URN (Paper): urn:nbn:de:gbv:ilm1-2014iwk-014:7

58th ILMENAU SCIENTIFIC COLLOQUIUM
Technische Universität Ilmenau, 08 – 12 September 2014
URN: urn:nbn:de:gbv:ilm1-2014iwk:3

FUZZY-ADAPTED LANE ASSIST OF VEHICLES WITH UNCERTAINTIES

Andreas Noll / Carsten Behn

Department of Technical Mechanics
Faculty of Mechanical Engineering
Ilmenau University of Technology
Max-Planck-Ring 12 (Building F)
98693 Ilmenau, Germany
andreas.noll@tu-ilmenau.de / carsten.behn@tu-ilmenau.de

ABSTRACT

Inspired by the motivation to find new possibilities for creating adaptive mechanical motion systems, this work is a contribution to the research on fuzzy adaptive control concepts. An already existing high-gain adaptive control strategy is extended by a fuzzy-adaptation law using various fuzzy logics. In succession, several strategies are developed, analyzed and simulated when they are firstly applied to control a biologically inspired sensor system (fast-adapting receptor cells). The controllers are more effective than other ones from literature. In a next step, the best working fuzzy-adaptation strategy is used to adaptively control a lane assist of vehicles based on a single-track model in state space. To find an optimal solution, the classic control theory and fuzzy logic are combined. In order to assess the effectiveness of the fuzzy-adaptation model, other lane assists which are already known from literature are compared to show the Pros and Cons and to find out which efficiency the attempt has.

Index Terms— fuzzy logic, fuzzy system, fuzzy-adaptive control, λ-tracking, linear state-feedback, lane assist of vehicles, single-track model.

1. INTRODUCTION

The requirements of modern motor vehicles are continuously increased in order to fulfill the expectation of the driver. The basic goal is to increase the driving comfort without compromising safety. This includes systems such as

- cruise control (CC),
- adaptive cruise control (ACC), and
- lane departure warning system (LDW).

For the implementation of these systems, modern controllers are required, which do not only fulfill their function but also follow the needs of the driver. Unwanted intrusive in the driving behavior is not only unpleasant for the driver, but can also endanger the driving safety.
On the basis of the modern video technology and image processing and the incentive for autonomous driving, lane assists of vehicles are more often in the focus of development. There are basically two principles, based on the human driver, for the controller designs: one is oriented to the road behavior and the other one on other vehicles. But, what the human being can judge bad and what is not included in the controller design, are low road inclination changes and fluctuating crosswind. In addition, the vehicle models for a classical controller design with linear state feedback (pole placement) can be mapped linearly only at a constant vehicle velocity, [6], [18].
For these reasons, the following investigations start. The goal is a **linear state-feedback** to improve a **P-controller with fuzzy adaptation**, to keep the **vehicle** with a specified lateral disturbance in a **λ-tube** around an ideal roadway centerline, **regardless** of the vehicle velocity.

1.1. MOTIVATION

Owing to the increasing electrification in a variety of industrial applications, classical regulatory approaches reach more and more often their technical limits. Nonlinear control strategies are a possibility for solving the problem. But, these have the disadvantages that they are usually designed for specific applications and they cannot be analyzed with classical methods of linear control theory. Therefore, another possibility for solving the problem is an adaptive control strategy. With its help it is possible to optimize classical linear control approaches and make them available to a larger field of tasks. The advantage of these methods is also that one can use many conventional analytical procedures. For this reason, the focus is on the adaptive λ-tracking control in the following, see Section 2. With the help of the adaptation strategy, linear control laws can be adapted in different situations to generate a higher control quality. Most of the works from literature usually use classical adaptive approaches. Therefore, in this work, a non-conventional adaptation is developed. With the help of the fuzzy logic, the adaptation task is going to verbalize, in order to improve the quality of adaptation compared to the approaches in literature.

1.2. FUZZY-CONTROL

In general language area, there are many statements that cannot be clearly described with *right/wrong*, *yes/no* or *belonging to/not belonging to*. In contrast to classical logic or binary logic, where the truth value of a statement is $x_i \; / \; \}0, 1|$, the fuzzy logic provides a continuous transition between membership and non-membership. Therefore, the possibility is offered to attributable a statement j the membership value $x_j \; / \; (0, 1)$, [16]. The origin of the unsharp logic and of the resulting unsharp control, known today as *fuzzy control*, goes back to [17]. In 1965, the author advanced a set theory by a fuzzy set mapping and so coined the term *fuzzy logic*.
By using the unsharp logic, a simple fuzzy system, consisting of a system of rules and inference-scheme, can be derived, which creates a static, nonlinear functional relationship $y = f(\underline{u})$ between scalar (sharp) input variables u_i with $i = 1, \ldots, m$, and a scalar (sharp) output y, see Fig. 1.

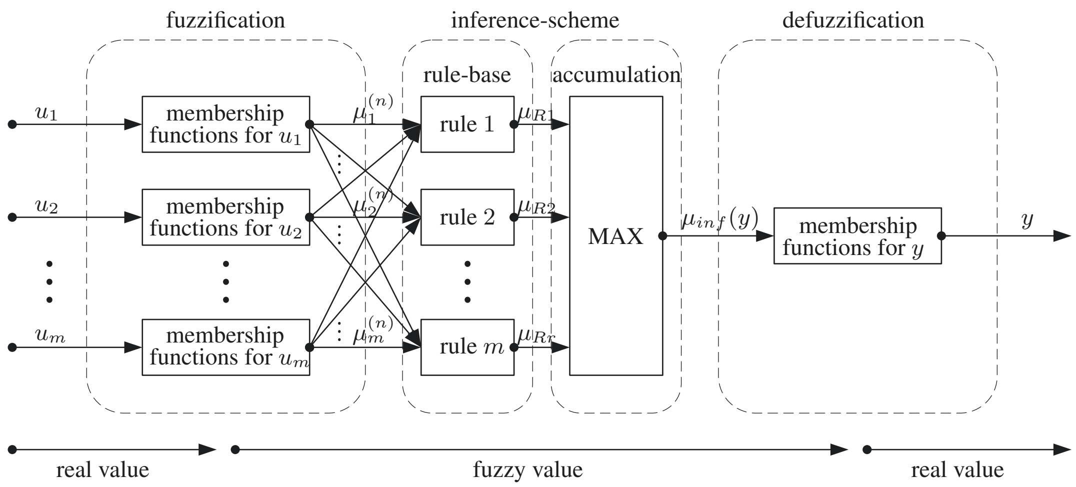

Figure 1. Scheme of a fuzzy system, adapted from [1].

The in-/output interfaces of a fuzzy system are generally referred to as *fuzzification* and *defuzzification*. They are used to convert the real values into fuzzy values. The processing of the fuzzified input variables u_i is then in the inference system, consisting of the *rule-base* with r rules and the *accumulation* which represents the union of all rule outputs.
The task of the fuzzification is to identify the membership degree of the different statements (e.g., “low”, “medium”, “high”) for the linguistic input variables (e.g., “error”, “velocity”). In this context, the membership function for each input variable

$$\mu_i^{(1)}(u_i), \mu_i^{(2)}(u_i), \ldots, \mu_i^{(k)}(u_i), , \quad k = \text{number of statements},$$

is evaluated and an inference-scheme is provided.
The fuzzy inference forms the core of each fuzzy system. It consists of a rule-base with $j = 1, \ldots, \; r$ rules in which

the causal in-/output context is defined and the accumulation which linked the different membership degrees of the rules r. Each rule is subject to a specific "if-then"-scheme:

IF premise j, THEN conclusion j
(e.g., IF error "high" and/or velocity "high", THEN gain factor "high")

Because premise and conclusion are based on the statements of fuzzy in-/output variables, such rules are referred in accordance to the classical implication as *fuzzy implication*.
The result of the fuzzy inference is initially a resulting fuzzy set of the output with the membership function $\mu_{Inf}(y)$. The task of defuzzification is then to win from the given fuzzy set a real output value of y. This is usually done with help of the gravity method, see [16]:

$$y_s = \frac{\int_{-\infty}^{+\infty} \mu_{inf}(y) \cdot y \, \mathrm{d}y}{\int_{-\infty}^{+\infty} \mu_{inf}(y) \, \mathrm{d}y} \quad \text{or} \quad y_s = \frac{\sum_{j=1}^{r} \mu_{Rj} \cdot y}{\sum_{j=1}^{r} \mu_{Rj}}$$

2. STATE OF THE ART

As a result of a large variety of adaptive control systems, the main focus is on the adaptive ***λ-tracking"***-control [7] in this paper, see Fig. 2.

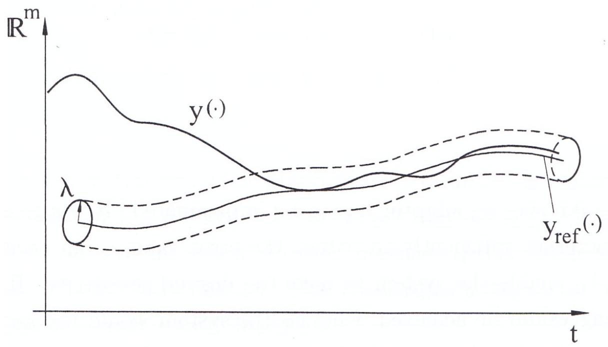

Figure 2. λ-tracking objective, [2].

The objective of the λ-tracking is:

- to force the controlled variable $y(\cdot)$ as quick as possible into the λ-tube, which is a around the setpoint trajectory $y_{ref}(\cdot)$,
- to keep the controlled variable $y(\cdot)$ in the λ-tube with the lowest possible control signal $u(\cdot)$, and
- to control $y(\cdot)$ under (supposed) unknown system parameters (i.e., uncertainty).

Remark 2.1. *An implementation of a λ-tracking control does not necessarily yield in a stable closed-loop system. The λ-tracking controller only achieves the desired object from above.*

Remark 2.2. *The λ-tracking objective also includes two special cases: firstly, the λ-stabilization where $\lambda \neq 0$ and $y_{ref}(\cdot) \leq 0$, and stabilization[1] with $\lambda \leq 0$ and $y_{ref}(\cdot) \leq 0$, [2], [11].*

[1]Although stabilization is also part of other adaptation methods, it is not discussed in this work. Further information on additional content can be found in the corresponding literature.

2.1. CLASSICAL λ-TRACKING USING A P- AND A PD-CONTROLLER

First attempts for solving this problem are done in 1994. In [8], the basic approach of a conventional P-controller was taken up and provided with a gain factor $k(t)$, which increases depending on the size of the control error $e(t)$, [8]:

$$\left.\begin{aligned} e(t) &:= y_{ref}(t) - y(t) \\ u(t) &= k(t)e(t) \\ \dot{k}(t) &= \max\{0, \|e\| - \lambda\}^2 , \quad \lambda > 0 \end{aligned}\right\} \qquad (1)$$

There results are extended in [2] and [3] to a PD-controller for multiple-input multiple-output (MIMO) systems:

$$\left.\begin{aligned} e(t) &:= y_{ref}(t) - y(t) \\ u(t) &= k(t)e(t) + k(t)\dot{e}(t) \\ \dot{k}(t) &= \max\{0, \|e\| - \lambda\}^2 , \quad \lambda > 0 \end{aligned}\right\} \qquad (2)$$

2.2. MODIFIED λ-TRACKING ADAPTATION OF A PD-CONTROLLER

Later, the authors of [5] use the approach of [8] (see Subsection 2.1) to control a biologically inspired sensor system, see also [4]. But, it turned out that the basic structure of the conventional λ-tracking controller is not sufficient for this system. An appropriate modification of the approach of [8] was necessary to get the required performance, [3], [5]:

$$\left.\begin{aligned} e(t) &:= y_{ref}(t) - y(t) \\ u(t) &= k(t)e(t) + \kappa k(t)\dot{e}(t) \\ \dot{k}(t) &= \begin{cases} \gamma(\|e(t)\| - \lambda)^2 & , \lambda + 1 \geq \|e(t)\| \\ \gamma(\|e(t)\| - \lambda)^{0.5} & , \lambda \geq \|e(t)\| < \lambda + 1 \\ 0 & , \|e(t)\| < \lambda \wedge t - t_e < t_d \\ -\sigma\left(1 - \frac{\|e(t)\|}{\lambda}\right) k(t) & , \|e(t)\| < \lambda \wedge t - t_e \geq t_d \end{cases} \\ &\lambda > 0 , \quad \kappa > 0 , \quad \sigma > 0 , \quad \gamma \to 1 , \quad t_d > 0 \end{aligned}\right\} \qquad (3)$$

By the reduction of the gain factor $\dot{k}(t) < 0$, it is possible to respond to a decreasing disturbance and to ensure an optimal utilization of the λ-tube. The result is, that the energy fed to the plant is reduced, so that the system can nearly free oscillate in the λ-tube.

2.3. INTERIM CONCLUSION

The modified λ-tracking control strategy (3) represents a process which is adapted from the classical PD-controller, to get an optimal result of the controlled system. Compared to the approach of [8], the quality of adaptation can be significantly improved. But until reaching the desired behavior, many simulations or optimization procedures are necessary to determine the free parameters. Despite this deficit, the modified λ-tracking control strategy (3) is used in the next steps of this investigation to compare the classical method with the fuzzy-adaptation strategy.

3. GOAL OF THIS WORK

The modified λ-tracking control strategy (3) in [5] represents an important improvement also including a fuzzy-adaptation law, but a large number of preliminary investigations are necessary to reach an optimum of performance. More precisely, the starting point is to find out the potential of a fuzzy-adaptation strategy. With the help of the fuzzy logic, the following questions should be answered:

1. Can a λ-tracking adaptation be performed with the addition of the fuzzy logic?
2. Can similarly good result, as in the approach of [5], be achieved or even more?
3. Can the number of free parameter be reduced (in the best case equal to zero) in order to increase the direct application to a diversity of systems and save costly simulation studies?
4. Can the designed fuzzy-adaptation strategies be used for a lane assist of vehicles to improved the feedback control?

4. FUZZY λ-TRACKING ADAPTATION

In the following subsections, various possibilities are presented for fuzzy-adaptation of mechanical motion systems. The most promising approaches are used later for the fuzzy adapted P-controller and for the comparison to the concept of [5], see Subsection 2.2.

In contrast to the approach of [5], the gain factor $k(t)$ is influenced by the fuzzy logic instead of an analytical adaptation law, where as the PD-structure of the feedback law is kept the same:

$$\begin{aligned} e(t) &:= y_{ref}(t) \quad y(t) \\ u(t) &= k(t)e(t) + \kappa k(t)\dot{e}(t) \\ \dot{k}(t) &= \text{error-dependent fuzzy-adaptation} \\ &\quad\ \text{or} \\ k(t) &= \text{event-dependent fuzzy-adaptation} \end{aligned}$$

To distinguish the various fuzzy systems, the following definition is introduced:

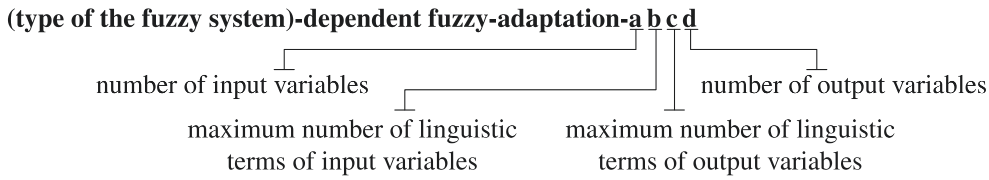

Figure 3. Definition of the fuzzy system designation.

4.1. ERROR-DEPENDENT FUZZY λ-TRACKING ADAPTATION

The first way to implement a λ-tracking adaptation with the help of the fuzzy logic is the change of the gain factor $k(t)$ dependent on the error $e(t)$, see Fig. 4.

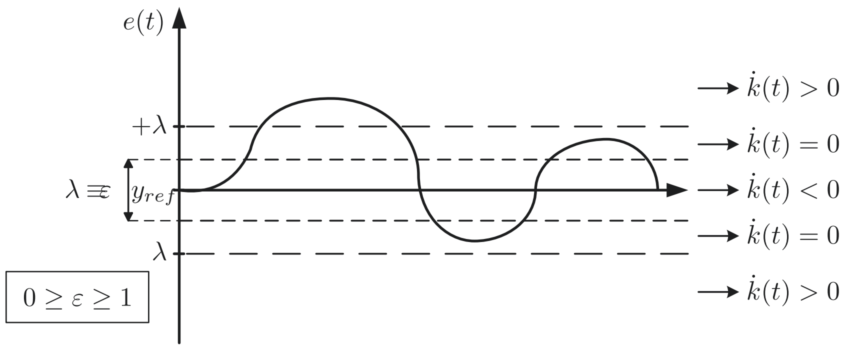

Figure 4. Error-dependent fuzzy λ-tracking adaptation.

GAIN AREA ($e(t)$ $\ll\lambda$):
Is the error $e(t)$ outside the λ-tube, the gain factor $k(t)$ is increased by evaluating the degrees of membership and the rule i:

$$\textit{rule } i: \qquad \textit{IF “ } e(t) \ > \lambda\textit{”, THEN “}\dot{k}(t) > 0\textit{”}$$

MINIMIZATION AREA ($e(t)$ $\geq \lambda\varepsilon$):
If the error $e(t)$ is within the $\lambda\varepsilon$-area, the gain factor $k(t)$ is reduced by evaluating the degrees of membership and the rule j:

$$\textit{rule } j: \qquad \textit{IF “ } e(t) \ \geq \lambda\varepsilon\textit{”, THEN “}\dot{k}(t) < 0\textit{”}$$

STABILIZATION AREA ($\lambda\varepsilon <$ $e(t)$ $< \lambda$):
If the error $e(t)$ does not belong to the presented intervals, the gain factor $k(t)$ is stabilized by evaluating the degrees of membership and the rule m:

rule m: *IF* "$\lambda \cdot \varepsilon < |e(t)| < \lambda$", *THEN* "$\dot{k}(t) = 0$"

Remark 4.1. *A too low stabilization area ($\varepsilon \Rightarrow 1^-$) leads to a continuous adaptation and early reduction of the gain factor $k(\cdot)$. For this reason, the minimization area is limited by $\varepsilon = 1/4$.*

4.1.1. ERROR-DEPENDENT FUZZY-ADAPTATION-1331

The easiest way to implement a error-dependent fuzzy adaptation is the solely use of the three basic rules i, j and m, see Subsection 4.1. For this, the fuzzy system requires only one input variable and one output variable with three linguistic terms (explained in Fig. 5) and the three fuzzified basic rules:

rule 1: *IF* "$|e(t)|$ *is high*", *THEN* "$\dot{k}(t)$ *is positive*"
rule 2: *IF* "$|e(t)|$ *is low*", *THEN* "$\dot{k}(t)$ *is negative*"
rule 3: *IF* "$|e(t)|$ *is medium*", *THEN* "$\dot{k}(t)$ *is zero*"

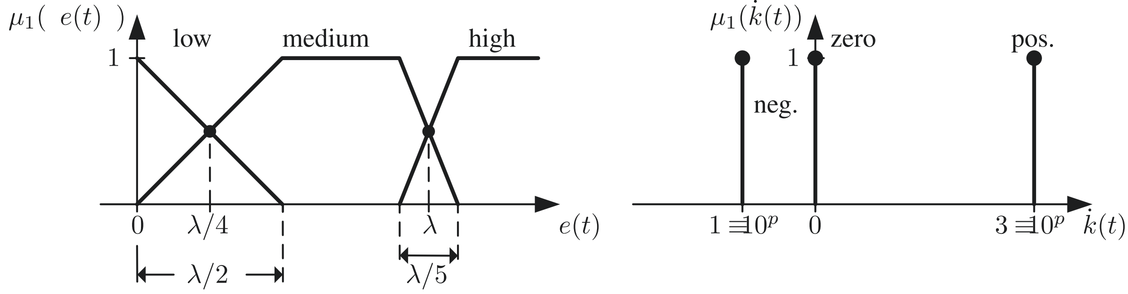

Figure 5. Membership functions of the error-dependent fuzzy-adaptation-1331 with the design factor p.

An advantage in using the fuzzy logic is to overlap the membership functions of the input variable depending on λ. Therefore, a sharp separation of the three specific areas (see Subsection 4.1) does not take place immediately. For selecting the overlapping membership functions, various simulations were performed. It turned out that

- a small overlap of the linguistic terms of the input variable produces an oscillating or chaotic adaptation performance, and
- a large overlap of the linguistic terms of the input variable generates a permanent adaptation of $k(\cdot)$ and partly an unstable system.

By defining the input membership functions dependent on λ, this results in the arrangement of the three singletons at the output for the remaining degrees of freedom of the fuzzy system. Because of the stabilization area, one singleton of the output variable $\dot{k}(t)$ must be zero. For the other two we have a ratio of $3 : 1$ between gain and minimization of $\dot{k}(t)$ by means of further simulations. This case is based on that

- a large positive value of the singleton *„positive"* produces a fast adaptation of the gain factor $k(\cdot)$ when leaving the λ-tube, and
- a small negative value of the singleton *„negative"* avoids a too rapid minimization of the gain factor $k(\cdot)$ and, therefore, a lower, partly stable, new adaptation is achieved.

Remark 4.2. *From the above statements it follows that the degree of freedom $DoF = 1$ for the fuzzy system with $p \hat{=} const$. This means, that in contrast to the approach in [5], only one parameter p must be specified to optimize the adaptation performance. Therefore, the necessary simulation studies will go down, when using this fuzzy system.*

4.1.2. ERROR-DEPENDENT FUZZY-ADAPTATION-1441

The second way to implement a error-dependent fuzzy-adaptation is the use of the two basic rules j and m (see Subsection 4.1), but the third basic rule i is divided into:

rule 1*:*	*IF "* $e(t)$ *is high", THEN "*$\dot{k}(t)$ *is positive"*
rule 2*:*	*IF "* $e(t)$ *is very high", THEN "*$\dot{k}(t)$ *is positive high"*
rule 3*:*	*IF "* $e(t)$ *is low", THEN "*$\dot{k}(t)$ *is negative"*
rule 4*:*	*IF "* $e(t)$ *is medium", THEN "*$\dot{k}(t)$ *is zero"*

This modification makes it possible to divide the increase of the gain factor $\dot{k}(t)$ into two areas. This provides a faster adaptation with a large initial deviation and a constant adaptation performance in the further course when the controlled variable is near the λ-tube. In addition to the subdivision of the third rule, one linguistic term of the input and output variable must be added, see Fig. 6.

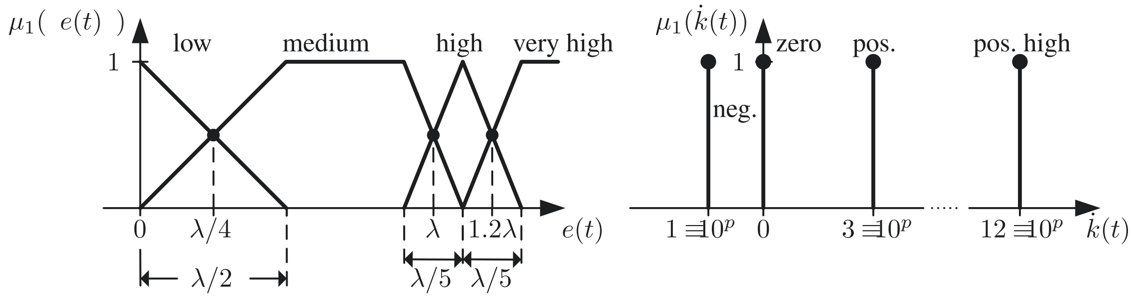

Figure 6. Membership functions of the error-dependent fuzzy-adaptation-1441 with the design factor p.

Remark 4.3. *With the help of further simulations a ratio of 1 : 4 between the two linguistic terms "positive" and "positive high" of the output variable* $\dot{k}(t)$ *is determined.*

By means of this adaptation structure, it is possible to respond quickly to large deviation between the controlled variable $y(t)$ and the λ-tube, without affecting the sensibility of the adaptation performance in the vicinity of the target area. The disadvantage of this system is to react quickly to small or reduced disturbances. Here is a shift of the singleton *"negative"* (see Fig. 6) inefficient, because the gain factor $\dot{k}(t)$ is reduced by a pass through the λ-tube. Furthermore, the adaptation performance is destabilized with a shift of the singleton.

4.1.3. ERROR-DEPENDENT FUZZY-ADAPTATION-3451

By referring to Subsection 4.1.2, it could be demonstrated, that a simple error-dependent fuzzy-adaptation is not sufficient to optimally control a system under disturbances into the λ-tube. The reason is the low information content, which is associated with the norm of the control error $e(t)$, so that the following fuzzy-adaptation-3451 has two new input variables, see Fig. 7.

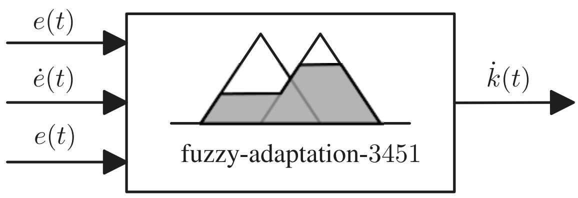

Figure 7. Extended fuzzy system-3451.

With the help of the control error $e(t) := y_{ref}(t) \quad y(t)$ and its derivative $\dot{e}(t)$ as new inputs of the fuzzy system, it is possible, taking into account the corresponding rule-base, to adapt the gain factor $k(t)$ in dependence on the

movement direction and the speed of the controlled variable $y(t)$.
For the selection of membership functions μ_i of the input variables $\underline{u}(t) = [e(t), \dot{e}(t),\ \ e(t)\ \ ]^{\mathrm{T}}$ or singletons of the output variables $\dot{k}(t)$, there are different approaches. One possible implementation is shown in the Figure 8.

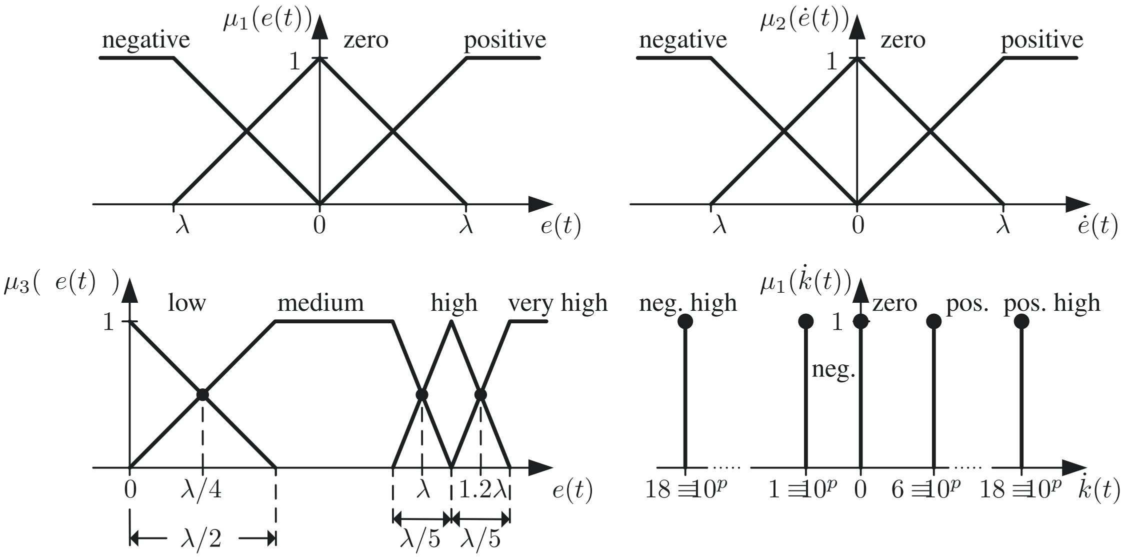

Figure 8. Membership functions of the error-dependent fuzzy-adaptation-3451 with the design factor p.

The linguistic statements and membership functions of the input variable $\ e(t)\ $ were taken from the error-dependent fuzzy-adaptation-1441, see Subsection 4.1.2. In the determination of new input variables $e(t)$ and $\dot{e}(t)$, it was found, with the help of various simulations, that a "simple" approach with three linguistic terms }"negative", "zero", "positive"| and the reference to λ-value is sufficient to achieve the desired adaptation performance.
On the other hand, it turned out that is very difficult to design the rule-base. Because of the different adaptation scenarios and the evaluation of three input signals at the same time, it had to be derived from the basic rules i, j and m, which described in the Subsection 4.1.

The rules used in the error-dependent fuzzy adaptation-3451 are summarized in the Table 1.

Table 1. Rule-base of the error-dependent fuzzy-adaptation-3451.

R01: *IF "* $\ e(t)\ $ *is low,* $e(t)$ *is zero and* $\dot{e}(t)$ *is zero", THEN "*$\dot{k}(t)$ *is negative high"*
R02: *IF "* $\ e(t)\ $ *is low,* $e(t)$ *is negative and* $\dot{e}(t)$ *is positive", THEN "*$\dot{k}(t)$ *is negative"*
R03: *IF "* $\ e(t)\ $ *is low,* $e(t)$ *is positive and* $\dot{e}(t)$ *is negative", THEN "*$\dot{k}(t)$ *is negative"*
R04: *IF "* $\ e(t)\ $ *is medium", THEN "*$\dot{k}(t)$ *is zero"*
R05: *IF "* $\ e(t)\ $ *is high,* $e(t)$ *is negative and* $\dot{e}(t)$ *is negative", THEN "*$\dot{k}(t)$ *is positive"*
R06: *IF "* $\ e(t)\ $ *is high,* $e(t)$ *is negative and* $\dot{e}(t)$ *is zero", THEN "*$\dot{k}(t)$ *is zero"*
R07: *IF "* $\ e(t)\ $ *is high,* $e(t)$ *is positive and* $\dot{e}(t)$ *is positive", THEN "*$\dot{k}(t)$ *is positive"*
R08: *IF "* $\ e(t)\ $ *is high,* $e(t)$ *is positive and* $\dot{e}(t)$ *is zero", THEN "*$\dot{k}(t)$ *is zero"*
R09: *IF "* $\ e(t)\ $ *is very high,* $e(t)$ *is negative and* $\dot{e}(t)$ *is negative", THEN "*$\dot{k}(t)$ *is positive high"*
R10: *IF "* $\ e(t)\ $ *is very high,* $e(t)$ *is negative and* $\dot{e}(t)$ *is zero", THEN "*$\dot{k}(t)$ *is positive"*
R11: *IF "'* $\ e(t)\ $ *is very high,* $e(t)$ *is negative and* $\dot{e}(t)$ *is positive", THEN "*$\dot{k}(t)$ *is positive"*
R12: *IF "* $\ e(t)\ $ *is very high,* $e(t)$ *is positive and* $\dot{e}(t)$ *is positive", THEN "*$\dot{k}(t)$ *is positive high"*
R13: *IF "* $\ e(t)\ $ *is very high,* $e(t)$ *is positive and* $\dot{e}(t)$ *is zero", THEN "*$\dot{k}(t)$ *is positive"*
R14: *IF "* $\ e(t)\ $ *is very high,* $e(t)$ *is positive and* $\dot{e}(t)$ *is negative", THEN "*$\dot{k}(t)$ *is positive"*

With the help of these rules, the fuzzy system is able to optimally assess the current situation. As soon as the disturbance is decreased and before there is a low utilization of the λ-tube, the gain factor $k(t)$ is quickly reduced in order to avoid an unnecessarily high energy fed to the plant.
In contrast to the error-dependent fuzzy-adaptation-1331 and -1441 (see Subsections 4.1.1 and 4.1.2), the optimal

fuzzy system-3451 is able to respond optimal to a reduction of the disturbance. Therefore, the result of the energy fed to the system is reduced substantially and the λ-tube is fully utilized.
The error-dependent fuzzy-adaptation-3451 cannot only respond to a reduced disturbance, but it has also other intelligent features:

- A reduction of the gain factor $k(t)$ always takes place when the controlled variable $y(t)$ is in a certain area and moves in the direction of the setpoint trajectory $y_{ref}(t)$. Therefore, the risk of premature reduction of $k(t)$ can be counteracted.

- An increase of the gain factor $k(t)$ takes place in different intensities. Is the controlled variable outside the λ-tube and moves away from the setpoint trajectory $y_{ref}(t)$, then the gain factor $k(t)$ is significantly increased. But, is the direction of motion in the opposite way, only a weak amplification takes place to avoid an unnecessarily high gain factor $k(t)$.

Remark 4.4. *In [3], [5] and [10], a velocity-dependent change of the gain factor, called a "non-classical-feedback", is specified and analyzed.*

4.1.4. INTERIM CONCLUSION

The error-dependent fuzzy-adaptation-3451 is based on the evaluation of

- the norm of the control error $\|e(t)\|$,

- the control error $e(t)$, and

- the speed of the control error $\dot{e}(t)$,

and is able to assess the situation of the plant and to adjust the gain factor $k(t)$. Through the evaluation of additional information, significant advantages could be achieved over the others presented error-dependent fuzzy-λ-tracking-adaptation, see Subsections 4.1.1 and 4.1.2. For this reason, only the error-dependent fuzzy-adaptation-3451 ($p = 1$) is used in further studies.

4.2. EVENT-DEPENDENT FUZZY-ADAPTATION-2221

Based on the already presented error-dependent fuzzy-adaptations and the modified λ-tracking control strategy (3) as well as time-dependent fuzzy-adaptation [13], at this point a possibility is to show which combines the advantages of all three strategies:

- simple and efficient structure, such as the time-dependent fuzzy-adaptation-1221-T_i [13];

- individual increase of the gain factor $k(t)$, such as the error-dependent fuzzy-adaptation-3451 (Subsection 4.1.3);

- simple reduction part to reduce the gain factor $k(t)$, such as the modified λ-tracking control strategy (3) (Subsection 2.2).

In order to generate a simple fuzzy system, two new input variables are introduced, see Fig. 9:

$$\begin{aligned} A_{out} &= \int_{i=1}^{N} A_i^{out} < A_{max} \\ t_{out} &= \int_{i=1}^{N} t_i^{out} < t_{max}, \end{aligned} \tag{4}$$

where A_{max} and t_{max} are limit values.
These two input variables are limited to describe the characteristic of the controlled variable $y(t)$ outside of the λ-tube. Therefore, the event-dependent fuzzy-adaptation-2221 may be adapted depending on the situation, similar to the error-dependent fuzzy-adaptation-3451, see Subsection 4.1.3. In this context, the use of the norm of the control error $\|e(t)\|$, the control error $e(t)$ and its derivative $\dot{e}(t)$ are redundant.

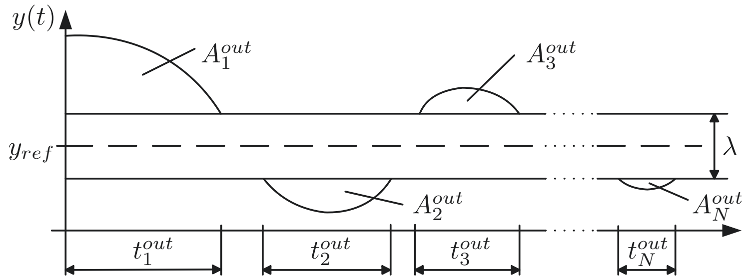

Figure 9. Definition of the event-dependent fuzzy-adaptation according to (4).

Furthermore, a substitute gain factor $\hat{k}(t) = k^{-1}(t)$ (based on the time-dependent fuzzy-adaptation-1221 [13]) is used to generate a high gain factor $k(t)$ in a short tim, see Fig. 10.

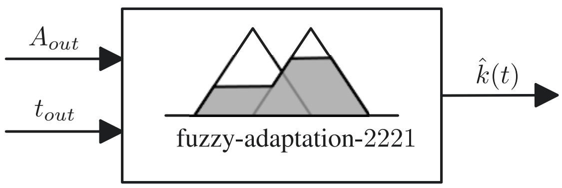

Figure 10. Fuzzy system-2221.

In the determination of membership functions of the input variables $\underline{u}(t) = [A_{out}(t), t_{out}(t)]^{\mathrm{T}}$ and singletons of the output variable $\hat{k}(t)$, it was found by different simulations, that a „simple" approach is sufficient. There are only two linguistic terms {"low","high"} at the inputs and two linguistic terms {"zero","one"} at the output needed, see Fig. 11.

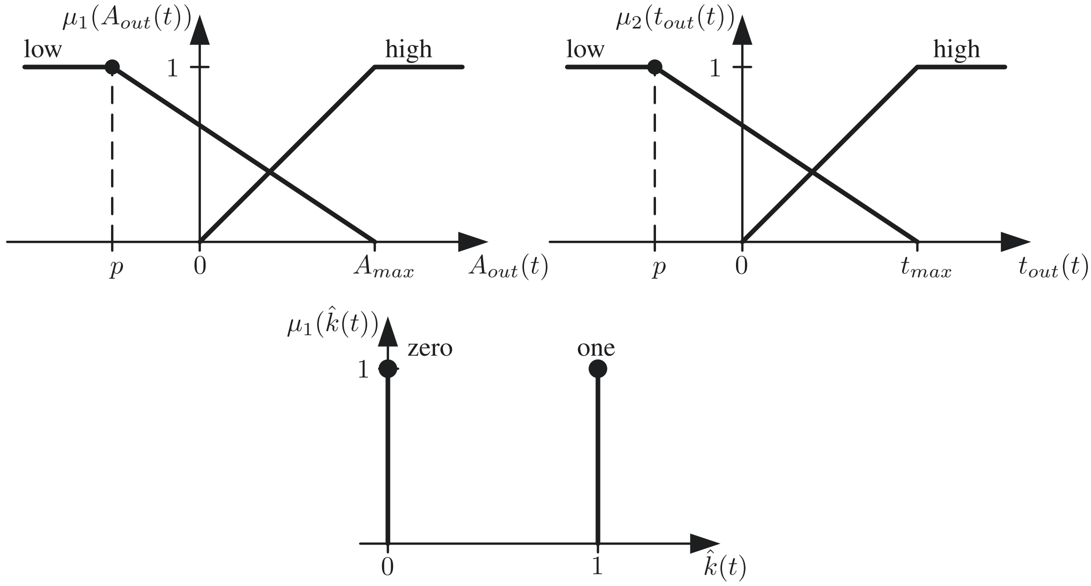

Figure 11. Membership functions of the event-dependent fuzzy-adaptation-2221 with the design factor p.

Owing to the simple structure of the fuzzy system and the increased level of abstraction by the new input variables, the rule-base can be very brief:

rule 1: *IF "A_{out} is low and t_{out} ist low", THEN "$\hat{k}(t)$ is one"*
rule 2: *IF "A_{out} is high or t_{out} is high", THEN "$\hat{k}(t)$ is zero"*

Last, the event-dependent fuzzy-adaptation-2221 needs a reduction part for the gain factor $k(t)$, to react to a decreasing disturbance and to reduced the energy fed to the plant. Different studies have shown that, at this point, the approach of [5] is very comfortable, because the effectiveness and easy combinability with the fuzzy system can be used:

$$\dot{k}(t) = \begin{cases} \text{fuzzy-adaptation-2221} & , \ \|e(t)\| \geq \lambda \\ 0 & , \ \|e(t)\| < \lambda \wedge t - t_e < t_d \\ -\sigma \left(1 - \frac{\|e(t)\|}{\lambda}\right) k(t) & , \ \|e(t)\| < \lambda \wedge t - t_e \geq t_d \end{cases} \tag{5}$$

$$\lambda \geq 0 \,, \quad \sigma > 0 \,, \quad t_d \geq 0$$

In this case the parameters σ and t_d (time duration of stay) represent the adaptation activities related to signal reduction and signal delay. Both parameters are determined according to the model of [5].

5. MODEL OF THE LANE ASSIST OF VEHICLES

The controllers showed their effectiveness in simulations of a fast-adaption receptor cell model, see [4]. Hence, they are now applied to a lane assist in this paper.
In a tracking control, a vehicle tries to follow a given course of the road by means of a change in steering angle of the front wheels (vehicle with front steering). Figure 12 shows the most commonly used measured variables for this controller.

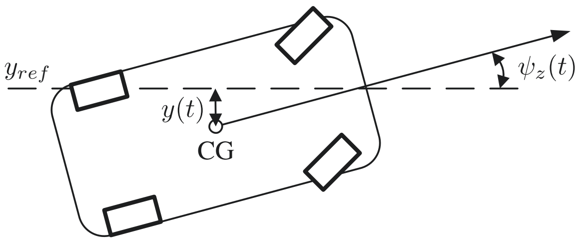

Figure 12. Measured variables of the vehicle for a tracking control.

The vertical distance of setpoint trajectory $y_{ref}(t)$ and the vehicle center of gravity (CG) is referred to as lateral deviation $y(t)$. The angle between the ideal lane centerline and the longitudinal axis of the vehicle is referred to as yaw error $\psi_z(t)$. In combination with the linear single-track model (see Fig. 13 and Tab. 2) and by using the principle of linear momentum, this system can be formulated in the state-space (the vehicle longitudinal velocity v_x must be const.), [14]:

$$\underbrace{\begin{bmatrix} v_y \\ \omega_z \\ \psi_z \\ y \end{bmatrix}^{\cdot}}_{\underline{\dot{x}}(t)} = \underbrace{\begin{bmatrix} \frac{C_v+C_h}{m v_x} & \frac{C_h l_h - C_v l_v}{m v_x^2} & v_x & 0 & 0 \\ \frac{C_h l_h - C_v l_v}{J_z v_x} & \frac{C_v l_v^2 - C_h l_h^2}{J_z v_x} & & 0 & 0 \\ 0 & 1 & & 0 & 0 \\ 1 & 0 & & v_x & 0 \end{bmatrix}}_{=:\underline{A}} \cdot \underbrace{\begin{bmatrix} v_y \\ \omega_z \\ \psi_z \\ y \end{bmatrix}}_{\underline{x}(t)} + \underbrace{\begin{bmatrix} \frac{C_v+C_h}{m v_x} \\ \frac{C_v+C_h}{m v_x} \\ 0 \\ 0 \end{bmatrix}}_{=:\underline{b}} \cdot \underbrace{\delta_v}_{u(t)} + \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{=:\underline{Z}} \cdot \ddot{f} \tag{6}$$

$$\underbrace{\begin{bmatrix} \psi_z \\ y \end{bmatrix}}_{\underline{y}(t)} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{=:\underline{C}} \cdot \underbrace{\begin{bmatrix} v_y \\ \omega_z \\ \psi_z \\ y \end{bmatrix}}_{\underline{x}(t)} \tag{7}$$

With the help of (6) and (7), the basic driving behavior of a vehicle can be sufficiently accurately described and analyzed. The state differential equation (6) is a linear system of differential equations of first order with the front wheel steering angle $\delta_v(t)$ as the input variable $u(t)$ and the state vector $\underline{x}(t)$ with the state variables:

$$\text{slip angle } \beta(t) \text{ and yaw rate } \dot{\psi}_z(t).$$

It should be noted, that some coefficients of matrix $\underline{A}$ and input vector $\underline{B}$ are dependent on the vehicle velocity v. This means that the driving characteristics are changed by the vehicle velocity v. If one chooses, for example, $v = 0$, then the geometric model results to the determination of the static Ackermann angle. On the other hand at high velocities, the vehicle may tend to oscillate depending on the suspension adjustment, [9].

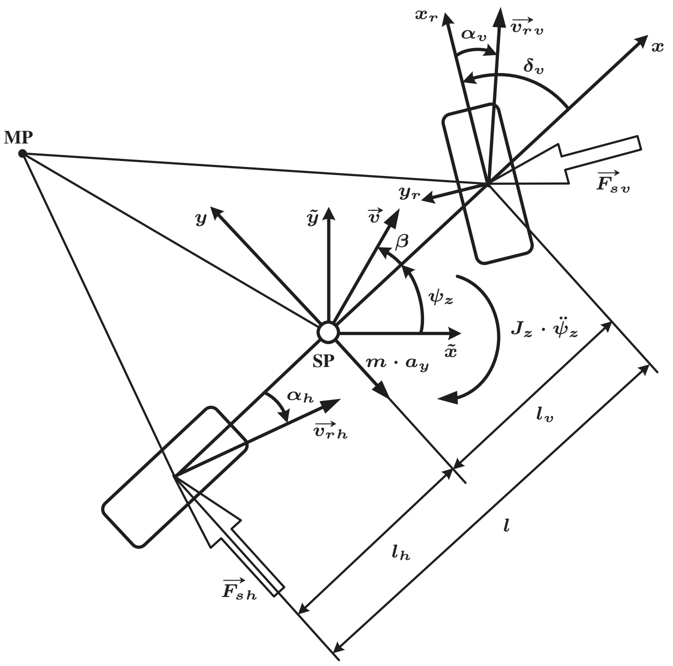

Figure 13. Linear single-track model.

Table 2. Parameters for the single-track model, [14].

Symbols	*Reference vehicle*	*Description*
m	$1700\,\mathrm{kg}$	*mass of vehicle*
J_z	$2500\,\mathrm{kg\,m^2}$	*mass moment of inertia (z-axis)*
C_v	$45\,300\,\mathrm{N/m}$	*cornering stiffness of front axis*
C_h	$65\,000\,\mathrm{N/m}$	*cornering stiffness of rear axis*
l_v	$1.33\,\mathrm{m}$	*distance CG to front axis*
l_h	$1.17\,\mathrm{m}$	*distance CG to rear axis*
v_x	$variable\ constant$	*longitudinal vehicle velocity*

6. CONTROL DESIGN

For the design of a tracking control, a linear state-feedback and a output-feedback with fuzzy-adaptation are developed, see Fig. 14. The goal is a combination of these two controllers to have a optimal control quality. For the basic control of the system, linear state-feedback with a vehicle velocity of $v_{x_{ZR}} = 27.78\,\mathrm{m/s}$ (equal $100\,\mathrm{km/h}$) of the linear single-track model is used. As an additional controller for the λ-tracking control, a fuzzy-adapted P-control is used, which is adapted depending on the lateral deviation $y(\Rrightarrow$ of the vehicle.
The corresponding differential equation is:

$$\underline{\dot{x}}(t) = \underbrace{\left(\underline{A} - \underline{b}\cdot\underline{k}^{\mathrm{T}}\right)\cdot\underline{x}(t)}_{\text{state-feedback control}} + \underbrace{f\left(y, k\right)}_{\substack{\text{fuzzy-adaptive}\\ \text{proportional control}}} \tag{8}$$

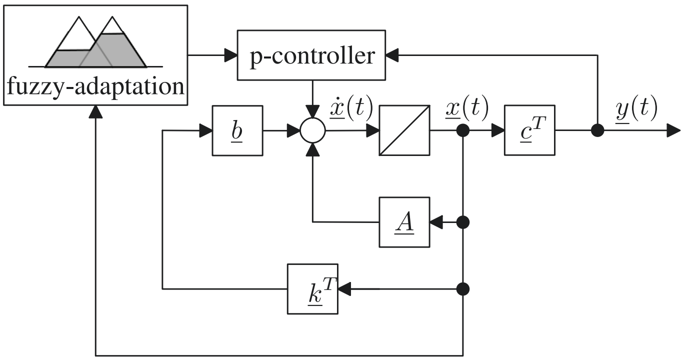

Figure 14. Scheme of the fuzzy-adaptation feedback control.

The reason for using of two controllers is the marginal stability[2] of the vehicle model, but it can be compensated by pole placement for the state-feedback, see Fig. 15.

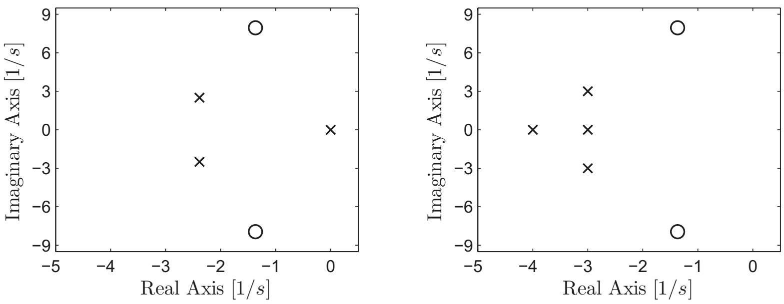

Figure 15. Pole zero map of the linear single-track model linear with (right) and without (left) linear state-feedback.

On the other hand, the fuzzy-adapted P-control can be used for readjustment to correct deviations of the vehicle (see Fig. 12) and model deviations and to control the vehicle into the λ-tube. This prevents that the fuzzy-adaptation affects directly the stability of the controlled system.

6.1. DESIGN OF THE STATE-FEEDBACK CONTROL

By using of pole placement in the controller design, the poles of the plant can be changed. In this way, unstable and marginally stable systems are transferred to stable ones. The precondition for the pole placement of linear state-feedback is a full controllability of all states. For this, the Kalman's criteria of controllability has to be fulfilled: the rank of the matrix

$$Q_s = \left[B \quad AB \quad A^2B \quad \cdots \quad A^{n-1}B\right] \tag{9}$$

has to be maximal n, or has n linearly independent column vectors, [12]. Insertion of the system matrix $\underline{A}$ and the input vector $\underline{B}$ (see (6)) into (9) yields:

$$rang(Q_s) = 4 = n$$

[2] "Marginal stability" means in this context that the model has a pole at the coordinate origin or a conjugated complex pole pair on the imaginary axis and the system can assume a value $\left|\lim_{t\to\infty} y(t)\right| \leq C < +\infty,\ C \in \mathbb{R}^n$.

That means, that the system is completely controllable and a linear state-feedback can be used.
In order to obtain an optimal correction behavior by the undisturbed vehicle model ($\ddot{f}(t) \equiv 0$), the poles P are chosen for the pole placement as follows:

$$P = \begin{bmatrix} -3+3i & -3-3i & -3 & -4 \end{bmatrix}.$$

With these values, a aperiodic corrective behavior for the undisturbed vehicle model ($\ddot{f}(t) \equiv 0$) is given, [13]. But, the transient response can be guaranteed only when the vehicle velocity is $v_x = v_{x_R} = 100\,\mathrm{km/h}$. For comparison, the author of [13] shows in a simulation that the oscillation is not more a aperiodic process when the vehicle velocity $v_x \neq v_{x_R}$ is changed.
In the later studies, there are also simulation results with other velocities $v_x \neq v_{x_R}$, so the fuzzy-adapted P-control is not only able to compensate the disturbance $\ddot{f}(t)$, but also model uncertainties of the linear state-feedback.

6.2. DESIGN OF THE FUZZY-ADAPTATION P-CONTROL

For the design of the fuzzy-adapted P-control according to (8), the following approach is chosen:

$$f\left(\underline{y}, k\right) = -k(t) \cdot \underline{b} \cdot \underline{p}^{\mathrm{T}} \cdot \underline{y}(t) = -k(t) \cdot \begin{bmatrix} \frac{C_v+C_h}{m v_x} \\ \frac{C_v+C_h}{m v_x} \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} p_1 & p_2 \end{bmatrix} \cdot \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\underline{C}} \cdot \underbrace{\begin{bmatrix} v_y \\ \omega_z \\ \psi_z \\ y \end{bmatrix}}_{\underline{x}(t)}$$

Substituting $p_1 = p_2 = 1$, so a destabilization of the system is counteracted, because the yaw error $\psi_z(t)$ is also multiplied with the gain factor $k(t)$. As a reminder, the gain factor $k(t)$ is only dependent on the lateral deviation $y(t)$. This prevents, that the vehicle turns too much to the setpoint trajectory $y_{ref}(t)$ for a high-gain factor $k(t)$ and causes an unstable vehicle behavior.

7. SIMULATIONS

Based on the previous Section 6, simulations of various driving scenarios with the following disturbance take place:

$$\ddot{f}(t) = 3\sin(t) + 10\cos(2t) \tag{10}$$

This disturbance $\ddot{f}(t)$ represents a lateral force on the vehicle, which is caused by fluctuating crosswind and lateral road inclinations, see Fig. 16.

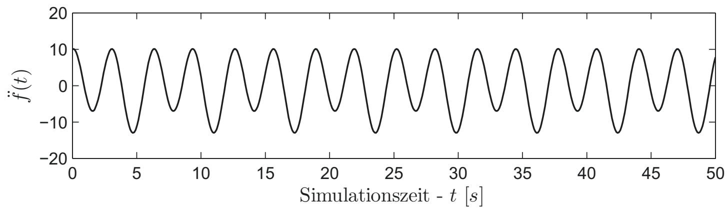

Figure 16. Graph of the lateral disturbance $\ddot{f}(t)$.

The vehicle speed is kept constant in the following three tests:

$$v_x \in \{100\,\mathrm{km/h}, 50\,\mathrm{km/h}, 175\,\mathrm{km/h}\},$$

The additional fuzzy-adapted P-control can control the vehicle into the λ-tube, although a part of the vehicle velocity v_x does not match with the velocity of the controller design ($v_{x_R} = 100\,\mathrm{km/h}$). A last study is performed to find out, how the system behaves in a unsteady ride. In this context, the vehicle is decelerated from an initial velocity $v_{x_0} = 175\,\mathrm{km/h}$ to a final velocity $v_{x_{end}} = 50\,\mathrm{km/h}$. Research has shown that this case is very critical to assess for a linear state-feedback, because the model is dynamically changing ($\Rightarrow$ non-linear single-track model).
Furthermore, the event-dependent fuzzy-adaptation-2221 is only used in the following simulations (Subsections 7.1 – 7.4) to illustrate the effectiveness of the new control strategy.

7.1. SIMULATION WITH $v_x = 100\,\mathrm{km/h}$ AND $\dot{v}_x \equiv 0$

In a first investigation, we focus on the vehicle behavior with and without an additional fuzzy-adapted P-control at a vehicle velocity of $v_x = v_{x_R} = 100\,\mathrm{km/h}$. In the following Figure 17, the simulation is shown without a fuzzy-adapted P-control.

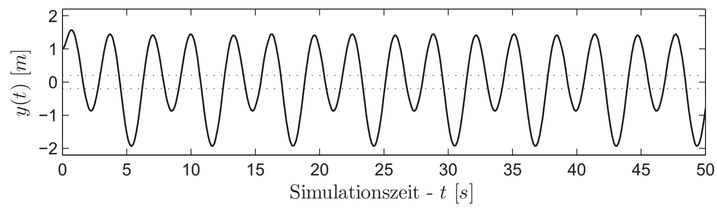

Figure 17. Simulation with $v_x = 100\,\mathrm{km/h}$ and without the event-dependent fuzzy-adaptation-2221.

It can be clearly seen, that the system is stable but the control signal $u(t)$ is not sufficient to control the vehicle into the λ-tube. A new pole placement is not the correct way, because the basic control behavior would change. Therefore, an additional control is required.
The modified control behavior with a fuzzy-adapted P-control is shown in the following Figure 18.

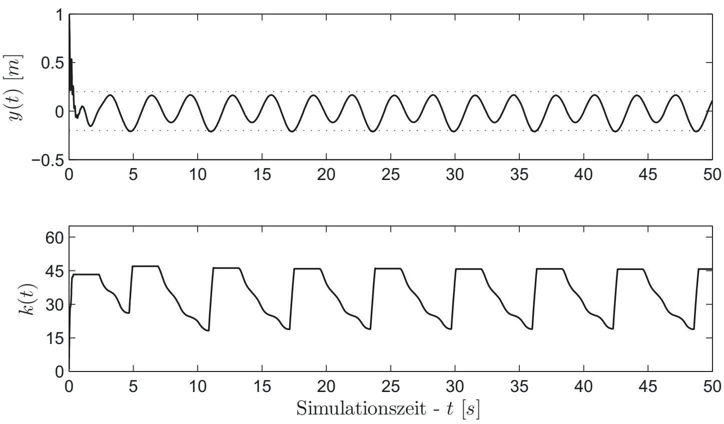

Figure 18. Simulation with $v_x = 100\,\mathrm{km/h}$ and with the event-dependent fuzzy-adaptation-2221.

Based on the Figure 18, one can see that the control behavior is improved. With the help of the event-dependent fuzzy-adaptation-2221, it is possible to control the vehicle into the λ-tube. At the beginning of the simulation, the gain factor $k(t)$ is increased. The result is a movement of the vehicle into the λ-tube. In the further course of the simulation, a permanent adapting of the gain factor $k(t)$ takes place by the event-dependent fuzzy-adaptation-2221. After entering of the controlled variable $y(t)$ into the λ-tube, the value for $k(t)$ is kept constant in the following time $(\Delta t < t_d)$. When $(\Delta t \ll t_d)$ and $y_{ref} \quad y(t) \quad \geq \lambda$, then the gain factor is reduced. But, if the controlled variable $y(t)$ leaves the λ-tube, then the gain factor $k(t)$ is increased again, repeatedly.

7.2. SIMULATION WITH $v_x = 50\,\mathrm{km/h}$ AND $\dot{v}_x \equiv 0$

A second simulation is carried out to find, how the vehicle behaves with and without an additional fuzzy-adapted P-control at a reduced vehicle velocity $v_x \gg v_{x_R}$. Owing to the additional model deviation, it is to be expected that the basic control behavior is changing by the linear state-feedback. Figure 19 shows the results without a fuzzy-adapted P-control.
Based on the Figure 19, one can see that the basic control behavior has changed in comparison to the first study, see Fig. 17. Owing to the low vehicle velocity $v_x = 50\,\mathrm{km/h}$ and despite of the model deviations, the linear state-feedback is capable to control the plant with smaller deviations to the setpoint trajectory $y_{ref}(t)$. Therefore, the requirements for the fuzzy-adapted P-control appear to be smaller than the first investigation. In [13], however, it

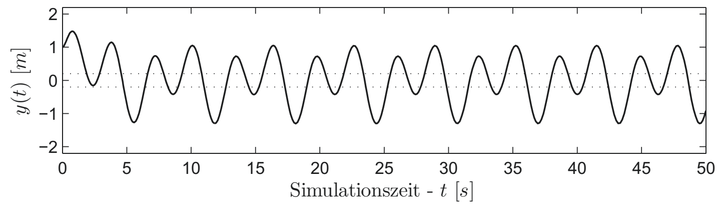

Figure 19. Simulation with $v_x = 50\,\mathrm{km/h}$ and without the event-dependent fuzzy-adaptation-2221.

could be demonstrated that a fuzzy adaptation method tends to incorrect interpretations, when the disturbance is below a certain value.
Figure 20 shows the result of simulation with a fuzzy-adapted P-control.

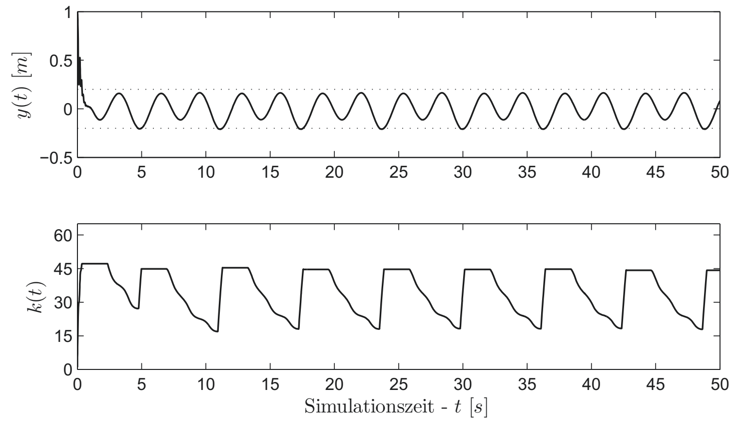

Figure 20. Simulation with $v_x = 50\,\mathrm{km/h}$ and with the event-dependent fuzzy-adaptation-2221.

With respect to Figure 20, one can see that here the plant is also optimally controlled with help of the event-dependent fuzzy-adaptation-2221. A peculiarity may be observed that the trend of the gain factor $k(t)$ is similar compared to the first investigation, see Fig. 18. The formulated problem of a faulty fuzzy-adaptation, owing to a incorrect interpretations by a small disturbance, is not relevant here. The reason is the lower yaw amplification factor[3] of the vehicle. This means that a larger steering wheel angle $\delta_v(t)$ and control signal $u(t)$ is necessary, respectively. Therefore, the event-dependent fuzzy-adaptation-2221 must spend similar values as in the first investigation.

7.3. SIMULATION WITH $v_x = 175\,\mathrm{km/h}$ AND $\dot{v}_x \equiv 0$

The last investigation is directed to a scenario with a constant vehicle velocity to find, how the vehicle behaves with and without an additional fuzzy-adapted P-control at an increased vehicle velocity $v_x \rightarrow v_{x_R}$. Owing to the dynamic characteristics of the vehicle at high velocities, an optimal control at this point is not only comfortably, but also safety critical. In the following Figure 21, the simulation result is shown without a fuzzy-adapted P-control.
With respect to Figure 21, one can see that the linear state-feedback is incapable to control the plant at a high vehicle velocity. It results in an unstable oscillation of controlled variable (⇒ unstable vehicle behavior) as a result of the disturbance $\ddot{f}(t)$. Therefore, an additional control is absolutely necessary.
The modified control behavior with a fuzzy-adapted P-control is shown in the following Figure 22.
Figure 22 shows a significantly better control behavior. Owing to the additional fuzzy-adapted P-control, the vehicle is not only stabilized but it is also controlled into the λ-tube. Compared to the previous simulation (see Fig. 21), the unstable oscillation of the vehicle is prevented. For this purpose, the gain factor $k(t)$ is comparatively strongly increased at the beginning, in order to compensate the model deviations and the control error. At the time $t > 15\,\mathrm{s}$, the gain factor $k(t)$ shows a periodic adaptation as in the studies before. The level which is reached at the end of

[3]"Yaw amplification factor" refers to the response of the vehicle (yaw velocity). This is dependent on the steering wheel angle and on the vehicle velocity v_x, [15], [14].

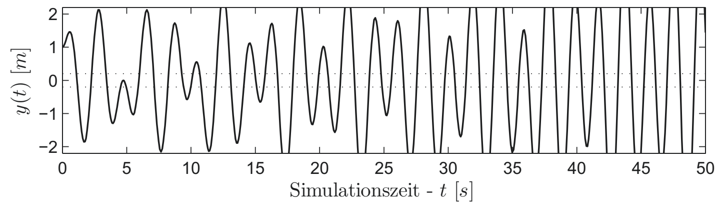

Figure 21. Simulation with $v_x = 175\,^{\mathrm{km}}/_{\mathrm{h}}$ and without the event-dependent fuzzy-adaptation-2221.

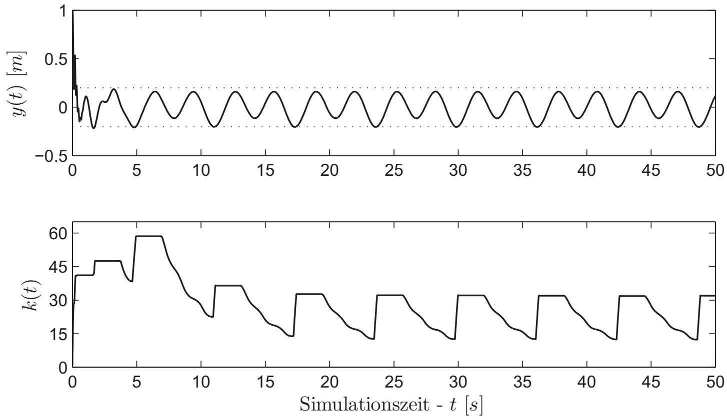

Figure 22. Simulation with $v_x = 175\,^{\mathrm{km}}/_{\mathrm{h}}$ and with the event-dependent fuzzy-adaptation-2221.

the simulation time is lower compared to the first two studies, see Fig. 18 and 20. The reason is the higher yaw amplification factor in this simulation.

7.4. SIMULATION WITH $v_{x_0} = 175\,^{\mathrm{km}}/_{\mathrm{h}}$ AND $v_{x_{end}} = 50\,^{\mathrm{km}}/_{\mathrm{h}}$

In a last study, we consider, how the vehicle behavior changes with and without an additional fuzzy-adapted P-control at a braking process. Owing to the non-linear system ($\dot{v}_x \nLeftarrow 0$), this case is especially interesting, because the state-feedback is based on a linear vehicle model and thereby a faulty control behavior could be produced. In the following Figure 23, the simulation is shown without a fuzzy-adapted P-control.

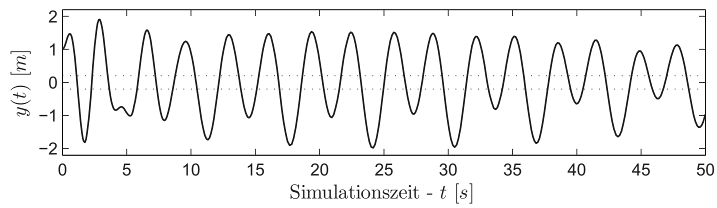

Figure 23. Simulation with $v_x = 175\,^{\mathrm{km}}/_{\mathrm{h}} \Rightarrow 50\,^{\mathrm{km}}/_{\mathrm{h}}$ and without the event-dependent fuzzy-adaptation-2221.

Based on the Figure 23, one can see that the system behavior is stable at the observed braking process. Therefore, the designed linear state-feedback is also able to compensate larger model deviations and certain model dynamics ($\underline{A}$(t) and $\underline{b}$(t), see (6)). However, a control into the λ-tube does not take place.

Figure 24 shows the result of the simulation with a fuzzy-adapted P-control. One can see that the additional fuzzy-adaptive control optimally forces the vehicle into the λ-tube, regardless to the vehicle velocity change $\dot{v}_x < 0$ and the resulting consequences for the linear state-feedback. By the pure evaluation of the current situation, the event-dependent fuzzy-adaptation-2221 is able to adapt the gain factor $k(t)$, to counteract model and control deviation.

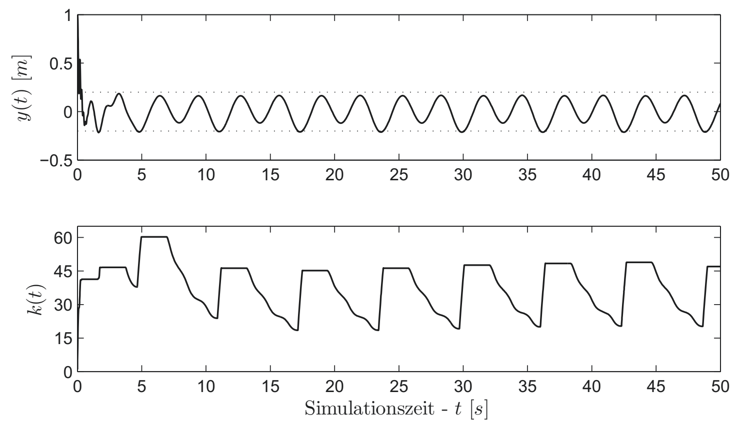

Figure 24. Simulation with $v_x = 175\,^{\mathrm{km}}/_{\mathrm{h}} \Rightarrow 50\,^{\mathrm{km}}/_{\mathrm{h}}$ and with the event-dependent fuzzy-adaptation-2221.

7.5. COMPARISON OF ADAPTATION STRATEGIES

In the previous Subsections 7.1 – 7.4, the advantages of an additional fuzzy-adapted P-control for a new lane assist of vehicle which control the vehicle in a λ-tube around the setpoint trajectory $y_{ref}(t)$ were shown. But, the goal of this subsection is to find out which of the presented fuzzy adaptation method:

- error-dependent fuzzy-adaptation-3451 (Subsection 4.1.3),
- event-dependent fuzzy-adaptation-2221 (Subsection 4.2)

and classical adaptation method:

- modified λ-tracking control strategy (3) (Subsection 2.2)

is the most effective. For this, the existing studies are used to analyze the different adaptation strategies and to determine the optimal method.
As the results of the controlled variable $y(t) = y_{ref} \quad e(t)$ are visually almost indistinguishable from each other, a performance index is introduced:

$$J = \underbrace{\frac{1}{2}\int q \cdot x(t)^2 dt}_{J_x} + \underbrace{\frac{1}{2}\int r \cdot u(t)^2 dt}_{J_u}$$

$$x(t) = \begin{cases} e(t) \quad \lambda & , \lambda \geq e(t) \\ 0 & , 0 \geq e(t) < \lambda \end{cases} \tag{11}$$

In choosing the weighting factors q and r, however, certain conditions must be considered:

- Owing to the various value ranges of $y(t)$ and $u(t)$ a scaling is necessary.
- For a fast control of the controlled variable into the λ-tube, $x(t)$ must be weighted more than $u(t)$.
- Quantity $x(t)$ makes no contribution when the controlled variable is in the λ-tube, so the simulation time t must be taken into account in the weighting factors q and r.

After a simulation study, the weighting factors have been found suitable for the performance index (11):

$$q = 200 \quad \text{and} \quad r = 1\,.$$

By this type of weighting, the course of controlled variable $y(t)$ outside the λ-tube is very severely punished. As soon as the controlled variable $y(t)$ is inside the target area, only the required control signal $u(t) = \delta_v(t)$ is taken into account.
The results of the performance indices for the various investigations from Subsections 7.1 – 7.4 are summarized in the Table 3. For a better overview, they are given in percent relative to the classical adaptation method in [5].

Negative values of the performance indices mean that the fuzzy-adaptation has reached a better value. For positive numbers, the reverse is true.

Table 3. Comparison of the performance index J_i of fuzzy-adaptations regarding to [5].

performance index $\boldsymbol{J = J_x + J_u}$		***vehicle velocity*** **100** $\mathbf{km/h}$	**50** $\mathbf{km/h}$	**175** $\mathbf{km/h}$	**175** $\mathbf{km/h} \in$ **50** $\mathbf{km/h}$
fuzzy-adaptation-3451	J_u	3.36 %	20.41 %	8.98 %	6.09 %
	J_x	12.54 %	4.26 %	34.06 %	8.08 %
	$\boldsymbol{J}$	**−10.01 %**	**−7.81 %**	**−19.27 %**	**−3.81 %**
fuzzy-adaptation-2221	J_u	0.06 %	3.09 %	0.81 %	0.56 %
	J_x	33.93 %	36.69 %	34.94 %	31.76 %
	$\boldsymbol{J}$	**−24.60 %**	**−27.95 %**	**−23.21 %**	**−22.36 %**

Based on the Table 3, one can generally say that no adaptation method in the carried out simulations appears negative. This is not always the case. The author of [13] shows with the help of a long-term study that the disturbance function $\ddot{f}($⇶ makes an essential contribution to the adaptation performance. Owing to the quasi zero-mean disturbance signal $\ddot{f}($⇶, all used adaptation methods can optimally work in these investigations.
Comparing the energy J_u fed to the plant (see Tab. 3), a clear trend can be seen: The event-dependent fuzzy-adaptation-2221 and the modified λ-tracking control strategy (3) have both similar results with a slight advantage of the fuzzy-adaptation method. In contrast, the error-dependent fuzzy-adaptation-3541 provides a fluctuating result. Depending on these investigations, a significantly better or poorer performance index J_u occurs. This is also confirmed by the study in [13]. It turned out that the error-dependent fuzzy-adaptation is highly dependent on the disturbance function $\ddot{f}($⇶ and, therefore, it cannot be used for all systems. For this reason, only the other two adaptation methods will be made in the further course.
When comparing the control performance, J_x also results an clear result: Despite the quasi identical energy fed to the plant, the factor is much better by the event-dependent fuzzy-adaptation-2221 compared to the classical adaptation method by [5]. On the average, the control performance J_x is lower by $1/3$ than the value of the comparison process. Thereby, the evaluation of the total performance index J is 25 % better when comparing the two methods. The findings obtained with the modified λ-tracking control strategy (3) must be judged critical at this point. Although this approach could not dominate in these studies, however, there was no fluctuating result as with the error-dependent fuzzy-adaptation-3451. Overall, it must be said that the modified λ-tracking control strategy (3) is suitable for many applications, however, a parameter optimization is always necessary in order to obtain similar results as a fuzzy method.

8. CONCLUSION AND OUTLOOK

The paper dealt with the development of various fuzzy-adaptation laws for λ-tracking control of a lane-assist of vehicles. With the help of the various investigations, it was shown which positive influence has an additional fuzzy-adaptation. A pure linear state-feedback can be applied only for a velocity v_x of the vehicle and a specific behavior. But, a fluctuating disturbance $\ddot{f}($⇶ may lead to different control behavior at different vehicle velocities $v_x($⇶, which has to be compensated. Therefore, the additional use of a fuzzy-adapted P-control could eliminate the crucial disadvantages. The result is a complete control system which allows a optimal λ-tracking control for various constant vehicle velocities v_x or vehicle accelerations $\dot{v}_x$.
Furthermore, the comparison of the various adaptation methods show that all presented methods can indeed adapt the control behavior of the vehicle, but only one method was able to prevail. With the help of the event-dependent fuzzy-adaptation-2221 it could be shown, how much potential lies in an additional adaptation to the controlled system.
Based on the findings in this paper, further investigations of the fuzzy-adapted lane assist are directed to:

- Investigations of a fuzzy-adaptation behavior with other lateral disturbances, see [13];
- Investigations of the fuzzy-adapted lane assist of vehicles with respect to a lateral disturbance and a cornering, evasive or overtaking maneuvers at the same time;
- Investigations of the fuzzy-adaptation behavior in combination with other control structures, such as a PID-controller or fuzzy controller.

9. REFERENCES

[1] Ament, C. (2012): *Skript zur Vorlesung Fuzzy Control im WS 2012/2013 (Lecture Notes of Fuzzy Control of the WS 2012/2013)*; Script, TU Ilmenau (Germany).

[2] Behn, C. (2005): *Ein Beitrag zur adaptiven Regelung technischer Systeme nach biologischen Vorbild (A contribution to the adaptive control of technical systems according to biological model)*; PhD thesis, Cuvillier, Göttingen (Germany).

[3] Behn, C. (2013): *Mathematical modeling and control of biologically inspired uncertain motion systems with adaptive features*; Habilitation thesis, Faculty of Mechanical Engineering, TU Ilmenau (Germany).

[4] Behn, C. (2013): Modeling the Behavior of Hair Follicle Receptors as Technical Sensors using Adaptive Control; *in Proceedings 10th International Conference on Informatics in Control, Automation and Robotics (ICINCO 2013)*, Reykjavík (Island), pp. 336–345, SciTePress.

[5] Behn, C. and P. Loepelmann (2012): Adaptive vs. Fuzzy Control of Uncertain Mechanical Systems; *International Journal of Applied Mechanics (IJAM)* **4**(2), 2012, DOI 10.1142/S1758825112500196.

[6] Brüdigam, C. (1994): *Intelligente Fahrmanöver sehender autonomer Fahrzeuge in autobahnähnlicher Umgebung (Intelligent driving maneuvers of autonomous vehicles in highway-like surroundings)*; PhD thesis, University FAF Munich (Germany).

[7] Ilchmann, A. (1993): *Non-Identifier-Based High-Gain Adaptive Control*; 1. Edition, Springer, London (UK).

[8] Ilchmann, A. and E. P. Rayn (1994): Universal λ-tracking for nonlineary-perturbed systems in the presence of noise; *Automatica* **30**, pp. 337–346.

[9] Kölbl, C. (2011): *Darstellung und Potentialuntersuchung eines integrierten Quer- und Längsreglers zur Fahrzeugführung (Representation and potential investigation of an integrated lateral and longitudinal controller for vehicles)*; PhD thesis, Cuvillier, Göttingen (Germany).

[10] Krüger, M. (2013): *Beiträge zum Entwurf und Analyse neuartiger Regelungskonzepte für biologisch inspirierte, mechanische Bewegungssysteme (Contributions to the Design and Analysis of novel Control Concepts for biologically inspired mechanical Motion Systems)*; Bachelor thesis, Faculty of Mechanical Engineering, TU Ilmenau (Germany).

[11] Loepelmann, P. (2011): *Design and Analysis of Control Strategies for biologically inspired Sensor Systems*; Master thesis, Faculty of Mechanical Engineering, TU Ilmenau (Germany).

[12] Lunze, J. (2013): *Regelungstechnik 2 – Mehrgrößensysteme (Control Engineering 2 – MIMO-Systems)*; 7. Edition, Springer Vieweg, Berlin.

[13] Noll, A. (2013): *Beiträge zur Analyse, Entwurf und Implementation von Fuzzy-Regelungskonzepten für mechanische Bewegungssysteme (Contributions to the Analysis, Design and Implementation of Fuzzy Control Concepts for Mechanical Motion Systems)*; Project thesis, Faculty of Mechanical Engineering, TU Ilmenau (Germany).

[14] Mitschke, M. and H. Wallentowitz (2004): *Dynamik der Kraftfahrzeugtechnik (Dynamics of Motor Vehicle Technology)*; 4. Edition, Springer, Berlin.

[15] Schindler, F. (2013): *Fahrdynamik: Grundlagen des Lenkverhaltens und ihre Anwendung für Fahrzeugregelsysteme (Vehicle Dynamics: Basics of steering behavior and its application for vehicle control systems)*; 2. Edition, Expert Verlag, Renningen (Germany).

[16] Unbehauen, H. (2008): *Regelungstechnik 1 (Control Engineering 1)*; 15. Edition, Vieweg, Wiesbaden (Germany).

[17] Zadeh, L. (1965): Fuzzy Sets; *Information and Control* **8**, pp. 338–353.

[18] Zapp, A. (1988): *Automatische Fahrzeugführung durch Rechnersehen (Automatic Vehicle Control by Computer Vision)*; PhD thesis, University FAF Munich (Germany).