviations of the ADC values D_{A} and D_{B}. A standard uncertainty of $u_1(N(t)) = 5$ digits was determined by measurements.

Furthermore, the uncertainties of the demodulation values depend on the offset and amplitude deviations and phase difference of the interference signals. When measuring movements in all coordinate directions and at different speeds, the ADC values were recorded and the position-dependent changes with ellipse regressions over 1000 consecutive values evaluated. The offset values (≤ 0.2 digits or 0.04 % relative to the signal amplitude) and phase difference ($\leq$0.02°) varied only marginally, while the amplitudes changed by about 1 % and the changes had a correlation of $\geq$0.96. These amplitude deviations are caused by dirt and tilting of the corner mirror used in the measurement as well as mirror coating inhomogeneities and laser power fluctuations. The amplitudes $\hat{U}_{\mathrm{A}}$ and $\hat{U}_{\mathrm{B}}$ in equation (6) must be expanded with the correlated amplitude deviations $\Delta\hat{U}_{\mathrm{A}}$ and $\Delta\hat{U}_{\mathrm{B}}$ in order to separate the uncorrelated and correlated amplitude deviations.

$$\varphi = \arctan\left(\frac{\left(\hat{U}_{\mathrm{A}} + \Delta\hat{U}_{\mathrm{A}}\right)\sin(\gamma + \alpha) + \bar{U}_{\mathrm{A}}}{\left(\hat{U}_{\mathrm{B}} + \Delta\hat{U}_{\mathrm{B}}\right)\cos(\gamma) + \bar{U}_{\mathrm{B}}} \right) \tag{11}$$

The partial derivatives of this equation can be used to determine uncertainty. The uncertainty of the demodulation phase φ or the demodulation value $N(t)$ can be calculated with these sensitivity coefficients using equation (12). This leads to a significant reduction of uncertainty $u_2(N(t))$ because of the significantly smaller sensitivity coefficients.

$$u_2(N(t)) = \frac{k_{\mathrm{TF}}}{2\pi} \sqrt{\left(\frac{\partial\varphi}{\partial\alpha}\right)^2 u^2(\alpha) + \left(\frac{\partial\varphi}{\partial\hat{U}_{\mathrm{A}}}\right)^2 u^2(\hat{U}_{\mathrm{A}}) + \left(\frac{\partial\varphi}{\partial\hat{U}_{\mathrm{B}}}\right)^2 u^2(\hat{U}_{\mathrm{B}}) + \left(\frac{\partial\varphi}{\partial\bar{U}_{\mathrm{A}}}\right)^2 u^2(\bar{U}_{\mathrm{A}}) + \left(\frac{\partial\varphi}{\partial\bar{U}_{\mathrm{B}}}\right)^2 u^2(\bar{U}_{\mathrm{B}}) + \left(\frac{\partial\varphi}{\partial\Delta\hat{U}_{\mathrm{A}}}\right)^2 u^2(\Delta\hat{U}_{\mathrm{A}}) + \left(\frac{\partial\varphi}{\partial\Delta\hat{U}_{\mathrm{B}}}\right)^2 u^2(\Delta\hat{U}_{\mathrm{B}}) + 2\frac{\partial\varphi}{\partial\Delta\hat{U}_{\mathrm{A}}}\frac{\partial\varphi}{\partial\Delta\hat{U}_{\mathrm{B}}} u(\Delta\hat{U}_{\mathrm{A}}) u(\Delta\hat{U}_{\mathrm{B}}) r(\Delta\hat{U}_{\mathrm{A}}, \Delta\hat{U}_{\mathrm{B}})} \tag{12}$$

The maximum and minimum uncertainties are $u_2(N(t)) = 18.5$ digits and $u_2(N(t)) = 15$ digits, respectively. The combined uncertainty for the demodulation can be calculated using equation (13).

$$u_{\mathrm{c}}(N(t)) = \sqrt{u_1^2(N(t)) + u_2^2(N(t))} \tag{13}$$

When the demodulation phase is unknown, the combined uncertainty of the demodulation value must be assumed to be at its maximum $u_{\mathrm{c}}(N(t)) = 19.3$ digits (x- and y-axes) and 19.5 digits (z-axis). The combined uncertainty of the demodulation value $u_{\mathrm{c}}(N(t))$ can be used in equation (10) for the uncertainties $u(N(t_2))$, $u(N(t_1))$ and $u(N(0))$, which allows us to calculate the measurement uncertainty contribution from the demodulation taking place in the measurements of length difference.

7. CONCLUSION

With the development of new demodulation electronics for the Nanomeasuring Machine, the measurement resolution has been improved to less than 0.1 nm. The additional use of the ADC values allows to compensate the static offset, amplitude and phase deviations of the interference signals through an off-line ellipse regression and adjustment and a subsequent analysis of the signals during the measurements. This was able to drastically reduce the uncertainty in the length measurement caused by the demodulation. Consideration of the correlation of the remaining amplitude deviations also resulted in a decreased uncertainty contribution.

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