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INVESTIGATION BY COMPUTER SIMULATION SOME ASPECT OF TRANSPORTATION SERVICE RELIABILITY IN URBAN PUBLIC TRANSPORT

1. Introduction

Some aspect of reliability of urban public transport is occurrence of events consisting in that on account of overloaded vehicle, a passenger in not able to board into tram or bus. He is obliged to wait for the next vehicle. Such situation should be taken into consideration in assesment of public transport service. Then the **factor of service unreliability on account of irregularity** is defined as following:
It is probability of the event, that the passenger finds departing vehicle fully filled, i.e. when all places seating and standing are occupied or condition of the travel, which would be not acceptable for passenger.
Overloading of vehicle is usually the result of occurrence jointly a few elements:

- irregularity of vehicle operation, appearing occurrence of very long headways in vehicle flow; proportionally greater is passenger flow arriving at stops to get into the late vehicle,
- random fluctuation of passenger flow,
- occurrence of micro-peaks in passenger flow,
- cancellation of vehicle, for instance as a result of an accident or break down,
- service frequency is badly fitted passenger flow (demand greater than supply) - average occupancy of vehicles is greater than their nominal capacity,
- dispatching of vehicles with differentiated capacity (for example: midi-buses and articulated buses) into the service for single line or for common bunch of lines.

Mentioned situations cause that carrying capacity to be exceeded by demand, especially on heavy loading section of the route.
From the mist of twenty reliability measures described in [4], someone similar that above defined is lacking.
Determination by analytical methods the probability of lack of a free place in public transport vehicle seams to be impossible and it would be at least numerical ineffective. Main difficulty lies in taking into consideration the situation when passenger, which does not find the place in first vehicle, increases number of passengers boarding into the next vehicle. Formulation this situation in a simulation model causes no difficulties.
As early as in the seventies, the computer simulation was found as a very effective method of investigation of public transport operation (see e.g. [1], [2], [6]).

2. Ground for simulation model

The process is observed in any given stop of single public transport line. This line does not cover with whichever ones, i.e. the passenger demand refers to one line.

2.1. Public transport vehicles flow

Operation of public transport vehicles in given point is described by distribution of headways. Of course, it is a random point process. For the rush hours period it can be treated as a stationary stochastic process [3]. On the base of own researches the investigated flow in the tangible majority is renewal process with Gamma distribution of headways.

Density function of probability is given:

$$f(h) = \frac{(\lambda \cdot k)^k}{\Gamma(k)} \cdot h^{k-1} \cdot e^{-\lambda \cdot k \cdot h} \quad (1)$$

where:
h is a headway in public transport vehicle flow,
λ is intensity of public transport vehicle flow,

$$\lambda = \frac{1}{\overline{h}} \quad (2)$$

$\overline{\mathbf{h}}$ is the average headway,
k is characteristic parameter of the Gamma distribution; it can be estimated as following:

$$k = \frac{\overline{h}^2}{s_h^{\;2}} \quad (3)$$

or $$k = \frac{1}{\nu^2}$$

ν is coefficient of variation:

$$\upsilon = \frac{s_h}{\overline{h}} \quad (4)$$

$\mathbf{s_h}$ is standard deviation of headways,
Γ(k) is the Gamma function of the ***k*** parameter.

In model the mutual independence of neighbouring headways has been assumed. In fact, in some cases there is stochastic relationship between these headways. It consists in that after the headway, the shorter than the average, occurs with high probability the longer one, and reciprocally. It did not succeed to estimate the general relationship, which could be apply for each cases of simulation procedure.

2.2. Passenger flow

According the presented paper in the IKM '97 Proceedings [5], passenger arrival flow at a stop is the Poisson process. Then number of passenger arriving during the period of a public transport vehicle headway is submitted to the Poisson distribution. In the same way number of boarding and alighting passengers for determined value of headway is under this distribution. Occupancy of public transport mean (the number of passenger in the vehicle) **C** at the ***j*** stop is given:

$$Q_j = \sum_{i=1}^{j} z_{b\,i} - \sum_{i=2}^{j} z_{a\,i} \quad (5)$$

where
$\mathbf{z_{bi}}$ is the number of boarding passenger at the ***i*** stop (i = 1,2,..,j)
$\mathbf{z_{ai}}$ is the number of alighting passenger at the ***i*** stop (i = 2,3,..,j)
Using methods of mathematical statistics it has been proved in the dissertation [6], that the occupancy for determined stop of single separated bus line during the period with approximate

unchanging passenger flow rate is submitted to the Poisson distribution. For great number of passengers in the vehicle (this case interests us) the Poisson distribution can be approximated by normal cut distribution.
Variance of number of passenger in the vehicle is equal the estimated average value.

2.3. Maximal capacity of a vehicle

The number of passenger at using up all seat places and all area for standing places, assuming 10 persons per one square meter was determined as the „maximal capacity" C_{max}. It is circa 30-35 % greater than the nominal capacity given by factory standards. It usually is at the level of 0,15 m^2/person. At this assumption, the„maximal capacity" for individual vehicle is objective and constant value. However, on account of overcrowding vehicle, the passenger backs out of getting into the vehicle even in cases though the number of passenger in the vehicle would be less than given „the maximal capacity". Likely, passenger can recognise that the condition of the travel would be not acceptable for him, what amounts to a service refuse. In fact „the maximal capacity" is random variable as well as can be interpreted as a fuzzy limit.
The typical vehicles operated in Polish cities were taken:
- single standard buses C_{max} = 130 and C_{max} = 150
- articulated buses C_{max} = 210
- tram with one car C_{max} = 180
- tram with two cars C_{max} = 360
- tram with three cars C_{max} = 540
- articulated tram C_{max} = 260

3. Simulation of respective processes and events

For public transport vehicle flow the succeeding headways are generated as independent variables according to the Gamma distribution with parameter
k = 1, 2, 4, 9, 25. 100, 500.
Mentioned values are connected with variation coefficient values, respectively
ν = 1,00 ; 0,71 ; 0,50 ; 0,33 ; 0,20 ; 0,10 ; 0,04 .
In last three cases the Gamma distribution was generated using normal distribution as an approximation. Each generated value of headways was checked in consideration the of maximal value of the public transport vehicle headway. This was assumed as:

$$h_{max} = \bar{h} + 3 \cdot s_h \qquad (6)$$

Expected value of number of passengers in a vehicle is proportional to the length of the current headway. Average occupancy $\mathbf{Q_{av}}$ was taken as fraction of maximal occupancy. The simulation included seven cases of this fraction
Q_{av}/C_{max} = 0,3 , 0,5 , 0,6 , 0,7 , 0,8 , 0,85 , 0,90 , 0,95 , 1,0
Generated occupancy **Q** as normal cut distribution value was integered and compared with minimal and maximal values

$$Q_{min} = Q_{av} - 2\, s_Q \qquad (7)$$
$$Q_{max} = Q_{av} + 3\, s_Q \qquad (8)$$

$\mathbf{s_Q}$ is standard deviation of passengers number in the vehicle:

$$s_Q = \sqrt{Q_{ar}} \qquad (9)$$

If actually generated occupancy value **Q** exceeds range (Q_{min} ; Q_{max}), its limit value is assumed.
If **Q** is greater than the maximal capacity $\mathbf{C_{max}}$ then the number of passengers exceeding of $\mathbf{C_{max}}$ to the next vehicle is added.

The states, when generated number of passengers exceeds public transport vehicle capacity were added up. This value related to whole number of simulated states determines the probability of service refusal.
Moreover cases described in the **2.3** item, the simulation was done for the round values of the „maximal capacity”
C_{max} = 70, 100, 200, 300, 500.
Altogether 12 case of capacity were investigated.

Each simulation run consisted 10 000 cycles. Each cycle included the arrival of a vehicle.
Then estimated error of simulation is order of magnitude one percent

$$\frac{1}{\sqrt{n}} = \frac{1}{\sqrt{10000}} = 0{,}01 \qquad (10)$$

756 of simulation runs considering all combinations of parameters were made. The value of parameters were changed automatically in sequence way. The computer IBM PC 486 realised total calculation in the time of 50 minutes.

4. Results of simulation

As main direct results of simulation, the probability of lack of a free place in public transport vehicle were obtained. They are shown at the Figure 1 which as an example was prepared for the vehicle with „the maximal capacity” $C_{max.}$ = 130 passengers.

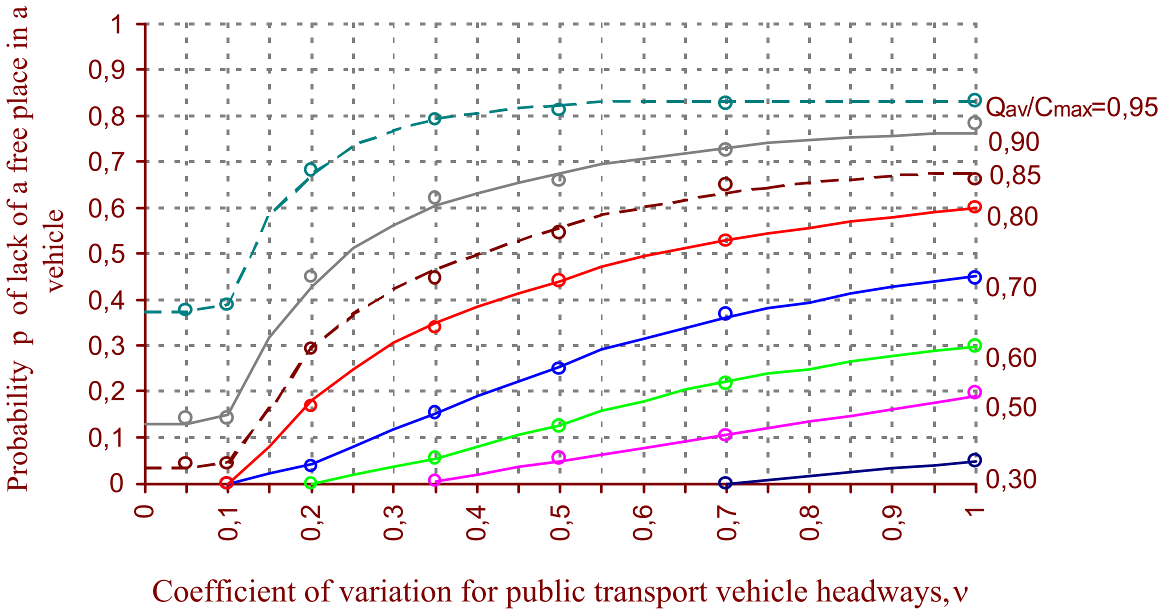

Fig. 1 Probability of lack of a free place in a vehicle as a function of variation coefficient for public transport vehicle flow on background of result of simulation for set of occupancy of vehicle. Maximal capacity of the vehicle: C_{max} = 130 passengers, Q_{av} - is average occupancy of vehicle.

It is seen that the rapid increase of probability occurs in the range of variation coefficient between 0,1 and 0,3 . This tendency is especially visible for great value ratio $Q_{av}/C_{max.}$
The carried comparison of probability curves for various the „maximal capacity” differentiates essentially only in the range of small value of variation coefficient of headways (0,04 - 0,20). The stabilising impact of greater capacity of vehicle is appeared.

The probability of lack of a free place in public transport vehicle can represent self-dependent factor describing of service reliability. It can also go into more synthetic measures of service performance, e.g. waiting time or own proposal so called „the equivalent passenger travel time". Moreover mentioned factor is useful for design of optimal average occupancy of public transport vehicle. To present wider this issue, the Figure 2 has been prepared. It determines the permitted occupancy of vehicle for three values of the probability of service refusal.

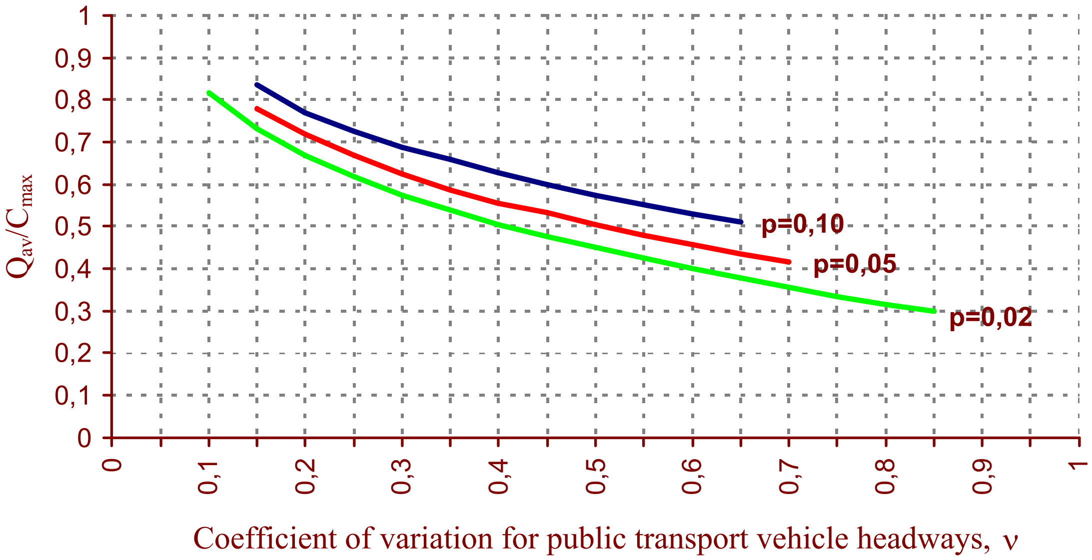

Fig. 2 Permitted occupancy (Q_{av}/C_{max}) of public transport vehicle as a function of variation coefficient for public transport vehicle headways given for three values of probability p of service refusal lack of a free place in a vehicle
Maximal capacity of the vehicle: Q_{max} = 130 passengers, Q_{av} - is average occupancy of vehicle.

It is seen, that the better is regularity of public transport vehicle flow (i.e. the value of variation coefficient is lower) the greater one can allow the average occupancy of the vehicle (ratio Q_{av}/C_{max}). For instance: let probability of service refusal p = 0,02 and for ν = 0,2 (good regularity) one can permit on the average of 70 % utilisation (Q_{av}/C_{max} = 0,7) of the „maximal capacity". But for an irregular operation (e.g. ν = 0,8) only circa 30% of the „maximal capacity" could be use, if probability p = 0,02 of lack of a free place in the vehicle should not exceeded. It explains loss of potential carriage capacity of public transport means caused lack of regularity.
In the practice for public transport users, the values of probability p given at the Figure 2 correspond with:

- p = 0,10: one event of refusal service per week,
- p = 0,05: two events of refusal service per month,
- p = 0,01: one event of refusal service per month.

The above value of probability could be recommended for the average public transport headway, respectively:

- below 6 minutes,
- between 6 and 12 minutes,
- more than 12 minutes.

5. Remarks

The described model is difficult to verify, since:

– situations of large occupancy usually occur only in the city centre where buses/trams operate bunchly, i.e. it in impossible to separate single line,
– events of lack of a free place in the vehicle are rare events; to collect relative large sample one need very long period of observations, what can appear troublesome at requirement of stability of operation parameters (including demand and supply),
– seeing that passengers demand in the last years decreased essentially but supply of transportation means only slightly, then average utilisation of vehicle capacity is currently lower than before.

Other measure of unreliability can be high percentile of headway (for instance 95 or 99). It is connected with occurrence of very long headways (for instance from 3 to 5 time greater than average headway. In such situation is high probability that the passenger finds departing vehicle fully filled, what additionally extends his waiting time. It means, that continuity of operation has been practically broken, namely the service happened reliable.

The described factor of service unreliability on account of irregularity is useful for cases of relative high frequence of buses/trams operation. For a rare service, e.g. with time table headway equals 30 minute, it would be more convinient the factor of service unreliability on account of unpunctuality. It can be defined as probability that a passenger has to wait for the next vehicle owing to earlier departure of a vehicle ahead of the scheduled time.

6. Conclusions

1. Randomness of passenger arrival flows, dispersion of bus/tramway headways, in circumstances of insufficient supply of place in public transport vehicle - all produce cases of overcrowded vehicles. Then one and more passengers can not get in, what expresses unreliability of public transport service. Probability of such event has been named as a factor of service unreliability on account of irregularity.
2. The computer simulation was found as a credible and very effective tool for estimation such defined factor.
3. To avoid cases of overcrowded vehicles, their maximal capacity can be utilised only partially. The worse service regularity decreases this possibility. In the conditions of mean service disturbances, the average occupancy of vehicle can not exceed 50% of the „maximal capacity”, e.i. circa 65-70% of the nominal capacity (with the standard level of 0,15 m^2 of standing area per one person). Irregularity means loss of potential carriage capacity of public transport.

References

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