Multimodel Numerical Analysis of the Elasto-Visco-Plastic Deformation of Materials and Constructions

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1. Introduction

The increasing demands for reliability and durability of structures with simultaneous material economy have stimulated improvement of constitutive equations for description of inelastic deformation processes. This has led to the development of phenomenological modeling of complex phenomena of irreversible deformation including history-dependent and rate-dependent effects. During the last several decades many works have been devoted to the development of plastic and viscoplastic models, in order to better predict the material behavior under combined variable thermomechanical loading. There are more than the ten theories of viscoplasticity and their modifications. Among the possible explanations of observed diversity of models can be pointed out on the two reasons of fundamental and applied character. The first cause is connected with variety of viscoplastic properties manifestation, reflecting diversity of physical micromechanisms of inelastic deformation. The second reason is brought about variety of requirements in complexity and accuracy for real engineering applications.

The results of various viscoplastic models predictions often are considerably differed from the experimental data in some cases of the non-proportional loading. Therefore the problem of choice of the more suitable theory of plasticity is actual for the strength analysis of structures under combined loading. The suggested multimodel approach, which is based on the creation of hierarchical consistency of the models, whose fields of reliable application partially coinciding mutually supplement each other, is most rational. Application of the concrete model must correspond to the complexity of being considered loading processes.

The developed strategy of multimodel analysis consists of the following features:

- creation of the ***plastic and viscoplastic models library***, providing solution of the wide spectrum of non-elastic problems,
- determination of the ***selection criteria system***, realizing the choice of the simplest variant of theory sufficient for correct problem solution,
- caring out of ***multivariant sequential clarifying computations***.

Computation with using of several different theories should be performed in the most responsible cases of strength analysis. Coincidence of the results according different models demonstrates the correctness of computations. Difference between obtaining results for structure demands determination of the most adequate model. This choice can be carried out on the base of comparison between numerical results and experimental data for more simple geometrical object - element of material, subjected to the same history of loading as the most stressed point of structure.

The basic ideas and application of multimodel analysis for the rate-independent material behavior was described in previous work of authors [1-3]. Present analysis is devoted to rate-dependent deformation.

2. Library of plastic and viscoplastic models

The developed library of material models represents generalized data set, including information about limitations on field application, basic experiments for material parameters determination, continuous mathematical model, discrete numerical model, computational algorithm, implementation into finite element program, recommendations about computation strategy.

The set of following criteria has been taken into consideration:

- conformity to a general principles of physics (thermodynamic laws, principle of determinism, principle of fading memory, principle of local action, tensorness of all relations and others),
- experimental verification of models for various classes of loading,
- possibility of micromechanical interpretation,
- complexity of determination of the material parameters,
- algorithmic effectiveness.

2.1 Plastic models

At the present time the developed and implemented into finite element program library of rate-independent (plastic) models includes:

- Plastic flow theories with the various isotropic-kinematic laws of hardening [4]-[8]. The relations of this "classical" models belong to the first-order linear tensorial equations convenient for computations.
- Structural (rheologic) models theories [9-10]. They possess a clarity of properties, thermodynamic basis, obvious creation and improvement.
- Multisurface theory with one active surface of plastic compliance [11-12]. The model provides high accuracy of the description for the complex paths of passive loading.
- Endochronic theory of plasticity [13-15]. The equations can be applied for a wide class of materials from a metal to a soil. This theory does not use the existence of a yield surface and employs the same equation for the loading and unloading processes.

2.2 Viscoplastic models

The our library of the rate-dependent (viscoplastic) models includes:

- Technical theories of creep
  (ageing theory [16], flow theory [16], hardening theory [16]).
  These models are convenient for the primary express analysis and are applicable for weakly variable loading. They are simplest models with least set of the necessary experimental data.
- Elastic/viscoplastic models
  (Perzyna [17], Chaboche [8,18], Robinson [19], Bodner-Partom [20], Miller [21], Krempl [22], Hart [23], Gilman [24], Gilat [25] models at el.).
  There are most popular in computations class of models. They demonstrate the viscous effects only after exceedition of static yield limit.
- Viscoelastic/viscoplastic models
  (Naghdi-Murch theory [26]).
  They demonstrate the viscous effects always as before as after exceedition of yield limit.
- Elastoviscoplastic models
  (Endochronic theory [13,15,27], Nonlinear heredity theory [28,29]).
  These models don't possess pronounced yield limit and demonstrate simultaneously elastic, viscous andplastic properties. They represent extension of viscoelasticity.
- Structural (rheological, fraction, sublayer) models
  (Bingham [30], Shvedov [30], Besseling [31], Gokhfeld-Sadakov [32], Palmov [10], Ivlev [33], Partom-Keren-Gennusov [34] et al.).
  They allow to create and easy to modify models with wide spectrum of elastic, viscous and plastic properties in clarified and obvious way.

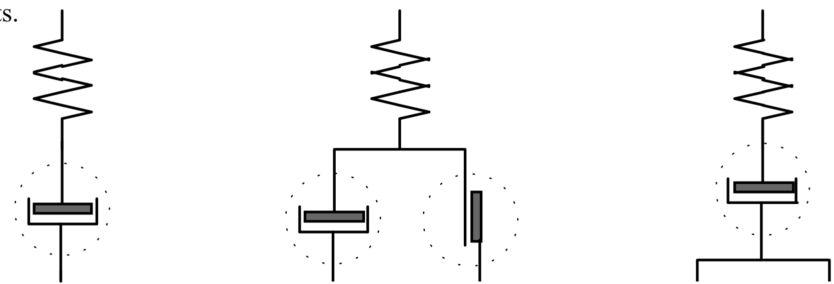
The detailed reviews of the inelastic constitutive equations have been presented in [35-38].

2.3 Regulation criteria of viscoplastic models

Viscoplastic deformation of materials demonstrates wide range of nonlinear time-dependent effects such as primary and secondary creep, relaxation, rate-dependence of the stress-strain relations, ageing, recovery, restoration, creep delay after partial unloading, cyclic softening and hardening, ratcheting, additional hardening in non-proportional loading, cross hardening, coupling of plasticity and viscoplasticity and et al. Diversity of mentioned effects generates the various viscoplastic models with different underlying conceptions. For the regulation of viscoplastic models can be considered the following criteria and corresponding selected groups of models:

1. Restrictiveness of viscous properties manifestation:
   - elastic/viscoplastic models,
   - viscoelastic/viscoplastic models.

In the first case the viscous effects manifest theyself only in combination with plastic properties after exceeding of yield limit. In the second case the viscous effects are observed as in elastic as in plastic range. It permits to describe more wide phenomena class, but leads to the considerable complication of mathematical description and computation. Schematic representation of the difference between these approaches is illustrated in Fig. 1 by means of rheological models. Elastic/viscoplastic models correspond to the cases b) and d) in Fig. 1, when viscous element is interlocked by parallel-connected plastic element .Viscoelastic/viscoplastic models are shown in Fig. 1, a) and c). Use of the generalized plastic and viscous elements (marked in Fig. 1 by dashed circles) allow to extend capability of considered models and to display purged peculiarity of strain decomposition. Generalized plastic (viscous) element is arbitrary combinations of arbitrary numbers of perfect plastic (linear viscous) and elastic elements.



a) b) c)

Fig. 1. Distinctive viscoplastic rheological models with using generalized elastic, viscous and plastic elements.

2. Plasticity-viscoplasticity interaction

- uncoupled models, governed by equation $\varepsilon = \varepsilon^e + \varepsilon^v + \varepsilon^p$ (Fig. 1, a, c),
- unified models, governed by equation $\varepsilon = \varepsilon^e + \varepsilon^{vp}$ (Fig. 1, b).

The first conception is founded on the existence of different time scale corresponding to different processes. Possibility of such decomposition is defined by variability and duration of loading process, temperature level.

3. Yield conditions

- static yield conditions,
- dynamic yield conditions,
- absent of yield condition.

Description of rate influence on material behavior and loading-unloading conditions can be formulated on the basis of quasistatic or dynamic stress-strain curve. This generates different approaches in viscoplasticity.

4. Hardening laws

- perfect viscoplasticity law,
- isotropic hardening,
- kinematic hardening (linear and nonlinear rules),
- anisotropic hardening.

Description of the complex history of combined loading demands complication of the hardening law.

2.4 Internal state variables approach

The uniform representation of constitutive equations is actual for the creation of inelastic models library with the purpose to simplify program realization and to perform comparative analysis. The thermodynamic approach with internal state variables provides a powerful tool for representing of the constitutive equations of plasticity and viscoplasticity. All considered here models of inelastic material can be written in common quite general mathematical form.

The inelastic strain rate tensor $\dot{\varepsilon}^{vp}$ is assumed as function of the stress tensor σ, a set suitably defined internal state variables $\chi^{(k)}$, k=1,...,n and temperature T and can be defined in form

$$\dot{\varepsilon}^{vp} = \dot{p}\,\mathbf{a}(\sigma, \chi^{(k)}, T) \tag{1}$$

In the most cases tensor $\mathbf{a}$ is defined as gradient of dissipation potential and p is determined from consistency plastic condition for rate-independent behavior or from uniaxial creep-relaxation experiments for rate-sensitivity behavior. Determination of these values in the endochronic and multisurface theory with one active surface is based on another concepts. The internal state variables $\chi^{(k)}$ can be either second-order tensors or scalars. Evolution laws for these internal variables can be represented in the form:

$$\dot{\chi}^{(k)} = \dot{p}\,\mathbf{b}(\sigma, \chi^{(l)}, T) \tag{2}$$

All members of plastic and viscoplastic models library fit into the frame (1)-(2).

3. Selection criteria system

The determination of the selection criteria system, based on classification of inelastic theories and their domain of advantageous applicability, is one of the main problem in multimodel analysis. Selection criteria system (see Fig. 2) generates necessary conditions for material model on the basis of information concerning external actions, available experimental data, discrete model of structure. The choice of rational model, which is the simplest among models satisfied necessary conditions, may be corrected by clarifying sequential computational experiments.

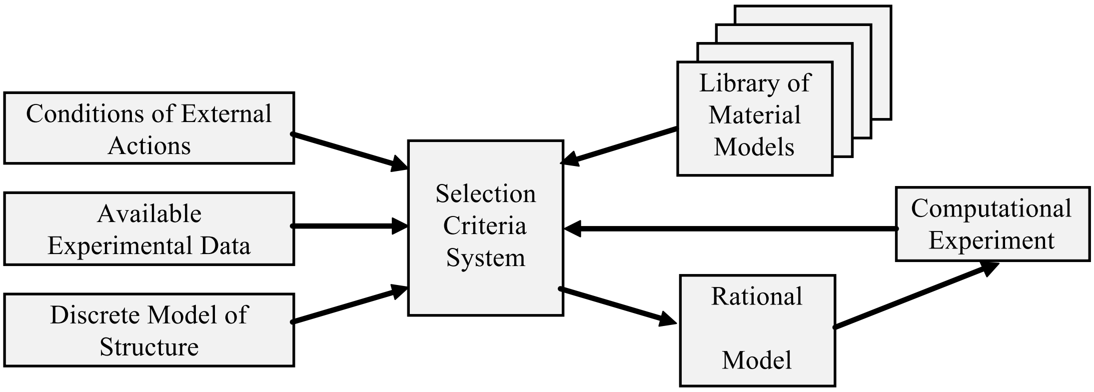


Fig. 2. Selection criteria system in multimodel analysis.

Necessity to take into consideration time-dependent effects is defined by the parameters of external actions such as temperature level, duration and rate loading and specific material properties. The viscoplastic analysis succeeds the criteria of plastic analysis [3] and adds some new. Some common viscoplastic criteria has been considered in section 2.3.

The classical examples of plastic criteria are degree of plastic strains development in comparison with elastic strain and proportionality of loading path. The first criterion determines choice between the Prandtl-Reuss and Levy-Mises theories of plastic flow, when the possibility to neglect elastic strain arises. The second criterion defines the conditions of possibility of application of the Hencky-Ilyushin or Prandtl-Reuss theory. The another selection criterion suggested by Ilyushin in [39] are founded on the path curvature of deformation process. Numerous stress state analysis of elasto-plastic behaviour of structures of different degree of complexity allows to formulate a new selection criterion. Suggested criterion is based on consideration of geometrical regulated levels of plastic deformation analysis. Similarly as in [40] we introduce the following levels:

- Body level **B** considers the body or complex structure as a whole. "Integral" analysis corresponds initial strength problem. In most cases zones of plasticity are local.
- Element level **E** is introduced for separate parts of the structure as fragment of structure, substructure, superelement or individual finite element. "Semi-integral" analysis is carried out in this case. In most of cases zones of plasticity are extensive.
- Point level **P** is the basic level, related to selected points of material continuum or to a model of structure. "Local" analysis is carried out for simplest geometrical object - element of material with homogeneous stress state. The whole object is a zone of plasticity.

Finally, the criterion can be formulated by following manner. Complexity of applying theory of plasticity must correspond to the level of the structure approximation. The levels **P** and **E** with more detail description of the structure geometry and with possibility of extensive zones of plasticity demand the more difficult variants of theory adequate to the loading process. Using of the simple models is sufficiently at the **B** level of the investigation, when the deformation of local zones of plasticity is smoothed by influence of extensive elastic region.

4. Results of multimodel computational analysis

Comparison of the results of numerical finite element analysis and experimental data for series investigated constructions corresponding to the first level **B** (frames, pipelines, vapor producing plant, gas generator, vessel of nuclear reactor) says about relative nearness of different theories predictions. However series of computations corresponding to the second level **E** of the structures considerations (fragment of rolling mill, fastenings of vapor producing plant, various fastening knots, socket, circular ring) have shown that the considerable differences of the prediction of stress-strain state by means of different theories of plasticity were displayed for a developed zone of plasticity and complex history of loading. Set of trials according level **P** carried out on tubular specimens of X18H10T steel under a wide range of the combined cyclic loading, including polygonal and circular paths of deformation. In general the results different theories corresponding to the level **P** can be essentially quality differed. Typical example of multimodel computations corresponding to the level **E** for thin circular ring being the part of more complex structure is shown in Fig. 3.

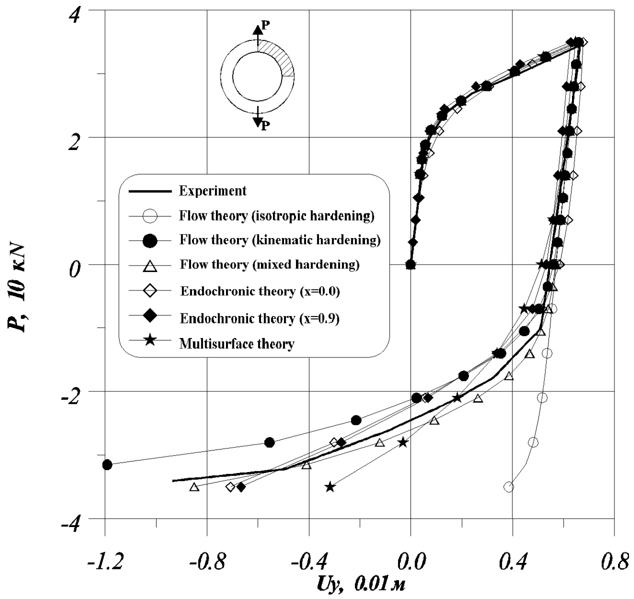


Fig. 3. Different models predictions for circular ring under axial tension-compression.

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