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Anatoly Chigarev, Klaus Zimmermann,
Vladimir Minchenya, Felix Becker

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AUTOMATIC STABILIZATION OF THE TRANSLATIONAL MOTION OF A MICROROBOT

Anatoly Chigarev[1], Klaus Zimmermann[2], Vladimir Minchenya[1], Felix Becker[2]

[1] Belorussian National Technical University Minsk, Department of Instrument-making
[2] Ilmenau University of Technology, Department of Technical Mechanics

ABSTRACT

The analog motion control of a microrobot with feedback requires a more complicated structure due to the corresponding sensors, microprocessors and actuators. Application of the digital control system allows the use of a control algorithm, such as the autopilot, without feedback, which allows us to simplify the system. We consider the stabilization of the motion of the microrobot on a horizontal surface along the given motion trajectory. Minimization of the deviation angle of the robot's axis from the tangent to the motion trajectory is a criterion for quality control and can be implemented by the digital device with a hysteresis characteristic.

Index Terms – micro robot, digital motion control

1. ANALYTICAL MODEL

Let's assume, that the robot is in the 0-X-Y-plane in some initial state, whene at a certain period of time the coordinate system $c\ x\ y$ has the values c_0, x_0, y_0, and variables h , $\dot{h}$ have the initial values, like it is presented in Figure 1. cx - is the longitudinal axis of the robot and $\vec{V}$ is the velocity vector of center c.

Let's mark h (t) as the distance from the center of masses of the robot c to the OY axis. Let the robot moves so, that the y-axis stays parallel to the Y axis, which helps to simplify the problem without considering the yaw angle.

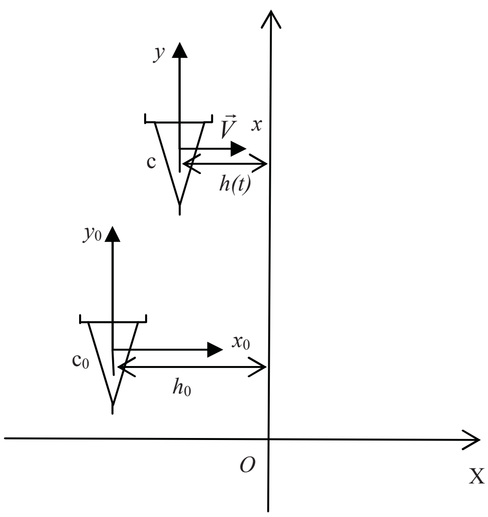

Figure 1: The analytical model of the robot motion

Equation (1) for the value h (t) can be written down in the form suitable for constructing the model of a digital control

$$\frac{d^2 h(t)}{dt^2} = r(\rho, V, \delta_1, \delta_2, \delta_3, \delta_4, \delta_5,) m(t) \quad (1)$$

Here h (t) – is the distance, m (t) – is the trajectory control of the robot approximation to the Y axis, r - is the parameter (function), which defines, in general, the influence of ρ - the environmental density, in which the robot moves, V - is the velocity and the parameters $\delta_1, \delta_2, \delta_3, \delta_4, \delta_5$ - describe aero-hydrodynamic properties of the medium.

As initial conditions we take

$$h(0) = |h_0|, \dot{h}(0) = 0 \quad (2)$$

We consider the velocity V to be constant, V = const. We believe that the effect of the environment on the robot cannot be accurately described and therefore we suppose that parameter r is random. We suppose that r is a random value and it is distributed evenly on any interval $\tau = T / N$, where T - is the time interval of the robot approaching with the Y axis, N is a number of measurements of the parameter on interval T.

We suppose that r takes value r^+ with probability p and value r^- with probability 1-p. Since the probability p is not known itself and must be calculated in the process of the control (a posteriori), then it is necessary to set some a priori probabilities. Let value p take value p_1 with a priori probability ξ and value p_2 with a priori probability $1 - \xi$.

The criterion of control: it is required to determine an algorithm of the optimal control, minimizing the mean square velocity of approaching at the moment of the coincidence of axes y and Y.

Denote $\frac{dh}{dt} = \gamma$, then (1) we write down in the form

$$\frac{d\gamma(t)}{dt} = rm(t) \quad (3)$$

Let's write down (3) in the finite form

$$\gamma_{k+1} = \gamma_k + r\tau m_k \quad (4)$$

where $\gamma_{k+1} = \gamma(k\tau)$, $m_k = m(k\tau)$

Thus, the problem of control of the final state is reduced to the determination of the control influence m, which minimizes the quality index

$$I = < \gamma^2(T) > = < \gamma^2(N\tau) > \quad (5)$$

where the angular brackets denote the operation of taking expectation

To calculate the means in (5) it is necessary to know the a posteriori probabilities, which we denote,

$$\alpha(\zeta) = P\{p = p_1 | r = r^+ \}, \ \beta(\varsigma) = P\{p = p_2 | r = r^{-1}\}.$$

According to the Bayes' theorem the calculation of a posteriori (after observation) probabilities is carried out according to the formulae

$$\alpha(\zeta)=\frac{P(p=p_1)P(r=r^+|p=p_1)}{P(p=p_1)P(r=r^+|p_1)+P(p=p_2)P(r=r^+|p=p_2)}=\frac{\zeta p_1}{\zeta p_1+(1-\zeta)p_2} \tag{6}$$

$$\beta(\zeta)=\frac{P(p=p_1)P(r=r^{-1}|p=p_1)}{P(p=p_1)P(\tau=\tau^{-1}|p=p_1)+P(p=p_2)P(r=r^{-1}|p=p_2)}=\frac{\zeta(1-p_1)}{\zeta(1-p_1)+(1-\zeta)(1-p_2)} \tag{7}$$

We denote the minimum criteria J through $f_N(\gamma_0,\zeta)$, where $\gamma_0=\dot{h}(0)$ - is the initial velocity of the robot approaching to the Y axis. This minimum depends on the initial velocity γ_0 and a priori probability ζ and is defined by the expression

$$f_N(\gamma_0\zeta)=\min_{\{m_k\}}<\gamma^2(T)> \tag{8}$$

In an arbitrary k +1 sample from the observations value γ_{k+1} can take two values

$$\gamma_{k+1}^+=\gamma_k+r^+\ \tau m_k \tag{9}$$

$$\gamma_{k+1}^-=\gamma_k+r^-\tau m_k \tag{10}$$

Let us suppose that in (9), (10) k = 0, then we obtain

$$\gamma_1^+=\gamma_0+r^+\tau m_0 \text{ with probability } p_0 \tag{11}$$

$$\text{and } \gamma_1^-=\gamma_0+r\tau m_0 \text{ with probability } 1-p_0 \tag{12}$$

where p_0 - the expected value p, calculated by the formula

$$p_0=\zeta p_1+(1-\zeta)p_2 \tag{13}$$

Hence, for N = 1

$$f_1(\gamma_0,\zeta)=\min_{m_0}\{p_0(\gamma_1^+)^2+(1-p_0)(\gamma_1^-)^2\ \} \tag{14}$$

As a result of the first solution, the process can be transferred into one of two possible states γ_1^+ and γ_1^- with probabilities p_0 and $1-p_0$ respectively. If the process is transferred into state γ^+, then the a posteriori probability $\alpha(\zeta)$ is calculated, if the process is transferred into state γ, then the a posteriori probability $\beta(\zeta)$ is calculated.

The optimal solution for one-step process is found by the differentiation of the function (14) according to m_0 and equating of the partial derivative to zero. As a result we obtain

$$p_0r^+\tau(\gamma_0+r^+\tau m_0)+(1-p_0)r^{-1}\tau(\gamma_0+r\ \ \tau m_0)=0$$

Hence we obtain

$$m_0=-\frac{D(r)}{D(r^2)}\gamma_0 \tag{15}$$

where

$$D(r) = p_0 r^+ \tau + (1 - p_0) r \tau \tag{16}$$

and

$$D(r^2) = p_0 (r^+)^2 \tau^2 + (1 - p_0)(r^-)^2 \tau^2 \tag{17}$$

Let's denote

$$D_i(r) = \left[p_i r^+ + (1 - p_i) r^- \right] \tau, \quad i = 1,2 \tag{18}$$

Then $D(r)$ is represented as

$$D(r) = \zeta D_1(r) + (1 - \varsigma) D_2(r) \tag{19}$$

Similarly, denoting

$$D_i(r^2) = \left[p_i (r^+)^2 + (1 - p_i)(r^-)^2 \right] \tau^2, i = 1,2 \tag{20}$$

we obtain that $D(r^2)$ can be written down through ζ as

$$D(r^2) = \zeta D_1(r^2) + (1 - \zeta) D_2(r^2) \tag{21}$$

Taking into account formulae (16) - (21) we can write down expression (14) in the form

$$f_1(\gamma_0, \zeta) = W_1 \gamma_0^2 \tag{22}$$

where W_1 has the form

$$W_1(\zeta) = 1 - \frac{D(r^2)}{D(r)} \tag{23}$$

From (23) taking into account (15) we obtain

$$m_0 = g_0(\zeta) \gamma_0 \tag{24}$$

where $g_0(\zeta)$ has the form

$$g_0(\zeta) = -\frac{D(r)}{D(r^2)} \tag{25}$$

Let's consider the case $N \geq 2$, accepting the principle of optimality, we write down

$$f_N(\gamma_0, \zeta) = \min_{m_0} \left\{ p_0 f_{N-1} \left[\gamma_1^+, \alpha(\zeta) \right] + (1 - p_0) f_{N-1} \left[\gamma_1^-, \beta(\zeta) \right] \right\} \tag{26}$$

We apply the method of mathematical induction to (22), we write down

$$f_k(\gamma_0, \zeta) = W_k(\zeta) \gamma_0^2 \tag{27}$$

We obtain from (27)

$$f_k \left[\gamma_1^+, \alpha(\zeta) \right] = W_k \left[\alpha(\zeta) \right] (\gamma_0 + r^+ \tau m_0)^2$$

$$f_k\left[\gamma_1^-, \beta(\zeta)\right] = W_k\left[\beta(\zeta)\right]\ (\gamma_0 + r^- \tau m_0)^2$$

The minimum for $(k+1)$ step process is

$$f_{k+1}(\gamma_0, \zeta) = \min_{m_0}\left\{D(\zeta)W_k\left[\alpha(\zeta)(\gamma_0 + r^+\tau m_0)^2\right] + \left[1 - D(\zeta)\right]W_k\left[\beta(\zeta)\right](\gamma_0 + r^-\tau m_0)^2\right\} \quad (28)$$

$k = 1,2...N-1$

From this recurrent correlation we find the optimal solution in the form

$$m_0 = g_k(\zeta)\gamma_0, \quad (29)$$

where

$$g_k(\zeta) = -\frac{D\left[rW_k(\zeta)\right]}{D\left[r^2W_k(\zeta)\right]} \quad (30)$$

$$D(rW_k(\zeta)) = \zeta D_1\left[rW_k(\zeta)\right] + (1-\zeta)D_2\left[rW_k(\zeta)\right] \quad (31)$$

$$D_i\left[rW_k(\zeta)\right] = \left\{p_i r^+ W_k\left[\alpha(\zeta)\right] + (1-p_i)r^- W_k\left[\beta(\zeta)\right]\right\}\tau \quad (32)$$

$$D\left[r^2W_k\zeta)\right] = \zeta D_1\left[r^2W_k(\zeta)\right] + (1-\zeta)D_2\left[r^2W_k(\zeta)\right] \quad (33)$$

$$D_i\left[r^2W_k(\varsigma)\right] = \left\{p_i(r^+)^2W_k\left[\alpha(\zeta)\right] + (1-p_i)(r^-)^2W_k\left[\beta(\zeta)\right]\right\}\tau^2, i = 1,2 \quad (34)$$

From equations (28), (29) we obtain

$$f_{k+1}(\gamma_0, \zeta) = W_{k+1}(\zeta)\gamma_0^2, \quad (35)$$

where

$$W_{k+1}(\zeta) = D\left[W_k(\zeta)\right] - \frac{D^2\left[rW_k(\zeta)\right]}{D\left[r^2W_k(\zeta)\right]} \quad (36)$$

$$D\left[W_k(\zeta)\right] = \zeta D_1\left[W_k(\zeta)\right] + (1-\zeta)D_2\left[W_k(\zeta)\right] \quad (37)$$

$$D_i\left[W_k(\zeta)\right] = p_iW_k\left[\alpha(\zeta)\right] + (1-p_i)W_k\left[\beta(\zeta)\right],\ i = 1,2 \quad (38)$$

Equations (9), (10) (23) (24) give recurrent algorithms for finding the minimum $f_N(\gamma_0, \zeta)$ for N - step control process.

Let's consider the control process of microrobots. In the initial state γ_0 with initial information ζ the first optimal solution will be

$$m_0 = g_{N-1}(\zeta)\gamma_0, \quad (39)$$

where $g_{N-1}(\zeta)$ we find from equations (30) – (34), (37) – (38) when $k = N-1$

The second optimal solution is taken after observing the random variable at the first stage of the solution. If as a result of observations it is found that $r = r^{+}$, then a posteriori probability $\zeta_1 = \alpha(\zeta)$ and a new state

$$\gamma_1^{+} = \gamma_0 + r^{+}\tau m_0 \tag{40}$$

are used as the initial information and the initial state for the remaining $N-1$ steps. Then the second optimal solution is

$$m_1 = g_{N-1}[\beta(\zeta)]\gamma_1^{-} \tag{41}$$

Thus, in time interval τ after the first sample the processor should calculate a posteriori probability $\alpha(\zeta)$ or $\beta(\zeta)$, a new state γ_1 and adopt the second optimal solution m_1.

If the observed value of variable r after the second solution is r^{+}, then a posteriori probability $\zeta_2 = \alpha(\zeta_1)$ and a new state

$$\gamma_2^{+} = \gamma_1 + r^{+}\tau m_1 \tag{42}$$

Are used as the initial information and the initial state for the remaining $N-2$ steps. If in expression (42) $\gamma_1 = \gamma_1^{+}$, then $\zeta_1 = \alpha(\zeta)$ and m_1 is defined by formula (41). If in expression (40) $\gamma_1 = \gamma_1^{-}$, then $\zeta_1 = \beta(\zeta)$ and m_1 should be expressed according to (41).
The third optimal solution is similarly found by formula

$$m_2 = g_{N-3}[\alpha(\zeta_1)]\gamma_2^{+} \tag{43}$$

If observed value r after the second solution is r^{-1}, then a posteriori probability $\zeta_2 = \beta(\zeta_1)$ and a new state

$$\gamma_2^{-} = \gamma_1 + r^{-}\tau m_1 \tag{44}$$

are used to find the third optimal solution, which is calculated by formula

$$m_2 = g_{N-2}[\beta(\zeta_1)]\gamma_2^{-} \tag{45}$$

Thus, the optimal control strategy $\{m_0, m_1, m_2 .... m_{N-1}\}$ for the problem of approaching of the robot to the Y axis is found through repeated observations and calculations according to recurrent algorithms mentioned above. Each optimal solution is calculated by the processor when using the new information from the observations of random variable r. In general, the relay characteristic depends on the angle, the angular velocity and the angular acceleration of the robot in the plane of parameters of the aerohydrodynamic damping medium and the ratio of the velocity control.

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