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FIXED CENTRODE OF THE COMPLIANT ISOSCELES SLIDER – CRANK MECHANISM

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Abstract

Compliant mechanisms gain some or all of their mobility from the relative flexibility of their joints rather than from rigid-body joints only. Compliant mechanisms can provide many benefits in the solution of design problems: they have less wear, weight, noise and backlash than their rigid-body counterparts. They can be manufactured from one piece of material, and therefore they are suitable to be applied in micromechanics. On the other hand, nonlinearities introduced by the large deflection of elastic segments further complicate the analysis of compliant mechanisms.

This paper takes into consideration the isosceles slider–crank mechanism and its compliant counterpart mechanism, being developed on the basis of the rigid-body mechanism. The design of the compliant slider-crank mechanism with notch joints (circular flexure hinges) has been shown in this paper. The methods of determining of the "fixed centrode" has been also presented for the compliant isosceles slider–crank mechanism. The aim of the paper is to suggest a method of determining of the "fixed centrode" of the compliant mechanism.

Nachgiebige Mechanismen bekommen ihre Beweglichkeit von ihre elastischen Gelenken. Sie können aus einem Stück hergestellt werden und deswegen sind sie geeignet für die Anwendungen in der Mikromechanik. Die nachgiebige Mechanismen haben viele Vorteile. Sie sind leicht, spielfrei, reibungsarm, verschleißarm (es gibt nur innere Reibung) und geräuscharm.

Anderseits, große nichlineare Verformungen von elastischen Segmenten führen die Schwierigkeiten in Analyseverfahren von nachgiebiger Mechanismen ein.

Dieser Beitrag befasst sich mit der gleichschenkligen Schubkurbel und der nachgiebigen Kopie von gleichschenkligen Schubkurbel. In diesem Beitrag wird die nachgiebige gleichschenklige Schubkurbel mit Kerbgelenken entworfen und die "Rastpolbahn" des geführten Segments ermittelt. Das Ziel des Beitrags ist die Vorgehenssweise für die Bestimmung der "Rastpolbahn" bei den nachgiebigen Mechanismen vorzuschlagen.

1 Introduction

Compliant mechanisms gain some or all of their mobility from the relative flexibility of their joints rather than from rigid-body joints only [1]. These mechanisms may be made, for example, from modern-day plastics by an injection moulding process that gives them the desired resiliency and strength. Compliant mechanisms can provide many benefits in the solution of design problems. They are desirable because they have less wear, weight, noise and backlash than their rigid-body counterparts. By reducing the number of required interconnections, the reliability of a design can be improved. The field of compliant mechanisms is expected to continue to grow as materials with superior properties are developed.

Although there are many advantages, the inclusion of compliance provides several challenges in mechanism analysis and design. Nonlinearities introduced by the large deflection of elastic segments further complicate the analysis of compliant mechanisms.

There are many papers dealing with the general compliant mechanisms, as well as considering the structure of the compliant slider-crank mechanisms.

The papers [2], [3], [4] deal with rectilinear guiding of a coupler point of the compliant four-bar linkages. The paper [5] introduces some new designs of compliant mechanisms being able to realize translating planar displacement of the link.

The paper [6] introduces a pseudo-rigid-body constant-force slider mechanism, as well as a compliant slider mechanism with three flexural pivots. The paper [7] presents a general pseudo-rigid-body slider mechanism, as well as a compliant slider mechanism with flexible input crank and coupler. The paper [8] deals with a method of vibration control of a slider-crank mechanism with the flexible connecting-rod. The paper [9] introduces a design of compliant slider-crank mechanism with compliant joints designed as circular arc small length flexural pivots. The paper [10] deals with the modelling and guiding accuracy of a compliant Scott-Russel mechanism with small length flexural pivots.The paper [11] presents a new type of partially compliant spatial four-bar mechanism.

The paper [12] introduces a method for determining the limit positions of compliant mechanisms for which an appropriate pseudo-rigid-body model may be created. The paper [13] deals with the mobility of some compliant four-bar linkages.

However, there is neither paper dealing with the compliant slider-crank mechanism with compliant joints designed as circular flexural hinges, nor paper dealing with the determining of the "fixed centrode" of the compliant slider-crank mechanism.

This paper takes into consideration the isosceles slider–crank mechanism and its compliant counterpart mechanism, being developed on the basis of the rigid-body mechanism. The design of the compliant slider-crank mechanism with notch joints (circular flexure hinges) has been shown in this paper.

The aim of the paper is to suggest a method of determining of the "fixed centrode" of the compliant mechanism, as well as to apply suggested method to determine the "fixed centrode" for the compliant isosceles slider-crank mechanism.

2 Design of a compliant Scott-Russel mechanism with notch joints

Fig. 1 shows a rigid-body Scott-Russel mechanism (the isosceles slider–crank mechanism: $a = \overline{A_0A} = \overline{AB} = \overline{AC}$). The coupler point C is guided along an exact rectilinear vertical path.

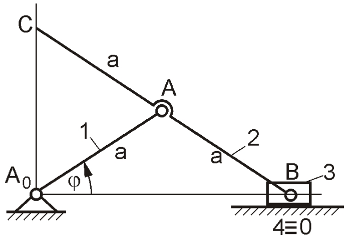


Fig. 1: A rigid-body Scott-Russel mechanism

On the basis of the rigid-body Scott-Russel mechanism, a compliant Scott-Russel mechanism with compliant joints designed as circular flexure hinges will be developed.

Fig. 2 shows a notch joint (circular flexure hinge) as a characteristic type of the compliant joint. This compliant joint is fully determined by two parameters: the width of relatively rigid segments w_R and the width of relatively elastic segments w_E.

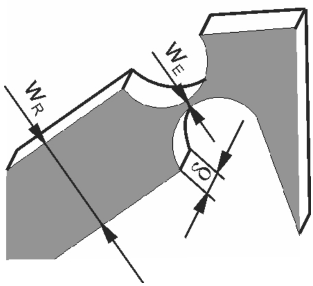
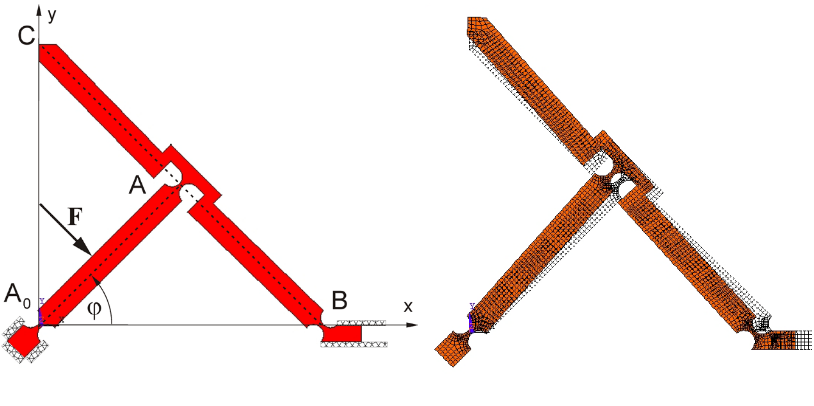


Fig. 2: A notch compliant joint

On the basis of the rigid-body Scott-Russel mechanism, a compliant Scott-Russel mechanism with notch joints was developed (Fig. 3a). The compliant joints A_0 and A have been oriented in the direction of the rigid-body input crank A_0A, while the compliant joint B has been oriented in the direction of the rigid-body coupler CAB.

The input force F acts in the middle of the "input crank". The positions of the links of the rigid-body counterpart have been presented with dashed lines. The deformed and undeformed position of the compliant slider-crank mechanism with notch joints are shown in Fig. 3b.



a) b)

Fig. 3: The compliant Scott-Russel mechanism with notch joints

The symmetry axes of the compliant notch joints cross each other at the point 1 (Fig. 4) corresponding to revolute joint of the rigid-body counterpart. The other characteristic keypoints of a compliant notch joint (Fig. 4) can be calculated by using the set of equations:

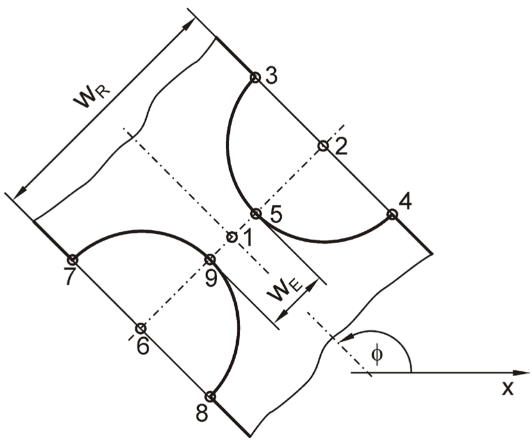


Fig. 4: Characteristic key points of a compliant notch joint

$$\vec{r}_2 = \vec{r}_1 + \frac{w_R}{2} e^{i\left(\phi - \frac{\pi}{2}\right)} \qquad \vec{r}_3 = \vec{r}_2 + \frac{w_R - w_E}{2} e^{i\phi}$$

$$\vec{r}_4 = \vec{r}_2 + \frac{w_R - w_E}{2} e^{i(\phi - \pi)} \qquad \vec{r}_5 = \vec{r}_2 + \frac{w_R - w_E}{2} e^{i\left(\phi + \frac{\pi}{2}\right)} \tag{2.1}$$

$$\vec{r}_6 = \vec{r}_1 + \frac{w_R}{2} e^{i\left(\phi + \frac{\pi}{2}\right)} \qquad \vec{r}_7 = \vec{r}_6 + \frac{w_R - w_E}{2} e^{i\phi}$$

$$\vec{r}_8 = \vec{r}_6 + \frac{w_R - w_E}{2} e^{i(\phi - \pi)} \qquad \vec{r}_9 = \vec{r}_6 + \frac{w_R - w_E}{2} e^{i\left(\phi - \frac{\pi}{2}\right)}$$

The angle ϕ determines the orientation of a compliant joint in reference to the x-axis (Fig. 4).

3 Determination of the fixed centrode of the compliant mechanisms

The fixed centrode is a locus traced on a fixed lamina by the instantaneous centre of velocity (instantaneous pole) of a coplanar moving lamina.

The fixed centrode of the rigid-body slider-crank mechanism can be obtained by defining of the instantaneous pole positions (coordinates x_p and y_p) for different values of the input crank angle φ (Fig. 5):

$$x_P = 2a\cos\varphi \tag{2.2a}$$

$$y_P = 2a\sin\varphi \tag{2.2b}$$

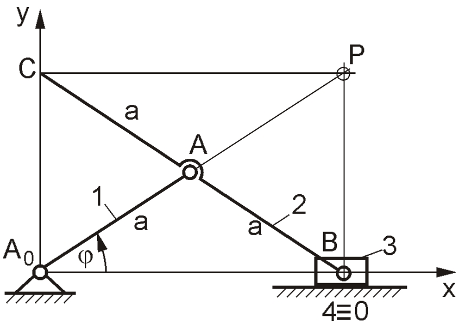


Fig. 5: Instantaneous pole of the isosceles rigid-body slider-crank mechanism

In order to obtain the instantaneous pole position of the above mentioned compliant slider-crank mechanism, it is nesseccary to obtain the trajectories of the two coupler points (A_S and B_S) locating at the relatively rigid segment of the "coupler" (Fig. 6).

The equations defining the motion geometry of the compliant mechanisms are not feasible due to complexity of their kinematic model structure. Therefore in this paper the positions of the corresponding points A_S and B_S are calculated using FEM program ANSYS for different values of the driving force. The trajectory of points A_S and B_S obtained in this way could be approximated by the second-order curve equation:

$$y_A = Ax^2 + Bx + C \tag{3.1a}$$

$$y_B = Dx^2 + Ex + F \tag{3.1b}$$

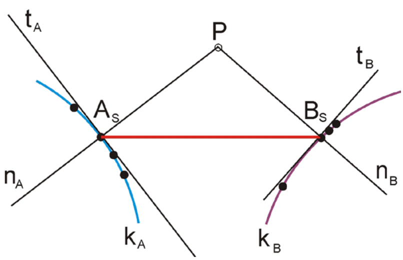


Fig. 6: Trajectories of the coupler points A_S and B_S and the instanteneous pole P

The equations of the straight lines n_A and n_B, being orthogonal to the tangent lines on the trajectory at the corresponding points $A_S(x_A, y_A)$ and $B_S(x_B, y_B)$ respectively, are:

$$y - y_A = k_{n_A}(x - x_A) \tag{3.2a}$$

$$y - y_B = k_{n_B}(x - x_B) \tag{3.2b}$$

where the gradients of the straight lines n_A and n_B are respectively:

$$k_{n_A} = -\frac{1}{2Ax_A + B} \tag{3.3a}$$

$$k_{n_B} = -\frac{1}{2Dx_B + E} \tag{3.3b}$$

The straight lines n_A and n_B cross each other at the point $P(x_P, y_P)$ representing the instanteneous pole of the "coupler" of the compliant mechanism, where the coordinates of the point P can be calculated as:

$$x_P = \frac{k_{n_A}x_A - k_{n_B}x_B + y_B - y_A}{k_{n_A} - k_{n_B}} \tag{3.4a}$$

$$y_P = k_{n_A}(x_P - x_A) + y_A = k_{n_B}(x_P - x_B) + y_B \tag{3.4b}$$

The compliant mechanism are assumed to be made of piacryl (modulus of elasticity E = 3700 N/mm^2, bending strength σ_{bs} = 90 N/mm^2, material thickness of δ = 4 mm). The calculation was performed for the elements with a rectangular cross-sectional area using Two-dimensional-eight-node Structural Solid as a characteristic ANSYS element type. For different values of the driving force F, within the possible motion area of the compliant mechanism, the instantaneous pole locus of the compliant mechanism has been obtained. The maximal permissible bending stress $\boldsymbol{\sigma_{max} < \sigma_{bs}}$ determines the constraint positions of the links, that is, the limits of their displacement (mobility) and the maximal permissible acting force.

This method of determining of the "fixed centrode" can be applied to all compliant mechanism with concentrated compliance (relatively rigid segments of the compliant mechanism are clearly separated from the relatively elastic segments, that is, compliant joints).

In order to apply this method on the compliant isosceles slider-crank mechanism with notch joints, two pairs of "coupler" points (A_S and B_S, that is D_S and B_S), locating at the relatively rigid segment of the coupler, are chosen as it is shown in the Fig. 7. It has been already mentioned that the symmetry axes of the compliant notch joints cross each other at the points corresponding to revolute joint of the rigid-body counterpart. It means that the positions of the points corresponding to revolute joint of the rigid-body counterpart are located in the middle of compliant joints, so they do not belong to the relatively rigid segment of the compliant mechanism and they are not suitable to be used as reference points for determining the "fixed centrode".

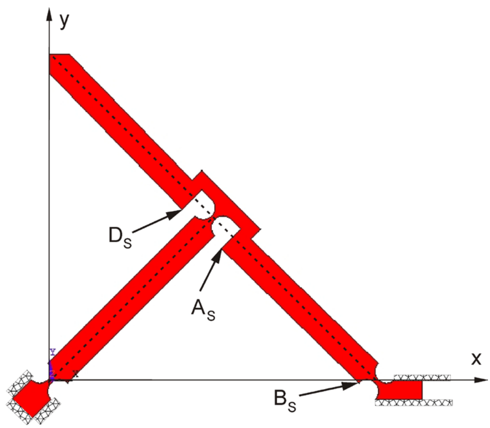


Fig. 7: The chosen "coupler" points A_S, B_S and D_S for determining the "fixed centrode" of the compliant isosceles slider-crank mechanism

The "fixed centrodes" of the compliant isosceles slider-crank mechanism with notch joints and the corresponding rigid-body mechanism within the possible motion area of the compliant mechanism have been presented in Fig. 8.

The parameters defining the rigid body, that is, compliant mechanism, are: $a = 50$ mm, $w_R = 6$ mm, $w_E = 1$mm, $\varphi = 45^o$.

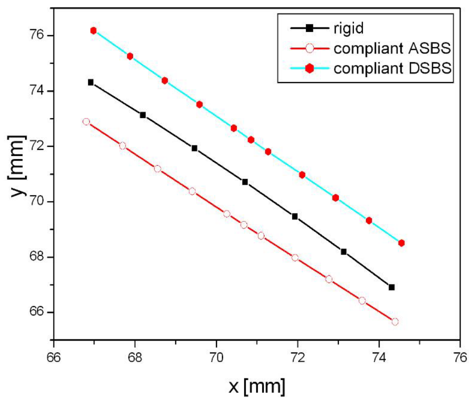


Fig. 8: The fixed centrodes of the rigid body and the compliant counterpart of the isosceles slider-crank mechanism, determined by the trajectory of the pair of "coupler" points A_S and B_S, that is, D_S and B_S

It is obvious that the "fixed centrodes" of the compliant isosceles slider-crank mechanism is similar to the fixed centrode of the rigid body counterpart. The relative deviations between fixed centrodes of the rigid body mechanism and compliant counterpart mechanism are up to 2.2%.

However, the design of compliant Scott-Russel mechanism gives another way of determining of the "fixed centrode" of the compliant isosceles slider-crank mechanism. The coupler point C of the rigid body isosceles slider-crank mechanism is guided along an exact rectilinear vertical path (Fig. 5), so it could be assumed that the coupler point C of the compliant isosceles slider-crank mechanism would be guided along an approximate rectilinear vertical path. It means that the straight line n_C, being orthogonal on the trajectory line of the coupler point C_S (Fig. 9), could be also used for determining of the fixed centrode of the compliant mechanism.

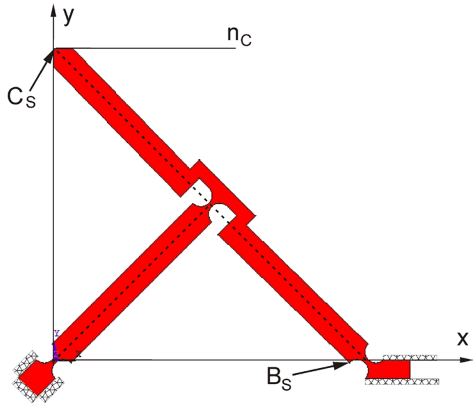


Fig. 9: The chosen "coupler" points B_S and C_S for determining the "fixed centrode" of the compliant slider-crank mechanism

The "fixed centrode" of the compliant isosceles slider-crank mechanism, determined by the coupler points B_S and C_S, are presented in Fig. 10.

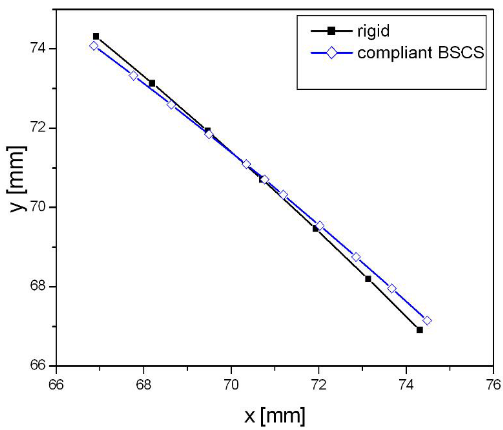


Fig. 10: The fixed centrodes of the rigid body and the compliant counterpart of the isosceles slider-crank mechanism, determined by the trajectory of the "coupler" points B_S and C_S

The deviatons between two fixed centrodes are negligible.

The fixed centrode of the rigid body mechanism defines the motion of all points belonging to the coupler plane.

The deviation between fixed centrodes of the rigid body mechanism and compliant counterpart mechanism is a consequence of the choice of the coupler points. In Fig. 7 the chosen coupler points A_S and D_S belong to the area located nearby the compliant joint (relatively elastic segment of the mechanism), and therefore there is a deviation between fixed centrodes of the rigid body mechanism and compliant counterpart mechanism. In Fig. 9 the chosen coupler point C_S belongs to the area located away from any compliant joint, and therefore there is a negligible deviation between fixed centrodes of the rigid body mechanism and compliant counterpart mechanism. On the basis of the results shown in Fig. 8 and Fig. 10 it can be concluded that there would not be unique solution for the instantaneous pole for all points of the "coupler" of the compliant mechanism.

In both cases, on the basis of similarity between the "fixed centrodes" of rigid body mechanism and its compliant counterpart, it can be concluded that the kinematic characteristics of the compliant mechanism will be similar to the ones of its rigid body counterpart.

4 Conclusions

The introduction of compliant joints in the mechanism structure is desirable, because compliant mechanisms have less weight, wear, clearance, friction (there is only inner friction) and noise than their rigid-body counterparts. On the other hand, the mobility of the compliant mechanisms is limited, that is, they can realize relatively small displacements. Another limitation to their use is the fatigue failure at the elastic joints.

This paper takes into consideration the isosceles slider–crank mechanism and its compliant counterpart mechanism, being developed on the basis of the rigid-body mechanism. The design of the compliant slider-crank mechanism with notch joints (circular flexure hinges) has been introduced in this paper.

The aim of the paper is to suggest a method of determining of the "fixed centrode" of the compliant mechanism, as well as to apply suggested method to determine the "fixed centrode" for the compliant isosceles slider-crank mechanism.

The fixed centrode is a locus traced on a fixed lamina by the instantaneous centre of velocity (instantaneous pole) of a coplanar moving lamina.

The fixed centrode of the rigid-body slider-crank mechanism can be obtained by defining of the instantaneous pole positions for different values of the input crank angle. In order to obtain the "fixed centrode" of the compliant slider-crank mechanism, it is nesseccary to obtain the trajectories of the two "coupler" points locating at the relatively rigid segment of the coupler.

The equations defining the motion geometry of the compliant mechanisms are not feasible due to complexity of their kinematic model structure. Therefore in this paper the corresponding "coupler" point positions are calculated using FEM program ANSYS for different values of the driving force within the possible motion area. The trajectory of "coupler" points are approximated by the second-order curve equation. The straight lines, being orthogonal to the tangent lines on the trajectory curves at the corresponding points, cross each other at the point representing the instanteneous pole of the "coupler" of the compliant mechanism.

The suggested method of determining of the "fixed centrode" can be applied to all compliant mechanism with concentrated compliance (relatively rigid segments of the compliant mechanism are clearly separated from the relatively elastic segments, that is, compliant joints).

However, dissimilar forms of the "fixed centrode" have been obtained by the choice of different pairs of the "coupler" points of the compliant mechanism. Therefore it can be concluded that there would not be unique solution for the instantaneous pole for all points of the "coupler" of the compliant mechanism.

On the basis of similarity between the fixed centrodes of rigid body mechanism and its compliant counterpart, it can be concluded that the kinematic characteristics of the compliant mechanism will be similar to the ones of its rigid body counterpart.

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