THE NUMERICAL MODELLING AND ANALYSIS OF RC CRACKED STRUCTURES

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1. NON-LINEAR BEHAVIOUR OF CONCRETE AND STEEL.

Reinforced concrete structures exhibit very complicated behaviour differs widely from the results of a linear elastic computation. The structural system is composed of different materials, such as cement, steel bars, aggregate, etc. Moreover each material shows various physical phenomena. The non-linear behaviour of entire structures can be considered to be accumulated from cracking of concrete, non-linear material properties of concrete under compression and tension, time-dependent deformations due to creep and shrinkage of concrete, bond behaviour, yielding and strain hardening of steel essentially. Progressive cracking of concrete is surely the most important component of the non-linear response of reinforced concrete structures in normal service state.

The experiments [2,3] show that under cyclic loads concrete and reinforced concrete structure response like linear-elastic materials. The linear-elastic behaviour of concrete elements under cyclic loads is the base to assume that total deformation is the sum of elastic, residual, plastic and creep deformations as show on Fig. 1.

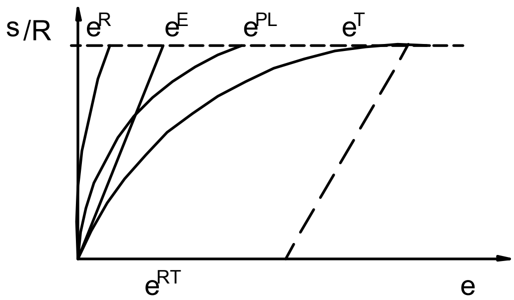

Fig. 1. Stress-strain relationship for concrete.

The total residual e^{RT} deformation is the sum:

$$\varepsilon^{RT} = \varepsilon^{R} + \varepsilon^{PL} + \varepsilon^{R\varphi} \tag{1}$$

where: e^{RT} - residual deformation,
e^{PL} - plastic deformation,
$e^{R\varphi}$ - creep deformation.

The creep deformation was described using Rush [1] theory. It was assumed that residual creep deformation is 80% of total deformation. The non-linear residual deformation e^{R} (because of plane cross section), gives "self-stresses" s^{R} inside the element:

$$\int \sigma^{R} \, dF = 0 \,, \qquad \int \sigma^{R} z \, dF = 0 \quad . \tag{2}$$

Using the equilibrium of forces and moments in the cross section of element (Eq. 2), the constants A, B and "self-stresses" were calculated:

$$\sigma^{R} = E_b \varepsilon^{ED} = E_b (A + Bz - \varepsilon^{R}) \quad . \tag{3}$$

We can write the total stress as a sum of self-stresses σ^{R} and linear elastic stress σ^{E}:

$$\sigma^{RT} = \sigma^{R} + \sigma^{E} \quad . \tag{4}$$

The steel reinforcement is stressed only in one direction. The material is represented by a bilinear model that may either be elastic, perfectly plastic or strain-hardening, as shown in Fig. 2.

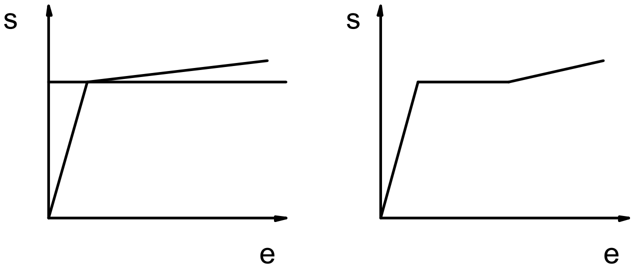

Fig. 2. Stress-strain model for steel.

2. SELF-STRESSES IN CONCRETE ELEMENT.

The self-stresses were calculated for bending concrete beam with rectangular cross-section - Fig. 3.

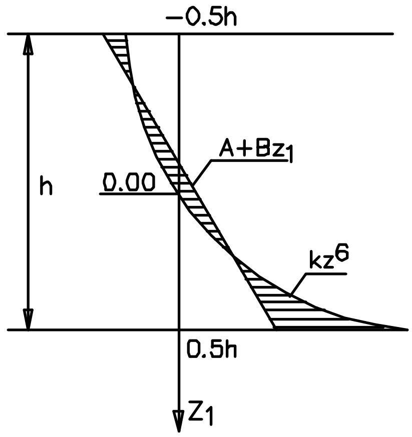

Fig. 3. Self-stresses in concrete beam.

The course of residual strain was assumed as known and described as follows:

$$\varepsilon^{R} = \begin{cases} -kz^{6} & \text{for } z<0 \\ kz^{6} & \text{for } z>0 \end{cases} \quad , \tag{5}$$

where: $z = s/f_c$ for $s<0$,
$z = s/f_{ct}$ for $s>0$.

Using the equilibrium of forces and moments (Eq. 2) the constants A and B were calculated:

$$\int \sigma^{R}\, dF = \int \left[A + Bz_1 - \varepsilon^{R}(z_1)\right] b\; dz_1 = 0 \quad ,$$

$$\int \sigma^{R} z_1\, dF = \int \left[A + Bz_1 - \varepsilon^{R}(z_1)\right] b z_1\, dz_1 = 0 \quad , \tag{6}$$

$$A = 1/F_b \int \varepsilon^{R}(z_1) b\, dz_1 \;\; ; F_b = bh \quad ,$$

$$B = 1/I_b \int \varepsilon^{R}(z_1) z_1 b\, dz_1 \;\; ; I_b = bh^3/12 \quad . \tag{7}$$

Some calculations for different concrete were done. The results of calculations of self-stresses, elastic stresses and total stresses are shown on Fig. 4.

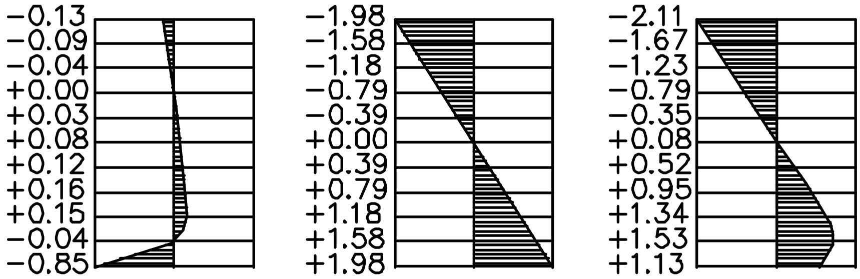

Fig. 4. Calculation results of the self-stresses, elastic stresses and total stresses in concrete element (concrete B-15 - f_c=15.0 Mpa, f_{ct}=1.40 Mpa, E_c=23.1*10^3MPa).

3. SELF-STRESSES IN REINFORCEMENT CRACKED CONCRETE ELEMENT.

The self-stresses in reinforcement concrete bending beam were calculated in the same way as shown above for concrete element - Fig. 5.

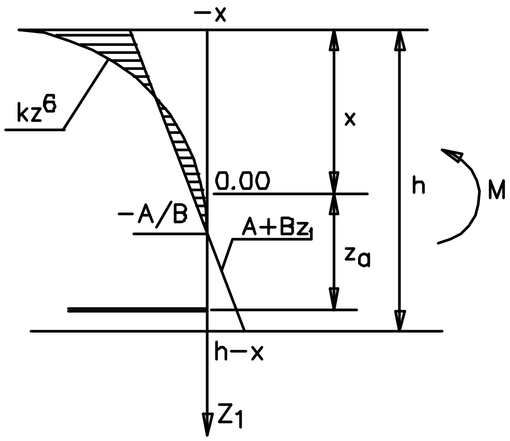

Fig. 5. Self-stresses in reinforced concrete beam with crack.

The residual strain-stress relationship described the same equals (Eq. 4) like for concrete element. Using condition (2) the constants A and B were calculated as follows:

$$E_b \int (A + Bz_1 - \varepsilon^R(z_1))\,\mathrm{dF} + \mathrm{E_a} F_a (A + Bz_a - \varepsilon_a^R) = 0 \quad ,$$

$$E_b \int (A + Bz_1 - \varepsilon^R(z_1))\,\mathrm{z_1 b\, dz_1} + \mathrm{E_a} F_a (A + Bz_a - \varepsilon_a^R) z_a = 0 \quad , \tag{8}$$

where: e_a^R - residual strain for steel.

Some numerical calculations for different causes were done. The results of this calculation are demonstrated on Fig. 6.

4. NON-LINEAR FINITE ELEMENTS METHOD ANALYSIS

The Finite Elements Method was used to calculate the non-linear effects in reinforced concrete elements. The rectangular elements were used with stiffness matrix calculated by Rockey [4]. The first steep is taken as the linear solution using known relationship of FEM. [5]:

$$[K]\{d\}-\{R\}=0 \quad , \tag{9}$$

where: [K] - stiffness matrix,

{d} - displacement,

{R} - external forces.

In Eq. 9 the linear behaviour of materials was assumed:

$$\{s\}=[D](\{e\}-\{eo\})+\{so\} \quad , \tag{10}$$

$$F(\{s\},\{e\})=0 \quad . \tag{11}$$

The non-linear effects were calculated using iteration and changes of the external forces {R}. The external forces were calculated on the basis of initial strain and initial stress that describes the cracks or non-linear material behaviour. This method needs no necessity of changes of the stiffness matrix. The reinforcement was described as linear elements that are added to the stiffness matrix. Before the cracks appear the concrete strain and the steel strain is equal. After cracking the steel elements take over the stresses from the concrete. These stresses are added to the external forces as initial stresses.

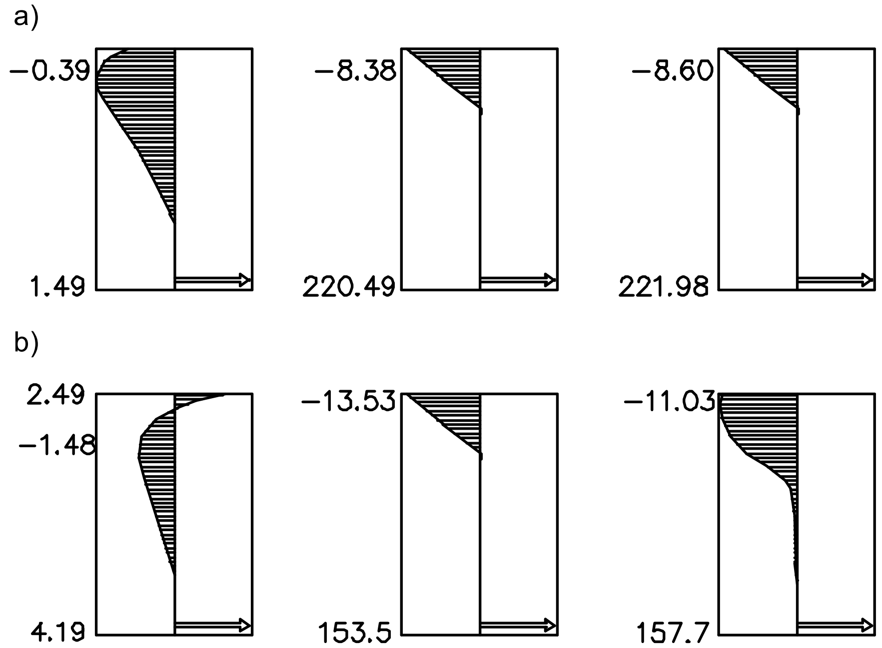

Fig. 6. Stresses in cracked concrete element,
concrete B-15 (see Fig. 4), steel f_y = 220 MPa.
a) μ = 0.5 %, M = 29 kNm, b) μ = 2 %, M = 75 kNm

5. ANALYSIS OF RC DISK TESTED BY LEONHARDT AND WALTHER [7]

The purpose method was examined on calculations of the RC panel (Fig. 7) tested by Leonhardt and Walther [6]. The same panel was calculated using FEM by Floegl [7], Buyukozturk [8], Lewiñski [9] and using BEM by Minch [10]. So, there is the material to compare the results of analysis.

The results of numerical calculations are shown on Fig. 8-11. Figure 8 shows the propagation of cracks under different loading. The first crack was observed for P=400 kN. In each level of the loading the number of cracks and the width of the cracks vary as shows the figure.

In Fig. 9 the relationship between loading and displacement of the panel as compared with other finding. The good compatibility was notice.

Figure 10 and Fig. 11 show the comparison of stresses in steel bars calculated by authors with experimental findings and other calculations.

4. CONCLUSIONS.

The mathematical idealization of cracked reinforced concrete structure is very difficult. The presented model of non-linear behaviour of reinforced concrete can be used to numerical analysis with the finite element method. It may give relatively quick solution because of no necessity to change the stiffness matrix and solve equations several times. Description of all non-linear behaviour of materials, cracks, self-stresses treated as initial strain's gives only different right sides of standard equals. Therefore the iteration of cracks and loads do not change the stiffness matrix. All that may preference the finite element method to numerical analysis.

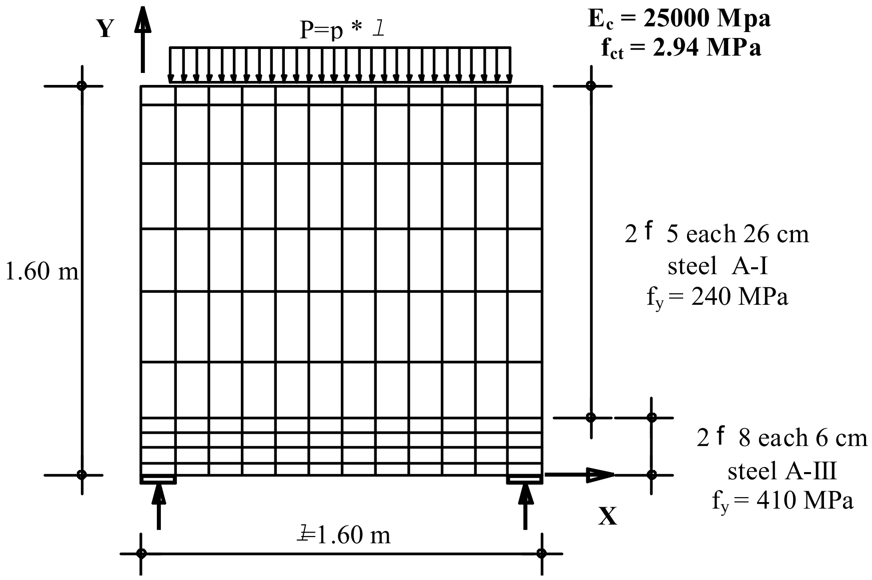

Fig. 7. The schema of calculated RC panel

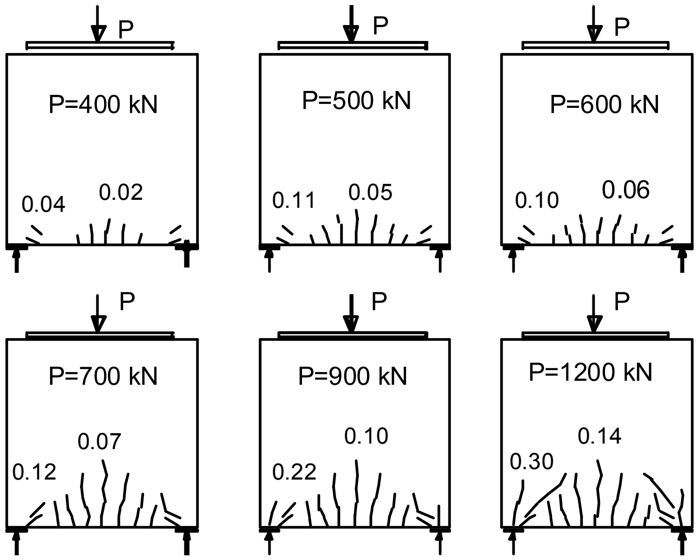

Fig. 8. The propagation of the cracks under different level of loading

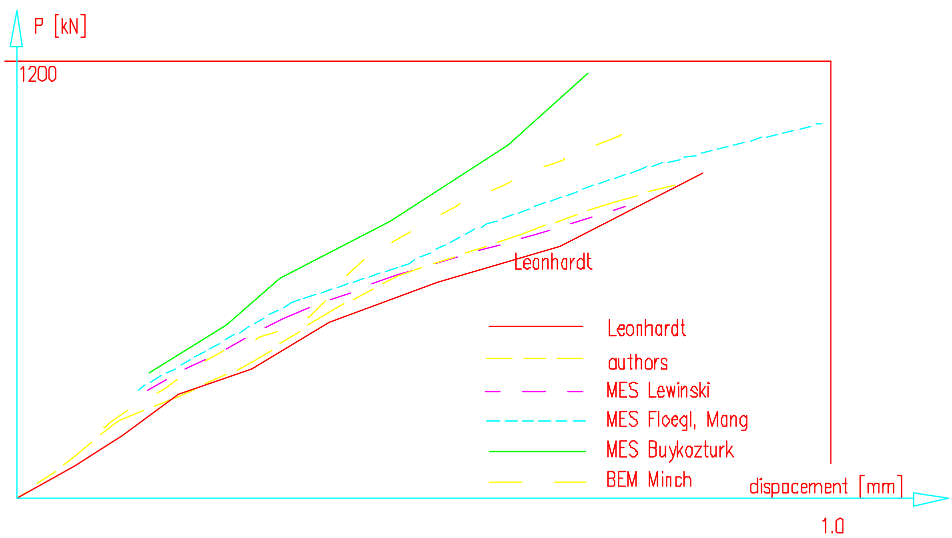

Fig. 9. Comparison of calculated load-midspan deflection relations.

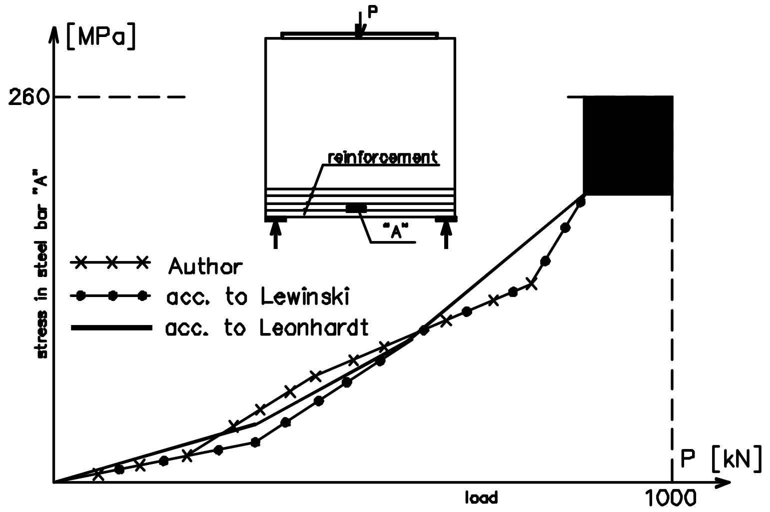

Fig. 10. Comparison of the stresses in reinforcement of the panel with other findings.

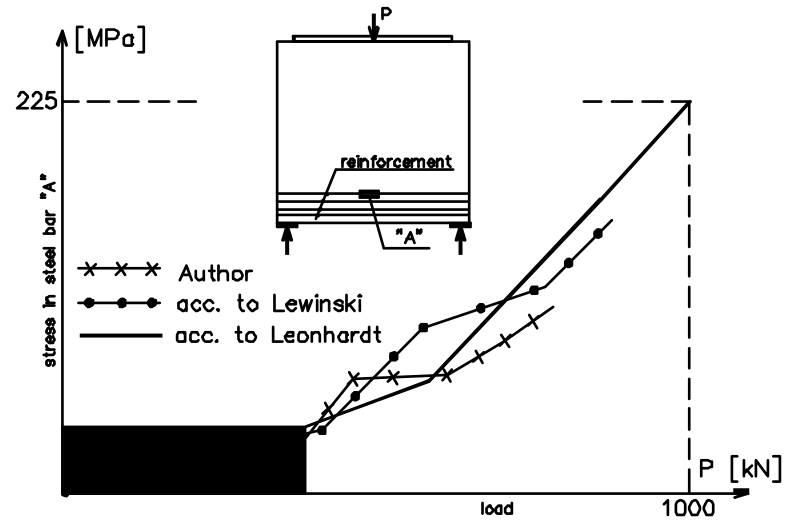

Fig. 11. Comparison of the stresses in reinforcement of the panel with other findings.

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