NUMERICAL ALGORITHMS AND COMPUTER MODELING FOR NONLINEAR ANALYSIS OF SHELL STRUCTURES

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Shells are now widely used in construction and mechanical engineering as critical components of machinery and 3-D structures, possessing of high durability and lightness, that allows the most effectively to decide a material reduction problem of structures. At the same time the shell-type structures are while in service frequently exposed to action of intensive dynamic loadings, resulting in occurrence of complex oscillatory movements. These fluctuations of shells are accompanied by several specific effects and physical phenomenas, peculiar to nonlinear mechanical systems. Their behaviour under static and dynamic loadings is described by system of deep nonlinear differential equations. Solution of these equations can be received with assistance of technique basing on a modern numerical algorithms and computer modeling. In given work an effective technique and software for research of nonlinear fluctuations and stability of thin shells under force and kinematic excitations based on joint application of methods of numerical and tensor analysis, projective method and parameter prolongation of solution method is presented.

Let us determine in general an algorithm for construction and research the trajectories of stationary solutions of nonlinear equations, describing steady-state modes of forced fluctuations of compound shells of revolution. The fluctuations can be initiated by power periodic on time loading, and kinematic excitation of the basis, on which the shell is rigidly fixed. We shall consider only those influences, which are characterized by symmetry to axis of rotation of an shell. In such case at absence of initial imperfaction of the form of a surface the steady-state fluctuation of shells at small meanings of excitation parameter will be realized on axisymmetric forms with frequency, equal to frequency of excitation. With increase of intensity of power or kinematic excitation at fixed frequency of forced fluctuations their symmetric modes can be transformed in cyclically symmetric. Besides under certain conditions the steady-state fluctuations of shells, remaining axisymmetric, can sharply increase amplitude and to change the form of fluctuations. The enumerated factors characterize loss of stability of steady- state movement and to them there correspond the special points on stationary states curves. Therefore the research of stability of a steady-state movement on the basis of offered approach is connected to construction of stationary states curves, completely describing evolution of fluctuations, and finding on them critical (limiting and bifurcation) points.

The construction of stationary states curve of shells will be realized under circuit, which substance we shall consider on example of a nonlinear functional equation

$$F(x) = \Psi(x) - q = 0 \quad , \tag{1}$$

describing motion modes x under action of loadings q. To an equation (1) is put in conformity operator

$$G(x, \lambda) = \Psi(x) - \lambda q, \qquad (x \in X; 0 \le \lambda \le 1) \tag{2}$$

with the meanings in Y such, that

$$G(x,1) = \Psi(x) - q \equiv F(x), \qquad G(x,\lambda) = 0 \quad \text{when} \quad x = x^0.$$

We shall consider operational equation

$$G(x, \lambda) = 0 \tag{3}$$

and we shall assume, that it has the continuous solution $x = x(\lambda)$, certain at $0 \le \lambda \le 1$. Proceeding of any known solution x^0, it is possible to construct the family of the solutions *x(λ)* and, changing step by step parameter λ, to look after corresponding evolution of dynamic states, described by an equation (1). We shall for this purpose break an interval $[0,1]$ by points

$$\lambda(0) = 0 \angle \lambda(1) \angle \ldots . \angle \lambda(m) = 1$$

and operator $\Psi(x)$ in vicinity of a known state $x = x_n$ shall present by Tailor's series

$$\begin{gathered}\Psi_{(n)} + \Psi'_{(n)}(x_{(n+1)} - x_{(n)}) + \frac{1}{2!}\Psi''_{(n)}(x_{(n+1)} - x_{(n)})^2 + \ldots \\ + \frac{1}{r!}\Psi^{r}_{(n)}(x_{(n+1)} - x_{(n)})^r \approx \lambda_{(n+1)} q\end{gathered} \tag{4}$$

Keeping in decomposition (4) two first components, we shall receive a recurrence ratio

$$x_{(n+1)} = x_{(n)} + \left[\Psi'_{(n)}\right]^{-1}\left(\lambda_{(n+1)} q - \Psi_{(n)}\right),$$

which allows approximately to define $x_{(n+1)}$, possessing by element $x_{(n)}$. For numerical realization the expression (4) is more convenient for presenting in kind

$$\Psi'(x_{(n)})(x_{(n+1)} - x_{(n)}) = \lambda_{(n+1)} q - \Psi\left(x_n\right). \tag{5}$$

We shall assume now, that an equation (1) describes stationary (steady-state) movement of an shell, raised by periodic on time excitation

$$q(t) = q(t + T),$$

where T - period of excitation. Let $x(t)$ - some vector-function of generalized coordinates of an shell, describing it steady-state fluctuations with frequency $\omega = 2\pi/T$, which satisfies to a state of periodicity

$$x(t) = x(t + kT). \tag{6}$$

Then standing in left-hand part of equality (5) differential Freshe represents a linearized in vicinity T-periodic function, constructed in view of accumulated in shell membrane stresses and bending moments. The states of periodicity (6) are equivalent to boundary states

$$x(0) = x(T),$$

that allows to execute transition to Coshi problem. During solution of this problem with help of algorithm (5) at continuous change of parameter λ the solutions x of an equation (3) in general case correspond to a continuous sequence of the periodic solutions of system (1) in space of states, describing the forms of steady-state movement of shell, received in result of its forced fluctuations, proceeding from non-perturbed state. In vicinity of points, describing a dynamic critical state, there is the degeneration of the operator Ψ' thus to small increment of loading parameter λ there correspond large changes of periodic solutions $x(t)$. As the linearized operator in these points degenerate, in them is possible occurrence of branching solutions. For their construction it is necessary by methods of the theory of branchings to determine a number of branching solutions and their direction, and then to continue the solution on each of directions. In present work this problem is not considered. A problem of construction of the periodic solutions of equations of forced fluctuations of shells, finding on them the special points and determination of the forms, describing unstable modes of a movement is put.

In result of application of numerical methods for solution of linearized equation (5), operational equation (3) of a boundary problem is reduced to set of nonlinear equations

$$f_i(x_1, x_2, \ldots, x_n) = \lambda q_i = 0, \qquad (i=1,2,\ldots s) \tag{7}$$

and Freshe derivative $\Psi'(x)$ - to Jacobi matrix

$$J = \left\| a_{ij} \right\| = \left\| \frac{\partial f_i (x_1 , x_2 , \ldots , x_s)}{\partial x_j} \right\| . \tag{8}$$

If at solution of a problem in vicinity of some point $x^*_{(r)}$ jacobian turnes into zero, i.e..

$$det\, J(x^*_{(r)}) = 0,$$

the system (6) becomes inconsistent and the process of construction of stationary states curve cannot be continued in vicinity of this point. In this case to find $x_{(r+1)}$ new parameter is entered and the solution is under construction in enough small vicinity of $x^*_{(r)}$. Limiting meaning of excitation intensity parameter λ^*, at which jacobian $J(x^*)$ changes a sign, characterizes top dynamic critical load or bifurcation point.

The elaborated algoritm is applied to investigation of nonlinear fluctuations and stability of shells with various shapes of surface at action of periodic on time and uniformly-distributed power loadings [1]. The solutions of vibration stability problems of toroidal shells of circular and ellipsoidal sections, but also compound shell-type designs, assembled from cylindrical and conic fragments and ring plates, connected by toroidal belts of various radius are received. As example of such designs can serve investigated in work the shell of a torocylinder tank, which is used in rocket engineering as fuel tank. For considered shells the peculiarities of their loss of stability are discovered, the dependences of critical values of force excitation intensity from fluctuation frequencies and variationÿ of geometrical parameters of designs are determined .

Solutions of a number of stability problems of kinematically excited fluctuations for spherical and ellipsoidal segments, torospherical shells, cylindrical reservoirs with spherical, ellipsoidal and torospherical caps are received [2]. Critical amplitudes of kinematical excitation depending on geometric ratios of shells and frequencies of steady-state fluctuations are determined, forms of loss of stability are found. As particular object a stability of equilibrum state of cooling tower shell under uniformly distributed external pressure and own weight loading are investigated. A stability problem of fluctuations of cooling tower shell at kinematic excitation of the basis is resolved [3]..

A construction of set of nonlinear differential equations of fluctuations of thin axisymmetric shells at translational vibrating movement of connected with it basis and it rotation around of axis of symmetry with constant angular speed is formulated. The solutions of eigen fluctuations problems for rotating axisymmetric shells of canonical form and connected from separate fragments are found [4]. The values of lowest eigen frequencies are received and corresponding to them forms of fluctuations are constructed for cylindrical shell, truncated cone, sphere, ellipsoid, torospherical shell, for cylinder, connected with spherical, ellipsoidal and torospherical caps. Stability of nonlinear kinematically excited fluctuations of rotating axisymmetric shells is investigated. Values of critical amplitudes of kinematic excitation are found at various meanings of angular speed of rotation. For considered objects the most characteristic forms of compelled nonlinear fluctuations at various meanings of angular speed of rotation are constructed . Influence of rotation to frequencies and forms of eigen fluctuations of considered designs, but also on values of critical amplitudes of excitation of shells and forms of their compelled fluctuations is investigated.

For realization of numerical algorithms and technique of construction of the periodic solutions and analysis of their critical states, describing loss of stability of shells a problem-oriented computer complex SEVSOR has been developed. This software is intended for determination of stability parameters of shell-type structures, used in engineering and building practice, at operational modes under dynamic loadings. Frame output of motion forms in real time or either in decelerated or accelerated time scales for creating cartoons or video films is used for analysis of the compound dynamic process in shell-type structures.

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