THE VIRTUAL WEIGHT

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ABSTRACT

A virtual weight is a numerical representation of a physical mass weight, describing the metrological behavior under certain measurement conditions. Based on a Monte Carlo simulation, it considers all significant influence variables with their respective uncertainty distribution. For the virtual weight this includes the ambient conditions, the cleaning status, the form, and material properties of the weight. It distinguishes between random and systematic errors and can thus be used to correct a measurement result for the task-specific prevailing ambient conditions. The virtual weight is part of the so-called Planck-Balance, a self-calibrating precision balance for industrial applications, currently under development in a cooperation of the PTB and the TU-I. Enhanced with calibration data from a dedicated weight, the virtual weight becomes the digital twin of this specific weight. A first implementation of a digital twin of a mass weight has been set up as a demonstrator.

Index Terms – measurement uncertainty, Monte Carlo simulation, mass metrology

1. INTRODUCTION

Stating the correct uncertainty for a measurement processes is an important step to define the quality and usefulness of the measurement results. The common approach described in "Evaluation of measurement data – Guide to the expression of uncertainty in measurement methods" (GUM) [1] relies on an uncertainty budget composed from type A and type B uncertainties. Type A uncertainties come from repeated observations of the measurement process and a statistical evaluation of the results and type B uncertainties come from previous knowledge, like manufacturer specifications or data sheets. The established GUM approach for the correct combination of the respective uncertainties to a general uncertainty of the measurement result has drawbacks for complicated or non-linear model functions. This is the case with the Planck-Balance, a type of user-friendly Kibble balance tailored for an industrial environment, currently under development by the PTB and the TU-I [2].
Supplement 1 of the GUM describes the uncertainty estimation with a Monte Carlo simulation [3]. This allows the uncertainty estimation for complicated or non-linear measurement processes without increasing the complexity like the established approach of error propagation. With participation of the PTB the uncertainty determination via simulation has been introduced in the industrial practice for coordinate measurement machines [4]. This is called the virtual coordinate measurement machine (VCMM). For the Planck-Balance, analog to the VCMM, a virtual Planck-Balance (VPB) will be set up, to determine the measurement uncertainty of a weighing instrument. The VPB will consist of multiple subsystems, like an interferometer, a weighing cell, a voltmeter, a standard resistor, or a magnet system, that are modeled according to their behavior and connected to each other. In this article one of these

subsystems, the virtual weight, will be presented as an example for the virtual devices that are the core components of the VPB. The virtual weight is a behavioral representation of a physical weight, considering the systematic influence of the ambient conditions, under which it is used, in combination with physical properties of the weight.

One main application for the virtual weight will be in the digital twin of a weight. Because of the digitization of data acquisition, storage and processing in the metrology, digital twins will make it possible to increase the accuracy of uncertainty indication for measurement processes. Digital twins are a virtual representation of a dedicated object or measurement device. For metrology applications, they combine a generic behavioral model, the virtual device, with data and preferences specifically belonging to the object or device represented. The digital distribution and processing of the calibration data of the devices, including the calibration history, makes it possible to model the behavior of the dedicated device correctly, even considering long term effects.

2. UNCERTAINTY DETERMINATION WITH VIRTUAL DEVICES

A realistic model function of the measurement process is required for all uncertainty determinations. Figure 1 shows how the expectation value and the assessed uncertainty of a measurement process are obtained. The process is modeled by the function f, that depends on the input quantities $X_1, \ldots, X_m$ and yields the result Y. The uncertainty $u(Y)$ of the result can be estimated via two ways. One is the established method from the GUM of assessing an uncertainty budget containing type A and type B uncertainties. Another method is a Monte Carlo simulation as proposed in the GUM supplement 1 [3].
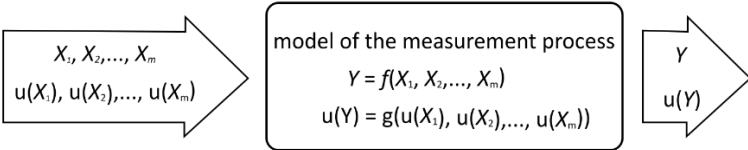

Figure 1: Uncertainty determination of a measurement process according to the GUM.

The supplement 1 to the GUM allows the uncertainty of a measurement being calculated by a Monte Carlo simulation. This means, the variation of the measurement result is determined by n repeated calculations using input variables with random variation. Size and shape of each individual uncertainty distribution of the input variables are adjusted to represent the application. The measurement uncertainty $u(Y)$ and the best estimate $\bar{Y}$ of the result is then derived from the distribution of the individual calculation results (figure 2). The shape of the uncertainty distribution for the input variables can be chosen in order to represent the behavior in a realistic manner. This allows a more precise uncertainty determination especially in complicated or non-linear model functions or correlated input data.
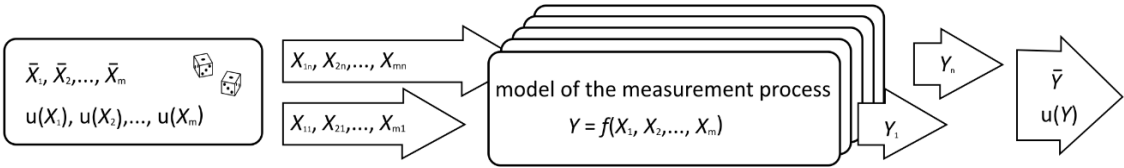

Figure 2: Uncertainty determination with n virtual experiments.

The input variables consist of the actual measurement values and the ambient conditions that influence the measurement. Influences can be modeled as three error types:

- known systematic errors,
- unknown systematic errors,
- random errors.

Known systematic errors are characterized as constant offsets from "true values". They can be determined and corrected in the measurement results and their corresponding measurement uncertainties. Unknown systematic errors remain unchanged during a measurement only. In opposite to known systematic errors, they can change in size and characteristics. For example, the position of the mass on the weighing pan can shift slightly each time it is lifted off and put back, and thus affect the measurement results due to varying off-center forces. In a situation where repeated measurements are done without lifting the weight, the actual positional deviation is random, but it does not change for the whole measurement set. The consideration of unknown systematic influences depends on the measurement procedure and the application. Consequently, this leads to a higher measurement uncertainty. The last type, random errors, describes stochastic error behaviours. Its systematic effects cannot be repeated with significant stability and therefore not be used for the correction of measurement results.

3. INFLUENCES MODELLED FOR THE VIRTUAL WEIGHT

In state-of-the-art mass metrology, the significant error sources affecting weights during a measuring process are investigated and considered in the measurement results. The biggest influence comes from buoyancy effects like fluctuations in the air density or deviations of the weight material density. The modeled influences will be extended in the future to account for more relevant parasitic influences on the apparent weight measured by a scale. This section explains the influence factors that are currently considered in the model function of the virtual weight.

3.1 Air Buoyancy

If the mass of an object is determined via a weight measurement or comparison, the fluid in which the measurement takes place will influence the resulting weight of the mass. Since in usual applications the weight measurement is done in air, the density of the displaced air must be considered for the mass calculation. The indicated mass m of an object is determined through the measured weight F, the gravitational acceleration g, the densities of moist air ρ_a and of the weight material ρ as

$$m = \frac{F}{g}\left(\frac{\rho - \rho_a}{\rho}\right). \tag{1}$$

The CIPM-formula:

$$\rho_a = \frac{pM_a}{ZRT}\left(1 - x_v\left(1 - \frac{M_v}{M_a}\right)\right), \tag{2}$$

is used to calculate the density of moist air ρ_a with p being the air pressure in Pa, T the temperature in K, x_v the molar fraction of water vapor, M_a the molar mass of dry air in g mol⁻¹, M_v the molar mass of water also in g mol⁻¹, Z the compressibility factor, and R the molar gas constant in J mol⁻¹ K⁻¹. The latest update of the CIPM formula from 2007 [7] is used as a starting

point to model the influences of the air buoyancy effects. For their use in the virtual weight, the molar mass of water and the molar gas constant are considered as constant values with a fixed measurement uncertainty. Z and x_v are functions of the ambient conditions: relative humidity, air pressure and temperature. The molar mass of dry air can be calculated from the values stated in [7] to be 28.96546 g mol^{-1}. The CO_2 fraction is assumed to be 400 ppm for practical applications [5]. In [7] it was found, however, that the CO_2 fraction present in different laboratories can deviate significantly from this assumption. To account for this, M_a is calculated by

$$M_a = \left(28.96546 + 12.011\left(x_{CO_2} - 0.0004\right)\right) \cdot 10^{-3}, \qquad (3)$$

considering the CO_2 fraction x_{CO2} [7]. Volume changes of the weight due to temperature changes have an influence on the volume of the air displaced by the weight [5]. The volumetric expansion coefficient α_v of the material at a reference temperature T_0 can be used to calculate the actual volume V of the weight for a given temperature T via

$$V = V(T_0) \cdot \left(1 + \alpha_v(T - T_0)\right). \qquad (4)$$

3.2 Surface Layers

Adsorbed water layers on the surface of the weight are another error source in weighing processes. The amount of water being physically adsorbed by the weight surface is not constant and changes with the surface condition and the humidity of the surrounding air. The cleaning status of the weight influences how much water is physically adsorbed on the surface. Based on the Brunauer-Emmet-Teller formula the mass of adsorbed water at the weight surface can be calculated with by

$$m_A = A \cdot \left(\mu_{h=0} + \frac{\mu_m c_B h}{(1-h)\left(1 + h \cdot (c_B - 1)\right)}\right). \qquad (5)$$

The mass m_A of the adsorbed water depends on the surface area A of the weight and the relative air humidity h. The BET constant c_B and the quantity of an adsorbed monolayer μ_m depend on the cleaning status of the weight. It is expressed with $\mu_{h=0}$, the surface adsorbed quantity for a humidity of 0% [5][6].

3.3 Gravity Gradient

The gravitational acceleration decreases by about 2.5…3.5 $\cdot$ 10^{-7} m^{-1} with increasing distance from the surface of the Earth [8]. Since the center of mass position of weights can be at different heights above the weighing pan with different nominal values or shapes, the resulting gravitational acceleration acting on the weights is different.

4. POTENTIAL APPLICATIONS

The virtual weight could be used as a core element in multiple configurations. The software interfaces must be well documented to make the integration in different software applications possible. A very important application of the virtual weight is in a digital twin, which is currently under development by the PTB. A first demonstrator has been implemented and will be presented at the end of this section.

4.1 On-line Uncertainty Determination in the Virtual Planck-Balance

The virtual weight can be used as part of an on-line uncertainty determination of the Virtual Planck-Balance (VPB). This concept is already established in the Virtual Coordinate Measuring Machine (VCMM) developed at the PTB [4]. The uncertainty of a single measurement or a measurement set is directly calculated by the software of device itself, considering all relevant input variables and error sources at the time the measurement takes place. This includes the ambient conditions during the measurement, the selected measurement mode, and influences from the specimen to be measured.

As shown in figure 3, the PB consists of multiple subsystems that can be modeled individually. The resulting model of the PB will be a combination of the subsystem models, including the virtual weight, with ambient data and the actual measurement data. These are all input data for a Monte Carlo simulation of the uncertainties of all input variables. The resulting uncertainty of a measurement of the PB can then be calculated with an analysis of the output value histogram considering the desired confidence interval.
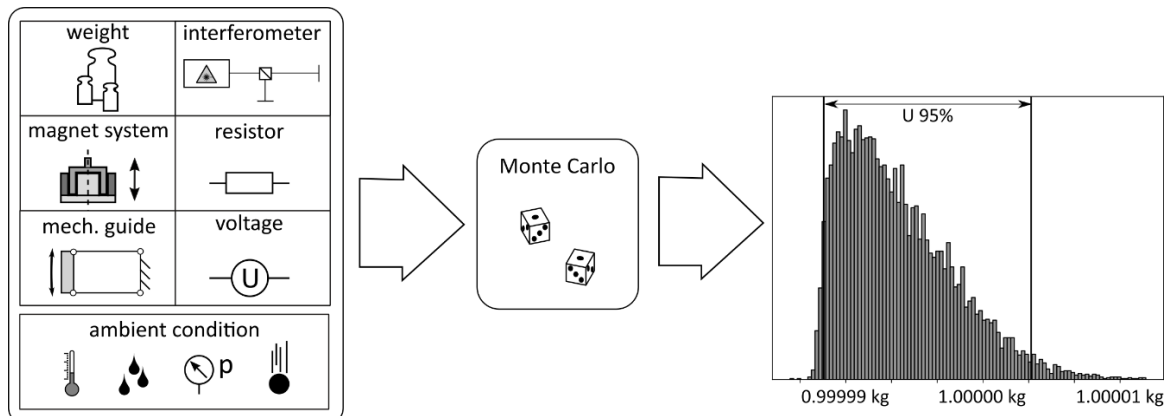

Figure 3: The complete measurement device (the PB) contains individual virtual subsystems. The uncertainty contribution of each subsystem is evaluated by a Monte Carlo simulation. The final measurement uncertainty is calculated from a combination of all subsystems to the user defined confidence level (here 95 %).

The benefit of an on-line uncertainty determination is that the actual ambient conditions are considered for the calculation. If the conditions shift during the measurement or are exceptionally stable, the results are not dismissed, but the measurement uncertainty is indicated according to the actual ambient conditions. The influence of different measurement modes on the measurement uncertainty is also considered in the calculation.

4.2 Uncertainty Simulation for Different Application Scenarios

The virtual weight can be used as a design tool for the evaluation of application scenarios. This could happen as a general evaluation how certain ambient conditions affect the measurement result and the corresponding uncertainty when using a specific type of weight. Thus, the quality of an application can be assessed and the influence and magnitude of possible improvements estimated. This helps with the selection of balances and measures of error correction or avoidance for a specific application. It enables the user for example, to evaluate the influence on the measurement uncertainty of an improved temperature stabilization measure. It is also possible to simulate the indicated mass of the weight including the corresponding measurement uncertainty when bringing it to another laboratory with different ambient conditions.

As part of the VPB the virtual weight will also be used to quantify sensitivities for ambient influences and optimize the measurement process accordingly. The sensitivity of the

measurement result for an ambient influence can be estimated by keeping all other influences constant and calculating the corresponding variation of the result. This can be used to identify the subsystems with the most potential for optimization.

4.3 Digital Twins

The digital twin is a virtual device that is dedicated to a specific physical device. It combines:

- a generic model function of the device (virtual device),
- calibration data,
- long term data storage and analysis, and
- a data interface for input and output data.

The interface reads in all input variables and data of the ambient conditions into the digital twin and provides the results of the computation for other digital twins or the user. The traceability of the results is guaranteed if the interfaces are encrypted and validated and the calibration information can only be accessed if correctly authorized. With a detailed modelling of aging processes, the digital twin can give out warnings to the user to perform maintenance tasks or do recalibrations of subsystems or the whole device.
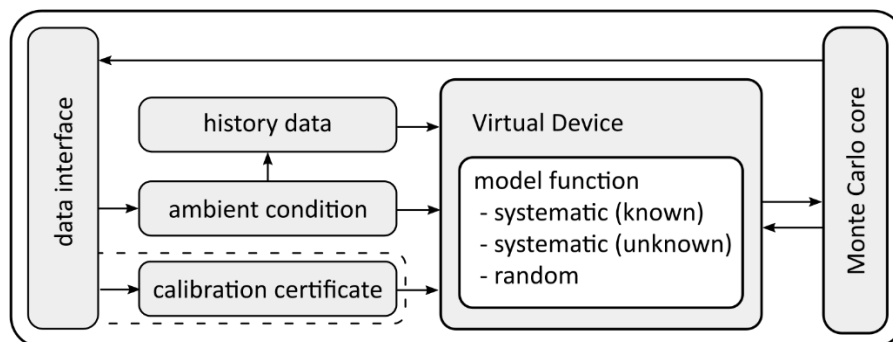

Figure 4: Structure of a digital twin of a measurement device.

Based on the model in figure 4, the PTB has implemented a digital twin of a weight containing the calibration data of the weight:

- mass at time of calibration,
- uncertainty of the calibration,
- volume of the weight, and
- uncertainty of the volume measurement,

along with other information specific for this weight:

- surface area and height of center of mass determined from the shape of the weight,
- long term drift of the calibration value, and
- cleaning status to determine the buildup of surface contamination.

With this information and the model from the virtual weight, the behavior of the weight for a given application and prevailing ambient conditions can be simulated. Figure 5 shows a screenshot of the graphical user interface of the currently implemented demonstrator.
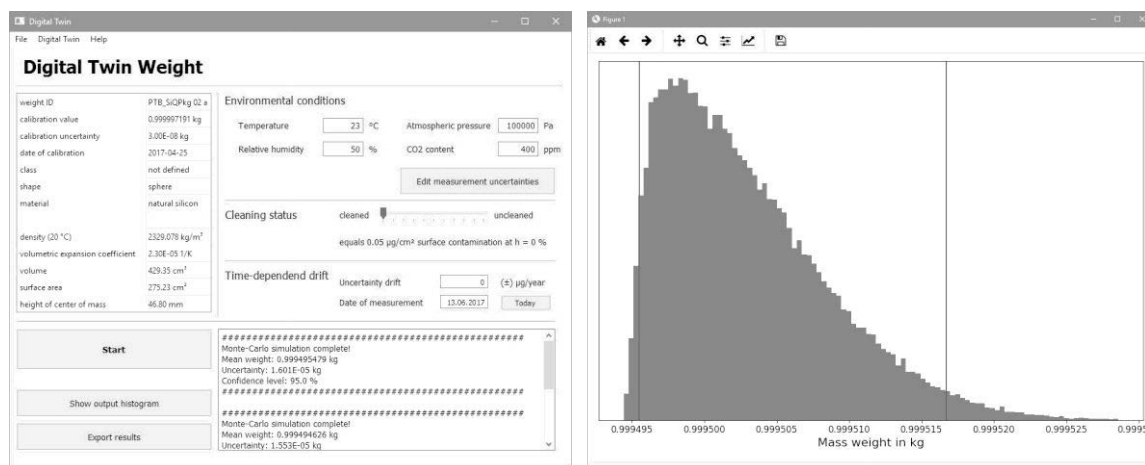
Figure 5: Screen shot of the main application and the histogram window of the digital twin of a weight.

The digital twin of a real weight can be used in a comparison on different balances or in different laboratories. The expected measurement result according to the characteristics of the weight, the instrument and the ambient conditions of the measurement is calculated with a digital twin. This result can then be used to assess the measurement results of the comparison to find measurement errors or to rate the quality of a measurement setup. It is also possible to equip the digital twin with a self-learning algorithm to improve its own model of the long-term behavior from past calibration and the appearing deviations.

The digital twin of a weight can be used as a part of the digital twin of a complex measurement device. This digital twin gets its input data from other connected digital twins communicating through a well-documented, standardized data interface. To perform the measurement, the digital twin of the device requests its input and the ambient condition data from the digital twins of the respective measurement devices. This leads to measurements with a very detailed expression of the measurement uncertainty, because the exact behavior and uncertainty of the involved measurement devices and artefacts are considered for the uncertainty determination.

5. CONCLUSION

The virtual weight represents the behavior of a mass artefact under given measurement conditions. Focus is given on the determination of the measurement uncertainty using a core algorithm based on a Monte Carlo simulation in accordance with the GUM supplement 1. This will allow for an improved estimation of the measurement uncertainty of the weighing process. Further applications are an on-line uncertainty determination during a weight measurement as well as the optimization of weighing instruments. The virtual weight is a first step towards a fully virtual weighing instrument, the virtual Planck-Balance, which will consist of a combination of many virtual subsystems.

Enhanced with the specific calibration and physical information the virtual weight becomes the digital twin of a dedicated weight. This digital twin can be used to simulate the current and future behavior of the specific weight in a realistic manner. In a common project of the PTB and the TU-I, a first demonstrator of a digital twin for a weight has been implemented. It covers the most important systematic errors and will be integrated in the efforts for the digitization of metrology. It is possible to use the digital twin in combination with the virtual Planck-Balance or to integrate a modified version into other state-of-the-art balances.

6. ACKNOWLEDGEMENT

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