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Prevention of solidifcation cracking during pulsed laser beam welding

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Abstract

We present a mathematical model to describe laser beam welding based on the heat equation. Since the material coeff cients depend on the temperature, this leads to a quasi-linear parabolic partial differential equation. It is our goal to prevent solidif cation cracking. We address this problem by means of optimal control. It is the intensity prof le of the laser beam which acts as the control function. The main challenge is the formulation of a suitable objective function. In particular, high velocities of the solidif cation interface need to be properly penalized in order to deal with and avoid cracking phenomena.

Keywords: laser beam, welding, heat equation, quasi-linear parabolic equation, solidif cation cracking, optimal control

1 Introduction

Hot cracking is one of the major challenges in laser welding of aluminum alloys of the 2XXX, 5XXX and 6XXX series. In comparison to continuous wave laser welding processes, pulsed laser welding has a higher tendency to generate hot cracks because the pulsed mode leads to higher cooling rates and thus shorter solidif cation times and higher strain rates [4].

A conventional pulsed laser welding beam is simply on or off, i.e., it is described by a simple rectangular shape of laser beam power vs. time. By contrast, temporal pulse shaping involves varying the laser beam power within the pulse duration [5]. In [5] pulse shaping is investigated and the laser pulses comprise a cooling time in which the laser power is gradually ramped down, see also [6].

Several studies showed that this method reduced hot cracking in pulsed laser welding by lowering the solidification rate of the weld pool [7]. Due to the additional cooling phase of the pulse shape, the welding speed decreases, which reduces the efficiency of the welding process [1]. It is our goal to design the laser pulse in such a way that the welding seam solidifies in a crack-free manner while high welding speeds are maintained. To this end we derive, proceeding differently than [8], a mathematical model to determine an optimal pulse form as the solution of an optimal control problem.

2 System Dynamics and Constraints

Let $\Omega \subset \mathbb{R}^3$ be an open cuboid whose surface $\partial\Omega$ is subdivided in the three disjoint areas A, B, and $\partial\Omega \backslash (A \cup B)$ as in the figure. The temperature distribution in the cuboid Ω is described by the heat equation

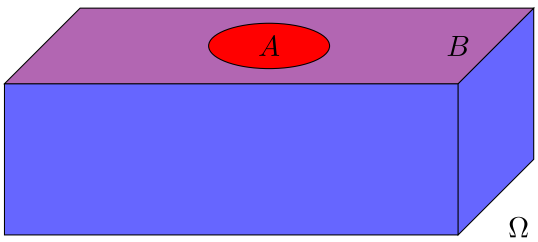
$$c(T(x,t))\,\rho(T(x,t))\,\frac{\partial T(x,t)}{\partial t} = \operatorname{div}\left(\lambda(T(x,t))\nabla T(x,t)\right), \quad \text{for } x \in \Omega,\ t \geq 0$$

with (temperature dependent) heat capacity c defined by $c(T) = 715 + 51\ln(T)$ in $\frac{J}{kg\,K}$ as well as material density ρ in $\frac{kg}{m^3}$ and thermal conductivity λ in $\frac{W}{m\,K}$ given by

$$\rho(T) = \begin{cases} 2753 - 0.2T & T_0 \leq T < 585\,^\circ C, \\ 4250 - 2.8T & 585\,^\circ C \leq T < 650\,^\circ C, \\ 2513 - 0.1T & 650\,^\circ C \leq T, \end{cases}$$

and

$$\lambda(T) = \begin{cases} 176 + 0.4T & T_0 \leq T < 585\,^\circ C, \\ 10^{-5} \cdot (1000 - 1.4T) & 585\,^\circ C \leq T < 650\,^\circ C, \\ 110 & 650\,^\circ C \leq T. \end{cases}$$



The initial condition is $T(x,0) = T_0 = 25\,^\circ C$ for all $x \in \Omega$. On $A \cup B$ we have

Robin boundary condition,

$$\lambda(T(x,t))\,\frac{\partial T(x,t)}{\partial n} = \begin{cases} P(t) & \text{on } A, \\ k\,(T_0^4 - T(x,t)^4) + h(T_0 - T(x,t)) & \text{on } B, \end{cases}$$

with laser intensity P in $\frac{W}{m^2}$ (to be determined) and coeff cients $k = 2.26 \cdot 10^{-9}$ in $\frac{W}{m^2\,K^4}$ and $h = 5$ in $\frac{W}{m^2\,K}$. On $\partial\Omega \setminus (A \cup B)$ the Dirichlet boundary condition $T(x,t) = T_0$ is used.

3 Interface Velocity

In contrast to a two-phase Stefan problem (cf., e.g., [3]), we have a temperature corridor during which the material solidif es from a liquid to a solid state. This transition allows us to eschew the *classical* Stefan condition in the description of the partial differential equation. Instead of an isothermal line as considered in the *classical* Stefan problem [2] we consider the boundary surface

$$C(t) := \{x \in A \cup B \,|\, 585^\circ \leq T(x,t) \leq 650^\circ\}$$

def ned by the temperature corridor of interest. For a given typical laser intensity $P : [0, t_f] \to [0, p_{\max}]$, the material is heated up before it starts to cool down after the melting process. There exists a time instant at which $C(t)$ performs its transition from the empty set at $t = 0$ to a two dimensional manifold (which temporarily contains a hole) before it contracts to a single point and, f nally, vanishes. Solidif cation cracks typically occur during the cooling phase due to the temperature gradient becoming too large within the surface $C(t)$. We therefore consider the interface velocity for each isothermal line within the temperature corridor of interest,

$$\frac{\mathrm{d}}{\mathrm{d}t} T(x(t),t) = T_x(x(t),t) \cdot \dot{x}(t) + T_t(x(t),t) = 0 \tag{3.1}$$

with $x(t) \in C(t)$. We are only interested in the component of the derivative $\dot{x}(t)$ which is perpendicular to the isothermal line. This results in

$$\dot{x}(t) = \alpha(x(t),t)\, T_x(x(t),t)^\top$$

where α is a scalar function. Hence, $\dot{x}(t)$ is parallel to the spatial derivative of the temperature distribution $T_x(x(t),t)$. Plugging this equation into Equation (3.1) yields

$$\alpha(x(t),t) \cdot \|T_x(x(t),t)\|^2 + T_t(x(t),t) = 0.$$

Since we are interested in the absolute value, we define $v(x(t), t) := \|\dot{x}(t)\|$, which leads to

$$v(x(t), t) = \frac{|T_t(x(t), t)|}{\|T_x(x(t), t)\|}. \tag{3.2}$$

4 Objective Functional

In addition to high velocities of thermal isolines near the melt temperature, excessive control efforts should be penalized. Moreover, two additional terms are added to the objective, which essentially ensure that the surface is heated up sufficiently so that a melting process is triggered and, secondly, take care that the cooling process is over at the time t_f. Hence the objective function consists of several summands.

Since we are indeed intested in the isoline velocities only during the cooling phase, we propose to use the uni-directional penalty term

$$\int_{A\cup B} \int_0^{t_f} \max\left\{ -\frac{T_t(x,t)}{\|T_x(x,t)\|} - v_{\max}, 0 \right\}^2 \cdot \chi(T(x,t))\, \mathrm{d}t\, \mathrm{d}x \tag{4.1}$$

in our cost functional. The indicator function χ is defined as

$$\chi(T) := \begin{cases} 1 & \text{if } 585^\circ \leq T \leq 650^\circ, \\ 0 & \text{otherwise.} \end{cases}$$

The second term in the objective function is

$$\frac{\beta_P}{2} \int_0^{t_f} P(t)^2\, \mathrm{d}t$$

with weighting factor $\beta_P > 0$, which penalizes the control effort. An alternative to the L^2-norm would be to penalize the absolute value, i.e.

$$\frac{\beta_P}{2} \int_0^{t_f} |P(t)|\, \mathrm{d}t.$$

Since the process is supposed to be completed at time t_f we add the following terms to the objective functional which ensure that both the melting and the cooling process have to occur during the time interval $[0, t_f]$:

$$\frac{\beta_f}{2} \int_\Omega \max\left\{T(x, t_f) - 585^\circ, 0\right\}^2 \mathrm{d}x + \frac{\beta_{x_0}}{2} \left(\|T(x_0, \cdot)\|_{L^p(0,t_f)} - 650^\circ \right)^2 \tag{4.2}$$

with weighting coefficients β_f and β_{x_0} and $p \gg 2$. The point x_0 has to be suitably chosen such that a temperature above the threshold of 650° at that point implies that the melting process is sufficiently triggered. In addition, (4.2) implies that a *leisurely* preheating of the material is excluded from being an optimal control strategy.

5 Conclusions and Future Work

A mathematical model considering temperature dependent material properties is presented with the intention of determining an optimal temporal laser pulse shape as the solution of an optimal control problem in order to prevent solidif cation cracking during pulsed laser beam welding. Further investigations will deal with the experimental validation of the mathematical model and the analysis and numerical solution of the proposed optimal control problem.

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