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Stability bounds for systems and mechanisms in linear descriptor form

Rick Voßwinkel[a], Jiancheng Tong[b], Klaus Röbenack[a], Naim Bajcinca[b]

[a]*Technische Universität Dresden, Faculty of Electrical and Computer Engineering, 01062 Dresden, Germany*
[b]*University of Kaiserslautern, Faculty of Mechanical and Process Engineering, 67663 Kaiserslautern, Germany*

Abstract

Mathematical models for simulation and control of systems and mechanisms naturally arise in a descriptor form. The stability analysis of descriptor systems, involving free parameters as uncertainties or design qualifiers is subject of this paper. Two approaches for the calculation of the stability boundaries in the underlying parameter space are discussed. The first one uses a quantifier elimination method, while the second one is based on the direct solution of the Lyapunov equation. The computational methods are exemplary demonstrated on Chua's circuit.

Index Terms — Stability, descriptor systems, quantifier elimination, Lyapunov functions

1. INTRODUCTION

A large class of systems in mechatronics feature algebraic conditions (e.g. holonomic and non-holonomic constraints in mechanics, Kirchoff's laws in networks etc.) in addition to differential equations, yielding a system of differential-algebraic equations (DAEs) as a modeling setting [3, 19,22]. Stability of such models continues to attract attention in control and modeling and builds the subject of the present paper.

More specifically, we study the stability of parameter-dependent descriptor systems. The parameter dependence may result from the presence of a control algorithm, but it could also describe inherent system uncertainties. Such classes of problems for ordinary state-space models have been already a subject of research for many years. A variety of methods have been developed, e.g. see [1], for a collection. Most of such methods are based on direct mapping of stability criteria into the parameter space which typically boils down to solving two nonlinear algebraic equations in two parameters. Yet the extensions of these methods to the class of descriptor models have been scarcely addressed. In the current paper, we make a step in this direction. On one hand, we make use of a recent Lyapunov-based approach proposed by the authors in [25], which is based on solving the Lyapunov equation for ordinary state-space models and results in four parameter-dependent determinant conditions. Thereby we adopt the result of [25] to the descriptor system formulations. On the other hand, one can consider the Lyapunov feasibility conditions as prenex formula as, by their very definition, they include existential and universal quantifiers. A related general computational framework called "Quantifier Elimination" (QE) attempts to produce corresponding equivalent quantifier-free algebraic expressions [8, 28]. Such expressions are just about what every parameter space method tends to end with. Hence, from this perspective the utilization of QE for computing the stable parameter regions in the sense of Lyapunov is a particularly appealing computational approach. Several recipes in applying this framework for solving the feasibility problem of Lyapunov function have been already published, e.g. [21]. However, nearly all methods address ordinary differential equations. In the present paper, we propose an extension to the descriptor system formulation.

The paper is organized as follows. In Section 2 some aspects of linear descriptor systems and their Lyapunov formulations are briefly introduced. This is followed by an introduction to QE and its usage in the context of Lyapunov stability. In Section 4 we address the first algebraic method based on the solutions of the Lyapunov equation. After introducing the two approaches in Sections 3 and 4, we demonstrate the two methods on stability analysis of a descriptor model of Chua's circuit in Section 5.

2. STABILITY ANALYSIS OF DESCRIPTOR LINEAR SYSTEMS

We consider the systems description in linear descriptor form

$$E(q)\dot{x} = A(q)x \tag{1}$$

with the matrices $E(q), A(q) \in \mathbb{R}^{n\times n}$, the generalized states $x \in \mathbb{R}^n$ and the free parameters $q \in \mathbb{R}^p$, see [3]. The descriptor system (1) is called regular if

$$\exists \gamma \in \mathbb{C}: \quad \det(\gamma E - A) \neq 0. \tag{2}$$

For regular descriptor systems described by a pair (E,A) or the matrix pencil $sE - A$, we can check its stability by determining the eigenvalues, which are the roots of the characteristic equation (for convenience, we drop from time to time the parameter dependency on q, but it will be there throughout the paper)

$$\det(sE - A) = 0. \tag{3}$$

If all real parts of these eigenvalues are negative, then the system (E,A) is (asymptotically) stable. Another important characteristic is impulse-freeness. The descriptor system (1) is said to be impulse-free if

$$\operatorname{rank}(E) = \deg\det(sE - A), \tag{4}$$

which means that the state response of system (1), does not contain impulsive terms. Formally, a regular system is impulse-free if and only if its index does not exceed one, $\operatorname{ind}(E,A) \leq 1$. Regularity and the index are structural properties, see [23, 24, 26]. A system which is stable and impulse-free is often referred to as admissible [11, 20].

Recall that our aim is calculation of the stability regions of (1) in the parameter space $\mathbb{R}^p$ of q. This domain describes the set of all stabilizing parameters q for the pair $(E(q),A(q))$. A straightforward idea is applying the frequency sweeping mapping techniques from [1] on the equation (3). However, one is afflicted thereby with solving nonlinear algebraic equations which is both time consuming and, typically, applicable to the parameters space of second order only. The recent alternative approach based on the quadratic Lyapunov formulations [25] is more natural for being more general and capable of dealing (at least principally) with parameter space of a larger order. Hence, in our study we rather follow such a "Lyapunov approach".

To this end, we consider two technical formulations of Lyapunov stability for differential algebraic equations [20]. The first approach is a natural extension to the ordinary state-space case.

If we consider a quadratic candidate Lyapunov function in the form

$$V_1(x) = \frac{1}{2} x^T E^T X_1 E x, \tag{5}$$

where $X_1 = X_1^T$, the Lyapunov equation results to be

$$A^T X_1 E + E^T X_1 A = -Y, \tag{6}$$

where $Y = Y^T$. Equation (6) may be unsolvable if the matrix pencil $sE - A$ has roots at infinity [27]. To overcome this constraint, we make use of the Weierstrass canonical form. Each regular system (E, A) can be transformed in the Weierstrass canonical form [11, 12]

$$QEP = \begin{pmatrix} I_{n_1} & 0 \\ 0 & N \end{pmatrix} \quad \text{and} \quad QAP = \begin{pmatrix} A_1 & 0 \\ 0 & I_{n_2} \end{pmatrix}, \tag{7}$$

with invertible matrices P and Q, while N is nilpotent and $n_1 + n_2 = n$. This transformation decomposes the system (1) into two subsystems

$$\dot{x}_1 = A_1 x_1, \tag{8}$$

$$N\dot{x}_2 = x_2. \tag{9}$$

The subsystems (8) and (9) are called slow and fast subsystem, respectively. It is important to emphasize that the stability of the descriptor system (1) is completely determined by the slow subsystem [11]. Using the transformation matrices P and Q, one can define the spectral projectors P_r and P_l onto the right/left finite deflating subspace of the matrix pencil $sE - A$ by

$$P_r = P \begin{pmatrix} I_{n_1} & 0 \\ 0 & 0 \end{pmatrix} P^{-1}, \quad P_l = Q \begin{pmatrix} I_{n_1} & 0 \\ 0 & 0 \end{pmatrix} Q^{-1}, \tag{10}$$

respectively. Considering this, we can now provide another formulation of the Lyapunov stability, see [20, 27].

Theorem 1. *The regular descriptor system* (1) *is asymptotically stable if and only if the equation*

$$A^T X_1 E + E^T X_1 A = -P_r^T Y P_r, \quad X_1 = P_l^T X_1 P_l = X_1^T \tag{11}$$

has a unique symmetric, positive semidefinite solution $X_1 = X_1^T \geq 0$ *for every symmetric, positive definite matrix* $Y = Y^T > 0$.

The additional condition $X_1 = P_l^T X_1 P_l = X_1^T$ guaranties the uniqueness of solution, while the solution of $A^T X_1 E + E^T X_1 A = -P_r^T Y P_r$ is not unique [27]. Nevertheless the existence of a positive definite matrix Y and a positive semidefinite matrix X_1 assures the stability of the pair (E, A), see [11]. If the system (E, A) is admissible, we can avoid the calculation of the projector and rather use the matrix E directly, as given by the following theorem [11, Theorem 3.16].

Theorem 2. *The regular descriptor system* (1) *is admissible if there exist matrices* $X_1 = X_1^T \geq 0$ *and* $Y = Y^T > 0$ *solving the equation*

$$A^T X_1 E + E^T X_1 A = -E^T Y E. \tag{12}$$

If the system (1) *is admissible, then for each $Y > 0$ there exists a $X > 0$ solving* (12).

The second approach to generate a Lyapunov formulation for linear descriptor systems assumes the candidate Lyapunov function in the form

$$V_2(x) = x^T E^T X_2 x, \tag{13}$$

with $E^T X_2 = X_2^T E$. Using this Lyapunov function, the related Lyapunov equation reads:

$$A^T X_2 + X_2^T A = -Y, \tag{14}$$

with $Y = Y^T > 0$. This formulation resembles the case of classical state-space systems. The two differences to the ordinary state-space formulation are the additional condition $E^T X_2 = X_2^T E$ and that there are no requirements on the symmetry and definiteness of X_2. This is summarized in the following [17, Lemma 2]

Theorem 3. *The regular descriptor system* (1) *is admissible if and only if for any $Y > 0$ there exists a matrix X_2 that solves*

$$A^T X_2 + X_2^T A = -Y, \tag{15}$$

$$E^T X_2 = X_2^T E \geq 0. \tag{16}$$

Based on these theorems two approaches to calculate the stability boundaries in the parameter space $\mathbb{R}^p$ are proposed in the following to sections.

3. QUANTIFIER ELIMINATION

The *quantifier elimination (QE) algorithm* can be used for the stability analysis and design of control systems, as shown, e.g. in [10, 15] for linear time-invariant systems in state-space form and in [21] for switched linear systems. Since A. Tarski had proven the existence of a solution to the QE problem over the reals and provided the first algorithmic technique for real quantifier elimination in the 1940s (see below), a variety of tools have been developed for efficient implementation of the QE procedures. For instance, the software packages QEPCAD [4], REDLOG [9], as well as the library of RegularChains [6] in Maple are commonly used. However, these algorithms issue unavoidably computational barriers in solving practical QE problems. In the worst case, their computational complexity is doubly exponential in terms of the number of variables [6].
The QE tools are used to simplify the formulas involving quantified variables and logic connectives. The quantifiers can be universal or existential. In order to introduce the QE approach, we review some basic definitions from [8].

Definition 1. *An atomic formula is defined as a polynomial expression of the form*

$$f(x_1, \cdots, x_k)\ \tau\ 0 \tag{17}$$

with $\tau \in \{>, =\}$ and $f \in \mathbb{Q}[x_1, \cdots, x_k]$, the latter representing the set of real polynomials in the variables $x_1, \ldots, x_k$ with rational coefficients.

Definition 2. *A formula is said to be quantifier-free, if it is a propositional combination of atomic formulas with the boolean operators $\vee, \wedge, \neg$ and $\Rightarrow$.*

Definition 3. *A prenex formula is a standard formula in the variables* $X = (x_1, \cdots, x_k)$ *and* $Y = (y_1, \cdots, y_l)$ *with the form*

$$PF(X,Y) := (Q_1 y_1) \cdots (Q_k y_l) F(X,Y), \tag{18}$$

where $Q_i \in \{\exists, \forall\}$ *and* $F(X,Y)$ *is a quantifier-free formula. When a quantifier is corresponding to a variable, then the variable is called quantified, or free otherwise.*

The goal of a quantifier elimination procedure is to output a quantifier-free formula $QF(X)$, which is equivalent to the prenex formula $PF(X,Y)$. Tarski has proved the solvability of QE problem over the real field in [28].

Theorem 4 (Quantifier Elimination over the Real Field)**.** *For any formula* $PF(X,Y)$ *over the real field, there always exists a quantifier-free formula* $QF(X)$ *such that, for any* $Y \in \mathbb{R}^l$, $QF(X)$ *is true if and only if* $PF(X,Y)$ *is true.*

One of the most efficient implementations of the QE procedure is the cylindrical algebraic decomposition (CAD), see [2, 5, 8]. Given a set of polynomials in $\mathbb{R}^n$, a CAD is a partition of $\mathbb{R}^n$ into finite number of cylindrically arranged cells. Two cells C_1 and C_2 are said to be cylindrically arranged if their projections in the subspaces $\mathbb{R}^k (1 \leq k \leq n)$ are equal or disjoint. Furthermore, each cell in the partition is a connected semi-algebraic subset, i.e., over such a cell, all the polynomials are sign-invariant. Thus, selecting a sample point in each cell is sufficient to determine the sign of the polynomials in the cell. For the prenex formula (18), we can apply CAD to the quantifier-free part $F(X,Y)$ and evaluate the truth value over each cell to perform quantifier elimination.
Coming back to our original problem, we first formulate the feasibility condition of Lyapunov function as a prenex formula, and then apply QE to get a quantifier-free formula which is a polynomial expression in terms of the parameters q. To this end, we make use of Theorem 3. Thereby, without loss of generality, we can set the matrix Y to be the identity matrix I and define:

$$U(q,X_2) := A^T(q)X_2 + X_2^T A(q) + I \stackrel{!}{=} 0, \tag{19}$$

$$V(q,X_2) := E^T(q)X_2 - X_2^T E(q) \stackrel{!}{=} 0, \tag{20}$$

$$W(q,X_2) := E^T(q)X_2 \stackrel{!}{\geq} 0, \tag{21}$$

where $U(q,X_2)$, $V(q,X_2)$ and $W(q,X_2)$ represent the matrix functions of the parameter q and the entries of X_2. Rewriting (19), (20) and (21) into the element-wise form, yields the equivalent descriptions of (15) and (16) in form of the quantified formula

$$\exists X_2 \left[\left(\bigwedge_{i=1}^{n} \bigwedge_{j=1}^{n} u_{ij}(q,X_2) = 0 \right) \wedge \left(\bigwedge_{i=1}^{n} \bigwedge_{j=1}^{n} v_{ij}(q,X_2) = 0 \right) \wedge \left(\bigwedge_{i=1}^{m} z_i(q,X_2) \geq 0 \right) \right] \tag{22}$$

where the polynomials $u_{ij} \in \mathbb{Q}[q,X_2]$ and $v_{ij} \in \mathbb{Q}[q,X_2]$ represent the ij-th entry of matrix U and V respectively, $z_i \in \mathbb{Q}[q,X_2]$ denotes the i-th principal minor of matrix W. Here the entries of X_2 are considered as existential quantified variables, and the elements of parameter q as free variables.

After introducing the second approach in the next section, we will apply this procedure to Chua's circuit.

4. ALGEBRAIC LYAPUNOV APPROACH

The basic idea of this approach lies in a reformulation of the Lyapunov equation (12) to a system of linear equations. To illustrate the procedure, we briefly discuss the case of ordinary state-space models given by

$$\dot{x} = A(q)x. \tag{23}$$

The stability of this system can be analyzed using the Lyapunov equation

$$A^T(q)X + XA(q) = -Y, \tag{24}$$

where both matrices $Y = Y^T$ and $X = X^T$ need to be strictly positive definite. Applying the Kronecker product we can rewrite equation (24) in vector form:

$$\underbrace{\left(I \otimes A^T(q) + A^T(q) \otimes I\right)}_{M(q)} \mathrm{vec}(X) = -\mathrm{vec}(Y), \tag{25}$$

see [16]. The solution of the linear system of equations (25) is then given by

$$\mathrm{vec}(P) = -M^{-1}\mathrm{vec}(Y). \tag{26}$$

Now, the stability is determined by the definiteness of P. The inverse M^{-1}, as well as P, have the determinant of M in their denominators. This determinant is connected to the system matrix A via [13]

$$\det(M(q)) = \prod_{i=1}^{n}\prod_{j=1}^{n}(s_i(q) + s_j(q)). \tag{27}$$

Eq. (27) shows that if an eigenvalue of A crosses the imaginary axis, the determinant of M becomes zero and if an eigenvalue of A tends to infinity, so does the determinant of M, too. Therefore, the stability information of (23) is encoded in the determinant of M. In fact, it turns out that $\det(M(q)) = 0$ and $\det(M(q)) \to \infty$ produce all the relevant stability boundaries in the q-parameter space, see [25]. On the basis of (12), we extend this idea to descriptor systems by posing the generalized form of (25):

$$\underbrace{\left(E^T \otimes A^T(q) + A^T(q) \otimes E^T\right)}_{M(q)} \mathrm{vec}(X) = -\mathrm{vec}(E^T Y E). \tag{28}$$

Note that, for convenience, we changed the notation of X_1 to X, because we do not need to discriminate between the two Lyapunov approaches here. Due to the singularity of E, the matrix M is non-invertible and the linear system of equations (28) is underdetermined.

Although the structure of (28) is similar to (25), we are not able to use the conditions based on the determinant of M. However, we can reshape M in a form

$$M = \begin{pmatrix} M^{r\times r} & 0^{r\times t} \\ 0^{t\times r} & 0^{t\times t} \end{pmatrix}, \tag{29}$$

with $r+t = n^2$ and $\text{rank}(M) = r$. The explicit values of r and t depending on rank deficit of E. The full-rank matrix $M^{r\times r}$ is associated with the matrix of the slow subsystem and hence we can use the the $\det(M)$ conditions, introduced for the state-space case. In contrast to the approach given in [29], we are here able to calculate the exact boundaries.
The workflow of the two proposed approaches are exemplary illustrated in the following section.

5. CHUA'S CIRCUIT

5.1 MODELING AND PROBLEM STATEMENT

As an example for the application of the procedures Chua's circuit [7] is used. The circuit diagram is shown in Fig. 1. Chua's circuit is an chaotic oscillator. The nonlinearity which generates this behavior is an voltage-controlled current source, called Chua's diode. This diode can be modeled by a piecewise linear function consisting of three segments [14]

$$\bar{\varphi}(u_1) = -G_0 u_1 - \frac{1}{2}(G_1 - G_0)\left[|u_1+1| - |u_1-1|\right], \tag{30}$$

which is shown in Fig. 2. The gains G_0 and G_1 have to be chosen such that the system has three equilibrium points, one in the origin and two symmetrically located around the origin. In opposite to the normal issue, the aim is to create a system with chaotic behavior and thus an unstable one. In particular, all equilibrium points of Chua's circuit have to be unstable. More precisely, the following selection is used in order to obtain the well-known chaotic *double scroll* [18]:

1. The gain G_0 in the outer region of $\bar{\varphi}$ is chosen such that the resulting linear system has an unstable conjugate pair of eigenvalues and one stable real eigenvalue.

2. The gain G_1 in the inner region of $\bar{\varphi}$ is chosen such that the linearization has a real unstable eigenvalue and a stable conjugate pair.

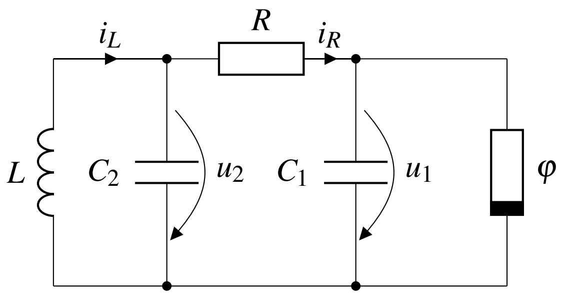

Figure 1: Chua's circuit

Therefore, we are looking for incremental gains of Chua's diode which destabilizes the system. This claim can be represented in a DAE parameter space analysis.

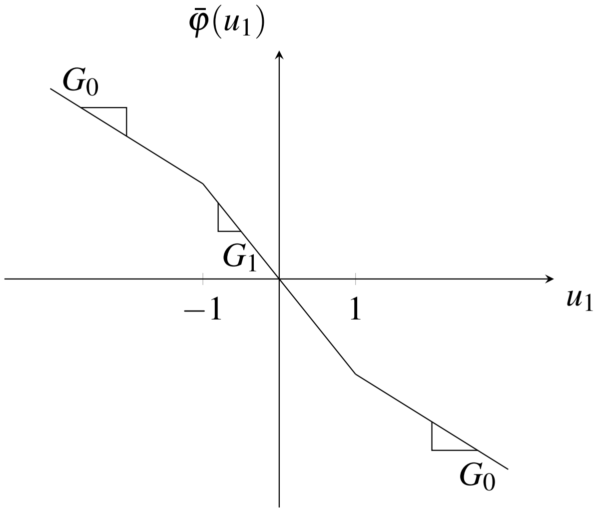

Figure 2: Characteristic curve of Chua's diode

Chua's circuit shown in Fig. 1 can be modeled using Kirchhoff's current and voltage law. Following the aim of determining the gains for Chua's diode which leads to an unstable behavior, we thus consider the piecewise linear nonlinearity $\bar{\varphi}$ in one the outer part as the linear function

$$\varphi(u_1) = -G_0 u_1. \tag{31}$$

The same approach can be used for the linear part around the origin with the gain G_1 instead of G_0. Using this approximation and $(u_1, u_2, i_L, i_R)^T$ as generalized variables it results the descriptor system

$$\underbrace{\begin{pmatrix} 0 & C_2 & 0 & 0 \\ C_1 & 0 & 0 & 0 \\ 0 & 0 & L & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}}_{E} \begin{pmatrix} \dot{u}_1 \\ \dot{u}_2 \\ \dot{i}_L \\ \dot{i}_R \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 0 & 1 & -1 \\ G_0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & \frac{1}{G} \end{pmatrix}}_{A} \begin{pmatrix} u_1 \\ u_2 \\ i_L \\ i_R \end{pmatrix}, \tag{32}$$

with $G = \frac{1}{R}$ being the conductance of the resistor R.
In order to use the Lyapunov formulations stated in Section 2, we need to verify the regularity and impulse-freeness of (32). Therefore we calculate the determinant

$$\det(sE - A) = \frac{s^3 + (10 - 9G_0)s^2 + (7 - 9GG_0)s + 63G - 63G_0}{63G}, \tag{33}$$

with $(C_1, C_2, L) = (\frac{1}{9}, 1, \frac{1}{7})$. The determinant (33) is not identically zero (in the sense of the zero polynomial), so the system is regular. Furthermore $\deg\det(sE - A) = \operatorname{rank} E = 3$ states the impulse-freeness of Chua's circuit. Therefore, we can use the simplified versions of the Lyapunov formulations.

5.2 QE BASED STABILITY BOUNDARY CALCULATION

We first apply the QE method to compute the feasible region for the considered design parameters (G, G_0) for the given system (32). To this end, the Maple package RegularChains from [5] is used. With the Lyapunov formulation from Theorem 3, we can easily calculate the following prenex formula in Maple, yielding

$$\begin{aligned} PF \;:=\; \exists X_2 \;\; & [(\bigwedge_{i=1}^{n}\bigwedge_{j=1}^{n} u_{ij}(G,G_0,X_2)=0)\wedge(\bigwedge_{i=1}^{n}\bigwedge_{j=1}^{n} v_{ij}(G,G_0,X_2)=0)\wedge \\ & (\bigwedge_{i=1}^{m} z_i(G,G_0,X_2)\geq 0\wedge(G>0)\wedge(G_0>0))], \end{aligned} \tag{34}$$

where u_{ij} and v_{ij} denote respectively the entries of matrix U and V as defined in (19) and (20), z_i is the i-th principal minor of matrix W. The entries of matrix X_2 are quantified variables and denoted by the symbol X_2, and the parameters (G, G_0) are free variables.

It should be noted that two additional atomic formulas $G > 0$ and $G_0 > 0$ are added in the prenex formula, because of the positive definiteness from the characteristics of conductance and Chua's diode. Applying the Quantifier Elimination function in RegularChains, the default output FF is a quantifier free formula formed by polynomial constraints and logical connectives, yielding

$$FF = (0 < G_0)\wedge(0 < G - G_0)\wedge(90GG_0 - 81G_0^2 < 7). \tag{35}$$

These boundaries and the resulting stability area are shown in Fig. 3. The linear system is (asymptotically) stable in region A; see the figure. In our application, we want to obtain an unstable linearization. For the conductance G we choose the normalized parameter value $G = 0.7$ as in [14]. This choice is sketched by a vertical blue line in Fig. 3. The parameter $G_0 = 0.5$ is taken from the region C, whereas the parameter $G_1 = 0.8$ is selected from region B. In both cases, we obtain an unstable behaviour, just as desired, see [14].

5.3 ALGEBRAIC STABILITY BOUNDARY CALCULATION

The procedure proposed in Section 4 before leads to a 16×16 matrix M of the form

$$M = \begin{pmatrix} M^{15\times 15} & 0 \\ 0 & 0 \end{pmatrix}. \tag{36}$$

The determinant of the submatrix $M^{15\times 15}$ is

$$\det(M^{15\times 15}) = \frac{8(G-G_0)(81G_0^2-90GG_0+7)^2}{3938980639167G^4} \tag{37}$$

and we are thus able to calculate the stability boundaries by analyzing this determinant. The conditions $\det(M) \to \infty$ and $\det(M) = 0$ of (37) can easily determined and gives us the four

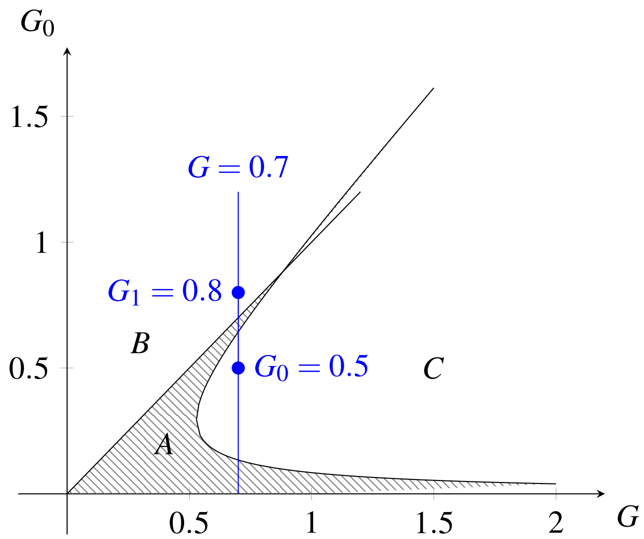

Figure 3: Stability boundaries of Chua's circuit

boundaries

$$G = G_0, \quad G = \frac{81G_0^2 + 7}{90G_0}, \quad G = 0, \quad G \to \infty. \tag{38}$$

Obviously, the only two conditions of interest are the two left ones.

6. CONCLUSION

We discuss the computation of feasibility regions for parameter-dependent Lyapunov functions for linear descriptor system descriptions. Therefore, we develop two approaches. The first one is indirect, and it deals with the inherent formulation of the existence condition of a Lyapunov function in form of a prenex formula. In this case, a proper formulation of the feasibility condition is utilized and standard quantifier elimination techniques are applied thereupon to compute quantifier-free expressions, i.e. explicit polynomials in terms of the free parameters, describing the stability boundaries in the parameter space. The second approach is direct in that it derives the mapping conditions from the algebraic solution of the resulting Lyapunov linear system of equations. While QE approach is principally a much more general approach, it has computational limitations. Nevertheless, the basic idea of the QE based approach is applicable to a large class of control and analysis problems. On the other hand, the direct approach based on Lyapunov equation is trimmed to the particular problem formulation of the paper and, hence, it is principally applicable to systems with more states and free parameters.

References

[1] J. Ackermann, A. Bartlett, D. Kaesbauer, W. Sienel, and R. Steinhauser. *Robust control: Systems with uncertain physical parameters*. Springer, 1993.

[2] S. Basu, R. Pollack, and M.-F. Roy. *Algorithms in Real Algebraic Geometry (Algorithms and Computation in Mathematics)*. Springer-Verlag, Berlin, Heidelberg, 2006.

[3] K. E. Brenan, S. L. Campbell, and L. R. Petzold. *Numerical Solution of Initial-Value Problems in Differential-Algebraic Equations*. SIAM, Philadelphia, 2nd edition, 1996.

[4] C. W. Brown. Qepcad b: a program for computing with semi-algebraic sets using cads. *ACM SIGSAM Bulletin*, 37(4):97–108, 2003.

[5] C. Chen and M. M. Maza. Cylindrical algebraic decomposition in the regular chains library. In *International Congress on Mathematical Software*, pages 425–433. Springer, 2014.

[6] C. Chen and M. M. Maza. Quantifier elimination by cylindrical algebraic decomposition based on regular chains. *Journal of Symbolic Computation*, 75:74–93, 2016.

[7] L. O. Chua. The genesis of Chua's circuit. *Archiv für Elektronik und Übertragungstechnik (AEÜ)*, 46(4):250–257, 1992.

[8] G. E. Collins. Quantifier elimination for real closed fields by cylindrical algebraic decomposition–preliminary report. *ACM SIGSAM Bulletin*, 8(3):80–90, 1974.

[9] A. Dolzmann and T. Sturm. Redlog: Computer algebra meets computer logic. *Acm Sigsam Bulletin*, 31(2):2–9, 1997.

[10] P. Dorato. Non-fragile controller design: an overview. In *American Control Conference, 1998. Proceedings of the 1998*, volume 5, pages 2829–2831. IEEE, 1998.

[11] G.-R. Duan. *Analysis and Design of Descriptor Linear Systems*. Springer, 2002.

[12] F. R. Gantmacher. *Theory of Matrices*. Chelsea, New York, 1959.

[13] G. T. Gilbert. Positive definite matrices and Sylvester's criterion. *The American Mathematical Monthly*, 98:44–46, 1991.

[14] M. P. Kennedy. Robust AMP realization of Chua's circuit. *Frequenz*, 3-4:66–80, 1992.

[15] R. Liska and S. Steinberg. Applying quantifier elimination to stability analysis of difference schemes. *The Computer Journal*, 36(5):497–503, 1993.

[16] M. Marcus and H. Minc. *A Survey of Matrix Theory and Matrix Inequalities*. Dover, 1992.

[17] I. Masubuchi, Y. Kamitane, A. Ohara, and N. Suda. H_∞ control for descriptor systems: a matrix inequalities approach. *Automatica*, 33(4):669–673, 1997.

[18] T. Matsumoto. A chaotic attractor from chuas circuit. *IEEE Trans. on Circuits and Systems*, 31(12):1055–1058, 1984.

[19] V. Mehrmann and T. Stykel. Descriptor Systems: A General Mathematical Framework for Modelling, Simulation and Control. *Automatisierungstechnik*, 54(8):405–415, 2006.

[20] P. C. Müller. Lyapunov matrix equations for the stability analysis of linear time-invariant descriptor systems. In S. Schöps, A. Bartel, M. Günther, E. J. W. ter Mater, and P. C. Müller, editors, *Progress in Differntial-Algebraic Equations*, pages 3–20. Springer, 2013.

[21] T. V. Nguyen, Y. Mori, T. Mori, and Y. Kuroe. Qe approach to common Lyapunov function problem. *Journal of Japan Society for Symbolic and Algebraic Computation*, 10(1):52–62, 2003.

[22] K. Panreck, M. Jahnich, and F. Dörrscheidt. Verwendung differetial-algebraischer Gleichungen zur verkopplungsorientierten Modellierung komplexer Prozesse. *Automatisierungstechnik*, 42(6):239–247, 1994.

[23] K. J. Reinschke, K. Röbenack, and G. Wiedemann. Strukturelle Analyse von Deskriptorsystemen mit Hilfe von Digraphen. *Automatisierungstechnik*, 46(1):22–31, 1998.

[24] K. Röbenack and K. J. Reinschke. Graph-theoretically determined Jordan-block-size structure of regular matrix pencils. *Linear Algebra and its Applications*, 263:333–348, 1997.

[25] F. Schrödel, E. Almodaresi, A. Stump, N. Bajcinca, and D. Abel. Lyapunov stability bounds in the controller parameter space. In *2015 54th IEEE Conference on Decision and Control (CDC)*, pages 4632–4636, Dec. 2015.

[26] B. Straube, K. Reinschke, W. Vermeiren, K. Röbenack, B. Müller, and C. Clauß. DAE-index increase in analogue fault simulation. In R. Merker and W. Schwarz, editors, *System Design Automation – Fundamentals, Principles, Methods, Examples*, pages 221–232. Kluwer, 2001.

[27] T. Stykel. Stability and Inertia Theorems for Generalized Lyapunov Equations. *Linear Algebra and its Applications*, 1-3(355):297–314, 2002.

[28] A. Tarski. A decision method for elementary algebra and geometry. In *Quantifier elimination and cylindrical algebraic decomposition*, pages 24–84. Springer, 1998.

[29] R. Voßwinkel, F. Schrödel, N. Denker, K. Röbenack, D. Abel, and H. Richter. Lyapunov stability bounds mapping for descriptor and switching systems. In *13th International Multi-Conference on Systems, Signals Devices (SSD)*, pages 376–381, Mar. 2016.

CONTACS

Rick Voßwinkel, M.Sc.	rick.vosswinkel@tu-dresden.de
Jiancheng Tong, M.Sc.	tong@mv.uni-kl.de
Prof. Dr.-Ing. habil. Dipl.-Math. Klaus Röbenack	klaus.roebenack@tu-dresden.de
Prof. Dr.-Ing. Naim Bajcinca	naim.bajcinca@mv.uni-kl.de