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Numerical Models for Speckle Fields

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Speckles appear as noisy steady light patterns caused by random phase disturbance of coherently propagating light. Their suppression and use in optical metrology make its simulation and evaluation necessary. This contribution addresses the issue of numerical propagation of speckle distributions.

1 Introduction

When we work on a practical setup in our lab with coherent light, grainy steady patterns, called speckles, appear. They are caused by light straying either on rough surfaces or within scattering volumes or can be generated synthetically by spatial light modulators. They are noisy, random light distributions and can be evaluated statistically. Therefore they are applied as test distributions in metrology, i.e. shearing interferometry, measurements of vibrations and rotations, visualization of aberrations and phase retrieval. If a speckle field is captured by a camera one gets an intensity image. Because of its noise, i.e. random characteristics statistical evaluation can be done as follows: either in a histogram where the counts for each intensity value are plotted and can be fitted to a special probability density function PDF, or by calculation of the autocorrelation function, which contains the information of the correlation length, i.e. the average speckle size *ASS*. The digital generation of complex amplitude $U(N,M)$ of speckle distributions is performed by adding two sets of random, Gaussian distributed numbers; one set represents the real and the other the imaginary part, the intensity is then calculated by the multiplication of its complex amplitude times its complex conjugate [1].

2 Evaluation of Speckles

In this contribution the evidence of speckle distribution is given by the answer of two questions:

1. Does the intensity distribution obey the negative exponential PDF?
2. Does the average speckle size follow the relationship of $\lambda z/2W$?

Both answers should be positive for the generation of fully developed speckle patterns and after its processing by numerical propagation [2].

The statistical behavior is analyzed by making up a histogram of intensity followed by the fitting of the curves by gamma as well as negative exponential PDF. The gamma PDF depends on 2 parameters of α and β, whereas the negative exponential PDF is described by the parameter of μ. If $\alpha=1$ and $\beta=\mu$, i.e. $\beta/\mu=1$, the gamma PDF reduces to negative exponential function what is a typical property of speckles.

The autocorrelation function delivers the correlation length, the distance between lateral spatial locations where the intensity values are still correlated statistically with each other. This corresponds to the average speckle size of the observed distribution and is evaluated by measuring the distance between the central global maximum and the first local minimum. Meanwhile this average speckle size follows the relationship of $ASS=\lambda z/2W$ with λ as the wavelength, z the propagation distance and $2W$ the aperture width for squared, but $ASS=1{,}22\cdot\lambda z/2R$ for circular apertures having a radius of R. Both criteria, i.e. the negative exponential PDF and the *ASS* are used for speckle validation [1].

The propagation of optical wavefields is represented by the Kirchhoff diffraction integral. This can be simplified to a Fresnel transform for the paraxial regime which corresponds to a Fourier transform of a multiplication of the complex amplitude distribution $U(x',y')$ in object plane times a parabolic phase propagator. This is called direct method (DM). The Kirchhoff diffraction integral can be seen as a convolution of $U(x',y')$ by the impulse response of free space which is generally performed in spectral domain by multiplication of its Fourier transforms. Then the spectral result is transformed back to the spatial domain by inverse Fourier transform. If this impulse response is approximated parabolically we call this spectral method (SM). The distributions and the following propagation can be only represented numerically by sampling. This results in 2 consequences: I) replicas appear due to propagation at a lateral replica period $RP'=\lambda z/\delta X'$ with $\delta X'$ as pixel size at object plane, II) there is a different pixel size δX in the diffraction plane for each method. For DM the pixel size scales with the distance by $\delta X=\lambda z/(N\delta X')$ with N as an integer number of samples while for SM δX remains const., i.e. $\delta X=\delta X'$ [2]. This behavior concerns both lateral coordinates of x,y.

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3 Observations

The observation of the numerical propagation is realized in 3 steps as follows:

I) the numerical evaluation of the propagation integrals using DM and SM respectively, without considering the sampling and pixel size,

II) the adaptation of the spatial extent $\Delta X=N\delta X$ to the replica period $RP'=\lambda z/\delta X'$. This works only for a unique distance. For further distances RP' increases. Holding $\delta X=\delta X'$, the spatial extent ΔX should be enlarged by zero padding: $\Delta X=RP'=M\delta X$.

III) the expansion of the spatial extent ΔX to the diffraction area [3]. For a given distance z and aperture size of $2W$, ΔX is enlarged to $\Delta X= 2W+\lambda z/\delta X'=RP=\lambda z/\delta X$. The new replica period RP can be achieved by a new (smaller) pixel size of $\delta X=\lambda z/(\lambda z/\delta X'+2W)$. The original distribution has to be interpolated followed by an appropriate zero padding to the diffraction area.

The numerical algorithms of DM and SM were regarded versus the propagation distance. First, the intensity statistics were evaluated concerning the negative exponential PDF, where one wishes to have the parameters $\alpha=1$ and $\beta/\mu=1$ as mentioned above. Second, the measured correlation length is compared to the relationship of average speckle size ASS.

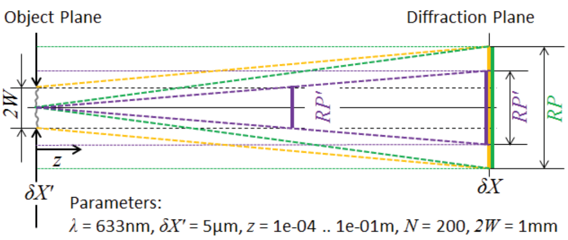

Fig. 1 *Observation: step 2 (violet), diffraction area (orange), step 3 (green).*

When just the DM and SM are programmed (step I), the calculated distributions are not comparable to each other, due to the different pixel sizes mentioned above. The statistics of the intensity distribution follows the expectation of the negative exponential PDF, but the reproduction of ASS fails. The measured size corresponds to one pixel representing the behavior of the pixel size for both DM (scaled with distance) and SM (remains const.) [s. **Fig. 1**].

The adaptation of the spatial extent to the replica period results in intensity statistics as expected and reproduces the functional behavior of the speckle size [s. **Fig. 2**]. It turned out that the ASS was only few pixels, so that an exact confirmation could not be reached. Here, the DM and SM yielded identical intensity distributions.

Considering the diffraction phenomenon under the conditions of step III) delivers once again identical distributions independent of SM or DM. However, the speckle intensities don't follow the negative exponential PDF anymore. But it should not be surprising because of the intensity decay at the boundaries due to diffraction. The expectation of the ASS could be confirmed similar to step II).

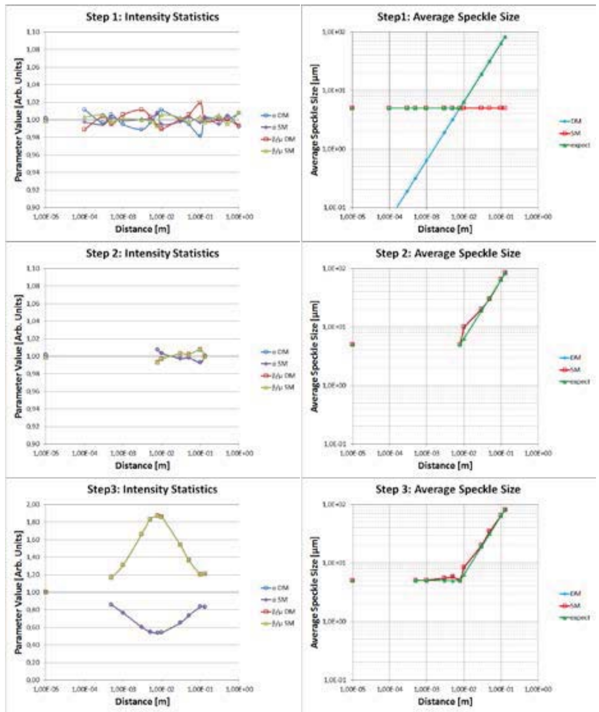

Fig. 2 *Intensity statistics (left) and average speckle size (right) vs. propagation distance.*

4 Conclusions

For the numerical propagation of speckle fields obeying the sampling criterion $\Delta X=RP'$ is inevitable. In a strict sense, this criterion is only given for one propagation distance. If one wished to propagate to further axial distances or to expand the lateral region, an appropriate zero padding as well as sampling (interpolation of pixel size) are available methods to adapt this new situation to the sampling criterion. Then, the result is independent on the applied method (DM, SM).

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