# Analysis, Design \& Applications of <br> Cryptographic Building Blocks 

Inauguraldissertation<br>zur Erlangung des akademischen Grades<br>doctor rerum naturalium (Dr. rer. nat.)

der Bauhaus-Universität Weimar
an der Fakultät Medien

vorgelegt von<br>Christian Forler

September 2014

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## Abstract

This thesis deals with the basic design and rigorous analysis of cryptographic schemes and primitives, especially of authenticated encryption schemes, hash functions, and password-hashing schemes.

In the last decade, security issues such as the PS3 jailbreak demonstrate that common security notions are rather restrictive, and it seems that they do not model the real world adequately. As a result, in the first part of this work, we introduce a less restrictive security model that is closer to reality. In this model it turned out that existing (on-line) authenticated encryption schemes cannot longer be considered secure, i.e., they can guarantee neither data privacy nor data integrity. Therefore, we present two novel authenticated encryption schemes, namely COFFE and McOE, which are not only secure in the standard model but also reasonably secure in our generalized security model, i.e., both preserve full data inegrity. In addition, McOE preserves a resonable level of data privacy.

The second part of this thesis starts with proposing the hash function TWISTER ${ }_{\pi}$, a revised version of the accepted SHA-3 candidate TWISTER. We not only fixed all known security issues of TwISTER, but also increased the overall soundness of our hash-function design.

Furthermore, we present some fundamental groundwork in the area of passwordhashing schemes. This research was mainly inspired by the medial omnipresence of password-leakage incidences. We show that the password-hashing scheme scrypt is vulnerable against cache-timing attacks due to the existence of a password-dependent memory-access pattern. Finally, we introduce Catena the first password-hashing scheme that is both memory-consuming and resistant against cache-timing attacks.

## Zusammenfassung

Diese Dissertation widmet sich dem Design und der Analyse von kryptographischen Primitiven und deren korrekte Anwendung. Im Mittelpunkt dieser Arbeit stehen daher das Design von beweisbar sichere Verfahren zur authentisierten Verschlüsselung, kryptographische Hashfunktionen sowie Passwort-Hashing-Algorithmen.

In der Vergangenheit haben Sicherheitslücken wie beispielsweise der PS3-Jailbreak gezeigt, dass die gängigen Sicherheitsmodelle zu restriktiv sind und daher die Praxis eher unangemessen widerspiegeln. Der erste Hauptteil dieser Arbeit beschäftigt sich daher mit der Einführung eines realitätsnäheren Sicherheitsmodelles unter dem praktisch alle bestehenden On-line-Verfahren zur authentisierten Verschlüsselung als unsicher anzusehen sind. Weder Vertraulichkeit noch Integrität der zu schützenden Daten können noch sichergestellt werden. Aus diesem Grund stellen wir in dieser Arbeit zwei neue On-line-Verfahren zur authentisierten Verschlüsselung vor. Bei dem ersten handelt es sich um COFFE. Dieser Betriebsmodus für Hashfunktionen schützt selbst in unserem realitätsnäheren Sicherheitsmodelles noch die Integrität der verarbeiteten Daten. Das zweite ist McOE. Es ist ein äusserst robustes Verfahren - basierend auf einer Blockchiffre - welches auch in dem neuen Modell vollständige Integrität und angemessene Vertraulichkeit von den verarbeiteten Daten sicherstellt.

Im zweiten Hauptteil dieser Arbeit wird als Erstes die kryptographische Hashfunktion $\mathrm{Twister}_{\pi}$ vorgestellt. Hierbei handelt es sich um eine überarbeitete Version von Twister (akzeptierter SHA-3-Kandidat). Zum einem behebt Twister $\pi_{\pi}$ alle bekannten Sicherheitsprobleme der ursprünglichen Version, zum anderen bietet es im Vergleich noch signifikant höhere Performance.

Weiterhin werden neue Anforderungen für Sicherheit und Funktionalität an Passwort-Hashing-Verfahren vorgestellt. Im Zuge dessen wurde ein akademischer Cache-TimingAngriff auf das derzeit führende Passwort-Hashing-Verfahren scrypt entwickelt, mit dessen Hilfe sich ein sehr effizienten Filter für Passwortkandidaten konstruieren lässt.

Letzlich stellen wir in dieser Arbeit noch Catena vor. Hierbei handelt es sich um das erste beweisbar sichere Passwort-Hashing-Verfahren welches sowohl speicherintensiv als auch sicher gegen Cache-Timing-Angriffe ist.

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## List of Abbreviations

| AEAD | Authenticated Encryption with Associated Data |
| :--- | :--- |
| AES | Advanced Encryption Standard |
| BTM | Bivariate Tag Mixing |
| CAESAR | Competition for Authenticated Encryption: Security, Applicability, <br> and Robustness |
| CBC | Cipher Block Chaining |
| CCA3 | Chosen-Ciphertext Attack 3 |
| CCFB | Counter-CipherFeedback |
| CCM | Counter with CBC-MAC |
| CFB | Cipher Feedback |
| CHM | CENC with Hash-based MAC |
| COFFE | Ciphertext Output Feedback Faithful Encryption |
| COTS | Commercial Off-The-Shelf |
| CTR | Counter |
| CTS | Ciphertext Stealing |


| CWC | Carter-Wegman Counter |
| :---: | :---: |
| cpb | Cycles per Byte |
| DAE | Deterministic Authenticated Encryption |
| DAG | Directed Acyclic Graph |
| DBG | Double Butterfly Graph |
| DBH | Double Butterfly Hashing |
| DDT | Difference-Distribution Table |
| DES | Data Encryption Standard |
| ECB | Electronic Codebook |
| EME | Encrypt-Mix-Encrypt |
| EtM | Encrypt-then-Mac |
| FFT | Fast Fourier Transform |
| GCM | Galois/Counter Mode |
| GPU | Graphical Processing Unit |
| HBS | Hash Block Stealing |
| HMAC | Hash-Based Message Authentication Code |
| IACBC | Integrity Aware Cipher Block Chaining Mode |
| IAPM | Integrity Aware Parallelizable Mode |
| IND-CCA | Indistinguishability under Chosen-Ciphertext Attack |
| IND-CPA | Indistinguishability under Chosen-Plaintext Attack |
| IND-OCCA | Indistinguishability under On-Line Chosen-Ciphertext Attack |
| IND-OCCA2 | Indistinguishability under On-Line Chosen-Ciphertext Attack 2 |
| IND-OPRP | Indistinguishability from an On-line Pseudorandom Permutation |


| IND-PRP | Indistinguishability from a Pseudorandom Permutation |
| :---: | :---: |
| INT-CTXT | Integrity of Ciphertext |
| IoT | Internet of Things |
| KDF | Key Derivation Function |
| LCP | Longest Common Prefix |
| LSB | Least Significant Bit |
| MAC | Message Authentication Code |
| MDS | Maximum Distance Separable |
| MMO | Matyas-Meyer-Oseas |
| MSB | Most Significant Bit |
| NDMA | Nonce- and Decryption-Misuse Attack |
| NIST | National Institute of Standards and Technology |
| OAE | On-line Authenticated Encryption |
| OCB | Offset Codebook |
| OCCA3 | On-line Chosen-Ciphertext Attack 3 |
| OFB | Output Feedback |
| ONDMA | On-line Nonce- and Decryption-Misuse Attack |
| OPerm | On-line Permutation |
| OPRP | On-line Pseudorandom Permutation |
| PBKDF2 | Password-Based Key Derivation Function 2 |
| PHC | Password Hashing Competition |
| PIN | Personal Identification Number |
| PRF | Pseudorandom Function |


| PRF-RKA | Pseudorandom Function under Related-Key Attacks |
| :--- | :--- |
| PRP | Pseudorandom Permutation |
| PRP-RKA | Pseudorandom Permutation under Related-Key Attacks |
| PRNG | Pseudorandom Number Generator |
| RPC | Related Plaintext Chaining |
| SIV | Synthetic Initialization Vector |
| TAE | Tweakable Authenticated Encryption |
| TDOWF | Trap-Door One-Way Function |
| TLS | Transport Layer Security |
| TS | Tag Splitting |
| TMTO | Time-Memory Tradeoff |
| WPA | Wi-Fi Protected Access |
| XCBC-XOR | eXtended Ciphertext Block Chaining with XOR |
| XEX | XOR-Encrypt-XOR |

## List of Mathematical Symbols

| $X \in\{0,1\}^{n}$ | $X$ is a bit string of $n$ bits |
| :--- | :--- |
| $X \in\{0,1\}^{*}$ | $X$ is a bit string of abitrary length |
| $X \in\{0,1\}^{+}$ | $X$ is a bit string of at least one bit |
| $X \in\left(\{0,1\}^{n}\right)^{m}$ | $X$ is a bit string of $n \cdot m$ bits |
| $X \in\left(\{0,1\}^{n}\right)^{+}$ | $X$ is a bit string of $k \cdot n$ bits with $k \geq 1$ |
| $X \underset{\leftarrow}{\leftarrow}\{0,1\}^{k}$ | $X$ is selected uniformly at random from the set $\{0,1\}^{k}$ |
| $A \\| B$ | Concatenation of two bit strings A and B |
| $\|A\|$ | Bit length of variable $A$ |
| $\bar{A}$ | Bitwise complement of the bit string $A$ |
| $A \oplus B$ | Modular addition of $A$ and $B$ |
| $A \oplus B$ | Exclusive-or $($ XOR $)$ of the first $\alpha$ MSB $;$ of $A$ and $B$ with |
|  | $\alpha=\min \{\|A\|,\|B\|\}$ |
| $0^{n}$ | String of $n$ zero bits |
| $0^{*}$ | String of abitrary number of Zero bits (zero padding) |

## Introduction

No amount of experimentation can ever prove me right; a single experiment can prove me wrong.

Albert Einstein

This thesis is dedicated to provable security wich is an essential part of modern cryptography. It deals with the formalization and the rigorous analysis of cryptographic schemes with the goal to turn an ancient art into a science.

Foundations of Provable Security. Provable security arised in the late 1970s and was shaped in the 1980s. In 1976, Diffie and Hellman pioneered the public-key cryptography when they presented a key-exchange protocol based on the foundations of abstract mathematics [79]. This enabled them to justify their security claims by mathematical arguments. Furthermore, they introduced the concept of a Trap-Door One-Way Function (TDOWF), a function which can only be inverted when knowing auxiliary (trap-door) information, i.e., the secret key. However, without giving an example. Two years later, in 1978, Rivest et al. introduced the first instance of a TDOWF; the famous RSA encryption scheme [202].

In 1982, Goldwasser and Micali showed that the concept of a TDOWF is not sound since it allows to leak certain information about the encrypted plaintext such as its parity [116]. Therefore, they introduced a superior security notion, probabilistic encryption 1 . This revolutionary work paved the way for the theory of provable security.

[^0]In 2012, the importance of their work was dignified by The Association for Computing Machinery Advancing Computing as a Science \& Profession with the Turing Award.

Polynomial Security. The security notions introduced by Goldwasser and Micali in [116] have their roots in complexity theory and the intractability of well-known and hard mathematical problems, e.g., the discrete logarithm problem or the integer factorization problem. This allows cryptographers to apply polynomial-time reductions to proof the security of an encryption scheme. However, such a reduction does not make any statements about the choice of the security parameter, e.g., the key length. Practitioner have to guess reasonable values. Therefore, polynomial security results are rather unsatisfactory for practical applications, but very crucial for the field of information-theoretic cryptography.

Concrete Security. In 1993, Bellare and Rogaway introduced the random oracle model [24] where a cryptographic primitive is modeled as a random oracel, i.e., a black box that always returns a random value that does not depend on its input. An adversary, allowing to ask queries to such an oracle, is modeled as a computationally unbounded algorithm which is only limited by the number of oracle queries. For the first time, this approach allowed to observe, in some very limited way, the behavior of an adversary during the attack by recording the queries to the oracle. Moreover, it allowed to upper bound the concrete success probability of adversaries in breaking encryption schemes.

Security proofs in the random oracle model are controversial. In 1998, Canetti et al. presented a separation result [59]: "There exist signature and encryption schemes which are secure in the Random Oracle Model, but for which ANY implementation of the random oracle results in insecure schemes". Nevertheless, concrete security bounds derived from security proofs in this model are meaningful since they enable practitioners to apply reasonable security parameters.

A much less controversial approach is the standard model where a cryptographic primitive, e.g., a hash function or a block cipher, is replaced by a (keyed) pseudorandom counterpart ,e.g., Pseudorandom Function (PRF) 114] or Pseudorandom Permutation (PRP) [20], instead of an ideal one. An adversary $\mathcal{A}$ is modeled as a (timeand) computationally bounded algorithm since an unbounded algorithm can apply an exhaustive search on the key space to reveal the secret key. Like in the random oracle model, $\mathcal{A}$ has black-box access to the cryptographic scheme $\Pi$. In the standard model, the concrete security of a scheme against an adversary $\mathcal{A}$ is determined by the success probability of $\mathcal{A}$ in breaking $\Pi$. A scheme is considered to be secure if the
maximum success probability over all adversaries that ask at most $q$ oracle queries is negligible. In 1994, Bellare et al. introduced concrete security in the standard model by presenting a security notion for Message Authentication Codes (MACk) [20]. In the following years, Bellare et al. introduced several standard-model security notions for all common cryptographic schemes such as digital signatures [26], symmetric encryption [18], and authenticated encryption schemes 21]. Starting from the last decade, it is costume to introduce a novel cryptographic scheme along with a concrete security claim supported by a security proof given either in the standard or in the stronger random oracle model.

Misuse Resistance. A provably secure cryptographic scheme provides rigorous security properties, e.g., integrity and confidentiality, only under well-defined assumptions against well-defined adversaries. Hence, the term secure is a placeholder for to protect something against a well-defined class of adversaries. In contrast to cryptographers, who exactly know what is meant by referring a cryptographic algorithm to be secure, regular users implicitly assume that a secure scheme matches any of their security requirements; without further investigation. This common misconception causes serious security issues and is typical for the human nature. Thomas Gray pointed this out by the felicitous idiom: "Where ignorance is bliss, 'tis folly to be wise ${ }^{2}$. Hence, re-education of all users is an enormous hard task for the cryptographic community. This virtuality leads to the conclusion that cryptographers should take responsiblility and should design their algorithms in a way that the naive usage of their algorithms should not end up into big security disasters.
But, this is easier said than done. On the one hand, it is not the task of a digital signature scheme to provide any kind of data privacy. On the other hand, a digital signature scheme should not reveal the secret key when one of its security assumption is violated once such as the ECDSA signature scheme [128]. It is a good starting point to design robust algorithms that still offer some decent level of security, even when a security assumption is occasionally violated. In this thesis we introduce the first authenticated encryption scheme that provides full security under standard assumptions, and still a reasonable level of security under much weaker assumptions. Therefore, such an algorithm provides a second line of defense in a misuse scenario, e.g., faulty random number generator.

[^1]
## Outline

In the first part of this work we introduce the concepts, security notions, and definitions needed to grasp the latter parts. Nevertheless, the experienced reader can directly start with the second part of this thesis.

The essential elements of this work, namely Part III and Part III, are partitioned as follows:

Part II: First, in Section 5 we introduce the concept of robust authenticated encryption schemes. Then we show in Section 6 that published authenticated encryption schemes are not robust, so far. In Section 7 we present COFFE, a partially robust On-line Authenticated Encryption (OAE) scheme. Finally, in Section $\square$ we introduce McOE, the first robust OAE scheme. Note that a preliminary version of MCOE was published before in [98, 100], and has been thoroughly revised.

Part III: First, in Section 9 we present Twister ${ }_{\pi}$, a family of cryptographic hash functions. It is a rigorously improved revision of the accepted SHA-3 candidate Twister [93]. Furthermore, in Section 10 we introduce Catena, a novel memory-consuming password scrambling framework that is based on a cryptographic hash function. Note that a preliminary version of Catena was published before in [103] and the extended abstract will be appear at ASIACRYPT'14 [104].

Further notable results of my studies that are not mentioned in this work can be found in [1-4, 92, 94, 95, 97, 99, 101, 102]. A complete list of my publications, so far, is given in Section 12.

## Part I

## Foundations

## Hash Functions

Nothing in life is to be feared, it is only to be understood. Now is the time to understand more, so that we may fear less.

## Marie Curie

The concept of hash functions was introduced in the early 1950s [142], and became vital in the field of modern cryptography. Informally, a hash function compresses an input of arbitrary length to a fixed-length output which is usually referred to as a hash value or message digest. Today, hash functions have many applications and are virtually used everywhere in, e.g., encryption schemes [25], digital signature schemes [26, 108], key derivation schemes [131, 191], key exchange protocols [109], and MAC; [17, 240]. Due to the wide range of use-cases, a good hash function should be both memory- and time-efficient to be applicable on restricted devices, e.g., wireless sensor nodes [244], trusted computing modules [228], and smart meters [172]. In this thesis we borrow the notion of unkeyed hash function that was presented by Rogaway in [207].

Definition 2.1 (Hash Function). An n-bit hash function $\mathcal{H}$ is a function

$$
\mathcal{H}:\{0,1\}^{*} \rightarrow\{0,1\}^{n}, \quad n \in \mathbb{N}^{+}
$$

In practice, the notion arbitrary-length input is usually interpreted as messages up
to $2^{r}$ bits for a reasonable large $r$ with $r \ll n / 2$. Cryptographic hash functions have this limitation since their expected security properties cannot longer be guaranteed if the length of an input exceeds $2^{n / 2}$ bits [238]. Considering the fact that contemporary hash functions have an output length of 256 or 512 bits, this observation is only of academical interest. Anyway, common hash functions follow an iterative approach to process a long message $M$; they divide $M$ into $m$ blocks of $n$ bits, i.e., $M=M_{1}, \ldots, M_{m}$, where $\left|M_{i}\right|=n$ for $i=1, \ldots, m$. Individual message blocks are processed iteratively by a compression function.

Definition 2.2 (Compression Function). A compression function $\mathcal{F}$ is a function

$$
\mathcal{F}:\{0,1\}^{h} \times\{0,1\}^{n} \rightarrow\{0,1\}^{h}, \quad h, n \in \mathbb{N}^{+}
$$

where $h$ denotes the size of the chaining value and $n$ the size of the message block.

In the area of cryptography, the majority of iterative hash functions are based on the Merkle-Damgård design [72, 171] where the message blocks are sequentially processed by a fixed-length compression function.

This approach obtained its popularity for being property-preserving, i.e., certain properties of the hash function are inherit from the compression function. In the light of the SHA-3 competition, the recent research has put a focus on designing constructions that preserve as many properties of the compression function as possible [36, 41].

### 2.1. Security Notions

### 2.1.1. Random Oracle Model

Ideally, a hash function should be indistinguishable from a random oracle [24] with fixed output size. A random oracle is an abstract and ideal primitive that returns a random bit string for each fresh input. Thus, the output of a random oracle is independent of the input, except that repeated queries are always treated consistently, i.e., the function property is always fulfilled. Furthermore, random oracles are atomic building blocks, i.e., they cannot be decomposed. In the context of provable security, random oracles are used for hiding implementation details, e.g., the insides of a specific hash function like MD5 [200] or SHA-256 [184]. Random oracles become handy when no known implementable function provides the mathematical properties
required for the proof - or when it gets too tedious to formalize these. A security proof of a cryptographic scheme using a random oracle as a component function is said to be in the random oracle model. From a theoretical point of view, it is clear that such a security proof is only a heuristic indication of the security of the scheme when instantiated with a specific hash function.
In fact, many recent separation results [16, 60, 81, 115, 163, 179] illustrate that various cryptographic schemes are secure in the random oracle model, but completely insecure for any efficient instantiation. According to [144], all such counterexamples are artificial and do not seem to attack any practically relevant scheme directly. Nevertheless, a security proof in the random oracle model is at least an indication for the soundness of the analyzed scheme.

### 2.1.2. Standard Model

Beside the random oracle model, the security of a hash function can also be determined under the three much weaker standard model assumptions: (1) collision resistance, (2) preimage resistance, and (3) 2nd-preimage resistance. The insecurity of a cryptographic function is quantified by the success probability of an optimal and resource-bounded adversary $\mathcal{A}$. Depending on the setting, different notions of success and different limitations of the resources apply for the adversary. Actually, the standard model does only work for families of hash functions $\mathfrak{H}$ where $\mathfrak{H}$ is considered to be secure if there exists no efficient adversary $\mathcal{A}$ that violates at least one out of the three standard assumptions for $\mathcal{H} \stackrel{\$}{\leftarrow} \mathfrak{H}$. The standard model is not suitable to prove the security of a single $n$-bit hash function $\mathcal{H}$ such as SHA-256 [184] since here $\mathcal{A}$ is not restricted in the access to SHA-256. Suppose $\mathcal{A}_{X, Y}$ with some fixed $X, Y \in\{0,1\}^{2 n}$ is an adversary that just outputs the two $2 n$-bit values $X$ and $Y$. By the pigionhole principle, there must be two values $X^{\prime}$ and $Y^{\prime}$ such that $\mathcal{H}\left(X^{\prime}\right)=\mathcal{H}\left(Y^{\prime}\right)$ and thus, there exists an efficient adversary, namely $\mathcal{A}_{X^{\prime}, Y^{\prime}}$. In 2006, Rogaway introduced a way to analyze the security of a single hash function in the standard model by bringing human ignorance into equation [207] which means that $\mathcal{H}$ is secure if there is no efficient algorithm which is known to man that violates at least one out of the three standard assumptions. Thus, a hash function is considered to be secure if mankind is unable to find an efficient adversary.

Next, we introduce a common hybrid model where an adversary has only restricted access to a hash function.

### 2.1.3. Hybrid Standard Model

In this thesis, any analyzed cryptographic system is an algorithm that uses (at least one) other component function - the primitive - inside. As the adversary is assumed to have no knowledge about the inner workings of these primitives - in the past always formalized by assuming a secret key - these are accessed by the adversary via an oracle interface. Such an oracle interface essentially formalizes the black-box mode of operation of an adversary towards the scheme or primitive being attacked. It provides a clearly defined set of exposed functions an adversary is able to send queries to and can expect to get an answer from. We always assume that such an adversary is an efficient algorithm, i.e., it has resource-bounded access to the compression or hash function. Next, we give formal definitions of the mentioned standard model assumptions.

Collision Resistance. A hash function $\mathcal{H}$ is collision resistant if it is hard to find two distinct inputs that are mapped to the same output. More formally, the advantage of an adversary $\mathcal{A}$ with oracle access to $\mathcal{H}$ is defined as follows:

Definition 2.3 (Collision Resistance). Let $\mathcal{H}$ be a hash function and $\mathcal{A}$ be an adversary. Then, the collision advantage of $\mathcal{A}$ against $\mathcal{H}$ is given by

$$
\operatorname{Adv}_{\mathcal{H}}^{\text {coll }}(\mathcal{A})=\operatorname{Pr}\left[\left(M, M^{\prime}\right) \leftarrow \mathcal{A}^{\mathcal{H}}: \mathcal{H}(M)=\mathcal{H}\left(M^{\prime}\right) \wedge M \neq M^{\prime}\right] .
$$

Note that the adversary $\mathcal{A}$ is only limited by the number of queries to its oracles. Thus, we write

$$
\operatorname{Adv}_{\mathcal{H}}^{\text {coll }}(q, t)=\max _{\mathcal{A}}\left\{\operatorname{Adv}_{\mathcal{H}}^{\text {coll }}(\mathcal{A})\right\}
$$

where the maximum is taken over all adversaries that ask at most $q$ oracle queries and run in time at most $t$.
For an $n$-bit hash function, the number of message pairs with $q$ messages is $\binom{q}{2}=$ $q(q-1) / 2 \approx q^{2} / 2$. An ideal $n$-bit hash function returns random $n$-bit strings. Since two of these are equal with probability $2^{-n}$, one needs $2^{n}$ pairs before a collision can be expected. More precisely with $q=2^{(n+1) / 2}$ queries, the probability of a collision is greater than 0.5 , i.e., $1-\frac{1}{e} \approx 0.63$. This generic attack works for any hash function and is commonly known as the birthday attack.

Preimage Resistance. A hash function $\mathcal{H}$ is preimage resistant if, given a hash value, it is hard to find a message that hashes to this value.

More formally, the advantage of an adversary $\mathcal{A}$ with oracle access to $\mathcal{H}$ is defined as follows:

Definition 2.4 (Preimage Resistance). Let $\mathcal{H}$ be a hash function and $\mathcal{A}$ be an adversary. Then, we define the preimage advantage of $\mathcal{A}$ against $\mathcal{H}$ as

$$
\operatorname{Adv}_{\mathcal{H}}^{p r e}(\mathcal{A})=\operatorname{Pr}\left[Y \stackrel{\$}{\leftarrow}\{0,1\}^{n}, M \leftarrow \mathcal{A}^{\mathcal{H}, Y}: \mathcal{H}(M)=Y\right]
$$

and

$$
\mathbf{A d}_{\mathcal{H}}^{p r e}(q, t)=\max _{\mathcal{A}}\left\{\mathbf{A d}_{\mathcal{H}}^{p r e}(\mathcal{A})\right\}
$$

as the maximum advantage over all preimage adversaries that ask at most $q$ oracle queries and run in time at most $t$.

A method for finding preimages that works for any hash function is the brute-force attack, i.e., one hashes random messages until the hash value $Y$ is reached. Assuming that the output of the hash function is uniformly balanced, an adversary is expected to try $2^{n}$ distinct messages in order to be successful.

2nd-Preimage Resistance. A hash function $\mathcal{H}$ is 2nd-preimage resistant if, given a hash value message pair $(Y, M)$ where $Y=\mathcal{H}(M)$, it is hard to find a fresh message that also produces the same hash value. More formally, the advantage of an adversary $\mathcal{A}$ with oracle access to $\mathcal{H}$ is defined as follows:

Definition 2.5 (2nd-Preimage Resistance). Let $\mathcal{H}$ be a hash function and $\mathcal{A}$ be an adversary. Then, the 2nd-preimage advantage of $\mathcal{A}$ against $\mathcal{H}$ for a random message $M \stackrel{\$}{\leftarrow}\{0,1\}^{*}$ is defined as

$$
\mathbf{A d v}_{\mathcal{H}}^{2 n d-p r e}(\mathcal{A})=\operatorname{Pr}\left[Y \leftarrow \mathcal{H}(M), M^{\prime} \leftarrow \mathcal{A}^{\mathcal{H}, M, Y}: \mathcal{H}\left(M^{\prime}\right)=Y \wedge M^{\prime} \neq M\right]
$$

and

$$
\mathbf{A d v}_{\mathcal{H}}^{2 n d-p r e}(q, t)=\max _{\mathcal{A}}\left\{\mathbf{A d v}_{\mathcal{H}}^{2 n d-p r e}(\mathcal{A})\right\}
$$

as the maximum advantage over all 2nd-preimage adversaries that ask at most $q$ oracle queries and run in time at most $t$.

### 2.2. Iterated Hash Functions

At CRYPTO'98, Damgård 72] and Merkle [171] proposed - independently from each other - an iterative approach to construct a collision resistant hash function based on a fixed-input length compression function. This idea has influenced the design of virtually all popular hash functions such as MD4 [201], MD5 [200], SHA$0 / 1$ [183, 185], and the SHA-2 family [184].

Definition 2.6 (Iterated Hash Function). Let $\mathcal{F}:\{0,1\}^{h} \times\{0,1\}^{n} \rightarrow\{0,1\}^{h}$ be a compression function and let $M=M_{1}, \ldots, M_{m}$ be a message with $M_{i} \in\{0,1\}^{n}$ for $i=1, \ldots, m$. For a fixed inital value $V_{0} \in\{0,1\}^{h}$, the iterated hash function $\mathcal{H}:\left(\{0,1\}^{n}\right)^{*} \rightarrow\{0,1\}^{h}$ is defined as

$$
V_{i} \leftarrow \mathcal{F}\left(V_{i-1}, M_{i}\right) \text {, where } Y=\mathcal{H}(M)=V_{m+1} \text { with } i=1, \ldots, m
$$

Usually, the message length in bits, denoted by $|M|$, is not necessarily a multiple of $n$. Thus, a padding procedure is required. Note that it can also be applied if the message length is already a multiple of $n$ bits since it can serve as a pre-processing function. This step is sometimes called message expansion. The most common padding procedure is the so called $10^{*}$-padding specified in [200].

Definition 2.7 (10*-Padding). Suppose $M$ is an $\ell$-bit input message. Then,

$$
b=n-(\ell+1) \quad(\bmod n)
$$

denotes the number of appended zero bits. And the padded message is computed by the following rule:

$$
M^{\prime} \leftarrow M\|1\| 0^{b},
$$

where ' 1 ' denotes a single one-bit and ' $0^{b}$, denotes a sequence of $b$ zero-bits.

There are numerous further padding rules known and the choice depends on the application. More examples are given in [123, 213, 221].

Next, we discuss why the length of the message might also be included into the padding as a security measure. Damgård and Merkle independently provided theorems in their papers that essentially show Theorem [2.8,

Theorem 2.8 (Merkle-Damgård Security [72, 171]). Suppose $\mathcal{H}$ is an iterated hash function as in Definition 2.6 and $\mathcal{F}$ its underlying compression function. If the initial chaining value $V_{0}$ is fixed and if the padding procedure includes the message length into the padding bits, it holds that

$$
\mathcal{F} \text { is collision resistant } \Longrightarrow \mathcal{H} \text { is collision resistant. }
$$

Fixing the initial value and adding a representation of the message length, is called MD-strengthening. Unfortunately, this result does not extend to pre- and 2ndpreimage resistance. Recent results highlight some intrinsic limitations of the MerkleDamgaird approach. This includes being vulnerable to multi-collision attacks [129], long 2nd-preimage attacks [135], and herding attacks [134]. Even though the practical relevance of these attacks is unclear, they highlight some security issues which designers should take care of. Therefore, in recent years, research has put a focus on designing constructions that preserve as many properties of the compression function as possible, e.g., $9,10,23,36,41,64,91]$.

### 2.3. Generic Attacks

On one hand, the iterative structure of cryptographic hash functions makes it possible to design time- and memory-efficient hash functions, and handling inputs of arbitrary length. On the other hand, iterative modes of operation for compression functions allow generic attacks; even for an ideal compression function, i.e., a random oracle. Next, we give a brief introduction to generic attacks on hash functions.

Length-Extension Attacks. Given a Merkle-Damgård-based hash function $\mathcal{H}$. If one can find a collision for two messages $M$ and $M^{\prime}$ with $M \neq M^{\prime}$, such that $\mathcal{H}(M)=$ $\mathcal{H}\left(M^{\prime}\right)$, then, one can apply a length-extension attack. For any message $M^{\prime \prime}$, one can easily produce a collision for $M \| M^{\prime \prime}$ and $M^{\prime} \| M^{\prime \prime}$.

Multi-Collision Attacks. Joux [129] found that when iterative hash functions are used, finding a set of $2^{k}$ message all colliding on the same hash value (a $2^{k}$-multicollision) is as easy as finding $k$ collisions for the hash function. After finding a collision in the compression function, one can find $k$ of such collisions each starting from the chaining value produced by the previous one-block collision. In other words, one
has to find two distinct messages blocks $M_{i}$ and $M_{i}^{\prime}$ with $\mathcal{F}\left(V_{i-1}, M_{i}\right)=\mathcal{F}\left(V_{i-1}, M_{i}^{\prime}\right)$, where $\mathcal{F}(\cdot, \cdot)$ represents the compression function and $V_{i}$ the chaining value. Then, it is possible to construct $2^{k}$ messages with the same hash value by choosing for block $i$ either the message block $M_{i}$ or $M_{i}^{\prime}$. This attack can find $2^{k}$-way internal multicollisions with a complexity of $k \cdot 2^{n / 2}$ compression function calls. Joux also showed that the concatenation of two different hash functions is not more secure against collision attacks than the strongest one.

Herding Attacks. The herding attack [134] works as follows: An adversary $\mathcal{A}$ takes $2^{k}$ chaining values which are fixed or randomly chosen. Then, $\mathcal{A}$ chooses $O\left(2^{n / 2-k / 2}\right)$ message blocks. Next, $\mathcal{A}$ computes the output of the compression function for each chaining value and each block. It is expected that for each chaining value there exists another chaining value, such that both collide to the same value. Then, $\mathcal{A}$ stores the message block that leads to such a collision in a table and repeats this process again with the newly found chaining values. Once the adversary has only one chaining value, it is used to compute the hash value to be published. To find a message whose chaining value is among the $2^{k}$ original values, the attacker has to perform $O\left(2^{n-k}\right)$ operations. For such a message, the attacker can retrieve from the stored messages the message blocks that would lead to the desired hash value. The time complexity of this attack is about $O\left(2^{n / 2+k / 2}\right)$ operations for the first and $O\left(2^{n-k}\right)$ operations for the second step.

Long 2nd-Preimage Attacks. Dean [74] found that fix points in the compression function $\mathcal{F}$, i.e., a point $\left(Y_{i}, M_{i}\right) \in\{0,1\}^{h} \times\{0,1\}^{n}$ with $Y_{i}=\mathcal{F}\left(Y_{i}, M_{i}\right)$, can be used for a 2nd-preimage attack against long messages in time $O\left(n \cdot 2^{n / 2}\right)$ and memory $O(n$. $2^{n / 2}$ ). Kelsey and Schneier [135] extended this result and provided an attack to find a 2nd-preimage on a Merkle-Damgård construction with MD-strengthening much faster than the expected workload of $O\left(2^{n}\right)$. The complexity of the attack is determined by the complexity of finding expandable messages. These are messages of varying sizes such that all these messages collide internally for a given initial value. Expandable messages can either be found using internal collisions or fix points between a singleblock message and a multi-block message. The complexity of the generic attack to find a 2 nd-preimage for a $2^{k}$-block message is about $k \cdot 2^{n / 2+1}+2^{n-k+1}$ compression function calls.

Andreeva et al. [8] showed that a combination of the attacks from [74, 134, 135] can be mounted on dithered hash functions, i.e., hash functions based on compression functions with an additional input, which gives an adversary $\mathcal{A}$ more control on the

2nd-preimage since $\mathcal{A}$ can choose about the first half of the message in an arbitrary way. This attack can be done in time $2^{n / 2+k / 2+2}+2^{n-k}$. Although, it is more expensive than the attack of Kelsey and Schneier [135]. As a hash function designer, one has to make the dithering as huge as possible, such that there are no small cycles.

Slide Attacks. Slide attacks are common in block-cipher cryptanalysis, but also applicable to hash functions. Given a hash function $\mathcal{H}$ and two messages $M$ and $M^{\prime}$, where $M$ is a prefix of $M^{\prime}$, one can find a slide pair of messages ( $M, M^{\prime}$ ) such that the last message block of the longer message $M^{\prime}$ performs only an additional blank round, e.g., for sponge constructions. These two messages are then slide by one blank round. This attack allows to recover the internal state of a slide pair of messages and even to perform backward computation, as shown in [118].

Differential Attacks. The essential idea of differential attacks on hash functions 61], as used to break MD5 [200] and SHA-0/1 [183, 185], is to exploit a high probability input/output differential over some component of the hash function, e.g., in the form of a 'perturb-and-correct' strategy for the latter functions, exploiting high probability linear/non-linear characteristics.

### 2.4. Keyed Hash Functions

In 1992, Tsudik introduced the concept of keyed hash functions which compress a $k$-bit key and an input of arbitrary length to an output of fixed length [230]. From start, they were used to generate MAC5 [17, 32, 223, 230].

Definition 2.9 (Keyed Hash Function). $A$ keyed hash function $\mathcal{H}$ is a function

$$
\mathcal{H}:\{0,1\}^{k} \times\{0,1\}^{*} \rightarrow\{0,1\}^{n}, \quad n, k \in \mathbb{N}^{+} .
$$

Any hash function $\mathcal{H}(\cdot)$ can be easily transformed into a keyed hash function $\mathcal{H}(\cdot, \cdot)$ by prepending the key to the message, i.e., $\mathcal{H}(K, M)=\mathcal{H}(K \| M)$ with $K \in\{0,1\}^{k}$ and $M \in\{0,1\}^{*}$. In the following we use $\mathcal{H}_{K}(M)$ and $\mathcal{H}(K, M)$ as synonymes.

Pseudorandom Function (PRF) Model. Let $\$_{n}:\{0,1\}^{*} \rightarrow\{0,1\}^{n}$ be a random oracle (random function). An $n$-bit keyed hash function $\mathcal{H}_{K}$, under a secret key
$K \stackrel{\$}{\leftarrow}\{0,1\}^{k}$, can be considered as secure, namelyPRF-secure, if it is indistinguishable from $\$$. This security notion can be formalized by giving an adversary $\mathcal{A}$ either blackbox (oracle) access to $\mathcal{H}_{K}$ (under a random key $K$ ) or to $\$$. Suppose the choice is based on the result of a fair coin toss. Let denote heads ('1') the case where $\mathcal{A}$ gets oracle access to the keyed hash function, and let denote tails ('0') the case where $\mathcal{A}$ gets oracle access to the random function. The task of $\mathcal{A}$ is to guess the result of the coin toss after a certain amount of oracle queries. More formally, the advantage of $\mathcal{A}$ can be defined as follows:
 tion and $\mathcal{H}:\{0,1\}^{k} \times\{0,1\}^{*} \rightarrow\{0,1\}^{n}$ be a keyed hash function. Then, we define the PRF advantage of an adversary $\mathcal{A}$ against $\mathcal{H}$ as

$$
\operatorname{Adv} \sqrt{\frac{P R F F}{\mathcal{H}}}(\mathcal{A})=\left|\operatorname{Pr}\left[K \stackrel{\$}{\leftarrow}\{0,1\}^{k}: \mathcal{A}^{\mathcal{H}(K, \cdot)} \Rightarrow 1\right]-\operatorname{Pr}\left[\mathcal{A}^{\S(\cdot)} \Rightarrow 1\right]\right|,
$$

and

$$
\operatorname{Adv}{\left.\sqrt{\frac{P R F}{\mathcal{H}}}(q, t)=\max _{\mathcal{A}}\left\{\operatorname{Adv} \frac{\sqrt{P R F}}{\mathcal{H}}(\mathcal{A})\right\}, ~\right\}}
$$

as the maximum advantage over all $\overline{P R F}$ adversaries that ask at most $q$ oracle queries and run in time at most $t$.

## Block Ciphers

An expert is a person who has made all the mistakes that can be made in a very narrow field.

## Niels Bohr

A block cipher is a cryptographic primitive with a fixed input and output size, e.g., 64 or 128 bit, to either encrypt or decrypt blocks of data. In modern cryptography, block ciphers are the most common building blocks for symmetric encryption schemes. They are omnipresent to provide confidentiality (data privacy) for both network traffic [58, 78, 138, 196] and data storage [49, 89, 160]. Furthermore, several MAC; are based on block ciphers [19, 46, 125].

Definition 3.1 (Block Cipher). Let $E:\{0,1\}^{k} \times\{0,1\}^{n} \rightarrow\{0,1\}^{n}$ be a family of functions for some $k, n \in \mathbb{N}^{+}$. We denote $E$ as a $(k, n)$-block cipher iff for any $K \in\{0,1\}^{k}$ it holds that

$$
E(K, \cdot) \text { is a permutation }
$$

We denote the first input as key, the second input as message or plaintext, and the output as ciphertext.

Mathematically, a $(k, n)$-block cipher $E$ is a keyed family of permutations, i.e., a set of $2^{k} n$-bit permutations. Since for a fixed key $K \in\{0,1\}^{k}, E(K, \cdot)$ is a

```
Algorithm 1 Random Permutation \(\mathcal{P}\) Implemented via Lazy Sampling
Init ()\(\quad \mathcal{P}(M) \quad \mathcal{P}^{-1}(C)\)
    \(X \leftarrow \perp \quad\) if \(X[M]=\perp\) then \(\quad\) if \(Y[C]=\perp\) then
    \(Y \leftarrow \perp \quad X[M] \stackrel{\$}{\leftarrow}\{0,1\}^{n} \backslash \mathfrak{R} \quad Y[C] \stackrel{\$}{\leftarrow}\{0,1\}^{n} \backslash \mathfrak{D}\)
    \(\mathfrak{D} \leftarrow \emptyset \quad Y[X[M]] \leftarrow M \quad X[Y[C]] \leftarrow C\)
    \(\mathfrak{R} \leftarrow \emptyset \quad \mathfrak{D} \leftarrow \mathfrak{D} \cup\{X[M]\} \quad \mathfrak{D} \leftarrow \mathfrak{D} \cup\{C\}\)
    \(\mathfrak{R} \leftarrow \mathfrak{R} \cup\{M\} \quad \mathfrak{R} \leftarrow \mathfrak{R} \cup\{Y[C]\}\)
    end if end if
        return \(C \leftarrow X[M] \quad\) return \(M \leftarrow Y[C]\)
```

bijection, it has an inverse, namely $E^{-1}(K, \cdot)$, i.e., for all $M \in\{0,1\}^{n}$ it holds that $M=E^{-1}(K, E(K, M))$. In this thesis we use $E(K, \cdot)$ and $E_{K}(\cdot)$ as synonyms. Furthermore, we denote $\operatorname{Block}(k, n)$ as the set of all $(k, n)$-block ciphers. Note that for each key, there exists $2^{n}$ ! $n$-bit permutations, and any permutation can be assigned to a given key. Thus, we have a huge set of $\left(2^{n}!\right)^{2^{k}}$ possible block ciphers.

### 3.1. Security Notions

Ideal Cipher Model. Let $\operatorname{Perm}_{n}$ denote the family of all possible $n$-bit permutations. We denote by $\mathcal{P} \stackrel{\$}{\leftarrow} \mathbf{P e r m}_{n}$ a random permutation. In the ideal cipher model [48, 87, 139], the block cipher is modeled as a family of $2^{k}$ random permutations $\mathfrak{P}$. Let denote $\mathcal{P}_{i}$ the i-th element of $\mathfrak{P}$. Ideally, under a secret key a $(k, n)$-block cipher should be computationally indistinguishable from $\mathfrak{P}$, i.e., it should not be possible to distinguish $E_{K}$ from $\mathcal{P}_{K}$. Similar to the random oracle model, there are also separation results published for the ideal cipher model [44]. Hence, a cryptographic scheme proven to be secure in the ideal cipher model does not preserve its security properties - such as collision resistance - when instantiated with a real block cipher like the Advanced Encryption Standard (AES) [176] or the Data Encryption Standard (DES) [186].

Pseudorandom Permutation (PRP) Model. Beside the artificial strong ideal cipher model, the security of a block cipher can also be determined by the common notion of a PRP A family $E_{K}$ of $n$-bit permutations is called PRP when the input-output behaviour of $E_{K}$ is computationally indistinguishable from that of an $n$-bit random permutation. Next, we introduce the security notion of (strong) Indistinguishability from a Pseudorandom Permutation (IND-PRP).

Suppose, depending on the result of a coin toss, an adversary $\mathcal{A}$ has either blackbox access to a block cipher $E \in \operatorname{Block}(k, n)$ under a secret key $K \stackrel{\$}{\leftarrow}\{0,1\}^{k}$, or to an $n$-bit random permutation $\mathfrak{P}$ which is independent from $K$. We denote $E_{K}$ as IND-PRP-secure when $\mathcal{A}$ cannot distinguish between these two scenarios. Let ' 1 ' denote the real scenario where $\mathcal{A}$ has access to the block cipher and ' 0 ' the random scenario, where $\mathcal{A}$ has access to a random permutation, which is usually implemented in an efficient way using the lazy-sampling technique (cf. Algorithm (1). Then, $\operatorname{Pr}\left[K \stackrel{\&}{\leftarrow}\{0,1\}^{k}: \mathcal{A}^{E_{K}(\cdot), E_{K}^{-1}(\cdot)} \Rightarrow 1\right]$ denotes the success probability that $\mathcal{A}$ guesses ' 1 ' when in the real scenario. Then, the formal definition of the IND-PRP. advantage is defined as follows.

Definition 3.2 (IND-PRP-Advantage). Let $E \in \operatorname{Block}(k, n)$ be a block cipher and $\mathcal{P} \stackrel{\&}{\leftarrow} \mathbf{P e r m}_{n}$ a random permutation. Then, we define the IND-PRP advantage of an adversary $\mathcal{A}$ as
and

$$
\operatorname{Adv} \sqrt{\frac{P R P}{E, E^{-1}}}(q, t)=\max _{\mathcal{A}}\left\{\mathbf{A d v} \frac{\left.\sqrt{\frac{P R P}{E, E^{-1}}}(\mathcal{A})\right\}}{}\right.
$$

as the maximum advantage over all IND-PRP adversaries that run in time at most $t$ and ask a total maximum of $q$ queries to the encryption and decryption oracles.

PRP under Related-Key Attacks. In a related-key scenario we assume that an adversary $\mathcal{A}$ has partial control over the secret key $K \stackrel{\$}{\leftarrow}\{0,1\}^{k}$ of a block cipher $E \in \operatorname{Block}(k, n)$. Following the security notions of Lucks 154], the partial control over the key is modeled as a key-transformation function $\varphi:\{0,1\}^{k} \times\{0,1\}^{k} \rightarrow$ $\{0,1\}^{k}$. In this thesis we assume that $\varphi(K, \cdot)$ is the XOR-operation. In contrast to the IND-PRP security model, in the Pseudorandom Permutation under RelatedKey Attacks (PRP-RKA) model the adversary $\mathcal{A}$ has either access to the related-key encryption oracle $E_{\varphi(K, \cdot)}(\cdot)$ or to a set of $2^{k}$ random permutations $\mathfrak{P} \in \operatorname{Perm}_{n}^{k}$, where $\mathbf{P e r m}_{n}^{k}=\underbrace{\operatorname{Perm}_{n} \times \ldots \times \mathbf{P e r m}_{n}}_{2^{k} \text { times }}$. Depending on the setting, the first input is either the key relation or an index that determines a specific random permutation, and the second input is the plaintext. The PRP-RKA advantage is defined as follows:

Definition 3.3 (PRP-RKA Advantage). Let $E \in \operatorname{Block}(k, n)$ be a block cipher and let $\mathfrak{P} \stackrel{\$}{\leftarrow} \operatorname{Perm}_{n}^{k}$ be a family of random permutations. Let $\varphi:\{0,1\}^{k} \times\{0,1\}^{k} \rightarrow$ $\{0,1\}^{k}$ is a key transformation function, and $K \stackrel{\$}{\leftarrow}\{0,1\}^{k}$. Then, we define the PRP-RKA-advantage of an adversary $\mathcal{A}$ as

$$
\mathbf{A d v} \sqrt{\frac{P R P-R K A}{E, E^{-1}}}(\mathcal{A})=\left|\operatorname{Pr}\left[\mathcal{A}^{E_{\varphi(K, \cdot)}(\cdot), E_{\varphi(K, \cdot)}^{-1}(\cdot)} \Rightarrow 1\right]-\operatorname{Pr}\left[\mathcal{A}^{\mathfrak{P}(\cdot \cdot \cdot), \mathfrak{P}^{-1}(\cdot, \cdot)} \Rightarrow 1\right]\right|
$$

and

$$
\mathbf{A d v} \sqrt{\frac{P R P-R K A}{E, E^{-1}}}(q, t)=\max _{\mathcal{A}}\left\{\mathbf{A d} \mathbf{v}_{E}^{\frac{P R P-R K A}{E}}(\mathcal{A})\right\}
$$

as the maximum advantage over all PRP-RKA adversaries that run in time at most $T$ and ask a total maximum of $q$ queries to the encryption and decryption oracles.

### 3.2. Tweakable Block Ciphers

The concept of tweakable block ciphers was introduced by Liskov et al. in [153]. The design is based on a common block cipher, which is extended by a so called tweak. A tweakable block cipher $\widetilde{E}:\{0,1\}^{k} \times\{0,1\}^{u} \times\{0,1\}^{n} \rightarrow\{0,1\}^{n}$ is defined as follows:

Definition 3.4 (Tweakable Block Cipher). Let $\widetilde{E}:\{0,1\}^{k} \times\{0,1\}^{u} \times\{0,1\}^{n} \rightarrow$ $\{0,1\}^{n}$ be a family of functions for some $k, u, n \in \mathbb{N}^{+}$. We denote $\widetilde{E}$ as a tweakable $(k, u, n)$-block cipher iff for any key tweak tuple $(K, U) \in\{0,1\}^{k} \times\{0,1\}^{u}$ it holds that

$$
\widetilde{E}(K, U, \cdot) \text { is a permutation. }
$$

We denote the first input as key, the second input as tweak and the third input as message or plaintext, and the output as ciphertext.

We denote $\widetilde{E}_{K}^{-1}(U, \cdot)$ as the inverse of $\widetilde{E}_{K}(U, \cdot)$, i.e., for all $M \in\{0,1\}^{n}$ it holds that $M=\widetilde{E}_{K}^{-1}\left(U, \widetilde{E}_{K}(U, M)\right)$. Furthermore, we denote $\operatorname{Block}(k, u, n)$ as the set of all tweakable $(k, u, n)$-block ciphers.

A tweakable block cipher $\widetilde{E} \in \operatorname{Block}(k, u, n)$ is considered to be secure if it is computationally indistinguishable from a family of $2^{u}$ random $n$-bit permutations.

The formal definition of the IND-PRP advantage for tweakable block ciphers is similar to Definition 3.3.

Definition 3.5 (IND-PRP Advantage). Let $\widetilde{E} \in \operatorname{Block}(k, u, n)$ be a tweakable block cipher and let $\mathfrak{P} \stackrel{\$}{\leftarrow}$ Perm $_{n}^{k}$ be a family of random permutations. Suppose that $K \stackrel{\$}{\leftarrow}\{0,1\}^{k}$. Then, we define the IND-PRP advantage of an adversary $\mathcal{A}$ as
 and
as the maximum advantage over all IND-PRP adversaries that run in time at most $t$ and ask a total maximum of $q$ queries to the encryption and decryption oracles.

## Authenticated Encryption Schemes

Any sufficiently advanced technology
is indistinguishable from magic.
Arthur C. Clarke

A common requirement for cryptographic applications is to establish a secure channel between a sender and a receiver - usually referred to as Alice and Bob - who share a secret key. It may well be the case that sender and receiver represent the same entity, e.g., the same person can first write sensitive data to an insecure storage, and later read these data. Usually, a secure channel should provide data privacy to prevent an eavesdropper from revealing any information about a message sent from Alice to Bob, except its length. The cryptographic technique to ensure this requierement is "encryption". Sometimes, data authenticity/integrity is required, i.e., an adversary should not be able to manipulate messages without being noticed. This is cryptographically ensured by "authentication". Nevertheless, most of the time, users need both encryption and authentication: authenticated encryption (AE). An authenticated encryption scheme is a special kind of an encryption scheme that encrypts plaintext to authenticated ciphertexts.

Nonce. Goldwasser and Micali [117] formalized encryption schemes as stateful or probabilistic: otherwise, the data privacy is lost. Rogaway [204, 206, 208] proposed a unified point of view, by defining a cryptographic scheme as an always-deterministic algorithm that takes a user-supplied state called nonce (a number used once). We assume that an adversary is nonce-respecting when not stated otherwise. This type of
adversary has full control over a nonce with the restriction to never choose the same value twice. This limitation only holds for encryption queries. Thus, an adversary is allowed to use a nonce multiple times when query the decryption oracle.

Deterministic Authenticated Encryption (DAE). In [209], Rogaway and Shrimpton addressed authenticated encryption schemes which provide security against repeated nonces. Furthermore, the authors shaped the notion of misuse-resistance and they proposed Synthetic Initialization Vector (SIV) as a solution. SIV and related schemes (Bivariate Tag Mixing (BTM) [126] and Hash Block Stealing (HBS) [127]) actually provide excellent security against nonce-reusing adversaries. Though, they are inherently off-line, i.e., for encryption, one must either keep the entire plaintext in memory, or read the plaintext twice. This renders such deterministic approaches only practical for small messages.

On-line Authenticated Encryption (OAE). It is folklore that application programmers are used to process messages in an on-line manner. Hence, to seamlessly integrate authenticated encryption schemes into a typical software architecture, they should be on-line, i.e., plaintexts and ciphertexts are split into conveniently-sized blocks, and the $i$-th ciphertext block can be written before the $(i+1)$-th plaintext block has to be read. An AE scheme fulfilling the on-line requirement is referred to as an OAE scheme.

### 4.1. Authenticated Encryption with Associated Data Schemes

An authenticated encryption scheme is a triple $\Pi=(\mathcal{K}, \mathcal{E}, \mathcal{D})$ of three algorithms.

1. The key-generation algorithm $\mathcal{K}$ takes no input and returns a randomly chosen key $K$ from the key space $\{0,1\}^{k}$.
2. The deterministic encryption algorithm

$$
\mathcal{E}:\{0,1\}^{k} \times\left(\{0,1\}^{n}\right)^{*} \times\left(\{0,1\}^{n}\right)^{*} \rightarrow\left(\{0,1\}^{n}\right)^{*} \times\{0,1\}^{r}
$$

maps a key-header-message tuple $(K, H, M)$ to a ciphertext-tag-tuple $(C, T)$
3. The deterministic decryption algorithm

$$
\mathcal{D}:\{0,1\}^{k} \times\left(\{0,1\}^{n}\right)^{*} \times\left(\{0,1\}^{n}\right)^{*} \rightarrow\left\{\left(\{0,1\}^{n}\right)^{*}, \perp\right\}
$$

maps a header-ciphertext-tag-tuple $(H, C, T)$ either to the authentic plaintext, if the input is valid, or returns $\perp$.

Usually, AE schemes operate on $n$-bit blocks, where $n$ is the block length of the underlying primitive, e.g., a block cipher. This is reflected by the notion $\left(\{0,1\}^{n}\right)^{*}$ where $M \in\left(\{0,1\}^{n}\right)^{m}$ implies that the message $M$ consists of $m$ message blocks, $m-1$ blocks of $n$-bit and a final message block $M_{m}$ that can contain less then $n$-bits, i.e., $M=M_{1}, \cdots, M_{m}$ with $\left|M_{i}\right|=n$ and $\left|M_{m}\right| \leq n$ with $i=1, \ldots, m-1$.

It always holds that $|M| \leq|C|+|T|$ where $C$ denotes the ciphertext and $T$ the (authentication) tag. Note that OAE schemes require a nonce $N \in\{0,1\}^{v}$. In our notation $N$ is a mandatory part of the header, whereas the optional part consists of associated data or meta data of the plaintext, e.g., the TCP/IP header. Usually, for the sake of simplification, the nonce size $v$ matches the block size $n$ of the underlying primitive, e.g., a $(k, n)$-block cipher.

An Authenticated Encryption with Associated Data (AEAD) scheme ensures privacy and integrity for the plaintext; in addition, it ensures the integrity of the header. This renders those schemes useful in settings where the associated data of messages is predictable.

### 4.2. Generic Composition

An AE scheme can be generated by combining a secure encryption scheme with a secure MAC. Given two independent keys $K$ and $L$, the common literature lists three construction approaches for such a generic composition [22].

Encrypt-and-Mac. Encrypt the plaintext $M$ and append a MAC of the plaintext to the ciphertext: $E_{K}(M) \| \mathrm{MAC}_{L}(M)$. Variants of this method are used in the transport layer of the SSH protocol [242].

Mac-then-Encrypt. Append a MAC of the plaintext to the plaintext and then encrypt them together. Here, the output is $E_{K}\left(M \| \mathrm{MAC}_{L}(M)\right)$. Variants of this method are used in the TLS protocol version 1.0 and 1.1 [76, 77].

Encrypt-then-Mac. Encrypt the plaintext to get a ciphertext and append a MAC of this ciphertext: $E_{K}(M) \| \mathrm{MAC}_{L}\left(E_{K}(M)\right)$. Variants of this method are used in the IPSec protocol [137].

Out of these, only the Encrypt-then-Mac scheme is free of weaknesses [22]. Note that this approach can fail trivially by key management errors: suppose the receiver side
only updates the authentication key, but not the encryption key. Then, Encrypt-then-Mac will decrypt a ciphertext into "authentic" random garbage. Therefore, it is less error-prone to use a dedicated authenticated encryption scheme and not a generic composition.

### 4.3. Security Notions

Authenticated encryption schemes require security notions for both privacy and integrity. Notions and their relations were introduced for deterministic schemes in 210] and for nonce-based schemes in [22, 27, 133, 204, 208]. In this thesis we adopt the notion of Chosen-Ciphertext Attack 3 (CCA3) security suggested in 210]. Similar to the security definition of IND-PRP (cf. Definition 3.2), a CCA3 adversary $\mathcal{A}$ has to distinguish between the real world, where it has oracle access to $\mathcal{E}_{K}(\cdot, \cdot)$ and $\mathcal{D}_{K}(\cdot, \cdot, \cdot)$ of an AE scheme $\Pi=(\mathcal{K}, \mathcal{E}, \mathcal{D})$, and the random world, where $\mathcal{A}$ has access to the oracles $\$(\cdot, \cdot)$ and $\perp(\cdot, \cdot, \cdot)$. The random oracle $\$(\cdot, \cdot)$ returns a string of random bits, whereas $\perp(\cdot, \cdot, \cdot)$ always returns $\perp$. Note that the equation $|\$(H, M)|=\left|\mathcal{E}_{K}(H, M)\right|$ holds for all header-plaintext tuple $(H, M)$. For the sake of simplification, we assume that an adversary never asks a query to which the corresponding answer is already known.

Definition 4.1 (CCA3 Advantage). Let $\Pi=(\mathcal{K}, \mathcal{E}, \mathcal{D})$ be an authenticated encryption scheme as described in Section 4.1. The advantage of an adversary $\mathcal{A}$ in breaking $\Pi$ is defined as
and

$$
\operatorname{Adv} \sqrt{\Pi C A S}(q, \ell, t)=\max _{\mathcal{A}}\left\{\operatorname{Adv}_{\Pi}^{\sqrt{C C A S}}(\mathcal{A})\right\}
$$

as the maximum advantage over all nonce-respecting CCA3-adversaries that run in time at most $t$, ask total maximum of $q$ queries to the encryption and decryption oracles, and whose total query length is at most $\ell$ blocks.

It is easy to see that we can rewrite the term given in Definition 4.1 as

$$
\begin{align*}
& \operatorname{Adv}^{\operatorname{CCA}( }(\mathcal{A})= \\
& \quad \mid \operatorname{Pr}\left[K \leftarrow \mathcal{K}: \mathcal{A}^{\mathcal{E}_{K}(\cdot, \cdot), \mathcal{D}_{K}(\cdot, \cdot, \cdot)} \Rightarrow 1\right]-\operatorname{Pr}\left[K \leftarrow \mathcal{K}: \mathcal{A}^{\mathcal{E}_{K}(\cdot, \cdot), \perp(\cdot, \cdot, \cdot)} \Rightarrow 1\right]  \tag{4.1}\\
& \quad+\operatorname{Pr}\left[K \leftarrow \mathcal{K}: \mathcal{A}^{\mathcal{E}_{K}(\cdot, \cdot), \perp(\cdot, \cdot, \cdot)} \Rightarrow 1\right]-\operatorname{Pr}\left[\mathcal{A}^{\$(\cdot, \cdot), \perp(\cdot, \cdot, \cdot)} \Rightarrow 1\right] \mid . \tag{4.2}
\end{align*}
$$

One can interpret (4.1) as the advantage that an adversary has on the integrity of the ciphertext and (4.2) as the advantage an adversary has on the privacy. We use this decomposition as a motivational starting point to define ciphertext integrity and what we mean by an Indistinguishability under Chosen-Plaintext Attack (IND-CPA) adversary against authenticated encryption schemes.

Indistinguishability under Chosen-Plaintext Attack (IND-CPA). Let $\Pi=(\mathcal{K}, \mathcal{E}, \mathcal{D})$ be an authenticated encryption scheme and $\mathcal{A}$ an IND-CPA adversary. The task of $\mathcal{A}$ is to distinguish the real world, where it is given oracle access to $\mathcal{E}_{K}(\cdot, \cdot)$ under a secret key $K \in\{0,1\}^{k}$, from the random world, where $\mathcal{A}$ has access to a random oracle $\$(\cdot, \cdot)$ which returns, consistent, random ciphertexts, as described earlier in this section. If no such adversary $\mathcal{A}$ can perform significant better than random guessing, then, $\Pi$ protects the privacy of encrypted messages. The IND-CPA advantage is defined as follows:

Definition 4.2 (IND-CPA Advantage). Let $\Pi=(\mathcal{K}, \mathcal{E}, \mathcal{D})$ be an authenticated encryption scheme as described in Section 4.1. Then, the IND-CPA advantage of a nonce-respecting adversary $\mathcal{A}$ is defined as

$$
\mathbf{A d v} \frac{I N D-C P A}{\Pi I}(\mathcal{A})=\left|\operatorname{Pr}\left[K \leftarrow \mathcal{K}: \mathcal{A}^{\mathcal{E}_{K}(\cdot, \cdot)} \Rightarrow 1\right]-\operatorname{Pr}\left[\mathcal{A}^{\$(\cdot, \cdot)} \Rightarrow 1\right]\right|
$$

and

$$
\mathbf{A d v} \frac{\sqrt{I N D-C P A}}{\Pi}(q, \ell, t)=\max _{\mathcal{A}}\left\{\mathbf{A d v} \frac{\sqrt{I N D-C P A}}{\Pi}(\mathcal{A})\right\}
$$

as the maximum advantage over all nonce-respecting IND-CPA-adversaries that run in time at most $t$, ask a total maximum of $q$ queries to the encryption oracle, and whose total query length is not more than $\ell$ blocks.

Integrity of Ciphertext (INT-CTXT). The security notion INT-CTXT is defined by the game-playing approach [29], where the advantage of an adversary is measured

```
Algorithm 2 INT-CTXT Game
Initialize() \(\quad \operatorname{Encrypt}(H, M)\)
    \(K \stackrel{\&}{\leftarrow} \mathcal{K} \quad(C, T) \leftarrow \mathcal{E}_{K}(H, M)\)
    win \(\leftarrow\) false \(\quad \mathfrak{Q} \cup\{(H, C, T)\}\)
    \(\mathfrak{Q} \leftarrow \emptyset \quad\) return \((C, T)\)
Finalize()
    return win
```

```
Verify \((H, C, T)\)
```

Verify $(H, C, T)$
$M \leftarrow \mathcal{D}_{K}(H, C, T)$
$M \leftarrow \mathcal{D}_{K}(H, C, T)$
if $\quad((H, C, T) \notin \mathfrak{Q}) \wedge(M \neq \perp)$
if $\quad((H, C, T) \notin \mathfrak{Q}) \wedge(M \neq \perp)$
then
then
win $\leftarrow$ true
win $\leftarrow$ true
end if
end if
return $M$

```
    return \(M\)
```

as the success probability of winning a (cryptographic) game $G$. Each game consists of three functions: An initialization function Initialize(), a finalization function Finalize(), and oracle functions. Any adversary $\mathcal{A}$ that is playing a game calls the Initialize() function first. In the following, $\mathcal{A}$ then makes some queries to the encrypt and decrypt oracles, and finally, $\mathcal{A}$ ends the game by invoking Finalize().

To $\mathcal{A}$, every function of a game is a black box, i.e., it has no access to internal variables. An adversary wins the game if and only if Finalize() returns true. We denote $\operatorname{Pr}\left[\mathcal{A}^{G} \Rightarrow 1\right]$ as the probability that the adversary wins the Game $G$.

An AE scheme $\Pi=(\mathcal{K}, \mathcal{E}, \mathcal{D})$ protects the ciphertext integrity against an adversary $\mathcal{A}$ when it is not able to come up with a fresh authentic ciphertext tuple ( $H, C, T$ ), i.e., $\mathcal{D}_{K}(H, C, T) \neq \perp$, where $(H, C, T)$ is not the result of a previous query of $\mathcal{A}$. The INT-CTXT advantage based on the the Game $G_{\text {INT-CTXT }}$ (cf. Algorithm (2) is formally defined as follows:

Definition 4.3 (INT-CTXT Advantage). Let $\Pi=(\mathcal{K}, \mathcal{E}, \mathcal{D})$ be an authenticated encryption scheme as introduced in Section 4.1, and let GINT-CTXT denote the game from Algorithm 园. Then, the INT-CTXT advantage of a nonce-respecting adversary $\mathcal{A}$ is defined as

$$
\operatorname{Adv} \frac{I N T-C T X T}{I I}(\mathcal{A})=\operatorname{Pr}\left[\mathcal{A}^{G_{I N T-C T X T}} \Rightarrow 1\right],
$$

and

$$
\operatorname{Ad} \sqrt{\frac{I N T-C T X T}{\Pi}}(q, \ell, t)=\max _{\mathcal{A}}\left\{\operatorname{Ad} \sqrt{\left.\frac{\sqrt{I N T-C T X T}}{}(\mathcal{A})\right\}, ~}\right.
$$

as the maximum advantage over all nonce-respecting INT-CTXT-adversaries that run in time at most $t$, ask a total maximum of $q$ queries to the encryption and decryption oracles, and whose total query length is at most $\ell$ blocks.

Upper-Bounding the CCA3 Advantage. Bellare and Namprempre showed in [22] that an authenticated encryption scheme that is both IND-CPA and INT-CTXTsecure, is also CCA3-secure. This notable observation is formalized as follows:

Theorem 4.4 (CCA3 Advantage [22]). Let $\Pi=(\mathcal{K}, \mathcal{E}, \mathcal{D})$ be an authenticated encryption scheme as introduced in Section 4.1, and let $\mathcal{A}$ be a nonce-respecting CCA3 $n$ adversary that runs in time $t$, and makes $q$ queries with a total length of at most $\ell$ blocks. Then, there exists an $I N D-C P A_{\Pi}$-adversary $\mathcal{A}_{p}$ and an INT-CTXT adversary $\mathcal{A}_{c}$ such that

$$
\mathbf{A d v} \sqrt{\frac{C C A 3}{\Pi}}(\mathcal{A}) \leq \mathbf{A d v} \frac{\sqrt{I N D-C P A}}{\Pi}\left(\mathcal{A}_{p}\right)+\mathbf{A d v} \frac{\sqrt{I N T-C T X T}}{\Pi}\left(\mathcal{A}_{c}\right)
$$

where both $\mathcal{A}_{p}$ and $\mathcal{A}_{c}$ run in time $O(t)$ and make at most $q$ queries.

Proof Sketch. By applying the triangle inequality on Definition4.1, we have $\operatorname{Adv} \sqrt{\Pi C A 3}(\mathcal{A})=$

$$
\begin{aligned}
& \left|\operatorname{Pr}\left[K \leftarrow \mathcal{K}: \mathcal{A}^{\mathcal{E}_{K}(\cdot, \cdot), \mathcal{D}_{K}(\cdot, \cdot, \cdot)} \Rightarrow 1\right]-\operatorname{Pr}\left[\mathcal{A}^{\$(\cdot, \cdot), \perp(\cdot, \cdot, \cdot)} \Rightarrow 1\right]\right| \\
\leq & \left|\operatorname{Pr}\left[K \leftarrow \mathcal{K}: \mathcal{A}^{\mathcal{E}_{K}(\cdot, \cdot), \mathcal{D}_{K}(\cdot, \cdot, \cdot)} \Rightarrow 1\right]-\operatorname{Pr}\left[K \leftarrow \mathcal{K}: \mathcal{A}^{\mathcal{E}_{K}(\cdot, \cdot), \perp(\cdot, \cdot, \cdot)} \Rightarrow 1\right]\right| \\
& +\left|\operatorname{Pr}\left[K \leftarrow \mathcal{K}: \mathcal{A}^{\mathcal{E}_{K}(\cdot, \cdot), \perp(\cdot, \cdot, \cdot)} \Rightarrow 1\right]-\operatorname{Pr}\left[\mathcal{A}^{\$(\cdot, \cdot), \perp(\cdot, \cdot,)} \Rightarrow 1\right]\right|
\end{aligned}
$$

For a key $K \stackrel{\$}{\leftarrow} \mathcal{K}$, we design two adversaries $\mathcal{A}_{p}$ and $\mathcal{A}_{c}$ so that

$$
\begin{aligned}
& \left|\operatorname{Pr}\left[\mathcal{A}^{\mathcal{E}_{K}(\cdot, \cdot), \perp(\cdot, \cdot, \cdot)} \Rightarrow 1\right]-\operatorname{Pr}\left[\mathcal{A}^{\mathbb{Q}(\cdot, \cdot), \perp(\cdot, \cdot, \cdot)} \Rightarrow 1\right]\right| \leq \mathbf{A d v} \frac{\sqrt{I N D-C P A}}{\Pi}\left(\mathcal{A}_{p}\right) \\
& \left|\operatorname{Pr}\left[\mathcal{A}^{\mathcal{E}_{K}(\cdot, \cdot), \mathcal{D}_{K}(\cdot, \cdot, \cdot)} \Rightarrow 1\right]-\operatorname{Pr}\left[\mathcal{A}^{\mathcal{E}_{K}(\cdot, \cdot), \perp(\cdot, \cdot, \cdot)} \Rightarrow 1\right]\right| \leq \operatorname{Adv}_{\Pi}^{I N T-C T X T}\left(\mathcal{A}_{c}\right) .
\end{aligned}
$$

$\mathcal{A}_{p}$ : Adversary $\mathcal{A}_{p}$ runs $\mathcal{A}$ and answers $\mathcal{A}$ 's queries to the function Encrypt and Decrypt by using its own Encrypt oracle or returning $\perp$, respectively. $\mathcal{A}_{p}$ outputs whatever $\mathcal{A}$ outputs.
$\mathcal{A}_{c}$ : Adversary $\mathcal{A}_{c}$ runs $\mathcal{A}$, and answers $\mathcal{A}$ 's queries to the function Encrypt by using its own Encrypt oracle. It submits $\mathcal{A}$ 's queries to the Decrypt oracle to its own Verify oracle and, regardless of the response, returns $\perp$. Note that the Verify oracle sets win to true if and only if a fresh Decrypt query of $\mathcal{A}_{c}$ is valid.

### 4.4. Game-Based Proofs

The majority of the upcoming proofs in this paper are based on common game-playing arguments. In this thesis, all games are written in a language similar to $\mathcal{L}$ that was introduced by Bellare and Rogaway in [28]. The basic concept of this proof technique is called game hopping. It is a formalized way to transform a cryptographic scheme into an ideal scheme, e.g., a random function by a series of minor modifications. We denote $G_{0}, \ldots, G_{n}$ as a series of games, where $G_{0}$ denotes the initial game and $G_{n}$ the final game. As usual, our adversary $\mathcal{A}$ has only black-box access to any Game $G_{i}$. Thus, the advantage of $\mathcal{A}$ to distinguish Game $G_{i}$ from Game $G_{j}$ is given by

$$
\operatorname{Adv}_{G_{i}}^{G_{j}}(A)=\left|\operatorname{Pr}\left[\mathcal{A}^{G_{i}} \Rightarrow 1\right]-\operatorname{Pr}\left[\mathcal{A}^{G_{j}} \Rightarrow 1\right]\right|
$$

Game-playing proofs become handy when it is hard to compute $\operatorname{Adv}_{G_{0}}^{G_{n}}(\mathcal{A})$ in a straightforward manner. The difference between subsequent games $G_{i}$ and $G_{i+1}$ is, by construction, easy to compute. Finally, from the common triangle inequality, we have $\operatorname{Adv}_{G_{i}}^{G_{i+2}}(\mathcal{A}) \leq \operatorname{Adv}_{G_{i}}^{G_{i+1}}(\mathcal{A})+\operatorname{Adv}_{G_{i+1}}^{G_{i+2}}(\mathcal{A})$, and thus,

$$
\operatorname{Adv}_{G_{0}}^{G_{n}}(\mathcal{A}) \leq \sum_{i=1}^{n} \operatorname{Adv}_{G_{i-1}}^{G_{i}}(\mathcal{A})
$$

## Misusing Authenticated Encryption Schemes

You are never too old to set another goal or to dream a new dream.

C. S. Lewis

During the past decade, many AE schemes were proposed - usually with a formal proof of their respectiveCCA3 security. Up to now, CCA3 proofs used to rely on two common assumptions: (1) nonce-respecting adversaries, and (2) secure underlying primitives. While both aspects are well-understood in theory, they are hard to guarantee in practice. Thus, security issues were overlooked or ignored in various cases and security applications were put at high risk. In this thesis we highlight two blind spots in the established security definitions: nonce misuse and decryption misuse.

### 5.1. Nonce Misuse

The standard requirement for encryption schemes - authenticated or not - is to prevent leakage of any information about the plaintext except for its length. A stateless deterministic authenticated encryption scheme cannot fulfill this security requirement since an adversary can easily detect, if a plaintext was encrypted multiple times or not. Thus, the user must provide a fresh additional auxiliary input (called nonce) for each encryption. We speak of a nonce misuse, if a nonce value is reused.

In theory, the concept of nonces is simple. In practice, it is challenging to ensure that nonces never repeat. Flawed implementations of nonces are ubiquitous 51, 122, 146, 214, 239], but, apart from implementation failures, cases exist where software
developers cannot always prevent nonce reuse. For example, a persistently stored counter that is increased and written back each time a new nonce is needed may be reset by a backup - usually after some previous data loss. Similarly, the internal and persistent state of an application may be duplicated when a virtual machine is cloned, etc.

Our analysis in Section 6.1]shows that almost all previously published OAE schemes cannot longer ensure the privacy, integrity, or both for encrypted messages when threatened by a nonce-ignoring adversary.

Ideally, an adversary that is given the encryption of two (equal-length) plaintexts $M^{1}$ and $M^{2}$ cannot even decide if $M^{1}=M^{2}$ or not. When a nonce is used more than once, deciding if $M^{1}=M^{2}$ becomes easy. Deterministic encryption schemes, such as SIV [209], ensure that they do not leak any other additional information about plaintexts, even when exposed to a nonce-reusing adversary. In the case of on-line encryption, where the $i$-th ciphertext block is independend of all message blocks $M_{j}$ with $j>i$, it is unavoidably to leak information beyond $M^{1}=M^{2}$. The adversary can compare any pair of ciphertexts for their Longest Common Prefix (LCP), and then derive the longest common prefix of their corresponding plaintexts. We propose to call an (on-line) AE scheme misuse resistant if the only information an adversary can obtain from ciphertexts are their lengths, and the LCP of its plaintexts. In the following we first formally define the length of the LCP.

Definition 5.1 (Length of the Longest Common Prefix (LLCP)). Let $M, M^{\prime} \in\left(\{0,1\}^{n}\right)^{*}$ denote two messages. Then, we define the length of the longest common $n$-prefix of $M$ and $M^{\prime}$ as

$$
\operatorname{LLCP}_{n}\left(M, M^{\prime}\right)=\max _{i}\left\{M_{1}=M_{1}^{\prime}, \ldots, M_{i}=M_{i}^{\prime}\right\} .
$$

For a non-empty set $\mathfrak{Q}$ of elements of $\left(\{0,1\}^{n}\right)^{*}$, we define

$$
\operatorname{LLCP}_{n}(M, \mathfrak{Q})=\max _{X \in \mathfrak{Z}}\left\{\operatorname{LLCP}_{n}(M, X)\right\} .
$$

On-line Permutation (OPerm). We aim for larger permutations that not only permute single blocks but can handle messages of multiple blocks. Such a permutation, from $\{0,1\}^{n a}$ to $\{0,1\}^{n a}$ for $a>1$, is ( $n$-)on-line if the $i$-th block of the output is completely determined by the first $i$ blocks of the input. Let OPerm ${ }_{h}$ denote the set

| Algorithm 3 Random On-Line Permutation Implemented via Lazy Sampling |  |  |
| :---: | :---: | :---: |
| Init () | $\mathfrak{P}(M)$ | $\mathfrak{P}^{-\mathbf{1}( }(M)$ |
| $X_{I} \leftarrow \perp$ | for $i \leftarrow 1, \ldots,\|M\| / n$ do | for $i \leftarrow 1, \ldots,\|C\| / n$ do |
| $Y_{I} \leftarrow \perp$ | $I \leftarrow M_{1}, \ldots, M_{i-1}$ | $I \leftarrow C_{1}, \ldots, C_{i-1}$ |
| $\mathfrak{D}_{I} \leftarrow \emptyset$ | $Z \leftarrow X_{I}\left[M_{i}\right]$ | $Z \leftarrow Y_{I}\left[C_{i}\right]$ |
| $\mathfrak{R}_{I} \leftarrow \emptyset$ | if $Z=\perp$ then | if $Z=\perp$ then |
|  | $Z \underset{\leftarrow}{\leftarrow}\{0,1\}^{n} \backslash \mathfrak{D}_{I}$ | $Z \leftarrow\{0,1\}^{n} \backslash \mathfrak{R}_{I}$ |
|  | $X_{I}\left[M_{i}\right] \leftarrow Z$ | $Y_{I}\left[C_{i}\right] \leftarrow Z$ |
|  | $Y_{I}[Z] \leftarrow M_{i}$ | $X_{I}[Z] \leftarrow C_{i}$ |
|  | $\mathfrak{R}_{I} \leftarrow \mathfrak{R}_{I} \cup\{Z\}$ | $\mathfrak{D}_{I} \leftarrow \mathfrak{D}_{I} \cup\{Z\}$ |
|  | $\mathfrak{D}_{I} \leftarrow \mathfrak{D}_{I} \cup\left\{M_{i}\right\}$ | $\mathfrak{R}_{I} \leftarrow \mathfrak{R}_{I} \cup\left\{C_{i}\right\}$ |
|  | end if | end if |
|  | $C_{i} \leftarrow Z$ | $M_{i} \leftarrow Z$ |
|  | end for | end for |
|  | return $\left(C_{1}, \ldots, C_{\|M\| / n}\right)$ | return $\left(M_{1}, \ldots, M_{\|C\| / n}\right)$ |

of all on-line permutations from $\left(\{0,1\}^{n}\right)^{*}$ to $\left(\{0,1\}^{n}\right)^{*}$. It is easy to extend the definition with a state space $\{0,1\}^{v}$. Let OPermp denote the set of all functions from $\{0,1\}^{v} \times\left(\{0,1\}^{n}\right)^{*} \rightarrow\left(\{0,1\}^{n}\right)^{*}$. Then, for each $\mathfrak{G} \in \mathbf{O P e r m}^{p}$ and $N \in\{0,1\}^{v}$, the function $\mathfrak{G}(N, \cdot)$ is an $(n$-)on-line permutation. We define an On-line Pseudorandom Permutation (OPRP) as a family of $n^{*}$-bit on-line permutations with the property that the input-output behaviour of a randomly chosen member of this family is computationally indistinguishable from a set of $2^{v} n^{*}$-bit random permutations $\mathfrak{P}(\cdot, \cdot) \stackrel{\$}{\stackrel{\text { OPerm}}{n}} \underset{h}{ }$. An efficient lazy-sampling implementation of a random on-line permutation is given in Algorithm 3. Note that in the first iteration the prefix $I$ is always set to the empty string $\epsilon$ since neither $M_{0}$ nor $C_{0}$ exists.
Since AE schemes do not only output ciphertexts but also authentication tags, our encryption oracle has to simulate the computation of an authentication tag by returning a random bitstring that matches the length of the tag. Thus, our encryption oracle $\mathcal{O}^{\mathfrak{F}}$ processes a header-message tuple $(H, M)$ as follows:

1. Compute $C \leftarrow \mathfrak{P}(H, M)$.
2. Compute $T \stackrel{₫}{\leftarrow}\{0,1\}^{r}$.
3. Append some random bits to $C$ if necessary so that the equation $\left|\mathcal{E}_{K}(H, M)\right|=$ $|C|+|T|$ always holds.
4. Finally, output the ciphertext-tag tuple ( $C, T$ ).

To achieve length preserving encryption, i.e., $|M|=|C|$ for all messages $M \in\{0,1\}^{*}$, OAE schemes usually have a special treatment for the last message block, e.g., ciphertext stealing where the final message block $M_{m}$ is padded with the ciphertext block $C_{m-1}$. Thus, it is quite easy to distinguish such an OAE scheme from the encryption oracle $\mathcal{O}^{\mathfrak{F}}$. At first, we can send any encryption query $(H, M)$ with $M=M_{1}, \ldots, M_{m}$ to the encryption oracle. Then we query ( $H, M^{\prime}$ ) where $M^{\prime}=M \| Z$ for any $Z \in\{0,1\}^{n}$. Let $(C, T)$ denote the output of our first query and $\left(C^{\prime}, T^{\prime}\right)$ the output of the second query. Then we output 1 if $C_{m} \neq C_{m}^{\prime}$, and 0 otherwise. Thus, we have to update the definition of the random encryption oracle $\mathcal{O}^{\mathfrak{F}}$ by an intermediate Step 1b: Replace the final ciphertext block $\left(C_{m}\right)$ by a random bitstring when $\mathcal{E}_{K}$ treats the final message block special.

Definition 5.2 (IND-OPRP Advantage). Let $\Pi=(\mathcal{K}, \mathcal{E}, \mathcal{D})$ be an OAE scheme, and let $\mathfrak{P} \stackrel{\$}{\leftarrow}$ OPerm $_{n}^{n}$. Then, we define the IND-OPRP advantage of a nonceignoring adversary $\mathcal{A}$ as

$$
\operatorname{Adv} \frac{\lfloor\mathbb{I N D - O P R P}}{\Pi}(\mathcal{A})=\left|\operatorname{Pr}\left[K \leftarrow \mathcal{K}: \mathcal{A}^{\mathcal{E}_{K}(\cdot, \cdot)} \Rightarrow 1\right]-\operatorname{Pr}\left[\mathcal{A}^{\mathcal{O}^{\mathscr{P}}(\cdot, \cdot)} \Rightarrow 1\right]\right|,
$$

and
as the maximum advantage over all IND-OPRP adversaries that run in time at most $t$, ask a total maximum of $q$ queries to the encryption oracles, and whose total query length is at most $\ell$ blocks.

In the spirit of the CCA3 security definition (cf. Definition 4.1), we introduce the notion of On-line Chosen-Ciphertext Attack 3 (OCCA3) security.

Definition 5.3 (OCCA3 Advantage). Suppose $\Pi=(\mathcal{K}, \mathcal{E}, \mathcal{D})$ is an OAE scheme, and let $\mathfrak{P} \stackrel{\$}{\leftarrow}$ OPerm $_{n}$ be a random on-line permutation. Then, we define the OCCA3 advantage of a nonce-ignoring adversary $\mathcal{A}$ as

$$
\left.\operatorname{Ad} \sqrt{I I}=|\operatorname{PrCA}(\mathcal{A})=| K \leftarrow \mathcal{K}: \mathcal{A}^{\mathcal{E}_{K}(\cdot \cdot,), \mathcal{D}_{K}(\cdot, \cdot, \cdot)} \Rightarrow 1\right]-\operatorname{Pr}\left[\mathcal{A}^{\mathcal{O}^{\mathfrak{F}}(\cdot, \cdot), \perp(\cdot, \cdot,)} \Rightarrow 1\right] \mid,
$$

and

$$
\operatorname{Adv} \sqrt{C_{I I}^{C C A S}}(q, \ell, t)=\max _{\mathcal{A}}\left\{\operatorname{Adv}_{\Pi}^{\sqrt{C C A S}}(\mathcal{A})\right\}
$$

as the maximum advantage over all nonce-ignoring $O C C A 3$ adversaries that run in time at most $t$, ask a total maximum of $q$ queries to the encryption and decryption oracles, and whose total query length is at most $\ell$ blocks.

Using similar arguments as in the proof of Theorem4.4, one can show that for any $(q, \ell, t)$-bounded adversary $\mathcal{A}$, there exists a $(q, \ell, O(t))$-bounded $\mathcal{A}_{p}$ such that

$$
\left|\operatorname{Pr}\left[K \leftarrow \mathcal{K}: \mathcal{A}^{\mathcal{E}_{K}(\cdot, \cdot), \perp(\cdot, \cdot)} \Rightarrow 1\right]-\operatorname{Pr}\left[\mathcal{A}^{\mathcal{O}^{P}(\cdot, \cdot), \perp(\cdot, \cdot)} \Rightarrow 1\right]\right| \leq \mathbf{A d v} \frac{\sqrt{I N D-O P R P}}{\Pi}(\mathcal{A})
$$

Corollary 5.4 (Bound for the OCCA3 Advantage). Suppose $\Pi=(\mathcal{K}, \mathcal{E}, \mathcal{D})$ is an OAE scheme. Then, it holds that

$$
\mathbf{A d v} \frac{O C C A 3}{\Pi}(q, \ell, t) \leq \mathbf{A d v} \frac{\sqrt{I N D-O P R P}}{\Pi}(q, \ell, t)+\mathbf{A d v} \frac{\sqrt{I N T-C T X T}}{\Pi}(q, \ell, t)
$$

### 5.2. Decryption Misuse

The decryption algorithm of an authenticated encryption scheme either outputs a plaintext, or the bot symbol $\perp$, depending on whether a ciphertext is authentic or not. A decryption misuse describes the event that information about the would-be plaintext of an invalid ciphertext leaks. An adversary might use this leaked information to break the privacy (or integrity) of an AE scheme. A generic way to get rid of this problem is the Decrypt-Then-Mask approach by Fouque et al. [105], where the would-be plaintext is blinded after decryption by XORing it with a pseudorandom sequence of bits generated by a Pseudorandom Number Generator (PRNG). After successful authentication, the blinding is removed. Unfortunately, this technique is not applicable in low-end environments since required temporary storage for the decrypted data may just not exist. In high-speed environments, e.g., optical networks, the increased latency for the waiting period that is required until the plaintext authenticity has been established may be prohibitive.

We strive for an authenticated encryption scheme where any change to a valid ciphertext causes its entire post-decryption plaintext to be pseudorandom. Such a scheme is clearly decryption-misuse resistant since the decryption of a manipulated ciphertext results in uncontrollable random noise. Unfortunately, this strong definition of decryption-misuse resistance and on-line encryption are mutually exclusive:

If an adversary manipulates the $i$-th block of a ciphertext, an OAE scheme leaves the previous $(i-1)$ blocks unchanged. Therefore, we will introduce two flavours of decryption-misuse resistance, one for deterministic AE schemes and one for OAE schemes.

Indistinguishability under Chosen-Ciphertext Attack (IND-CCA). For deterministic authenticated encryption schemes, we can adapt the CCA3 notion (see Definition 4.1) by slightly modifying the behavior of the decryption oracle. Let $\widehat{\mathcal{D}}$ denote the faulty version of the decrypt and verify algorithm $\mathcal{D}$, i.e., $\widehat{\mathcal{D}}$ omits the verification and always returns the decryption for authentic as well as for unauthentic ciphertexts. Thus, in the decryption-misuse setting, an adversary has to distinguish $\left(\mathcal{E}_{K}, \widehat{\mathcal{D}}_{K}\right)$ with $K \leftarrow \mathcal{K}$ from two independent random oracles $\left(\$^{\mathcal{E}}, \$^{\mathcal{D}}\right)$. Note that the equation $\left|\mathcal{E}_{K}(H, M)\right|=\left|\mathscr{S}^{\mathcal{E}}(H, M)\right|$ holds for header-message tuple $(H, M)$ and $\left|\widehat{\mathcal{D}}_{K}(H, C, T)\right|=\left|\$^{\mathcal{D}}(H, C, T)\right|$ holds for all header-ciphertext-tag tuples $(H, C, T)$. Furthermore, we assume that an adversary never asks a query for which the corresponding answer is already known. Then, we define the IND-CCA advantage as follows:

Definition 5.5 (IND-CCA Advantage). Let $\Pi=(\mathcal{K}, \mathcal{E}, \mathcal{D})$ be a determinisitc AE scheme, and let the faulty decryption oracle $\widehat{\mathcal{D}}$ be defined as above. Then, we define the IND-CCA advantage of a nonce-respecting adversary $\mathcal{A}$ in breaking $\Pi$ with $K \leftarrow \mathcal{K}$ as

$$
\mathbf{A d v} \sqrt{\Pi I N D-C C A}(\mathcal{A})=\left|\operatorname{Pr}\left[\mathcal{A}^{\mathcal{E}_{K}(\cdot, \cdot), \widehat{\mathcal{D}}_{K}(\cdot, \cdot, \cdot)} \Rightarrow 1\right]-\operatorname{Pr}\left[\mathcal{A}^{\mathscr{S}^{\mathcal{E}}(\cdot, \cdot), \$^{\mathcal{D}}(\cdot, \cdot, \cdot)} \Rightarrow 1\right]\right|
$$

and

$$
\operatorname{Adv} \frac{\frac{I N D-C C A}{\Pi}}{\|}(q, \ell, t)=\max _{\mathcal{A}}\left\{\mathbf{A d v} \frac{\sqrt{I N D-C C A}}{\Pi}(\mathcal{A})\right\}
$$

as the maximum advantage over all nonce-respecting IND-CCA adversaries that run in time at most $t$, ask a total maximum of $q$ queries to the encryption and decryption oracles, and whose total query length is at most $\ell$ blocks.

The following Lemma discloses the relation between the IND-CPA and the IND-CCA security notions.

Lemma 5.6 (IND-CCA $\Longrightarrow$ IND-CPA). Let $\Pi=(\mathcal{K}, \mathcal{E}, \mathcal{D})$ be a deterministic AE scheme, and let $\mathcal{A}$ be an IND-CPA adversary that runs in time $t$, and makes $q$ queries with a total length of at most $\ell$ blocks. Then, there exists an IND-CCA adversary $\mathcal{A}^{\prime}$ such that

$$
\mathbf{A d v} \frac{\sqrt{I N D-C P A}}{\Pi}(\mathcal{A}) \leq \mathbf{A d v} \frac{\square I N D-C C A}{\Pi I}\left(\mathcal{A}^{\prime}\right)
$$

Proof. The adversary $\mathcal{A}^{\prime}$ runs $\mathcal{A}$ and answers $\mathcal{A}$ 's queries to its encryption oracle. Furthermore, $\mathcal{A}^{\prime}$ outputs whatever $\mathcal{A}$ outputs.

Indistinguishability under On-Line Chosen-Ciphertext Attack (IND-OCCA). We can adapt the IND-CCA notion by replacing the random oracle for decryption $\left(\$^{\mathcal{D}}\right)$ by a random permutation-based oracle $\widehat{\mathcal{O}}^{P}$ with $\mathfrak{P} \stackrel{\$}{\leftarrow}$ OPerm $_{n}^{n+}$. The construction of this new oracle is quite similar to the construction of the OPerm-based encryption oracle $\mathcal{O}^{\mathfrak{P}}$ from Section 5.1. Thus, $\widehat{\mathcal{O}}^{\mathfrak{P}}$ processes a header-ciphertext-tag tuple $(H, C, T)$ as follows: (1) It computes the plaintext $M \leftarrow \mathfrak{P}(H, C)$ and if necessary, replaces the final message block with random bits. (2) It appends random bits to $M$ if necessary, so that the equation $\left|\widehat{\mathcal{D}}_{K}(H, C, T)\right|=|M|$ always holds, and finally, outputs $M$.

Definition 5.7 (IND-OCCA Advantage). Let $\Pi=(\mathcal{K}, \mathcal{E}, \mathcal{D})$ be an OAE scheme. Then, we define for $\mathfrak{P} \stackrel{\$ \text { OPerm }_{n}^{n+}}{\leftarrow}$ the IND-OPRPadvantage of a nonce-respecting adversary $\mathcal{A}$ as

$$
\mathbf{A d} \sqrt{I N D-O C C A}(\mathcal{A})=\left|\operatorname{Pr}\left[K \leftarrow \mathcal{K}: \mathcal{A}^{\mathcal{E}_{K}(\cdot, \cdot), \widehat{\mathcal{D}}(\cdot, \cdot,)} \Rightarrow 1\right]-\operatorname{Pr}\left[\mathcal{A}^{\mathbb{I}(\cdot, \cdot), \widehat{\mathcal{O}} \mathfrak{F}(\cdot, \cdot, \cdot)} \Rightarrow 1\right]\right|
$$

and

$$
\mathbf{A d} \sqrt{\frac{I N D-O C C A}{\Pi}}(q, \ell, t)=\max _{\mathcal{A}}\left\{\mathbf{A d v} \sqrt{\frac{I N D-O C C A}{\Pi}}(\mathcal{A})\right\}
$$

as the maximum advantage over all $\overline{I N D-O C C A}$ adversaries that run in time at most $t$, ask a total maximum of $q$ queries to the encryption and decryption oracles, and whose total query length is at most $\ell$ blocks.

Remark. In 2014, Andreeva et al. introduced the notion of integrity of unverified plaintext release INT-RUP [6]. In their model, an adversary $\mathcal{A}$ has access to an AE
scheme $\Pi=(\mathcal{K}, \mathcal{E}, \mathcal{D}, \mathcal{V})$ with separate decryption and verification algorithms $\mathcal{D}$ and $\mathcal{V}$, respectively. $\mathcal{A}$ wins if it can forge. Furthermore, the authors also introduced the notion of plaintext awareness PA , where an adversary has to distinguish between the real and a simulated world, where the decryption oracle is replaced by a simulator $S$ that has no access to the secret key. In addition, Andreeva et al. propose two different notion of plaintext awareness, named PA1 and PA2. Both differ in the fact that the simulator has aceess to the query history of $\mathcal{E}_{K}$ in the former notion. Note that PA2 security implies PA1 security since every PA2-simulator is also a PA1-simulator.

We want to emphasize that despite the close relations of our work to that by [6], both are results of completely independent efforts. Analyzing the relations among our and their notions and unifying them is still an open research topic.

### 5.3. Robustness

This section concludes this chapter by answering the question whether an authenticated encryption scheme in this thesis is considered robust or not.

Deterministic Authenticated Encryption Schemes. It is not hard to tell if a deterministic AE scheme $\Pi=(\mathcal{K}, \mathcal{E}, \mathcal{D})$ is robust or not. It is robust iff it is CCA3. secure in the nonce-respecting and nonce-misuse setting, and IND-CCA-secure in the decryption-misuse setting. The following corollary upper bounds the Nonce- and Decryption-Misuse Attack (NDMA) advantage of a nonce-ignoring adversary $\mathcal{A}$. It can be derived from Theorem 4.4, which tells us that IND-CPA plus INT-CTXT security implies CCA3 security, and Lemma 5.6, which states that IND-CCA security implies IND-CPA security. Thus, NDMA security implies robustness.

Corollary 5.8 (NDMA Advantage). Let $\Pi=(\mathcal{K}, \mathcal{E}, \mathcal{D})$ be a deterministic $A E$ scheme. Let $\mathcal{A}$ be a nonce-ignoring NDMA adversary that runs in time $t$, and asks at most $q$ queries with a total length of at most $\ell$ blocks. Then, there exists a nonceignoring IND-CCA adversary $\mathcal{A}_{p}$ and a nonce-ignoring INT-CTXT adversary $\mathcal{A}_{c}$ such that

$$
\mathbf{A d v} \frac{{ }_{\Pi D M A}^{\Pi}}{}(\mathcal{A}) \leq \mathbf{A d v} \frac{\sqrt{I N D-C C A}}{\Pi}\left(\mathcal{A}_{p}\right)+\mathbf{A d v} \frac{\sqrt{I N T-C T X T}}{\Pi}\left(\mathcal{A}_{c}\right)
$$

where both $\mathcal{A}_{p}$ and $\mathcal{A}_{c}$ run in time $O(t)$ and make at most $q$ queries with a total length of at most $\ell$ blocks.

On-Line Authenticated Encryption Schemes. We denote an OAE scheme $\Pi=$ $(\mathcal{K}, \mathcal{E}, \mathcal{D})$ robust iff it is (1)CCA3-secure in the nonce-respecting setting, (2) OCCA3secure in the nonce-misuse setting, and (3) IND-OCCA-secure in the decryptionmisuse setting. In the following we introduce the definition of an Indistinguishability under On-Line Chosen-Ciphertext Attack 2 (IND-OCCA2) advantage, which is a generalisation of the IND-OCCA advantage by replacing the nonce-respecting adversary with a nonce-ignoring adversary. It is basically the same as the generalisation of the IND-CPA advantage by introducing the IND-OPRP advantage. For the following definition, we borrow the notion of the encryption oracle $\mathcal{O}^{P}$ and the decryption oracle $\widehat{\mathcal{D}}$ from the Sections 5.1 and 5.2, respectively.

Definition 5.9 (IND-OCCA2 Advantage). Let $\Pi=(\mathcal{K}, \mathcal{E}, \mathcal{D})$ be an OAE scheme. Then, we define for $\mathfrak{P} \stackrel{\$}{\leftarrow}$ OPerm $_{h}^{+}$the IND-OCCA2 advantage of $a$ nonce-ignoring adversary $\mathcal{A}$ as
$\operatorname{Adv} \frac{\text { IND-OCCAQ}}{1 I}(\mathcal{A})=\left|\operatorname{Pr}\left[K \leftarrow \mathcal{K}: \mathcal{A}^{\mathcal{E}_{K}(\cdot, \cdot), \widehat{\mathcal{D}}(\cdot, \cdot,)} \Rightarrow 1\right]-\operatorname{Pr}\left[\mathcal{A}^{\mathcal{O}^{P}(\cdot, \cdot), \widehat{\mathcal{O}}^{P}(\cdot, \cdot,)} \Rightarrow 1\right]\right|$, and

$$
\operatorname{Adv} \sqrt{\frac{I N D-O C C A 2}{}}(q, \ell, t)=\max _{\mathcal{A}}\left\{\operatorname{Adv}_{\Pi}^{\left.\frac{\sqrt{I N D-O C C A 2}}{}(\mathcal{A})\right\}, ~}\right.
$$

as the maximum advantage over all IND-OCCA2 adversaries that run in time at most $t$, ask a total maximum of $q$ queries to the encryption and decryption oracles, and whose total query length is at most $\ell$ blocks.

It is easy to see that IND-OCCA2 securtiy implies IND-OPRP security by using similar arguments as in the proof of Lemma 5.6. Therefore, the notion of IND-OCCA2 covers nonce-misuse restricted to data privacy. Now, we put all bits and pieces of this Chapter together and unite them to the following definition of the On-line Nonceand Decryption-Misuse Attack (ONDMA) advantage:

Definition 5.10 (ONDMA Advantage). Let $\Pi=(\mathcal{K}, \mathcal{E}, \mathcal{D})$ be an OAE scheme. Then, we define the ONDMA advantage of a nonce-ignoring adversary $\mathcal{A}$ as

$$
\operatorname{Adv} \frac{O N D M A}{\Pi}(\mathcal{A})=\operatorname{Ad} \sqrt{\Pi I N D-O C C A 2}(\mathcal{A})+\operatorname{Adv} v_{\Pi}^{\frac{I N T-C T X T}{}}(\mathcal{A}),
$$

and
as the maximum advantage over all $O N D M A$ adversaries that run in time at most $t$, ask a total maximum of $q$ queries to the encryption and decryption oracles, and whose total query length is at most $\ell$ blocks.

Note that any random OPerm which is only queried once cannot be distinguished from a random function since it is impossible to exploit the common-prefix characteristic of an on-line permutation. Thus, for any nonce-respecting adversary, it is impossible to distinguish the IND-CPA from the IND-OPRP setting. This implies that we can call an ONDMA-secure OAE scheme robust.

## Part II

## Design, Analysis, and Usage of Authenticated Encryption Schemes

## Robustness of Authenticated Encryption Schemes

To invent, you need a good imagination and a pile of junk.

Thomas A. Edison

### 6.1. Nonce-Misuse Resistance

In this section we analyze the nonce-misuse resistance of existing OAE schemes. Note that none of them claims nonce-misuse resistance, and efficient misuse attack does not automatically invalidate the security of those schemes. Nevertheless, ensuring that an implemented authenticated encryption scheme is resistant against misuse attacks is a difficult task for implementors, especially when the software is running in environments like inside a virtual machine.

A summary of our analysis including a brief discussion is given at the end of this chapter.

### 6.1.1. Generic Attacks

In the following we introduce two generic attack patterns on which the majority of the nonce-misuse attacks are based on.

Repeated-Keystream Attack Pattern. Assume that the encryption routine $\mathcal{E}$ of an on-line authenticated encryption scheme $\Pi=(\mathcal{K}, \mathcal{E}, \mathcal{D})$ generates a keystream $S=F_{K}(N)$ of length $|M|$, i.e., $|S|=|M|$, depending on a secret key $K$ and a nonce
$N$. The ciphertext of a message $M$ is typically computed by $C=S \oplus M$, where $S$ is generated by applying a block cipher in counter mode [30, 147, 164]. Assume that $\mathcal{A}$ is a nonce-ignoring IND-OPRP adversary, with access to an encryption oracle, that tries to distinguish real from random by comparing the difference of two single-block messages with that of their ciphertexts. It easy to see that $\mathcal{A}$ 's advantage is almost $1-2^{-n}$. A formal definition of $\mathcal{A}$ is given in Algorithm (4)

```
Algorithm 4 Repeated-Keystream Adversary
    \((C, T) \leftarrow \mathcal{O}(M, N) \quad\left\{\right.\) first encryption query with \(\left.\left.(N, M) \stackrel{\$}{\leftarrow}\{0,1\}^{v} \times\{0,1\}^{n}\right)\right\}\)
    \(\left(C^{\prime}, T^{\prime}\right) \leftarrow \mathcal{O}\left(M^{\prime}, N\right) \quad\left\{\right.\) second encryption query with \(\left.\left(M^{\prime} \stackrel{\$}{\leftarrow}\{0,1\}^{n} \backslash M\right)\right\}\)
    return \(\left(M \oplus M^{\prime}=C \oplus C^{\prime}\right)\)
```

Linear-Tag Attack Pattern. Common stateful authenticated encryption schemes such as Galois/Counter Mode (GCM) [164] or Counter with CBC-MAC (CCM) 85], apply the Encrypt-then-Mac paradigm (cf. Section4.2), i.e., they compute a tag $T$ by:

$$
T=F_{K}(N) \oplus G_{K}(M)
$$

where $N$ is the nonce, $M$ is the plaintext, $F_{K}$ is a key-dependent function and $G_{K}$ is a key-dependent permutation with $C=G_{K}(M)$. Suppose $\Pi=(\mathcal{K}, \mathcal{E}, \mathcal{D})$ is such a stateful scheme. This enables an efficient nonce-ignoring adversary $\mathcal{A}$ to mount an INT-CTXT attack with an advantage of 1 by solving a simple linear equation system. A formal definition of $\mathcal{A}$ is given in Algorithm 5 ,

```
Algorithm 5 Linear-Tag Adversary
    \((C, T) \leftarrow \mathcal{E}(N, M) \quad\left\{\right.\) first encryption query with \(\left.\left.(N, M) \stackrel{\$}{\leftarrow}\{0,1\}^{v} \times\{0,1\}^{n}\right)\right\}\)
    \(\left(C^{\prime}, T^{\prime}\right) \leftarrow \mathcal{E}\left(N^{\prime}, M^{\prime}\right)\left\{\right.\) second encryption query with \(\left.\left(N^{\prime} \neq N\right) \wedge\left(M^{\prime} \neq M\right)\right\}\)
    \(\left(C^{\prime \prime}, T^{\prime \prime}\right) \leftarrow \mathcal{E}\left(N, M^{\prime}\right)\left\{\right.\) third encryption query with \(\left.T^{\prime \prime}=F_{K}(N) \oplus G_{K}\left(M^{\prime}\right)\right\}\)
    return \(\mathcal{D}\left(N^{\prime}, C, T \oplus T^{\prime} \oplus T^{\prime \prime}\right)\left\{\right.\) forgery since \(\left.T \oplus T^{\prime} \oplus T^{\prime \prime}=F_{K}\left(N^{\prime}\right) \oplus G_{K}(M)\right\}\)
```


### 6.1.2. Misuse Attacks against Previously Published Authenticated Encryption Schemes

CWC, GCM, CCM, EAX, and CHM. Usually, common two-pass OAE schemes, Carter-Wegman Counter (CWC) [147], GCM [164], CCM [85], EAX [30], and CENC
with Hash-based MAC (CHM) [124), use the Counter (CTR) mode as their underlying encryption operation $30,85,124,164]$. These schemes are vulnerable to repeated-keystream attacks. Four of them, CHM, CWC, GCM, and EAX, are designed according to the Encrypt-then-Mac paradigm, and are thus vulnerable to the linear-tag attacks. The designers of CCM follow the Mac-then-Encrypt approach, which seems to defend against linear-tag attacks. Though, forgery attacks against CCM were presented in [106].

RPC. Related Plaintext Chaining (RPC) [55] combines two coommen modes of operations: CTR and Electronic Codebook (ECB). Given an $n$-bit block cipher $E$ under a key $K$ and a $v$-bit nonce $N$, RPC takes an $(n-v)$-bit plaintext block $M_{i}$ and computes the ciphertext block

$$
C_{i} \leftarrow E_{K}\left(M_{i} \|(N+i) \bmod 2^{v}\right) .
$$

Authentication is performed locally for each ciphertext block: During decryption, RPC computes $\left(M_{i} \| X_{i}\right)=E_{K}^{-1}\left(C_{i}\right)$ and accepts $M_{i}$ as authentic iff

$$
X_{i}=(N+i) \bmod 2^{v} .
$$

In the nonce-misuse setting, the same sequence of counter values is used for different messages. This makes it easy to attack the privacy - especially when encrypting messages of $m \cdot(n-v)$-bit blocks. Then, RPC degrades into $m$ independent electronic code books.

More precisely, any adversary that obtains two authentic $m$-block ciphertexts, $\left(C_{1}^{0}, \ldots, C_{m}^{0}\right)$ and $\left(C_{1}^{1}, \ldots, C_{m}^{1}\right)$ with the same nonce $N$, can forge $2^{m}$ new authentic ciphertexs $\left(C_{1}^{\sigma(1)}, \ldots, C_{m}^{\sigma(m)}\right)$ with $\sigma(i) \in\{0,1\}$ since authenticity is verified locally for each $C_{i}^{\sigma(i)}$.

CCFB. Similar to RPC, the Counter-CipherFeedback (CCFB) mode [156] is a combination of CTR and Cipher Feedback (CFB) mode. Given an $(n-a)$-bit nonce $N$ and $(n-a)$-bit plaintext blocks $M_{1} \ldots, M_{m}$ CCFB works as follows:

Initial Step: $C_{0} \leftarrow N$
Encryption: For $i \in\{1, \ldots, m\}:\left(Z_{i} \| T_{i}\right) \leftarrow E_{K}\left(C_{i-1} \| i\right), \quad C_{i} \leftarrow M_{i} \oplus Z_{i}$
Authentication: $\left(*, T_{m+1}\right) \leftarrow E_{K}\left(C_{m} \| m+2\right), \quad T \leftarrow \bigoplus_{1}^{m+1} T_{i}$

Note that the first ciphertext block $C_{1}$ is essentially the plain encryption of $M_{1}$ in CTR Mode. Thus, a variant of the repeated-keystream attack (cf. Algorithm (4) is also applicable to CCFB. Moreover, the following variant of the linear-tag attack pattern (cf. Algorithm 5) applies to CCFB.

1. Encrypt the plaintext $M_{1}$ under $N$ to $\left(C_{1}, T\right)$.
2. Encrypt the plaintext $M_{1}^{\prime} \neq M_{1}$ under $N^{\prime} \neq N$ to $\left(C_{1}^{\prime}, T^{\prime}\right)$.
3. Set $M_{1}^{\prime \prime} \leftarrow M_{1}^{\prime} \oplus C_{1}^{\prime} \oplus C_{1}$. Encrypt $M_{1}^{\prime \prime}$ under $N^{\prime}$ to $\left(C_{1}^{\prime \prime}, T^{\prime \prime}\right)$. Observe $C_{1}^{\prime \prime}=C_{1}$.
4. The triple $\left(N, C_{1}^{\prime}, T \oplus T^{\prime} \oplus T^{\prime \prime}\right)$ is a valid forgery.

IAPM, OCB1-3, and TAE. Given a nonce $N$ and a secret key $K$, Integrity Aware Parallelizable Mode (IAPM) [130] encrypts a message $M=\left(M_{1}, \ldots, M_{m}\right)$ to a ciphertext $C=\left(C_{1}, \ldots C_{m}\right)$ and an authentication tag $T$ as follows.

Initial Step: Generate $m+2$ pseudorandom values $s_{0}, s_{1}, \ldots, s_{m+1}$ depending on $N$ and $K$, but not on the message $M$.

Encryption: For $i \in\{1, \ldots, m\}: C_{i} \leftarrow E_{K}\left(M_{i} \oplus s_{i}\right) \oplus s_{i}$.
Authentication: $T \leftarrow E_{K}\left(s_{m+1} \oplus \widehat{M}\right) \oplus s_{0}$ with $\widehat{M}=\bigoplus_{i=1}^{m} M_{i}$.
When encrypting messages of $m$ blocks, IAPM behaves like a set of $m$ independent instances of the common ECB mode. Hence, IAPM is vulnerable to the same forgery attack as the one that applies to RPC, An adversary who can encrypt two messages $M$ and $M^{\prime}$ under the same nonce only has to take care that they produce the same checksum $\widehat{M}=\widehat{M^{\prime}}$ to create a valid forgery since than we have

$$
T=E_{K}\left(s_{m+1} \oplus \widehat{M}\right) \oplus s_{0}=E_{K}\left(s_{m+1} \oplus \widehat{M^{\prime}}\right) \oplus s_{0}
$$

All three versions of the Offset Codebook (OCB) mode family (i.e., OCB1 208], OCB 2 [205], and OCB 3 [148]) and Tweakable Authenticated Encryption (TAE) [153] work similar to IAPM, and thus the same attack also applies to them.

IACBC. Given a nonce $N$ and a secret key $K$, Integrity Aware Cipher Block Chaining Mode (IACBC) [130] encrypts the message $M=\left(M_{0}, \ldots, M_{m}\right)$ to $C=$ $\left(C_{1}, \ldots, C_{m}\right)$ and an authentication tag $T$ as follows:

Initial Step: Generate $m+1$ values $s_{0}, s_{1}, \ldots s_{m}$ depending on $N$ and $K$, but not on the Message $M$ and compute $C_{0} \leftarrow x_{0} \leftarrow E_{K}(N)$.

Encryption: For $i \in\{1, \ldots, m\}: x_{i} \leftarrow E_{K}\left(M_{i} \oplus x_{i-1}\right), C_{i} \leftarrow x_{i} \oplus s_{i}$.
Authentication: $T \leftarrow E_{K}\left(x_{m} \oplus \widehat{M}\right) \oplus s_{0}$ with $\widehat{M}=\bigoplus_{i=1}^{m} M_{i}$.
The following attack distinguishes IACBC from a random on-line permutation and also provides an existential forgery.

1. Encrypt $M_{1}$ under the nonce 0 to $\left(C_{0}, C_{1}, T\right)$.
2. Encrypt the nonce 0 under $M=\left(C_{0}, C_{0}, C_{0}, C_{0}\right)$ to $\left(C^{\prime}, T^{\prime}\right)$.
3. Set $C^{\prime \prime}=\left(C_{0}, C_{1}^{\prime}, C_{2}^{\prime}, T^{\prime}\right)$

Note that $\left(C^{\prime \prime}, T\right)$ is a valid encryption of $M=\left(C_{0}, C_{0}\right)$ since it holds that $C_{0} \oplus C_{0}=C_{0} \oplus C_{0} \oplus C_{0} \oplus C_{0}$.

XCBC-XOR. Given a nonce $N$ and secret keys $K$ and $K^{\prime}$, eXtended Ciphertext Block Chaining with XOR (XCBC-XOR) [113] encrypts a message $M=\left(M_{1}, \ldots, M_{m}\right)$ to a ciphertext $C=\left(C_{1}, \ldots, C_{m}\right)$ and an authentication tag $T$ as follows:

Initial Step: Generate $m+1$ values $s_{0}, \ldots, s_{m}$ depending on $N$ and $K^{\prime}$, but not on the plaintext $\left(M_{1}, \ldots, M_{m}\right)$.

## Encryption:

1. $C_{0} \leftarrow E_{K}(N) ; x_{0} \leftarrow E_{K^{\prime}}(N) ;$
2. For $i \in\{1, \ldots, m\}: x_{i} \leftarrow E_{K}\left(M_{i} \oplus x_{i-1}\right), C_{i} \leftarrow\left(x_{i}+s_{i}\right) \bmod 2^{n}$.

Authentication: $T \leftarrow E_{K}\left(\widehat{M} \oplus x_{m}\right)+s_{0}\left(\bmod 2^{n}\right)$ with $\widehat{M}=x_{0} \cdot \bigoplus_{i=1}^{m} M_{i}$.
The following attack provides an existential forgery:

1. Encrypt the message $\left(0^{n}, 0^{n}, 0^{n}\right)$ under the nonce $N$ to $\left(C_{0}, C_{1}, C_{2}, C_{3}, T\right)$.
2. Then, $\left(C_{0}, C_{1}, C_{2}, T^{\prime}=C_{3}\right)$ is a valid forgery.

The best IND-PRP attack we found for XCBC-XOR has a workload of $O\left(2^{n / 4}\right)$ instead of $O(1)$. Note that for this reuse-nonce chosen-plaintext attack, we ignore the authentication tag.

1. Generate $2^{n / 4}$ encryptions of messages $M_{1}^{\alpha}$ under the same nonce $N$ to $C_{1}$. Let denote $C_{1}^{\alpha}$ the encryption of the $\alpha$-th message $M_{1}^{\alpha}$ with $\alpha=1, \ldots, n / 4$.

Statistically, we can expect one pair $\left(M_{1}^{i}, M_{1}^{j}\right)$ with $i \neq j$ such that the least significant $n / 2$ bits of $C_{1}^{i}$ and $C_{1}^{j}$ are equal.
2. Generate $2^{n / 4}$ encryptions of messages $\left(M_{1}^{i}, M_{2}^{\alpha}\right)$ and $\left(M_{1}^{j}, M_{2}^{\alpha}\right)$ under $N$, where the $n / 2$ least significant bits of all message blocks $M_{2}^{\alpha}$ are equal.

Statistically, we can expect one pair $\left(M_{2}^{k}, M_{2}^{\ell}\right)$ with $k \neq \ell$ such that $C_{2}^{k}=C_{2}^{\ell}$ holds.
3. Choose an arbitrary $M_{3}$. Encrypt $\left(M_{1}^{i}, M_{2}^{k}, M_{3}\right)$ and $\left(M_{1}^{j}, M_{2}^{\ell}, M_{3}\right)$ under $N$ to $\left(C_{1}^{i}, C_{2}^{k}, C_{3}^{i, k}\right)$ and $\left(C_{1}^{j}, C_{2}^{\ell}, C_{3}^{j, \ell}\right)$.

Observe $C_{3}^{i, k}=C_{3}^{j, \ell}$.

### 6.2. Decryption-Misuse Resistance

In this section we analyze the decryption-misuse resistance of previously published authenticated encryption schemes. Similar to the nonce-misuse setting, none of them claims decryption-misuse resistance. Hence, our presented attacks do not invalidate their claimed security.

BTM, CCM, CHM, CWC, EAX, GCM, HBS, and [SIV These schemes use the CTR mode as their underlying encryption operation. Thus, they are vulnerable to decryption-misuse attacks. Assume $(C, T)$ is the encryption of $M$. Then, we can determine the would-be plaintext $M^{\prime}$ of the unauthentic tuple $\left(C^{\prime}, T\right)$ since - for the same counter value - it must hold that $C \oplus C^{\prime}=M \oplus M^{\prime}$. This observation can be exploited by an efficient IND-CCA adversary.

IAPM, OCB1-3, RPC, and TAE. These OAE schemes behave like the ECB mode and are therefore vulnerable in the decryption-misuse setting. Assume two unauthentic ciphertexts that only differ in the $i$-th block. The decryptions of those produce two messages that also differ only in the $i$-th block. Thus, these schemes are not IND-CCA-secure.

CCFB. This mode generates a keystream based on the previous ciphertext block and a counter:

$$
C_{i}=E_{K}\left(C_{i-1} \| i\right) \oplus M_{i} .
$$

Thus, it must hold that $C_{i} \oplus C_{i}^{\prime}=M_{i} \oplus M_{i}^{\prime}$ for $C_{i-1}=C_{i-1}^{\prime}$. This observation can be exploited by an IND-CCA adversary.
$\triangle A C B C$ and XCBC-XOR Both schemes are based on the concept of the Cipher Block Chaining (CBC) mode, i.e., the $i$-th message block $M_{i}$ depends only on the ciphertext blocks $C_{i-1}$ and $C_{i}$. Therefore, the decryption of the two nonce-ciphertexttag triples $\left(N, C_{1}\left\|C_{2}\right\| C_{3}, T\right)$ and ( $\left.N, C_{1}^{\prime}\left\|C_{2}\right\| C_{3}, T\right)$ with $C_{1} \neq C_{1}^{\prime}$ produces two messages that share the same final message block. In general, schemes based on the CBC approach are not IND-CCA secure in the decryption-misuse setting.

COPA. In 2013, Andreeva et al. introduced COPA [7], a nonce-misuse resistant and parallelizable OAE scheme inspired by McOE (cf. Chapter 8). It combines the XOR-Encrypt-XOR (XEX) encryption with the Encrypt-Mix-Encrypt (EME) approach. Therefore, two block-cipher calls are needed to process a single message block.

Initial Step: $Y_{0} \leftarrow E_{K}(N), L \leftarrow E_{K}(0), \Delta_{0}=3 L$, and $\Delta_{1}=2 L$.
Encryption: For $i \in\{1, \ldots, m\}: X_{i} \leftarrow E_{K}\left(M_{i} \oplus 2^{i-1} \Delta_{0}\right), Y_{i} \leftarrow X_{i} \oplus Y_{i-1}$,

$$
C_{i} \leftarrow E_{K}\left(Y_{i}\right) \oplus 2^{i-1} \Delta_{1} .
$$

Tag Generation: $X_{m+1} \leftarrow E_{K}\left(\widehat{M} \oplus 2^{m-1} 3^{2} L\right), \quad Y_{m+1} \leftarrow X_{m+1} \oplus Y_{m}$ with $\widehat{M}=$ $\bigoplus_{i=1}^{m} M_{i}$, and

$$
T \leftarrow E_{K}\left(Y_{m+1}\right) \oplus 2^{m-1} 7 L
$$

Let $M_{a} \neq M_{b}$ be two distinct message blocks. Then, we define $Y_{a}=E_{K}\left(M_{a} \oplus \Delta_{0}\right) \oplus$ $L \oplus Y_{0}$ and $Y_{b}=E_{K}\left(M_{b} \oplus \Delta_{0}\right) \oplus L \oplus Y_{0}$.

1. Encrypt ( $N, M_{a}, M_{c}$ ) to $\left(C_{a}, C_{(a, c)}\right)$ with

$$
\begin{gathered}
X_{c}=E_{K}\left(M_{c} \oplus 2 \Delta_{0}\right), \\
Y_{(a, c)}=Y_{a} \oplus X_{c}, \text { and } \\
C_{(a, c)}=E_{K}\left(Y_{(a, c)}\right) \oplus 2 \Delta_{1} .
\end{gathered}
$$

2. Encrypt ( $N, M_{b}, M_{c}$ ) to $\left(C_{b}, C_{(b, c)}\right)$ with

$$
\begin{gathered}
X_{c}=E_{K}\left(M_{c} \oplus 2 \Delta_{0}\right), \\
Y_{(b, c)}=Y_{b} \oplus X_{c}, \text { and } \\
C_{(b, c)}=E_{K}\left(Y_{(b, c)}\right) \oplus 2 \Delta_{1} .
\end{gathered}
$$

3. Decrypt $\left(C_{a}, C_{(b, c)}\right)$ to $\left(M_{a}, M_{(a, b c)}\right)$. It applies that

$$
\begin{gathered}
Y_{(b, c)}=E_{K}^{-1}\left(C_{(b, c)} \oplus 2 \Delta_{1}\right), \text { and } \\
X_{(a, b c)}=Y_{(b, c)} \oplus Y_{a}=Y_{b} \oplus X_{c} \oplus Y_{a}
\end{gathered}
$$

4. Decrypt $\left(C_{b}, C_{(a, c)}\right)$ to $\left(M_{b}, M_{(b, a c)}\right)$. It applies that

$$
\begin{gathered}
Y_{(a, c)}=E_{K}^{-1}\left(C_{(a, c)} \oplus 2 \Delta_{1}\right), \text { and } \\
X_{(b, a c)}=Y_{(a, c)} \oplus Y_{b}=Y_{a} \oplus X_{c} \oplus Y_{b}=X_{(a, b c)} .
\end{gathered}
$$

From $X_{(a, b c)}=X_{(b, a c)}$ follows that $M_{(a, b c)}=M_{(b, a c)}$. This observation can be used to distinguish COPA from a random OPERM with probability $1-2^{-n}$.

In the following we extend the IND-OPRP attack on COPA into an INT-CTXT attack. The first three queries of this attack are identical to those in the IND-OPRP attack. With their help, we can form a collision in the chaining values for two messages $\left(M_{a}, M_{(a, b c)}\right)$ and $\left(M_{b}, M_{(a, b c)}\right)$ since it must hold that $X_{(a, b c)}=X_{(b, a c)}$. Thus, we can apply a common length-extension attack to create an existential forgery:
4. Encrypt $\left(M_{a}, M_{(a, b c)}, M_{d}\right)$ to $\left(C_{a}, C_{(a, b c)}, C_{d}, T\right)$, where $T$ is the authentication tag.
5. Then, craft the existential forgery $\left(C_{a}, C_{(b, a c)}, C_{d}, T\right)$.

### 6.3. Results Summary

As it turned out, we actually found nonce and decryption-misuse attacks for all previously published OAE schemes. Table 6.1 summarizes our results. For over a decade, it has been common knowledge how to design a secure, an efficient, and stateless MAC [17, 125]. Therefore, it is surprising that none of the analyzed online schemes provide integrity protection. Only CCM offers some weak integrity protection against nonce-ignoring adversaries, but it does not provide any privacy protection.

Outlook. In the following we present two novel families of OAE scheme: COFFE and McOE. Their designs were inspired by the results of our robustness studies that we summarized in this chapter. The former scheme is the first OAE scheme based on a hash function. It suits very well for resource-restricted devices and offers [INT-CTXT] security even in the nonce-misuse scenario. MCOE is the first published robust OAE scheme.

| Scheme |  | Nonce Misuse |  | Decryption Misuse |
| :--- | :--- | :--- | :--- | :--- |
|  |  | privacy | integrity |  |
| on-line |  |  |  |  |
| CCFB | $[156]$ | $O(1)$ | $O(1)$ | $O(1)$ |
| CHM | $[124]$ | $O(1)$ | $O(1)$ | $O(1)$ |
| COPA | $[7]$ | N/A | N/A | $O(1)$ |
| CWC | $[147]$ | $O(1)$ | $O(1)$ | $O(1)$ |
| EAX | $[30]$ | $O(1)$ | $O(1)$ | $O(1)$ |
| GCM | $[164]$ | $O(1)$ | $O(1)$ | $O(1)$ |
| IACBC | $[130]$ | $O(1)$ | $O(1)$ | $O(1)$ |
| IAPM | $[130]$ | $O(1)$ | $O(1)$ | $O(1)$ |
| OCB 1 | $[208]$ | $O(1)$ | $O(1)$ | $O(1)$ |
| OCB 2 | $[205]$ | $O(1)$ | $O(1)$ | $O(1)$ |
| OCB 3 | $[148]$ | $O(1)$ | $O(1)$ | $O(1)$ |
| RPC | $[55]$ | $O(1)$ | $O(1)$ | $O(1)$ |
| TAE | $[153]$ | $O(1)$ | $O(1)$ | $O(1)$ |
| XCBC-XOR | $[113]$ | $O\left(2^{n / 4}\right)$ | $O(1)$ | $O(1)$ |
| off-line |  |  |  |  |
| BTM | $[126]$ | N/A | N/A | $O(1)$ |
| CCM | $[85]$ | $O(1)$ | $\ll 2^{(n / 2)}[106]$ | $O(1)$ |
| HBS | $[127]$ | N/A | N/A | $O(1)$ |
| SIV | $[209]$ | N/A | N/A | $O(1)$ |

Table 6.1.: Workloads of our robustness studies on previously published authenticated encryption schemes. Almost all attacks achieve an advantage close to 1. The workloads cover the computational effort, the amount of required memory, as well as the time complexity. Note that we classify CCM as off-line because the message-encryption process requires prior knowledge of the message length.

## COFFE: Ciphertext Output Feedback Faithful Encryption

Insight is the first condition of Art.
George Henry Lewes

In this Chapter we aim to provide authenticated encryption in constrained implementation environments where communication security is required, such as devices connected to the Internet of Things (IoT). Designing a sound authenticated encryption scheme is in fact challenging, but designing sound authenticated encryption scheme for constraint environments is a science of its own. Typically, restricted IoT devices have (very) limited computational power and no direct hardware support for any cryptographic primitives, so that all cryptography must be implemented in software. There is only limited memory available to hold executable object code on these processors, so it is imperative to provide the needed cryptographic services in the most compact way possible. One way to achieve this compactness is through the careful implementation of cryptographic primitives. However, it is also possible to facilitate compactness for an overall system by minimizing the number of primitives that must be included in an implementation. In this work we present a design for an on-line authenticated encryption (AE) scheme suitable for restricted devices using a standardized or soon-to-be standardized hash function, e.g., SHA-1 [183], SHA-2 [184], or SHA-3 [34]. Implementations of this scheme can omit a block cipher mode of operation; this is a useful approach since the code size for the block cipher is typically greater than that of the hash function, and hash functions are used in public key cryptography as well.

We focus on the challenge of providing an authenticated encryption scheme that
is easily accessible to developers. To provide this accessibility, we take the approach of defining a hash function mode of operation. That is, our AE scheme uses a cryptographic hash function as its only primitive, and does not require direct access to any hash function internals such as the compression function. We chose this approach based on feedback from the practice community. Hash function implementations are widely available, but these implementations do not provide interfaces to the compression function.

Note that to provide data privacy and data integrity, we transform the given hash function into a keyed hash function (PRF) On systems using restricted devices, due to the limited resources, it is desirable to minimize the cost of the code and the circuits for encrypting a message block [11], i.e., keep the size of the cryptographic footprint small. This means that we want only one costly operation per message block. We denote a scheme satisfying this property as a Rate-1 scheme. For example, the GCM authenticated encryption mode is not a Rate-1 AE scheme, since it needs not only one block cipher call per message block, but also an additional galois field multiplication per message block, rendering GCM to be a Rate-2 AE scheme. Another example would be a Feistel-based scheme which requires at least three or four block cipher calls rendering such a scheme to a low-performance Rate-3 or Rate-4 scheme.
Since the implementation of an encryption scheme can be error prone (e.g., 51,122 , $146,214,239]$ it would be desirable to provide a second line of defense to minimize the security fallout. A further preferable goal for our construction is to provide built-in resistance against side-channel attacks. Actually, the overlaying protocols, using an AE scheme, are responsible to provide this goal in an adequate form, e.g., TLS [78] and IPsec 120,137$]$ generate a new key for each session minimizing the number of measurements which can be done on the secret key. Obviously, an adversary can do a certain amount of measurements (depending on the size of the message) on the session key, but revealing the session key only compromises security for this specific encryption/decryption/authentication. Note that it does not compromise the currently used secret key. But, nevertheless, we provide side-channel resistance even if a protocol may fail to provide this kind of security.

We started our research by analyzing existing authenticated encryption schemes, where the block cipher within these schemes can be easily replaced by a keyed hash function. Unfortunately, none of those fulfill our requirements (see Table 7.1). As one can see, SpongeWrap [38] seems to be a very promising candidate, since it only lacks of built-in side-channel resistance. But, it belongs to the class of compression function based AE schemes, which yields to the fact that the internal used compression function can be seen as the real primitive to be used both for hashing and for authenti-

| Scheme | On-line | Side-Channel Res. | Rate-1 |
| :---: | :---: | :---: | :---: |
| COFFE (this work) | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| CHM [124] | $\checkmark$ | X | X |
| CWC [147] | $\checkmark$ | X | X |
| EAX [30] | $\checkmark$ | X | X |
| GCM [164] | $\checkmark$ | X | X |
| Generic Composition [22] | $\checkmark$ | X | X |
| [HBS [127] | X | X | X |
| SIV [209] | X | X | X |

Table 7.1.: Comparison of selected authenticated encryption schemes that can be instantiated with a hash function.
cated encryption. This is basically not a technical problem, but, while cryptographers know what is meant by the internal compression function, typical standards, such as the SHA-2 standard [184], do not formally define it. So, without an explicit specification of a "new" cryptographic primitive, engineers (non-cryptographers) would not be likely to properly implement the authenticated encryption scheme. Also, while on many constrained devices "jumping" to the address of the internal compression function may be easy, this may be not the case for all such devices. In fact, we did consider this approach at the beginning of our research. It would even allow us to design a more efficient AE scheme than the one we actually propose. But, due to the reasons discussed here, we made a decision against a purely compression function based AE scheme in favour of a hash function based AE scheme.

Outlook. In Section 7.1 we give a formal specification of Ciphertext Output Feedback Faithful Encryption (COFFE). In Section 7.2 we introduce a practical instantiation based on SHA-224. In Section 7.3 we show that COFFE is secure in the standard PRF model and in addition, it provides INT-CTXT security in the noncemisuse setting. Finally, Section 7.4 summarize our contribution.

### 7.1. Specification

Ciphertext Output Feedback Faithful Encryption (COFFE) is inspired by the CFB and Output Feedback (OFB) modes of operation [84]. It uses the chaining value $V_{i}$


Figure 7.1.: Illustration of the encryption and authentication process of COFFE, where $C_{0}$ denotes the first $\alpha / 4$ post-decimal hex digits of the number $\pi$.
and the previous ciphertext block $C_{i-1}$ as inputs for the computation of the subsequent ciphertext block $C_{i}$ (see Figure 7.1). The integrity of the ciphertext does not depend on the uniqueness of a nonce, but only on the security of the underlying $n$-bit hash function $\mathcal{H}$. The definition of COFFE is given in Algorithm 6, both functions Encrypt and Decrypt consist of the following four steps:

Step 1: Session Key Generation. COFFE is following the domain separation approach. Domain 0 is used to generate the session key $S$ (short-term key) which is derived from the secret key $K$ (long-term key) and the nonce $N$ as shown in Lines 10 and 20 of Algorithm 6. Note that the lengths of the key $|K|$ and nonce $|N|$ are encoded as single- or two-byte values. The actual encoding depends on the size of the key. Nevertheless, the domain always describes the least significant byte of the input. For practical applications, we recommend to use a key size of $n$ bits, where $n$ denots the output size of the underlying hash function $\mathcal{H}$. The session-key generation provides a built-in side-channel resistance since hash function do not have key schedules. Thus, COFFE can update $K$ for every new message without additional performance costs.

The term ' $0^{*}$ ' - which is used in each call to the hash function $\mathcal{H}$ - denotes a zero-padding, where the number of zeros depends on (1) the input size of the underlying compression function and (2) the internal message padding. Thus, it is always chosen such that one needs only one compression function call for one hash function invocation, which complies with our Rate-1 design goal. Therefore, we consider a

```
Algorithm 6 COFFE
Encrypt(N,H,M)
    0:S\leftarrow\mathcal{H}(K|N | 0* || |K|||N|| 0)
    (x,V)}\mp@subsup{V}{0}{}\leftarrow\mathrm{ ProcessHeader (H)
    (C,Vm)\leftarrowProcessMessage (S,V,M, M, )
    T\leftarrow\mathcal{H}(S\oplus\mp@subsup{V}{m}{}|\mp@subsup{C}{m}{}|\mp@subsup{0}{}{*}|\mp@subsup{L}{T}{}||\mp@subsup{M}{m}{}|+5)
    return (C,T)
Decrypt(N,H,C,T)
```



```
    (x,V)})\leftarrow\mathrm{ ProcessHeader (H)
    (M, Vm)\leftarrow\operatorname{ProcessCiphertext}(S,\mp@subsup{V}{0}{},C,x)
    T
    if T\not=\mp@subsup{T}{}{\prime}}\mathrm{ then
        M\leftarrow\perp
    end if
    return M
```

constant $\sigma$-bit input for $\mathcal{H}$, producing an $n$-bit output with $\sigma \geq n$.

Step 2: Header Processing. In this step we describe the processing of the associated data $H$, which can be of arbitrary length. For the sake of optimization, we invoke the hash function $\mathcal{H}$ only if indispensable, i.e., when the header is larger than the output length of $\mathcal{H}$. This allows us to process messages with a small header, e.g., IP packets, much faster by simply applying the $10^{*}$-padding. The domain $x$ indicates the length of the original header before the processing. A formal description of our length-dependent header processing is given next:

$$
\left(x, V_{0}\right) \leftarrow G(H):= \begin{cases}1, H \| 10^{*} & \text { if }|H|<n, \\ 2, H & \text { if }|H|=n, \\ 3, \mathcal{H}(H) & \text { else } .\end{cases}
$$

The goal of $G$ is to achieve pair-wise distinct tuples $\left(x, V_{0}\right)$ for pair-wise distinct values $H$ and $H^{\prime}$. Under the assumption that there is no collision for the hash function $G$, we have

$$
H \neq H^{\prime} \Longrightarrow\left(x, V_{0}\right)=G(H) \neq G\left(H^{\prime}\right)=\left(x^{\prime}, V_{0}^{\prime}\right)
$$

not necessarily meaning that $x \neq x^{\prime}$.

```
Algorithm 7 ProcessMessage/ProcessCiphertext
ProcessMessage \(\left(S, V_{0}, M, x\right) \quad\) ProcessCiphertext \(\left(S, V_{0}, C, x\right)\)
    \(C_{0} \leftarrow \mathrm{ToHex}(\pi)\)
    \(V_{1} \leftarrow \mathcal{H}\left(S \oplus V_{0}\left\|C_{0}\right\| 0^{*} \| x\right)\)
    \(C_{1} \leftarrow M_{1} \oplus V_{1}\)
    for \(i=2, \ldots, m\) do
        \(V_{i} \leftarrow \mathcal{H}\left(S \oplus V_{i-1}\left\|C_{i-1}\right\| 0^{*} \| 4\right) \quad\) 24: \(\quad V_{i} \leftarrow \mathcal{H}\left(S \oplus V_{i-1}\left\|C_{i-1}\right\| 0^{*} \| 4\right)\)
        \(C_{i} \leftarrow M_{i} \oplus V_{i}\)
    end for
    return \(C\)
    \(V_{1} \leftarrow \mathcal{H}\left(S \oplus V_{0}\left\|C_{0}\right\| 0^{*} \| x\right)\)
    \(C_{1} \leftarrow C_{1} \oplus V_{1}\)
    for \(i=2, \ldots, m\) do
    25: \(\quad M_{i} \leftarrow C_{i} \oplus V_{i}\)
    end for
    return \(M\)
```

Step 3: Plaintext/Ciphertext Processing. COFFE is generating a keystream for either encryption or decryption. Since our scheme is designed to comply with the requirements of the use of standardized building blocks, it works with hash functions like SHA- 1 and SHA-2. Thus, the input of the compression function is usually limited to less than $2 n$ bits, due to the message padding. Note that the $n$-bit session key $S$ and the domain separation value are mandatory inputs and hence, we have only less then $n$ bits remaining for the message input. To provide adequate security against forgery attacks, we need to additionally process two out of three of the following values: keystream block $V_{i-1}$, message block $M_{i-1}$, and ciphertext block $C_{i-1}$. More precisely, if we only use $V_{i-1}$ in the next iteration step, the tag would become messageindependent, i.e., the tag would not provide any integrity at all. Furthermore, if we use only $C_{i-1}$ or $M_{i-1}$, omitting $V_{i-1}$, the tag value would only depend on the last ciphertext or plaintext block, respectively. We decided to use the inputs to $\mathcal{H}$ in the following manner:

- $n$-bit value $S \oplus V_{i-1}$
- $\delta$-bit domain-separation value
- $(\alpha<n-\delta)$-bit ciphertext block $C_{i-1}$.

Our approach puts the hash function under a lot of stress since it violates the PRFindependency assumption. Thus, we require $\mathcal{H}$ to be indistinguishable from a PRF in the related-key model. More precisely, an adversary has partial control over the key-input to $\mathcal{H}$, resulting in a chance to produce a collision $S \oplus V_{i-1}=S^{\prime} \oplus V_{j-1}$ for two distinct keys $S \neq S^{\prime}$. Our security analysis in Section 7.3 shows that our approach still satisfies the birthday-bound security.

Let $M=M_{1}, \ldots, M_{m}$ denote the message, where $m=\lceil|M| / \alpha\rceil$ is the number of message blocks processed. Here, all but the last blocks of $M$ and $C$ are of size $\alpha$ bits. The final blocks of $M$ and $C$ consist of at most $\alpha$ bit. Then, the encryption and decryption process of COFFE is defined in Algorithm 7, where $\mathbf{T o H e x}(\pi)$ (see Lines 10 and 20) outputs the first $\alpha / 4$ post-decimal numbers of $\pi$ interpreted as hex values $\left(C_{0}=0 x 1415926 \ldots\right)$.

Step 4: Tag Generation. In the final step we derive the authentication tag from the final chaining value $V_{m}$ and the final ciphertext block $C_{m}$ as shown in Lines 13 and 23 of Algorithm [6. Note that the length of the tag is constrained by the output size of $\mathcal{H}$, e.g., at most $n$ bits. The last domain allows a user to authenticate the header without any message to encrypt. Thus, $\left|M_{m}\right|$ can become zero, but for $\mathcal{H}$, $\left|M_{m}\right|+5$ is always in the range $[5, \ldots, n+5]$.

### 7.2. COFFE-SHA-224 - A Practical Instantiation

In this section we discuss a practical instantiation of COFFE using SHA-224 as the underlying hash function - called COFFESHA-224. First, we justify our usage of SHA-224 over SHA-256.

Hash Function Choice. For the practical instantiation of COFFE, we searched for a standardized hash function which is suitable for restricted devices, where the usual size of a register is at most 32 bits. Thus, we made our choice in favour of a 32-bit-optimized hash function, which renders $\mathrm{SHA}-224$ and SHA-256 reasonable candidates. Both SHA-224 and SHA-256 share the same compression function $f$ : $\{0,1\}^{256} \times\{0,1\}^{512} \rightarrow\{0,1\}^{256}$. It compresses a 256 -bit chaining value and a 512 -bit message block into a 256 -bit output value. These two hash-function standards differ in two properties: 1) they use different initial values, and 2) SHA-224 truncates the output of the final compression function invocation while SHA-256 does not. Following the Merkle-Damgård paradigm [72, 171], SHA-224 and SHA-256 apply the secure $10^{*}$-padding followed by a 64 -bit value encoding the message length. Thus, the maximum possible input size to fit our requirements would be $512-1-64=447$ bits. Due to the sake of simplification, we consider only byte-aligned values and we assume all values to be encoded as octet-strings. Thus, we can only process message blocks with a size up to only 440 bits, i.e., 55 bytes. Using SHA- 256 implies a 256 -bit chaining value and thus, only 184 bits were left for the remaining input, including the domain separation byte and the previous ciphertext block. Furthermore, the tag
generation step requires two additional input bytes - the length of the last message block $\beta$ and the tag length $|T|$. Hence, we can process 160 -bit message blocks. Since the size of the hash value of SHA-224 is reduced by 32 bits in comparison to the usage of SHA-256, we can process message blocks of 192 bits, which leads to an estimated performance speedup of about $20 \%$ in comparison to SHA-256. Furthermore, the 224-bit session key used in SHA-224 is sufficient to make practical attacks infeasible. This makes SHA-224 a logical choice for COFFE.

Parameter Choice. Here, we introduce a sound parameter choice for COFFESHA224 depending on the used hash function SHA-224. The first step is to replace the function $\mathcal{H}$ from Algorithm 6 by SHA-224. This obviously leads to a size of 224 bits for the chaining values $V_{i}$. Based on our discussion above, we can process message blocks of up to 192 bits, i.e., we need only one byte to encode the domain specifier for the tag generation $\left(\left|M_{m}\right|+5<256\right)$. On one hand, the internal state of COFFE is larger than those of other common published authenticated encryption schemes like GCM, OCB, or EAX, which usually support a block size of 128 bits. On the other hand, COFFE employs a slightly worse ratio between the block size and the size of the internal state. Nevertheless, due to the larger block size, the performance of COFFE is still reasonable, i.e., approximately $85 \%$ of SHA-224. To ensure an adequate security, we set the default parameter of the size of the secret key to 224 bits. Therefore, we have up to 192 bits left for the nonce.

### 7.3. Security

This section describes the CCA3 security (cf. Definition 4.1) of COFFE considered under the reasonable assumption that the size of the secret key $K$ can be larger or equal to the size of the session key $S$, i.e., $|K| \geq|S|$. Therefore, at first we show the IND-CPA security of COFFE when considering a nonce-respecting adversary (cf. Definition 4.2), and then, we proof the INT-CTXT-security of COFFE against generalize the adversary by allowing it to reuse a nonce (cf. Definition 4.3).

Note that the length of the secret key $K$ can differ from the length of the session key $S$. If this is the case, we can partition $\mathcal{H}$ into two keyed hashfunctions: $\mathcal{H}_{1}:\{0,1\}^{|K|} \times$ $\{0,1\}^{*} \rightarrow\{0,1\}^{n}$ for the generation of the session key and $\mathcal{H}_{2}:\{0,1\}^{|S|} \times\{0,1\}^{*} \rightarrow$ $\{0,1\}^{n}$ for the processing of the message and the generation of the authentication tag. Due to the domain separation, the partitioning of $\mathcal{H}$ is still valid if $|K|=|S|$, i.e., the domain of the session key generation is always 0 and the domain of the message
processing is always 4 . In this section, for simplification, we define

$$
\mathbf{A d v} \frac{\sqrt[P R F]{\mathcal{H}_{*}}}{}(q+\ell, O(t))=\max \left\{\mathbf{A d} \mathbf{v} \frac{\sqrt[P R F]{\mathcal{H}_{1}}}{}(q, O(t)), \mathbf{A d} \mathbf{v} \frac{\sqrt[P R F-R K A]{\mathcal{H}_{2}}}{}(q+\ell, O(t))\right\}
$$

Theorem 7.1 (CCA3 Security of COFFE). Suppose $\Pi=(\mathcal{K}, \mathcal{E}, \mathcal{D})$ is the COFFE scheme as defined in Algorithm 6, i.e., $\mathcal{K}$ is the key derivation function, $\mathcal{E}=$ EncryptAndAuthenticate and $\mathcal{D}=$ DecryptAndVerify. Then,

$$
\begin{aligned}
\mathbf{A d v} \sqrt{\frac{C C A 3}{\Pi}}(q, \ell, t) & \leq \frac{5 \ell^{2}+3 q^{2}}{2^{n}}+2 \cdot \mathbf{A d v} \frac{\sqrt[P R F]{\mathcal{H}_{*}}}{}(q+\ell, O(t)) \\
& +\frac{2 \ell^{2}+3 q^{2}}{2^{n}}+\frac{q}{2^{|T|}}+2 \cdot \mathbf{A d v} \frac{\overline{P R F}}{\mathcal{H}_{*}}(q+\ell, O(t)) \\
& \leq \frac{7 \ell^{2}+6 q^{2}}{2^{n}}+\frac{q}{2^{|T|}}+4 \cdot \mathbf{A d v} \frac{\overline{P R F}}{\mathcal{H}_{*}}(q+\ell, O(t))
\end{aligned}
$$

Proof. The proof follows from Lemma 7.2 and Lemma 7.3 ,

Lemma 7.2 (IND-CPA-security of COFFE). Let $\Pi=(\mathcal{K}, \mathcal{E}, \mathcal{D})$ denote the COFFE scheme as defined in Algorithm 6. Then,

$$
\begin{aligned}
\mathbf{A d} \mathbf{v} \frac{\sqrt{I N D-C P A}}{\Pi}(q+\ell, t) & \leq \frac{(\ell+q)^{2}+2 \ell^{2}+2 q^{2}}{2^{n}}+2 \cdot \mathbf{A d} \mathbf{\sqrt { \frac { P R F } { \mathcal { H } _ { * } } } ( q + \ell , O ( t ) )} \\
& \leq \frac{5 \ell^{2}+3 q^{2}}{2^{n}}+2 \cdot 2 \cdot \mathbf{A d} \mathbf{v} \frac{\sqrt{P R F}}{\mathcal{H}_{*}}(q+\ell, O(t))
\end{aligned}
$$

Proof. This proof is using common game-playing arguments. At first, we replace the function $\mathcal{H}_{1}$ by a random $n$-bit function. The advantage therefore can be upper bounded by

$$
\mathbf{A d} \sqrt{\frac{P R F}{\mathcal{H}_{1}}}(q, O(t))
$$

Furthermore, we can also replace the function $\mathcal{H}_{2}$ by a random $n$-bit function since the adversary has partial control over the key $S \oplus V_{i}$. The advantage of this can be upper bounded by the PRF-RKA Advantage which is defined similar to the PRP-RKA Advantage (see Definition (3.3). Thus, we have

$$
\mathbf{A d v} \sqrt{\frac{P R F-R K A}{\mathcal{H}_{2}}}(q+\ell, O(t))
$$

In the following we always consider the full output length $n$ of the tag generation step, i.e., even if $|T|$ is smaller than $n$, we skip the truncation step for the proof. This is valid since showing IND-CPA security for the tag generation step without truncation implies IND-CPA security for the tag generation with truncation. From each adversary $\mathcal{A}$ with an advantage of $\epsilon$ attacking the truncated version we can construct an adversary $\mathcal{A}^{\prime}$ with the same advantage $\epsilon$ attacking the untruncated version. The algorithm $\mathcal{A}^{\prime}$ is a simulator that forwards all queries from $\mathcal{A}$ to the encryption oracle, truncates the tag output of the oracle responses before forwarding them to $\mathcal{A}$, and returns the same result as $\mathcal{A}$.

In the following we denote $V_{i}^{j}$ as the $i$-th keystream block of the $j$-th query and $m^{j}$ as the length of the $j$-th message in blocks. Let $\mathfrak{Q}$ be the query history of the adversary, where the subset $\mathfrak{Q}_{\mid V_{i}, T}$ consists of all the output values of $\mathcal{H}_{2}$, i.e., all chaining values $V_{i}^{j}$ and authentication tags $T^{j}$ with $i=1, \ldots, m^{j}$ and $j=1, \ldots, q$. We can say that COFFE is [IND-CPA-secure if the produced keystream and the tag values within the query history are indistinguishable from a sequence of distinct $n$-bit random values, where the length of this sequence is limited to $\ell+q$. It is easy to see that the probability of a collision between two values can be upper bounded by

$$
\frac{(\ell+q)^{2}}{2^{n}} .
$$

To complete our proof, we have to estimate the probability $\operatorname{Pr}[D i s t]$ that all values within the list $\mathfrak{Q}_{\mid V_{i}, T}$ are distinct. Therefore, we upper bound the probability $\operatorname{Pr}[$ Coll $]$ for a collision of at least two of the values within this list since

$$
\operatorname{Pr}[\text { Dist }]=1-\operatorname{Pr}[\text { Coll }] .
$$

To upper bound $\operatorname{Pr}[\mathrm{Coll}]$, we first consider the input parameter of $\mathcal{H}_{2}$ represented by the quadruple $z_{i}^{j}=\left(S^{j}, V_{i}^{j}, C_{i}^{j}, d_{i}^{j}\right)$ where the domain $d_{i}^{j}$ is either 4 or $\left|M_{m}\right|+5$. Note that we ignore the $0^{*}$-padding which leads to a higher success probability for an adversary. Let $(i, j)$ and $\left(i^{\prime}, j^{\prime}\right)$ be two distinct input tuples. In the following, we refer to the event $z_{i}^{j}=z_{i^{\prime}}^{j^{\prime}}$ as input collision. This event implies that either a collision for $\mathcal{H}_{2}$ occured or we have found a collision for the values $S^{j} \oplus T_{i}^{j}=S^{j^{\prime}} \oplus T_{i^{\prime}}^{j^{\prime}}$.

For our case analysis (cf. Table 7.2), we encode the difference between two input tuples $z_{i}^{j}$ and $z_{i^{\prime}}^{j^{\prime}}$ using a five-bit value. For example, the value " 10110 " defines the

| Case | Event | Case | Event | Case | Event | Case | Event |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 00000 | trivial | 01000 | - | 10000 | 1 | 11000 | 2,4 |
| 00001 | 3 | 01001 | - | 10001 | 1 | 11001 | 3 |
| 00010 | 3 | 01010 | - | 10010 | 1 | 11010 | 3 |
| 00011 | 3 | 01011 | - | 10011 | 1 | 11011 | 3 |
| 00100 | - | 01100 | 3 | 10100 | 3 | 11100 | 1,3 |
| 00101 | - | 01101 | 3 | 10101 | 3 | 11101 | 3 |
| 00110 | - | 01110 | 3 | 10110 | 3 | 11110 | 3 |
| 00111 | - | 01111 | 3 | 10111 | 3 | 11111 | 3 |

Table 7.2.: This table illustrates the case analysis for the proof of Lemma 7.2 where each case with a non-zero probability is covered by at least one event. The case " 11000 " is covered by two events depending on the considered domains (Event 2 covers the domain 1,2, and 3; Event 4 covers all other domains). The second special case " 11100 " is covered by Event 1 if $S^{j}=S^{j^{\prime}}$ and by Event 3 if $S^{j} \neq S^{j^{\prime}}$.
following case:

$$
10110:=\left\{\begin{array}{l}
j \neq j^{\prime} \\
V_{i}^{j}=V_{i^{\prime}}^{j^{\prime}} \\
S^{j} \oplus V_{i}^{j} \neq S^{j^{\prime}} \oplus V_{i^{\prime}}^{j^{\prime}} \\
C_{i}^{j} \neq C_{j^{j^{\prime}}}^{j^{\prime}} \\
d_{i}^{j}=d_{i^{\prime} \prime^{\prime}} .
\end{array}\right.
$$

Note that Table 7.2 contains a complete case analysis since all possible cases are covered. The cases which occur with a zero probability are obviously impossible and marked by ' - '. The reason for the occurrence of these cases is a violation of the XOR-relation between the values $S^{j}$ and $V_{i}^{j}$ or $S^{j^{\prime}}$ and $V_{i^{\prime}}^{j^{\prime}}$, respectively. For example, $\left(S^{j}=S^{j^{\prime}} \wedge V_{i}^{j}=V_{i^{\prime}}^{j^{\prime}}\right) \wedge\left(S^{j} \oplus V_{i}^{j} \neq S^{j^{\prime}} \oplus V_{i^{\prime}}^{j^{\prime}}\right)$ is an impossible case. Case " 00000 " implies that a collision must have happened before in the same query and is already covered by other cases. In the following we analyze four events which cover all cases with a non-zero probability from Table 7.2.

After asking at most $q$ queries, we check the query history $\mathfrak{Q}$ of the adversary which contains all queries and their results - for the occurrence of bad events. We let the adversary win immediately if one of the bad events becomes true. Let denote $A_{\alpha}$ the $\alpha$-th event. The occurrence of an event $A_{\alpha}$ implies that no event $A_{\beta}$ with $\beta \in\{1, \ldots, \alpha-1\}$ occurred before. Hence, the order of the events matters.

Event 1: Collision of two Session Keys. The first case describes the scenario where an adversary finds two values $S^{j}$ and $S^{j^{\prime}}$, generated using $\mathcal{H}_{1}$, with $j \neq j^{\prime}$ and $S^{j}=S^{j^{\prime}}$. The probability for this event can be upper bounded by

$$
q^{2} / 2^{n} .
$$

Event 2: Input Collision - Associated Data. In this case we consider an adversary which finds two colliding pairs $\left(V_{0}^{j} \oplus S^{j}, x^{j}\right)$ and $\left(V_{0}^{j^{\prime}} \oplus S^{j^{\prime}}, x^{j^{\prime}}\right)$ with $j \neq j^{\prime}$. A pair collides if it holds that $\left(V_{0}^{j} \oplus S^{j}=V_{0}^{j^{\prime}} \oplus S^{j^{\prime}}\right) \wedge\left(x^{j}=x^{j^{\prime}}\right)$. The occurence of this event leads to two colliding inputs for $\mathcal{H}_{2}$ in the first iteration. Observe that if no collision occurs, all $V_{i}^{j}$ are independent random values. The probability for this case can be upper bounded by

$$
\frac{q^{2}}{2^{n}}
$$

Event 3: Output Collision. For this case we consider an adversary which finds two values $V_{i}^{j}=V_{i^{\prime}}^{j^{\prime}}$ with $(i, j) \neq\left(i^{\prime}, j^{\prime}\right)$. The probability for this event can be upper bounded by

$$
\ell^{2} / 2^{n}
$$

Event 4: Input Collision - Message and Tag. Here, we consider an adversary which finds two tuples $\left(V_{i}^{j}, S^{j}\right)$ and $\left(V_{i^{\prime}}^{j^{\prime}}, S^{j^{\prime}}\right)$ with $V_{i}^{j} \oplus S^{j}=V_{i^{\prime}}^{j^{\prime}} \oplus S^{j^{\prime}}$. This leads to two colliding inputs for $\mathcal{H}_{2}$. Note that we assume that the adversary did not find an output collision before. The probability for this event can be upper bounded by

$$
\ell^{2} / 2^{n} .
$$

Our claim follows by adding up the individual bounds.

Lemma 7.3 (INT-CTXT Security of COFFE). Let $\Pi=(\mathcal{K}, \mathcal{E}, \mathcal{D})$ be the COFFE scheme as defined in Algorithm 6. We assume the adversary to be nonceignoring, i.e., it is able to choose two nonces $N^{j}=N^{j^{\prime}}$ with $j \neq j^{\prime}$. Then,

Proof. Our bound is derived by game-playing arguments. Consider Games $G_{1}-G_{3}$ of Figure 7.2 and Figure 7.3, and a fixed adversary $\mathcal{A}$ asking at most $q$ queries with a

```
```

Initialize ()

```
```

Initialize ()
$K \stackrel{\$}{\leftarrow} \mathcal{K}()$
$K \stackrel{\$}{\leftarrow} \mathcal{K}()$
$\mathfrak{Q}, \mathfrak{B}_{0}, \mathfrak{B}_{1}, \mathfrak{B}_{2}, \mathfrak{B}_{3}, \mathfrak{B}_{4}, \mathfrak{B}_{5} \leftarrow \emptyset$
$\mathfrak{Q}, \mathfrak{B}_{0}, \mathfrak{B}_{1}, \mathfrak{B}_{2}, \mathfrak{B}_{3}, \mathfrak{B}_{4}, \mathfrak{B}_{5} \leftarrow \emptyset$
win $\leftarrow$ false
win $\leftarrow$ false
Encrypt $(N, H, M)$ Game $G_{1}$
Encrypt $(N, H, M)$ Game $G_{1}$
$S \leftarrow \mathcal{H}\left(K\|N\| 0^{*}\||K|\||N| \mid 0\right)$
$S \leftarrow \mathcal{H}\left(K\|N\| 0^{*}\||K|\||N| \mid 0\right)$
$\left(x, V_{0}\right) \leftarrow G(H)$
$\left(x, V_{0}\right) \leftarrow G(H)$
$C_{0} \leftarrow \operatorname{ToHex}(\pi)$
$C_{0} \leftarrow \operatorname{ToHex}(\pi)$
$I \leftarrow S \oplus V_{0}$
$I \leftarrow S \oplus V_{0}$
$V_{1} \leftarrow \mathcal{H}_{2}\left(I\left\|C_{0}\right\| 0^{*} \| x\right)$
$V_{1} \leftarrow \mathcal{H}_{2}\left(I\left\|C_{0}\right\| 0^{*} \| x\right)$
$C_{1} \leftarrow V_{1} \oplus M_{1}$
$C_{1} \leftarrow V_{1} \oplus M_{1}$
for $i=2, \ldots, m$ do
for $i=2, \ldots, m$ do
$I \leftarrow S \oplus V_{i-1}$
$I \leftarrow S \oplus V_{i-1}$
$V_{i} \leftarrow \mathcal{H}_{2}\left(I\left\|C_{i-1}\right\| 0^{*} \| 4\right)$
$V_{i} \leftarrow \mathcal{H}_{2}\left(I\left\|C_{i-1}\right\| 0^{*} \| 4\right)$
$C_{i} \leftarrow V_{i} \oplus M_{i}$
$C_{i} \leftarrow V_{i} \oplus M_{i}$
$I \leftarrow S \oplus V_{m}$
$I \leftarrow S \oplus V_{m}$
$T \leftarrow \mathcal{H}_{2}\left(I\left\|C_{m}\right\| 0^{*} \||T|| | M_{m} \mid+5\right)$
$T \leftarrow \mathcal{H}_{2}\left(I\left\|C_{m}\right\| 0^{*} \||T|| | M_{m} \mid+5\right)$
$\mathfrak{Q} \leftarrow(N, H, C, T)$
$\mathfrak{Q} \leftarrow(N, H, C, T)$
return $(C, T)$

```
```

    return \((C, T)\)
    ```
```

```
```

```
Finalize ()
```

```
```

Finalize ()

```
```

```
Finalize ()
    return win
    return win
    return win
```

Decrypt $(N, H, C, T)$ Game $G_{1}$

```
Decrypt \((N, H, C, T)\) Game \(G_{1}\)
```

Decrypt $(N, H, C, T)$ Game $G_{1}$

```
Decrypt \((N, H, C, T)\) Game \(G_{1}\)
    \(S \leftarrow \mathcal{H}\left(K\|N\| 0^{*}\||K|\||N| \mid 0\right)\)
    \(S \leftarrow \mathcal{H}\left(K\|N\| 0^{*}\||K|\||N| \mid 0\right)\)
    \(S \leftarrow \mathcal{H}\left(K\|N\| 0^{*}\||K|\||N| \mid 0\right)\)
    \(S \leftarrow \mathcal{H}\left(K\|N\| 0^{*}\||K|\||N| \mid 0\right)\)
    \(\left(x, V_{0}\right) \leftarrow G(H)\)
    \(\left(x, V_{0}\right) \leftarrow G(H)\)
    \(\left(x, V_{0}\right) \leftarrow G(H)\)
    \(\left(x, V_{0}\right) \leftarrow G(H)\)
    \(C_{0} \leftarrow \operatorname{ToHex}(\pi)\)
    \(C_{0} \leftarrow \operatorname{ToHex}(\pi)\)
    \(C_{0} \leftarrow \operatorname{ToHex}(\pi)\)
    \(C_{0} \leftarrow \operatorname{ToHex}(\pi)\)
    \(I \leftarrow S \oplus V_{0}\)
    \(I \leftarrow S \oplus V_{0}\)
    \(I \leftarrow S \oplus V_{0}\)
    \(I \leftarrow S \oplus V_{0}\)
    \(V_{1} \leftarrow \mathcal{H}_{2}\left(I\left\|C_{0}\right\| 0^{*} \| x\right)\)
    \(V_{1} \leftarrow \mathcal{H}_{2}\left(I\left\|C_{0}\right\| 0^{*} \| x\right)\)
    \(V_{1} \leftarrow \mathcal{H}_{2}\left(I\left\|C_{0}\right\| 0^{*} \| x\right)\)
    \(V_{1} \leftarrow \mathcal{H}_{2}\left(I\left\|C_{0}\right\| 0^{*} \| x\right)\)
    \(M_{1} \leftarrow V_{1} \oplus C_{1}\)
    \(M_{1} \leftarrow V_{1} \oplus C_{1}\)
    \(M_{1} \leftarrow V_{1} \oplus C_{1}\)
    \(M_{1} \leftarrow V_{1} \oplus C_{1}\)
    for \(i=2, \ldots, m\) do
    for \(i=2, \ldots, m\) do
    for \(i=2, \ldots, m\) do
    for \(i=2, \ldots, m\) do
        \(I \leftarrow S \oplus V_{i-1}\)
        \(I \leftarrow S \oplus V_{i-1}\)
        \(I \leftarrow S \oplus V_{i-1}\)
        \(I \leftarrow S \oplus V_{i-1}\)
        \(V_{i} \leftarrow \mathcal{H}_{2}\left(I\left\|C_{i-1}\right\| 0^{*} \| 4\right)\)
        \(V_{i} \leftarrow \mathcal{H}_{2}\left(I\left\|C_{i-1}\right\| 0^{*} \| 4\right)\)
        \(V_{i} \leftarrow \mathcal{H}_{2}\left(I\left\|C_{i-1}\right\| 0^{*} \| 4\right)\)
        \(V_{i} \leftarrow \mathcal{H}_{2}\left(I\left\|C_{i-1}\right\| 0^{*} \| 4\right)\)
        \(M_{i} \leftarrow V_{i} \oplus C_{i}\)
        \(M_{i} \leftarrow V_{i} \oplus C_{i}\)
        \(M_{i} \leftarrow V_{i} \oplus C_{i}\)
        \(M_{i} \leftarrow V_{i} \oplus C_{i}\)
    \(I \leftarrow S \oplus V_{m}\)
    \(I \leftarrow S \oplus V_{m}\)
    \(I \leftarrow S \oplus V_{m}\)
    \(I \leftarrow S \oplus V_{m}\)
    \(T^{\prime} \leftarrow \mathcal{H}_{2}\left(I\left\|C_{m}\right\| 0^{*} \||T|| | C_{m} \mid+5\right)\)
    \(T^{\prime} \leftarrow \mathcal{H}_{2}\left(I\left\|C_{m}\right\| 0^{*} \||T|| | C_{m} \mid+5\right)\)
    \(T^{\prime} \leftarrow \mathcal{H}_{2}\left(I\left\|C_{m}\right\| 0^{*} \||T|| | C_{m} \mid+5\right)\)
    \(T^{\prime} \leftarrow \mathcal{H}_{2}\left(I\left\|C_{m}\right\| 0^{*} \||T|| | C_{m} \mid+5\right)\)
    if \(\left(T=T^{\prime}\right) \wedge(N, H, C, T) \notin \mathfrak{Q}\) then
    if \(\left(T=T^{\prime}\right) \wedge(N, H, C, T) \notin \mathfrak{Q}\) then
    if \(\left(T=T^{\prime}\right) \wedge(N, H, C, T) \notin \mathfrak{Q}\) then
    if \(\left(T=T^{\prime}\right) \wedge(N, H, C, T) \notin \mathfrak{Q}\) then
        win \(\leftarrow\) true
        win \(\leftarrow\) true
        win \(\leftarrow\) true
        win \(\leftarrow\) true
    return \(\perp\)
```

```
```

    return \(\perp\)
    ```
```

```
    return \(\perp\)
```

```
```

    return \(\perp\)
    ```
```

```
```

    ,\(\left.V_{0}\right) \leftarrow G(H)\)
    ```
```

    ,\(\left.V_{0}\right) \leftarrow G(H)\)
    ```
```

    ,\(\left.V_{0}\right) \leftarrow G(H)\)
    ```
```

    ,\(\left.V_{0}\right) \leftarrow G(H)\)
    ```
    路

Figure 7.2.: Game \(G_{1}\) for the proof of Lemma 7.3 ,
total length of at most \(\ell\) blocks. We assume that the adversary never asks a query for which the answer is already known. The functions Initialize and Finalize are identical for all games in this proof. Let \(G_{0}\) denote the INT-CTXT Game as defined in Algorithm 2 (cf. Section 4.3). Therefore, we have
\[
\operatorname{Adv} \frac{\sqrt{I T T-C T X T}}{\Pi T}(\mathcal{A}) \leq \operatorname{Pr}\left[\mathcal{A}^{G_{0}} \Rightarrow 1\right] .
\]

In \(G_{1}\), the encryption- and decryption-placeholders are replaced by their generic COFFE counterparts as of Algorithm 6 and, using similar arguments as in the proof for Lemma [7.2, we can partition \(\mathcal{H}\) into two independent PRFs \(\mathcal{H}_{1}\) and \(\mathcal{H}_{2}\). Thus,
\[
\operatorname{Pr}\left[A^{G_{0}} \Rightarrow 1\right] \leq \operatorname{Pr}\left[A^{G_{1}} \Rightarrow 1\right]+2 \cdot \operatorname{Ad} \mathbf{v} \frac{\sqrt[P R F]{\mathcal{H}_{*}}}{}(q+\ell, O(t)),
\]
where
\[
\operatorname{Ad} \sqrt{\frac{P R F}{\mathcal{H}_{*}}}(q+\ell, O(t))=\max \left\{\mathbf{A d v} \frac{\frac{P R F F}{\mathcal{H}_{1}}}{\left.(q, O(t)), \mathbf{A d} \sqrt{\frac{P R F-R K A}{\mathcal{H}_{2}}}(q+\ell, O(t))\right\} . . . ~}\right.
\]

We now discuss the differences between \(G_{1}\) and \(G_{2}\). The sets \(\mathfrak{B}_{0}, \ldots, \mathfrak{B}_{5}\) are initialized as empty sets (cf. Line 3 of Figure 7.2) and collect fresh values as follows:
- \(\mathfrak{B}_{0}\) collects all fresh values \(V_{0}\), where \(|H|>n\) in Lines 207 and 247 .
```

Algorithm 8 LLCP'
Input: $\mathfrak{Q}\{$ Query History $\}, N$ \{Nonce $\}, H$ \{Header $\}, M$ \{Message $\}$
$p \leftarrow 0$
for all $\left(N^{\prime}, H^{\prime}, M^{\prime}\right) \in \mathfrak{Q}$ do
if $\left(N=N^{\prime}\right) \wedge\left(H=H^{\prime}\right)$ then
$p \leftarrow \max \left\{p, \operatorname{LLCP}\left(M, M^{\prime}\right)\right\}$
end if
end for
return $p$

```
- \(\mathfrak{B}_{1}\) collects all fresh pairs \(\left(V_{0}, S, x\right)\) in Lines 212 and 252.
- \(\mathfrak{B}_{2}\) collects all fresh values \(I=V_{0} \oplus S\) in Lines 213 and 253 .
- \(\mathfrak{B}_{3}\) collects all fresh pairs \(\left(S \oplus V_{i}, C_{i}\right)\) with \(i=1, \ldots, m-1\). This is done in Lines 221 and 261
- \(\mathfrak{B}_{4}\) collects all fresh values \(V_{i}\) with \(i=1, \ldots, m\) in Lines \(215,225,255\), and 265.
- \(\mathfrak{B}_{\mathbf{5}}\) collects all fresh pairs \(\left(S \oplus V_{m}, C_{m}\right)\). This is done in Lines 233 and 270 .

In Lines 201 and 241, the \(L L C P^{\prime}\) oracle is inquired as defined in Algorithm 8 . Finally, the variable bad is set to true if one of the if-conditions in Lines 205, 210, \(219,223,228,245,250,259,263\), or 268 is true. None of these modifications affect the values returned to the adversary and therefore,
\[
\operatorname{Pr}\left[\mathcal{A}^{G_{1}} \Rightarrow 1\right]=\operatorname{Pr}\left[\mathcal{A}^{G_{2}} \Rightarrow 1\right]
\]

It follows that
\[
\begin{align*}
\operatorname{Pr}\left[\mathcal{A}^{G_{2}} \Rightarrow 1\right] & \leq \operatorname{Pr}\left[\mathcal{A}^{G_{3}} \Rightarrow 1\right]+\left|\operatorname{Pr}\left[\mathcal{A}^{G_{2}} \Rightarrow 1\right]-\operatorname{Pr}\left[\mathcal{A}^{G_{3}} \Rightarrow 1\right]\right| \\
& \leq \operatorname{Pr}\left[\mathcal{A}^{G_{3}} \Rightarrow 1\right]+\operatorname{Pr}\left[\mathcal{A}^{G_{3}} \text { sets bad }\right] . \tag{7.1}
\end{align*}
\]

We now proceed to upper bound the two terms contained in Equation (7.1) - in right to left order.

The success probability of Game \(G_{3}\) does not differ from the success probability of Game \(G_{2}\) unless one of the following cases occur, where each case causes a bad event, i.e., the variable bad is set to true. In the following, the indices \(j\) and \(j^{\prime}\) denote the \(j\)-th and \(j^{\prime}\)-th query with \(j, j^{\prime}=1, \ldots, q\), respectively.
```

$\underline{\operatorname{Encrypt}}(N, H, M)$ Game $G_{2}$ and $G_{3}$
$p \leftarrow \mathbf{L L C P}^{\prime}\left(\mathfrak{Q}_{\mid N, H, M},(N, H, M)\right)$
$S \leftarrow \mathcal{H}_{1}\left(K\|N\| 0^{*}\||K|\||N| \| 0\right)$
$\left(x, V_{0}\right) \leftarrow G(H)$
if $(x=3)$ then
if $\left(H \notin \mathfrak{Q}_{\mid H}\right.$ and $\left.V_{0} \in \mathfrak{B}_{0}\right)$ then
bad $\leftarrow$ true $V_{0} \stackrel{\$}{\leftarrow}\{0,1\}^{n} \backslash \mathfrak{B}_{0}$
$\mathfrak{B}_{0} \leftarrow \mathfrak{B}_{0} \cup\left\{V_{0}\right\}$
$C_{0} \leftarrow \operatorname{ToHex}(\pi)$
$I \leftarrow S \oplus V_{0}$
if $\quad\left(\left(V_{0}, S, x\right) \notin \mathfrak{B}_{1}\right.$ and $\left.I \in \mathfrak{B}_{2}\right)$
bad $\leftarrow$ true $I \stackrel{\$}{\leftarrow}\{0,1\}^{n} \backslash B_{2}$
$\mathfrak{B}_{1} \leftarrow \mathfrak{B}_{1} \cup\left\{\left(V_{0}, S, x\right)\right\}$
$\mathfrak{B}_{2} \leftarrow \mathfrak{B}_{2} \cup\{I\}$
$V_{1} \leftarrow \mathcal{H}_{2}\left(I\left\|C_{0}\right\| 0^{*} \| x\right)$
$\mathfrak{B}_{4} \leftarrow \mathfrak{B}_{4} \cup\left\{V_{1}\right\}$
$C_{1} \leftarrow V_{1} \oplus M_{1}$
for $i=2, \ldots, m$ do
$I \leftarrow S \oplus V_{i-1}$
if $\left(\left(I, C_{i-1}\right) \in \mathfrak{B}_{3}\right.$ and $\left.i>p\right)$ then
$\operatorname{bad} \leftarrow$ true $I \stackrel{\$}{\leftarrow}\{0,1\}^{n} \backslash \mathfrak{B}_{3}$
$\mathfrak{B}_{3} \leftarrow \mathfrak{B}_{3} \cup\left\{\left(I, C_{i-1}\right)\right\}$
$V_{i} \leftarrow \mathcal{H}_{2}\left(I\left\|C_{i-1}\right\| 0^{*} \| 4\right)$
if $\left(V_{i} \in \mathfrak{B}_{4}\right.$ and $\left.i>p\right)$ then
bad $\leftarrow$ true $V_{i} \stackrel{\$}{\leftarrow}\{0,1\}^{n} \backslash \mathfrak{B}_{4}$
$\mathfrak{B}_{4} \leftarrow \mathfrak{B}_{4} \cup\left\{V_{i}\right\}$
$C_{i} \leftarrow V_{i} \oplus M_{i}$
$I \leftarrow S \oplus V_{m}$
if $\left(\left(I, C_{m}\right) \in \mathfrak{B}_{5}\right)$ then
$\operatorname{bad} \leftarrow$ true $I \stackrel{\$}{\leftarrow}\{0,1\}^{n} \backslash \mathfrak{B}_{5}$
$\mathfrak{B}_{5} \leftarrow \mathfrak{B}_{5} \cup\left\{\left(I, C_{m}\right)\right\}$
$T \leftarrow \mathcal{H}_{2}\left(I\left\|C_{m}\right\| 0^{*}\||T|\|\left|M_{m}\right|+5\right)$
$\mathfrak{Q} \leftarrow(N, H, C, T)$
return $(C, T)$

```
\(40 \operatorname{Decrypt}(N, H, C, T)\) Game \(G_{2}\) and \(G_{3}\)
        \(p \leftarrow \mathbf{L L C P}^{\prime}\left(\mathfrak{Q}_{\mid, N, H, C},(N, H, C)\right)\)
        \(S \leftarrow \mathcal{H}_{1}\left(K\|N\| 0^{*}\||K|\||N| \| 0\right)\)
        \(\left(x, V_{0}\right) \leftarrow G(H)\)
        if \((x=3)\) then
        if \(\left(H \notin \mathfrak{Q}_{\mid H}\right.\) and \(\left.V_{0} \in \mathfrak{B}_{0}\right)\) then
            bad \(\leftarrow\) true \(V_{0} \stackrel{\$}{\leftarrow}\{0,1\}^{n} \backslash \mathfrak{B}_{0}\)
    \(\mathfrak{B}_{0} \leftarrow \mathfrak{B}_{0} \cup\left\{V_{0}\right\}\)
    \(C_{0} \leftarrow \operatorname{ToHex}(\pi)\)
    \(I \leftarrow S \oplus V_{0}\)
    if \(\quad\left(\left(V_{0}, S, x\right) \notin \mathfrak{B}_{1}\right.\) and \(\left.I \in \mathfrak{B}_{2}\right)\)
        \(\begin{aligned} & \text { if }\left(\left(V_{0}, S, x\right)\right.\left.\notin \mathfrak{B}_{1} \text { and } I \in \mathfrak{B}_{2}\right) \\ & \quad \mathrm{bad} \leftarrow \text { true } I \stackrel{\$}{\leftarrow}\{0,1\}^{n} \backslash \mathfrak{B}_{2}\end{aligned}\)
    \(\mathfrak{B}_{1} \leftarrow \mathfrak{B}_{1} \cup\left\{\left(V_{0}, S, x\right)\right\}\)
    \(\mathfrak{B}_{2} \leftarrow \mathfrak{B}_{2} \cup\{I\}\)
    \(V_{1} \leftarrow \mathcal{H}_{2}\left(I\left\|C_{0}\right\| 0^{*} \| x\right)\)
    \(\mathfrak{B}_{4} \leftarrow \mathfrak{B}_{4} \cup\left\{V_{1}\right\}\)
    \(M_{1} \leftarrow V_{1} \oplus C_{1}\)
    for \(i=2, \ldots, m\) do
        \(I \leftarrow S \oplus V_{i-1}\)
            if \(\quad\left(\left(I, C_{i-1}\right) \in \mathfrak{B}_{3}\right.\) and \(\left.i>p\right)\) then
            bad \(\leftarrow\) true \(I \stackrel{\$\{0,1\}^{n} \backslash \mathfrak{B}_{3}}{ }\)
            \(\mathfrak{B}_{3} \leftarrow \mathfrak{B}_{3} \cup\left\{\left(I, C_{i-1}\right)\right\}\)
            \(V_{i} \leftarrow \mathcal{H}_{2}\left(I\left\|C_{i-1}\right\| 0^{*} \| 4\right)\)
            if \(\left(V_{i} \in \mathfrak{B}_{4}\right.\) and \(\left.i>p\right)\) then
            bad \(\leftarrow\) true \(V_{i} \stackrel{\$}{\leftarrow}\{0,1\}^{n} \backslash \mathfrak{B}_{4}\)
            \(\mathfrak{B}_{4} \leftarrow \mathfrak{B}_{4} \cup\left\{V_{i}\right\}\)
            \(M_{i} \leftarrow V_{i} \oplus C_{i}\)
    \(I \leftarrow S \oplus V_{m}\)
    if \(\left(\left(I, C_{m}\right) \in \mathfrak{B}_{5}\right)\) then
            bad \(\leftarrow\) true \(I \stackrel{\$\{0,1\}^{n} \backslash \mathfrak{B}_{5}}{ }\)
    \(\mathfrak{B}_{5} \leftarrow \mathfrak{B}_{5} \cup\left\{\left(I, C_{m}\right)\right\}\)
    \(T^{\prime} \leftarrow \mathcal{H}_{2}\left(I\left\|C_{m}\right\| 0^{*} \||T|| | C_{m} \mid+5\right)\)
    if \(\left(T=T^{\prime}\right)\) and \((N, H, C, T) \notin \mathfrak{Q}\) then
            win \(\leftarrow\) true
        return \(\perp\)

Figure 7.3.: Games \(G_{2}\) and \(G_{3}\) for the proof of Lemma 7.3,

Case 1 (Collision - Initial Chaining Value): In Lines 206 and 246 the initial chaining value \(V_{0}\) is set to a new random value if the function \(G\) returns the same \(V_{0}\) twice for two distinct values \(H^{j} \neq H^{j^{\prime}}\) with \(j \neq j^{\prime}\) and \(H^{j}, H^{j^{\prime}}>n\), i.e., in the case when \(x=3\). The probability for such a collision can be upper bounded by
\[
q^{2} / 2^{n}
\]

Case 2 (Input Collision - Domain 1, .., 3): In Lines 211 and 251 the value \(I\) is set to a new random value if there is a non-trivial input collision between two input values \(I^{j}=S^{j} \oplus V_{0}^{j}\) and \(I^{j^{\prime}}=S^{j^{\prime}} \oplus V_{0}^{j^{\prime}}\) with \(x^{j}=x^{j^{\prime}}\), so that \(I^{j}=I^{j^{\prime}}\) with \(j \neq j^{\prime}\). We can upper bound the success probability for this case by
\[
q^{2} / 2^{n}
\]

Case 3 (Input Collision - Domain 4): In Lines 219 and 259 we test for a non-trivial input collision \(\left(S^{j} \oplus V_{i}^{j}, C_{i}^{j}\right)=\left(S^{j^{\prime}} \oplus V_{i^{\prime}}^{j^{\prime}}, C_{i^{\prime}}^{j^{\prime}}\right)\) with \((i, j) \neq\left(i^{\prime}, j^{\prime}\right)\). The success probability for this case can be upper bounded by
\[
\ell^{2} / 2^{n}
\]

Case 4 (Output Collision - Domain 4): In Lines 223 and 263 we test if the adversary has found a non-trivial collision of the form \(V_{i}^{j}=V_{i^{\prime}}^{j^{\prime}}\) with \((i, j) \neq\left(i^{\prime}, j^{\prime}\right)\). The success probability can be upper bounded by
\[
\ell^{2} / 2^{n}
\]

Case 5 (Input Collision - Domain 5): In Lines 228 and 268 we test for a non-trivial input collision \(\left(S^{j} \oplus V_{m}^{j}, C_{m}^{j}\right)=\left(S^{j^{\prime}} \oplus V_{m^{\prime}}^{j^{\prime}}, C_{m^{\prime}}^{j^{\prime}}\right)\) with \(j \neq j^{\prime}\). We can upper bound the success probability for this case by
\[
q^{2} / 2^{n}
\]

By adding up the individual bounds, it follows that
\[
\operatorname{Pr}\left[\mathcal{A}^{G_{3}} \text { sets } \mathrm{bad}\right] \leq \frac{2 \ell^{2}+3 q^{2}}{2^{n}}
\]

The adversary wins Game \(G_{3}\) iff the variable win is set to true, i.e., the if-condition in Line 272 holds. This implies that the adversary can win only with a fresh query to the Decrypt oracle, which leads to \(T=T^{\prime}\), where \(T^{\prime}\) is computed as shown in

Line 271. Lines 268 and 269 ensure that the input for the hash function \(\mathcal{H}_{2}\) in Line 271 is always a fresh value, i.e., it was never asked before. Since \(\mathcal{H}_{2}\) is a PRF the probability for \(T=T^{\prime}\) can be upper bounded by
\[
1 / 2^{|T|} .
\]

As we allow the adversary to ask at most \(q\) queries, the success probability for Game \(G_{3}\) can be upper bounded by
\[
\operatorname{Pr}\left[\mathcal{A}^{G_{3}} \Rightarrow 1\right] \leq q / 2^{|T|} .
\]

Our claim follows by adding up the individual bounds.

\subsection*{7.4. Results Summary}

In this Chapter we presented COFFE a novel hash function based OAE scheme which, to the best of our knowledge, is the first scheme that fulfills our stated requirements. It can be part of a minimal cryptographic suite that includes hashing and digital signatures. Because it is an AEAD scheme, it could be used in the AEAD interface of the Datagram Transport Layer Security (TLS) protocol [198] that has been identified by the IETF Constrained Application Working Group as suitable for applications for the [oT] In the standard model, COFFE provides the regular INT-CTXT and IND-CPA security plus INT-CTXT security in the nonce-misuse setting. Finally, it is resistant against side-channel attacks, which is usually a matter of the implementation of a cryptosystem, rather than of the cryptosystem itself. Nevertheless, we provide side-channel resistance even if an implementation lacks to provide this kind of security.

McOE: A Family of Robust On-Line Authenticated Encryption Schemes

\author{
If I have seen further it is by standing on the shoulders of Giants.
}

Isaac Newton
In recent years, cryptographers developed misuse-resistant schemes for authenticated encryption [126, 127, 210]. These guarantee excellent security even against general adversaries which are allowed to reuse nonces. Their disadvantage is that encryption can be performed in an off-line way, only. In this chapter we introduce a novel family of robust OAE schemes called McOE. Apart from the generic composition Encrypt-then-Mac (EtM), none of the ISO/IEC 19772:2009 schemes - in fact, no previously published authenticated encryption scheme at all - achieves both to be on-line and robust (cf. Table 8.11). In this table we classify a variety of provably secure block cipher based authenticated encryption schemes with respect to their on-line-ability and against which adversaries (nonce-respecting vs. nonce-ignoring) they are proven to be secure.

Encrypt-then-Mac (EtM). Since EtM is not a concrete scheme but a generic construction, there are some challenges left in order to make it fully on-line-secure: First, an appropriate on-line cipher has to be chosen. Second, a suitable, on-linecomputable, secure, and deterministic MAC must be selected. And, third, the generic EtM scheme requires at least two independent keys to be secure. Since two schemes are used in parallel, it is likely to squander resources in terms of run time and - im-
\begin{tabular}{lll}
\hline Type & CCA3-secure & Robust \\
\hline on-line & CCFB CHM COFFE CWC EAX GCM IACBC IAPM & McOE-X \\
& McOE-G McOE-X OCB1-3 RPC TAE XCBC & McOE-G \\
\hline off-line & CCM BTM HBS SIV & \\
\hline
\end{tabular}

Table 8.1.: Classification of provably secure block cipher based authenticated encryption schemes.
portant for hardware designers - in terms of space. Since EtM first has to be turned into an OAE scheme by making the appropriate choices, we do not consider it in our analysis.

Design Principles for AE Schemes. The question of how to provide authenticated encryption (without stating that name), when given a secure on-line cipher, is studied in [15], the revised and full version of [14]. The first approach in [15] only provides security if all messages are of the same length. The second approach repairs that by prepending the length of the message, at the cost of being off-line since the length must be known at the beginning of the encryption process. Their approach is to prepend and append a random value \(N\) to a message \(M\) and then to perform the online encryption of \((N\|M\| N)\). This looks promising, but the same \(N\) is used for two different purposes, putting different constraints on the generation of \(N\). For privacy, it suffices that \(N\) behaves like a nonce, not requiring secrecy or unpredictability. Even if \(N\) is not a nonce, but the same \(N\) is used for the encryption of several messages, all the adversary can determine are the lengths of common plaintexts prefixes, as we required for nonce reuse. On the other hand, authenticity actually assumes a secret or unpredictable \(N\) rather than a nonce. If the adversary \(\mathcal{A}\) can guess \(N\) before choosing a message, \(\mathcal{A}\) asks for the authenticated encryption of \((M \| N)\). Then, \(\mathcal{A}\) can predict the authenticated encryption of \(M\) without actually asking for it.

The general structure of McOE is based on the Tweak Chain Hash (TCH) from [153] which itself is adapted from the Matyas-Meyer-Oseas (MMO) construction [170]. Thereby, McOE replaces the random \(N\) by a proper nonce and the key-dependent tag computation value \(\tau\), performing a nonce-dependent on-line encryption of \((M \| \tau)\). The encryption can also depend on some associated data, which turns McOE into a family of schemes for on-line AEAD.


Figure 8.1.: The generic McOE construction. \(T^{\alpha}\) and \(\tau^{\alpha}\) denote the lower \(\alpha\) bits and \(T^{\beta}\) and \(\tau^{\beta}\) denote the upper \(\beta\) bits of \(T\), respectively. Furthermore, \(S\) is given by \(\widetilde{E}_{K}\left(1^{n},|M|\right)\).

Outlook. In Section 8.1 we introduce the generic specification of the MCOE family based on a tweakable block cipher, and in Section 8.2 we analyze its security. Section 8.3 introduces two practical instance of the MCOE family, called McOE-X and McOE-G. In Section 8.4 we present some performance benchmarks of both instances, and finally, Section 8.5 gives a brief summary about our contribution.

\subsection*{8.1. Generic Specification}

The structure of McOE bases on the TC3 construction, a tweakable block cipher based encryption scheme, (cf. Figure 8.1), which was presented by Rogaway and Zhan [211]. A formal definition of McOE is given in Algorithm 9 , Both functions Encrypt and Decrypt consist of the following three steps:

Step 1: ProcessHeader. In this step we describe the processing of the header \(H\), which can be of arbitrary length. In the case when the length of the header is not a multiple of the block size \(n\), we apply the common \(10^{*}\)-padding. Furthermore, \(H\) has to consist of at least one block since the tag computation value \(\tau\) depends on it. Hence, the whole header can be seen as a nonce. In the following, the lowest \(n-M_{m}\) bits of \(\tau\) are denoted by \(\tau^{\alpha}\). A formal definition of the header processing is given in Algorithm 10

Step 2: Plaintext/Ciphertext Processing. The plaintext/ciphertext blocks (except for the final block, which is discussed in step 3) are processed in a straightforward way
```

Algorithm 9 McOE
$\operatorname{Encrypt}(H, M)$
$m \leftarrow|M| / n$
$(U, \tau) \leftarrow \operatorname{ProcessHeader}(H)$
for $i=1, \ldots, m-1$ do
$C_{i} \leftarrow \widetilde{E}_{K}\left(U, M_{i}\right)$
$U \leftarrow M_{i} \oplus C_{i}$
end for
$S \leftarrow \widetilde{E}_{K}\left(1^{n},\left|M_{m}\right|\right)$
$X \leftarrow\left(M_{m} \| \tau^{\alpha}\right) \oplus S$
$Y \leftarrow \widetilde{E}_{K}(U, X)$
$\left(C_{m} \| T^{\alpha}\right) \leftarrow Y \oplus S$
$U \leftarrow X \oplus Y$
12: $\left(T^{\beta} \| Z\right) \leftarrow \widetilde{E}_{K}(U, \tau)$
13: $T \leftarrow T^{\alpha} \| T^{\beta}$
4: return $\left(C_{1}, \ldots, C_{m}, T\right)$

```
```

Decrypt $(H, C, T)$
21: $m \leftarrow|C| / n$
22: $(U, \tau) \leftarrow$ ProcessHeader $(H)$
23: for $i=1, \ldots, m-1$ do
24: $\quad M_{i} \leftarrow \widetilde{E}_{K}^{-1}\left(U, C_{i}\right)$
25: $U \leftarrow M_{i} \oplus C_{i}$
26: end for
27: $S \leftarrow \widetilde{E}_{K}\left(1^{n},\left|C_{m}\right|\right)$
28: $Y \leftarrow\left(C_{m} \| T^{\alpha}\right) \oplus S$
29: $X \leftarrow \widetilde{E}_{K}^{-1}(U, Y)$
30: $\left(M_{m} \| \tau^{\prime}\right) \leftarrow X \oplus S$
31: $U \leftarrow X \oplus Y$
32: $\left(T^{\prime}| | Z\right) \leftarrow \widetilde{E}_{K}(U, \tau)$
33: if $\tau^{\prime}=\tau^{\alpha}$ and $T^{\beta}=T^{\prime}$ then
34: return $\left(M_{1}, \ldots, M_{m}\right)$
35: end if
36: return $\perp$

```
```

Algorithm 10 ProcessHeader
Input: $H$ \{Header\}
Output: U \{Tweak\}, $\tau$ \{Tag Computation Value\}
$U \leftarrow 0^{n}$
for $i=1, \ldots,|H| / n$ do
$\tau \leftarrow \widetilde{E}_{K}\left(U, H_{i}\right)$
$U \leftarrow H_{i} \oplus \tau$
end for
return ( $U, \tau$ )

```
by the underlying tweakable block cipher \(\widetilde{E}\) or its inverse \(\widetilde{E}^{-1}\) (cf. Algorithm@, Lines 3-6 and Lines 23-26). The tweak \(U\) is computed by XORing the previous ciphertext block \(C_{i-1}\) and plaintext block \(M_{i-1}\). The length of the final plaintext/ciphertext block is between 1 bit and \(n\) bits since MCOE allows to process arbitrary length messages. A tweakable block cipher allows only to process \(n\)-bit message blocks. Therefore, we process the final plaintext block \(M_{m}\) as follows: At first we pad it to \(n\) bits by appending \(\tau\), before we XOR the padded value with the encryption of \(M_{m}\) encoded as \(n\)-bit value with \(1^{n}\) as tweak (see Algorithm [9, Line 7). The final message block can be computed by inverting this procedure (see Algorithm [ Lines 27-28).

Step 3: Tag Generation/Verification. A common technique to support lengthpreserving encryption is Ciphertext Stealing (CTS) 69]. Unfortunately, this approach contradicts the on-line property of MCOE since it requires to process the final block before its predecessors. Therefore, we introduce a novel method, called Tag Splitting (TS), where the \(n\)-bit tag \(T\) is split into an upper part \(T^{\alpha}\) consisting of the \(\alpha\) Most Significant Bits (MSB;) of \(T\) and a lower part \(T^{\beta}\) consisting of the \(\beta\) Least Significant Bits (LSB); of \(T\). Note that \(\alpha+\beta=n\) always holds. Furthermore, \(\alpha\) can be 0 if \(M_{m}\) is already an full \(n\)-bit block.
The final ciphertext block \(C_{m}\) together with the upper part of the authentication tag \(T^{\alpha}\) is derived from the padded message block \(M_{m}^{\prime}=M_{m} \| \tau^{\alpha}\) by applying (the XEX block cipher mode introduced by Liskov et al. in [152]), i.e., \(\left(C_{m} \| T^{\alpha}\right)=\) \(\widetilde{E}_{K}\left(U, M_{m}^{\prime} \oplus S\right) \oplus S\), where \(U\) is the current chaining value and \(S=\widetilde{E}_{K}\left(1^{n},\left|M_{m}\right|\right)\) (see Algorithm 9 Lines 7-10). This step implements length-preserving encryption since it ensures that \(|M|=|C|\) always holds. Moreover, for the sake of optimization, the masking value \(S\) can often be computed in advance for all possible message lengths. This allows to process the final blocks without an additional invocation of the tweakable block cipher. The lower part of the authentication tag is computed by the encryption of \(\tau\) (see Algorithm 9, Line 12). The remaining \(\alpha\) bits of the encryption \((Z)\) are discarded. The straightforward verification of the authentication tag is given in Lines 29-34 of Algorithm 9 .

\subsection*{8.2. Security Analysis}

In this section we show that MCOE is a robust OAE scheme, in the standard model.

Theorem 8.1 (ONDMA Security). Let \(\Pi=(\mathcal{K}, \mathcal{E}, \mathcal{D})\) be the McOE scheme as defined in the previous section with \(\mathcal{E}=\) Encrypt and \(\mathcal{D}=\) Decrypt from Algorithm (9. Then, for \(q \leq 2^{n / 2-2}\), we have
\[
\begin{aligned}
\mathbf{A d} \mathbf{\|} \frac{O N D M A}{\Pi}(q, \ell, t) \leq & \frac{(q+\ell+3)^{2}+2(q+\ell)+q^{2}}{2^{n-1}-q}+\frac{q}{2^{n / 2}-q} \\
& +2 \mathbf{A d v} \frac{\sqrt{I N D-P R P}}{\tilde{E}, \tilde{E}^{-1}}(2 q+\ell, O(t))
\end{aligned}
\]

Proof. The proof follows from Lemmas 8.2 and 8.3 ,

Lemma 8.2 (IND-OCCA2 Security). Let \(\Pi=(\mathcal{K}, \mathcal{E}, \mathcal{D})\) be a MCOE scheme as defined in the previous section with \(\mathcal{E}=\) Encrypt and \(\mathcal{D}=\) Decrypt from Algorithm 9 where the tag verification process (Lines 31-33 and 35-36) is omitted. Then, for all \(M \in\left(\{0,1\}^{n}\right)^{*}\), we have
\(\mathbf{A d v} \sqrt{\frac{I N D-O C C A 2}{}}(q, \ell, t) \leq \frac{(q+\ell+3)^{2}+2(q+\ell)+q^{2}}{2^{n}-q}+\mathbf{A d v} \frac{\sqrt{\frac{I N D-P R P}{\tilde{E}, \tilde{E}-1}}(2 q+\ell, O(t)) . . . . ~ . ~}{\text {. }}\)

Proof. The basic idea of this proof is to show that the absence of non-trivial collisions in the tweak values \((U)\) implies IND-CCA-security.

This proof borrows ideas from [211], proof of Theorem 3, and moreover, uses common game-playing arguments. The advantage of an adversary \(\mathcal{A}\) to distinguish \(G_{i}\) from \(G_{j}\) is given by
\[
\operatorname{Adv}_{G_{i}}^{G_{j}}(\mathcal{A})=\left|\operatorname{Pr}\left[\mathcal{A}^{G_{i}} \Rightarrow 1\right]-\operatorname{Pr}\left[\mathcal{A}^{G_{j}} \Rightarrow 1\right]\right|
\]

Let the tuple \((\mathcal{E}, \mathcal{D})\) denote the initial Game \(G_{0}\). The Game \(G_{1}\) is equal to the game \(G_{0}\) except that the tweakable block cipher \(\widetilde{E}\) is replaced by a set of pseudo random permutations \(\mathfrak{P} \stackrel{\$}{\leftarrow} \mathbf{P e r m}_{n}^{n}\) - which can be implemented efficiently via lazy sampling. To process a total amount of \(q\) encryption and decryption queries requieres \(\ell+2 q\) unique invocations of the tweakable block cipher. Thus, we have
\[
\mathbf{A d v}_{G_{0}}^{G_{1}}(\mathcal{A}) \leq \mathbf{A d v} \frac{\sqrt{I N D-P R P}}{\tilde{E}}(2 q+\ell, O(t))
\]

Game \(G_{1}\) is transformed into Game \(G_{2}\) as follows. First, we add a set \(\mathfrak{Q}\), the query history, collecting all output tuples \((H, M, C, T)\) consisting of a header \(H\), the authentication tag \(T\), the message \(M\), and the corresponding ciphertext \(C\) (see Figure 8.2

Lines 226 and 251). Then, the LLCP-oracle is called to compute the LCP between the current query and all previously asked queries (Figure 8.2 Lines 203 and 233). This is required to determine if a specific part of the message is a common prefix. Furthermore we add three additional sets \(\mathfrak{B}_{1}--\mathfrak{B}_{3}\) which are initialized as follows: \(\mathfrak{B}_{1}=\left\{0^{n}, 1^{n}\right\}, \mathfrak{B}_{2}=\emptyset\), and \(\mathfrak{B}_{3}=\emptyset\). Set \(\mathfrak{B}_{1}\) collects all tweaks \(U\) that are computed during the encryption or decryption process (see Figure 8.2 Lines 207, 213, 224, 237 and 243 ). Set \(\mathfrak{B}_{2}\) collects the input tuple \((U, X)\) for the encryption of the final message block - consisting of the masked final message block \(X\) and the corresponding tweak \(U\) (see Figure 8.2, Line 219). Similarly, the set \(\mathfrak{B}_{3}\) collects the input tuples \((U, Y)\) for the decryption of the final message block - consisting of the masked final ciphertext block \(Y\) and the corresponding tweak \(U\) (see Figure 8.2 Line 249). Finally, the variable bad is set to true if one of the if-conditions in lines 205, 211, 217, 222, 236,241 or 247 is true. None of these modifications affect the values returned to the adversary, and therefore, we have
\[
\operatorname{Pr}\left[\mathcal{A}^{G_{1}} \Rightarrow 1\right]=\operatorname{Pr}\left[\mathcal{A}^{G_{2}} \Rightarrow 1\right] .
\]

In Game \(G_{3}\) we eliminate the effects of bad events - immediately after setting the variable bad to true. After every collision between two chaining values occurs, the current value is replaced by a fresh value (see Figure 8.2 Lines 206, 212, 223, 236, and 242). Furthermore, the encryption of the masked final plaintext block is replaced when an input collision for \(\mathfrak{P}\) occurs (see Figure 8.2 Line 218). Finally, the decryption of the masked final ciphertext block is replaced when an input collision for \(\mathfrak{P}^{-1}\) occurs (see Figure 8.2 Line 248). Since \(G_{2}\) and \(G_{3}\) only differ when the variable bad is set to true, we have
\[
\operatorname{Adv}_{G_{2}}^{G_{3}}(\mathcal{A})=\operatorname{Pr}\left[\mathcal{A}^{G_{3}} \text { sets } \mathrm{bad}\right] .
\]

We upper bound the probability for this event by a case analysis.
Collision in \(\mathfrak{B}_{1}\). In this case the adversary must have found either a collision for \(\mathfrak{P}(V, W) \oplus W\) (i.e., it has found two input tuples \((V, W) \neq\left(V^{\prime}, W^{\prime}\right)\) such that \(\left.\mathfrak{P}(V, W) \oplus W=\mathfrak{P}\left(V^{\prime}, W^{\prime}\right) \oplus W^{\prime}\right)\) or it must have found a preimage of \(0^{n}\) or \(1^{n}\). In both cases the variable bad would have been set to true, and it follows from [47] that the success probability for this event can be upper bound by
\[
\frac{(q+\ell+2)(q+\ell+3)}{2^{n}-q}+\frac{2(q+\ell)}{2^{n}-q}
\]
```

```
Encrypt \((H, M)\) Game \(G_{2}\) and \(G_{3}\)
```

```
Encrypt \((H, M)\) Game \(G_{2}\) and \(G_{3}\)
    \(m \leftarrow|M| / n\)
    \(m \leftarrow|M| / n\)
    \(h \leftarrow|H| / n\)
    \(h \leftarrow|H| / n\)
    \(p \leftarrow \operatorname{LLCP}\left(\mathfrak{Q}_{\mid H, M},(H, M)\right)\)
    \(p \leftarrow \operatorname{LLCP}\left(\mathfrak{Q}_{\mid H, M},(H, M)\right)\)
    \((U, \tau) \leftarrow \operatorname{ProcessHeader}(H)\)
    \((U, \tau) \leftarrow \operatorname{ProcessHeader}(H)\)
    if \(\left((p<h)\right.\) and \(\left.\left(U \in \mathfrak{B}_{1}\right)\right)\)
    if \(\left((p<h)\right.\) and \(\left.\left(U \in \mathfrak{B}_{1}\right)\right)\)
        bad \(\leftarrow\) true \(; U \stackrel{\$}{\leftarrow}\{0,1\}^{n} \backslash \mathfrak{B}_{1}\)
        bad \(\leftarrow\) true \(; U \stackrel{\$}{\leftarrow}\{0,1\}^{n} \backslash \mathfrak{B}_{1}\)
    \(\mathfrak{B}_{1} \leftarrow \mathfrak{B}_{1} \cup\{U\}\)
    \(\mathfrak{B}_{1} \leftarrow \mathfrak{B}_{1} \cup\{U\}\)
    for \(i=1, \ldots, m-1\)
    for \(i=1, \ldots, m-1\)
        \(C_{i} \leftarrow \mathfrak{P}\left(U, M_{i}\right)\)
        \(C_{i} \leftarrow \mathfrak{P}\left(U, M_{i}\right)\)
        \(U \leftarrow M_{i} \oplus C_{i}\)
        \(U \leftarrow M_{i} \oplus C_{i}\)
        if \(\left((p<h+i)\right.\) and \(\left.\left(U \in \mathfrak{B}_{1}\right)\right)\)
        if \(\left((p<h+i)\right.\) and \(\left.\left(U \in \mathfrak{B}_{1}\right)\right)\)
        bad \(\leftarrow\) true; \(U \stackrel{\$}{\leftarrow}\{0,1\}^{n} \backslash \mathfrak{B}_{1}\)
        bad \(\leftarrow\) true; \(U \stackrel{\$}{\leftarrow}\{0,1\}^{n} \backslash \mathfrak{B}_{1}\)
    \(\mathfrak{B}_{1} \leftarrow \mathfrak{B}_{1} \cup\{U\}\)
    \(\mathfrak{B}_{1} \leftarrow \mathfrak{B}_{1} \cup\{U\}\)
\(S \leftarrow \mathfrak{P}\left(1^{n},\left|M_{m}\right|\right)\)
\(S \leftarrow \mathfrak{P}\left(1^{n},\left|M_{m}\right|\right)\)
\(X \leftarrow\left(M_{m} \| \tau^{\alpha}\right) \oplus S\)
\(X \leftarrow\left(M_{m} \| \tau^{\alpha}\right) \oplus S\)
\(Y \leftarrow \mathfrak{P}(U, X)\)
\(Y \leftarrow \mathfrak{P}(U, X)\)
if \(\left((U, X) \in \mathfrak{B}_{2}\right)\)
if \(\left((U, X) \in \mathfrak{B}_{2}\right)\)
    bad \(\leftarrow\) true,\(Y \stackrel{\Phi}{\leftarrow}\{0,1\}^{n}\)
    bad \(\leftarrow\) true,\(Y \stackrel{\Phi}{\leftarrow}\{0,1\}^{n}\)
\(\mathfrak{B}_{2} \leftarrow \mathfrak{B}_{2} \cup\{(U, X)\}\)
\(\mathfrak{B}_{2} \leftarrow \mathfrak{B}_{2} \cup\{(U, X)\}\)
\(\left(C_{m} \| T^{\alpha}\right) \leftarrow Y \oplus S\)
\(\left(C_{m} \| T^{\alpha}\right) \leftarrow Y \oplus S\)
\(U \leftarrow X \oplus Y\)
\(U \leftarrow X \oplus Y\)
if \(\left(U \in \mathfrak{B}_{1}\right)\)
if \(\left(U \in \mathfrak{B}_{1}\right)\)
    bad \(\leftarrow\) true; \(U \leftarrow\{0,1\}^{n} \backslash \mathfrak{B}_{1}\)
    bad \(\leftarrow\) true; \(U \leftarrow\{0,1\}^{n} \backslash \mathfrak{B}_{1}\)
\(\mathfrak{B}_{1} \leftarrow \mathfrak{B}_{1} \cup\{U\}\)
\(\mathfrak{B}_{1} \leftarrow \mathfrak{B}_{1} \cup\{U\}\)
\(\left(T^{\beta} \| Z\right) \leftarrow \mathfrak{P}(U, \tau)\)
\(\left(T^{\beta} \| Z\right) \leftarrow \mathfrak{P}(U, \tau)\)
\(\mathfrak{Q} \leftarrow \mathfrak{Q} \cup\{(H, M, C, T)\}\)
\(\mathfrak{Q} \leftarrow \mathfrak{Q} \cup\{(H, M, C, T)\}\)
return \((C, T)\)
```

```
return \((C, T)\)
```

```
```

Decrypt $(H, C, T)$ Game $G_{2}$ and $G_{3}$
$m \leftarrow|C| / n$
$h \leftarrow|H| / n$
$p \leftarrow \operatorname{LLCP}\left(\mathfrak{Q}_{\mid H, C},(H, C)\right)$
$(U, \tau) \leftarrow$ ProcessHeader $(H)$
if $\left((p<h)\right.$ and $\left.\left(U \in \mathfrak{B}_{1}\right)\right)$
bad $\leftarrow$ true $; U \stackrel{\&}{\leftarrow}\{0,1\}^{n} \backslash \mathfrak{B}_{1}$
$\mathfrak{B}_{1} \leftarrow \mathfrak{B}_{1} \cup\{U\}$
for $i=1, \ldots, m-1$
$M_{i} \leftarrow \mathfrak{P}^{-1}\left(U, C_{i}\right)$
$U \leftarrow M_{i} \oplus C_{i}$
if $\left((p<h+i)\right.$ and $\left.\left(U \in \mathfrak{B}_{1}\right)\right)$
bad $\leftarrow$ true; $U \stackrel{\&}{\leftarrow}\{0,1\}^{n} \backslash \mathfrak{B}_{1}$
$\mathfrak{B}_{1} \leftarrow \mathfrak{B}_{1} \cup\{U\}$
$S \leftarrow \mathfrak{P}\left(1^{n},\left|C_{m}\right|\right)$
$Y \leftarrow\left(C_{m} \| T^{\alpha}\right) \oplus S$
$X \leftarrow \mathfrak{P}^{-1}(U, Y)$
if $\left((U, X) \in \mathfrak{B}_{3}\right)$
$\operatorname{bad} \leftarrow$ true $; X \stackrel{\$}{\leftarrow}\{0,1\}^{n}$
$\mathfrak{B}_{3} \leftarrow \mathfrak{B}_{3} \cup\{(U, Y)\}$
$\left(M_{m} \| \tau^{\alpha}\right) \leftarrow X \oplus S$
$\mathfrak{Q} \leftarrow \mathfrak{Q} \cup\{(H, M, C, T)\}$
return $M$

```

Figure 8.2.: The IND-OCCA2 games \(G_{2}\) and \(G_{3}\) for the proof of Lemma 8.2, Game \(G_{3}\) contains the code in the box while \(G_{2}\) does not.

Collision in \(\mathfrak{B}_{2}\). In this case we can assume that no bad event has occurred so far. Therefore, the adversary can only win if it finds two colliding message blocks \(M_{m}\) and \(M_{m}^{\prime}\) sharing the same common prefix, i.e., \(M=M_{1}, \ldots, M_{m-1}, M_{m}\) and \(M=M_{1}, \ldots, M_{m-1}, M_{m}^{\prime}\) with \(M_{m} \neq M_{m}^{\prime}\). Now, we have to upper bound the success probability for the event
\[
\left(M_{m} \| \tau^{\alpha}\right) \oplus \mathfrak{P}\left(1^{n},\left|M_{m}\right|\right)=\left(M_{m}^{\prime} \| \tau^{\alpha}\right) \oplus \mathfrak{P}\left(1^{n},\left|M_{m}^{\prime}\right|\right) .
\]

Note that the equation above can hold only for \(\left|M_{m}\right| \neq\left|M_{m}^{\prime}\right|\). Since \(\mathfrak{P}\left(1^{n}, \cdot\right)\) is
a random permutation, we can upper bound the success probability of \(\mathcal{A}\) by
\[
\frac{q^{2}}{2^{n-1}-q} .
\]

Collision in \(\mathfrak{B}_{3}\). This case is similar to the previous one, and therefore, we can upper bound the success probability of an adversary by
\[
\frac{q^{2}}{2^{n-1}-q}
\]

By adding up the individual bounds it follows that
\[
\operatorname{Pr}\left[\mathcal{A}^{G_{3}} \text { sets } \mathrm{bad}\right] \leq \frac{(q+\ell+3)^{2}+2(q+\ell)+q^{2}}{2^{n}-q} .
\]

Let the tuple \(\left(\mathcal{O}_{P, 1}^{1}, \widehat{O}_{P^{-1}, 1}^{1}\right)\) denote the final game \(G_{4}\) where \(\mathcal{O}_{P}^{1}\) and \(\widehat{\mathcal{O}}_{P^{-1}}^{1}\) comply with the encryption and decryption oracles from Definition 5.5. Note that in \(G_{3}\) the chaining value \(U\) cannot collide and it is not possible to compute a preimage for any query. This implies that \(\mathfrak{P}\) is always invoked with a fresh tweak input, except two queries share a common prefix. Furthermore, we ensure by Lines 218 and 223 that both the final message block tag value is always a fresh random value. The same arguments hold for the decryption oracle. Thus, \(G_{3}\) and \(G_{4}\) have identical input-output behaviours and we have,
\[
\operatorname{Adv}_{G_{3}}^{G_{4}}(\mathcal{A})=0
\]

Our claim follows by adding up the individual bounds.

Lemma 8.3 (INT-CTXT Security). Let \(\Pi=(\mathcal{K}, \mathcal{E}, \mathcal{D})\) be a McOE scheme as in the previous section with \(\mathcal{E}=\) Encrypt and \(\mathcal{D}=\) Decrypt from Algorithm 9 , Then, for \(q \leq 2^{n / 2-2}\) we have
\[
\begin{aligned}
\operatorname{Adv} \frac{\sqrt{I T T-C T X T}}{I}(q, \ell, t) \leq & \frac{(q+\ell+3)^{2}+2(q+\ell)+q^{2}}{2^{n}-q}+\frac{q}{2^{n / 2}-q} \\
& +\operatorname{Adv} \frac{\sqrt{\frac{I N D-P R P}{E}, \tilde{E}^{-1}}(2 q+\ell, O(t))}{}
\end{aligned}
\]

Proof. Our bound is derived with the help of game-playing arguments. Consider Games \(G_{1}-G_{3}\) of Figures 8.3 and 8.4 and a fixed adversary \(\mathcal{A}\) asking at most \(q\) queries
```

Initialize ()
$K \stackrel{\$}{\leftarrow} \mathcal{K}()$
$\mathfrak{B}_{1} \leftarrow\left\{0^{n}, 1^{n}\right\}$
Encrypt $(H, M)$ Game $G_{1}$
$m \leftarrow|M| / n$
$h \leftarrow|H| / n$
$U \leftarrow 0^{n}$;
for $i=1, \ldots, h$ do
$\tau \leftarrow \widetilde{E}_{K}\left(U, H_{i}\right)$
$U \leftarrow H_{i} \oplus \tau$
for $i=1, \ldots, m-1$ do
$C_{i} \leftarrow \widetilde{E}_{K}\left(U, M_{i}\right)$
$U \leftarrow M_{i} \oplus C_{i}$
$S \leftarrow \widetilde{E}_{K}\left(1^{n},\left|M_{m}\right|\right)$
$X \leftarrow\left(M_{m} \| \tau^{\alpha}\right) \oplus S$
$Y \leftarrow \widetilde{E}_{K}(U, X)$
$\left(C_{m} \| T^{\alpha}\right) \leftarrow Y \oplus S$
$\left(T^{\beta} \| Z\right) \leftarrow \widetilde{E}_{K}(U, \tau)$
$\mathfrak{Q} \leftarrow \mathfrak{Q} \cup\{(H, M, C, T)\}$
return $\left(C_{1}\|\ldots\| C_{m}, T^{\alpha} \| T^{\beta}\right)$

```
```

Finalize ()

```
Finalize ()
```

Finalize ()
return win
return win
return win
Verify $(H, C, T)$ Game $G_{1}$
Verify $(H, C, T)$ Game $G_{1}$
Verify $(H, C, T)$ Game $G_{1}$
$m \leftarrow|C| / n$
$m \leftarrow|C| / n$
$m \leftarrow|C| / n$
$h \leftarrow|H| / n$
$h \leftarrow|H| / n$
$h \leftarrow|H| / n$
$U \leftarrow 0^{n}$
$U \leftarrow 0^{n}$
$U \leftarrow 0^{n}$
for $i=1, \ldots, h$ do
for $i=1, \ldots, h$ do
for $i=1, \ldots, h$ do
$\tau \leftarrow \widetilde{E}_{K}\left(U, H_{i}\right)$
$\tau \leftarrow \widetilde{E}_{K}\left(U, H_{i}\right)$
$\tau \leftarrow \widetilde{E}_{K}\left(U, H_{i}\right)$
$U \leftarrow H_{i} \oplus \tau$
$U \leftarrow H_{i} \oplus \tau$
$U \leftarrow H_{i} \oplus \tau$
for $i=1, \ldots, m-1$ do
for $i=1, \ldots, m-1$ do
for $i=1, \ldots, m-1$ do
$M_{i} \leftarrow \widetilde{E}_{K}^{-1}\left(U, C_{i}\right)$
$M_{i} \leftarrow \widetilde{E}_{K}^{-1}\left(U, C_{i}\right)$
$M_{i} \leftarrow \widetilde{E}_{K}^{-1}\left(U, C_{i}\right)$
$U \leftarrow M_{i} \oplus C_{i}$
$U \leftarrow M_{i} \oplus C_{i}$
$U \leftarrow M_{i} \oplus C_{i}$
$S \leftarrow \widetilde{E}_{K}\left(1^{n},\left|M_{m}\right|\right)$
$S \leftarrow \widetilde{E}_{K}\left(1^{n},\left|M_{m}\right|\right)$
$S \leftarrow \widetilde{E}_{K}\left(1^{n},\left|M_{m}\right|\right)$
$Y \leftarrow\left(C_{m} \| T^{\alpha}\right) \oplus S$
$Y \leftarrow\left(C_{m} \| T^{\alpha}\right) \oplus S$
$Y \leftarrow\left(C_{m} \| T^{\alpha}\right) \oplus S$
$X \leftarrow \widetilde{E}_{K}^{-1}(U, Y)$
$X \leftarrow \widetilde{E}_{K}^{-1}(U, Y)$
$X \leftarrow \widetilde{E}_{K}^{-1}(U, Y)$
$\left(M_{m} \| \tau^{\prime}\right) \leftarrow X \oplus S$
$\left(M_{m} \| \tau^{\prime}\right) \leftarrow X \oplus S$
$\left(M_{m} \| \tau^{\prime}\right) \leftarrow X \oplus S$
$U \leftarrow X \oplus Y$
$U \leftarrow X \oplus Y$
$U \leftarrow X \oplus Y$
$\left(T^{\prime} \| Z\right) \leftarrow \widetilde{E}_{K}(U, \tau)$
$\left(T^{\prime} \| Z\right) \leftarrow \widetilde{E}_{K}(U, \tau)$
$\left(T^{\prime} \| Z\right) \leftarrow \widetilde{E}_{K}(U, \tau)$
if $\left(\tau^{\prime}=\tau^{\alpha}\right.$ and $T^{\beta}=T^{\prime}$ and
if $\left(\tau^{\prime}=\tau^{\alpha}\right.$ and $T^{\beta}=T^{\prime}$ and
if $\left(\tau^{\prime}=\tau^{\alpha}\right.$ and $T^{\beta}=T^{\prime}$ and
if $\begin{aligned} & \tau^{\prime}=\tau^{\alpha} \text { and } T^{\beta}=T \\ & \left.(H, C, T) \notin \mathfrak{Q}_{\mid H, C, T}\right)\end{aligned}$
if $\begin{aligned} & \tau^{\prime}=\tau^{\alpha} \text { and } T^{\beta}=T \\ & \left.(H, C, T) \notin \mathfrak{Q}_{\mid H, C, T}\right)\end{aligned}$
if $\begin{aligned} & \tau^{\prime}=\tau^{\alpha} \text { and } T^{\beta}=T \\ & \left.(H, C, T) \notin \mathfrak{Q}_{\mid H, C, T}\right)\end{aligned}$
win $\leftarrow$ true
win $\leftarrow$ true
win $\leftarrow$ true
$\mathfrak{Q} \leftarrow \mathfrak{Q} \cup\{(H, \perp, C, \perp)\}$
$\mathfrak{Q} \leftarrow \mathfrak{Q} \cup\{(H, \perp, C, \perp)\}$
$\mathfrak{Q} \leftarrow \mathfrak{Q} \cup\{(H, \perp, C, \perp)\}$
return $\perp$

```
return \(\perp\)
```

return $\perp$

```

Figure 8.3.: Game \(G_{1}\) for the proof of Lemma 8.3
with a total length of at most \(\ell\) blocks. The functions Initialize and Finalize are identical for all games in this proof. Let \(G_{0}\) denote as the INT-CTXT Game defined in Algorithm 2. Definition 4.3 states that
\[
\operatorname{Ad} \sqrt[{\sqrt{I N T-C T X T}}]{\Pi I}(\mathcal{A}) \leq \operatorname{Pr}\left[\mathcal{A}^{G_{0}} \Rightarrow 1\right] .
\]

In Game \(G_{1}\), the encrypt and verify placeholders are replaced by their MCOE counterparts including two mentionable tweaks which does not effect the success probability of any adversary. First, the oracle Verify returns win instead of ( \(M \neq \perp\) ), and second, the query history collects additional data which is required to compute the LCP in the Games \(G_{2}\) and \(G_{3}\).

We now discuss the differences between \(G_{1}\) and \(G_{2}\). The set \(\mathfrak{B}_{1}\) is initialized with \(\left\{0^{n}, 1^{n}\right\}\) before it collects all new tweak values \(U\) that are computed during the encryption or verification process (in Lines 210, 216, 227, 250, 256, and 267). Furthermore, set \(\mathfrak{B}_{2}\) collects the input tuples ( \(U, X\) ) consisting of the masked final message block \(X\) and the corresponding tweak \(U\) (see Figure 8.4, Line 222). Similarly, the set \(\mathfrak{B}_{3}\) collects the input tuples ( \(U, Y\) ) consisting of the masked final ciphertext
block \(U\) and the corresponding tweak \(U\) (see Figure 8.4 Line 262).
In Lines 203 and 243, the LLCP oracle is inquired. Finally, the variable bad is set to true if one of the if-conditions in Lines 208, 214, 220, 225, 248, 254, 260 or 265 holds. None of these modifications affect the values returned to the adversary and therefore
\[
\operatorname{Pr}\left[\mathcal{A}^{G_{1}} \Rightarrow 1\right]=\operatorname{Pr}\left[\mathcal{A}^{G_{2}} \Rightarrow 1\right] .
\]

For our further discussion, we require another Game \(G_{4}\) which is explained in more detail later in this proo \({ }^{11}\). It follows that
\[
\begin{aligned}
\operatorname{Pr}\left[\mathcal{A}^{G_{2}} \Rightarrow 1\right] \leq & \operatorname{Pr}\left[\mathcal{A}^{G_{3}} \Rightarrow 1\right]+\left|\operatorname{Pr}\left[\mathcal{A}^{G_{2}} \Rightarrow 1\right]-\operatorname{Pr}\left[\mathcal{A}^{G_{3}} \Rightarrow 1\right]\right| \\
\leq & \operatorname{Pr}\left[\mathcal{A}^{G_{3}} \Rightarrow 1\right]+\operatorname{Pr}\left[\mathcal{A}^{G_{3}} \text { sets bad }\right] \\
\leq & \operatorname{Pr}\left[\mathcal{A}^{G_{4}} \Rightarrow 1\right]+\left|\operatorname{Pr}\left[\mathcal{A}^{G_{3}} \Rightarrow 1\right]-\operatorname{Pr}\left[\mathcal{A}^{G_{4}} \Rightarrow 1\right]\right| \\
& +\operatorname{Pr}\left[\mathcal{A}^{G_{3}} \text { sets bad }\right] .
\end{aligned}
\]

Next, we upper bound the three terms in the line above from right to left. Using similar arguments as in the proof of Lemma 8.2 we can upper bound \(\operatorname{Pr}\left[\mathcal{A}^{G_{3}}\right.\) sets bad] by
\[
\frac{(q+\ell+3)^{2}+2(q+\ell)+q^{2}}{2^{n}-q} .
\]

Now, we describe the new Game \(G_{4}\), which is equal to \(G_{3}\) except that the block cipher \(\widetilde{E}\) is replaced by a set of random permutations \(\mathfrak{P} \stackrel{\mathscr{S}}{\leftarrow} \operatorname{Perm}_{n}^{n}-\) which can be implemented efficiently via lazy sampling. To process a total amount of \(q\) encryption and decryption queries implies at most \(\ell+2 q\) unique invocations of the tweakable block cipher. Thus, we have
\[
\operatorname{Adv}_{G_{0}}^{G_{1}}(\mathcal{A}) \leq \mathbf{A d} \frac{\sqrt{I N D-P R P P}}{\tilde{E}, \tilde{E}^{-1}}(2 q+\ell, O(t))
\]

Finally, we have to upper bound the advantage for the adversary \(\mathcal{A}\) to win the Game \(G_{4} . \mathcal{A}\) can win this game only if the condition in Line 269 (resp. Line 469 for Game \(G_{4}\) ) holds. Without loss of generality, we assume that \(\mathcal{A}\) does not ask a question if the answer is already known. This implies that \((H, C, T) \notin \mathfrak{Q}_{\mid H, C, T}\). We formally adjust Line 269 (i.e., choose as the tag computation operation either \(\mathfrak{P}\) or \(\mathfrak{P}^{-1}\) ) such that we always have enough randomness left for our result. For the sake of simplicity, we denote the two final chaining values by \(U_{m}\) and \(U_{m+1}\). For our analysis, we distinguish between two main scenarios: (1) \(\left|M_{m}\right|=n\), and (2) \(\left|M_{m}\right| \neq n\)

\footnotetext{
\({ }^{1}\) Since the difference is minor, we do not provide an extra figure.
}
```

Encrypt $(H, M)$ Game $G_{2}$ and $G_{3}$
$m \leftarrow|M| / n$
$h \leftarrow|H| / n$
$p \leftarrow \operatorname{LLCP}\left(\mathfrak{Q}_{\mid H, M},(H, M)\right)$
$U \leftarrow 0^{n}$
for $i=1, \ldots, h$
$\tau \leftarrow \widetilde{E}_{K}\left(U, H_{i}\right)$
$U \leftarrow H_{i} \oplus \tau$
if $\left(U \in \mathfrak{B}_{1}\right.$ and $\left.p<i\right)$ then
bad $\leftarrow$ true; $U \stackrel{\$}{\leftarrow}\{0,1\}^{n} \backslash \mathfrak{B}_{1}$
$\mathfrak{B}_{1} \leftarrow \mathfrak{B}_{1} \cup U$
for $i=1, \ldots, m-1$
$C_{i} \leftarrow \widetilde{E}_{K}\left(U, M_{i}\right)$
$U \leftarrow M_{i} \oplus C_{i}$
if $\left((p<h+i)\right.$ and $\left.\left(U \in \mathfrak{B}_{1}\right)\right)$
bad $\leftarrow$ true $; \quad U \stackrel{\$}{\leftarrow}\{0,1\}^{n} \backslash \mathfrak{B}_{1}$
$\mathfrak{B}_{1} \leftarrow \mathfrak{B}_{1} \cup\{U\}$
$S \leftarrow \widetilde{E}_{K}\left(1^{n},\left|M_{m}\right|\right)$
$X \leftarrow\left(M_{m} \| \tau^{\alpha}\right) \oplus S$
$Y \leftarrow \widetilde{E}_{K}(U, X)$
if $\left(\quad(U, X) \in \mathfrak{B}_{2}\right)$
bad $\leftarrow$ true $; Y \stackrel{\$}{\leftarrow}\{0,1\}^{n}$
$\mathfrak{B}_{2} \leftarrow \mathfrak{B}_{2} \cup\{(U, X)\}$
$\left(C_{m} \| T^{\alpha}\right) \leftarrow Y \oplus S$
$U \leftarrow X \oplus Y$
if $\left(U \in \mathfrak{B}_{1}\right)$
$\operatorname{bad} \leftarrow$ true $; \quad U \stackrel{\$}{\leftarrow}\{0,1\}^{n} \backslash \mathfrak{B}_{1}$
$\mathfrak{B}_{1} \leftarrow \mathfrak{B}_{1} \cup\{U\}$
$\left(T^{\beta} \| Z\right) \leftarrow \widetilde{E}_{K}(U, \tau)$
$\mathfrak{Q} \leftarrow \mathfrak{Q} \cup\{(H, M, C, T)\}$
return $\left(C_{1}\|\ldots\| C_{m}, T^{\alpha} \| T^{\beta}\right)$

```
```

Verify $(H, C, T)$ Game $G_{2}$ and $G_{3}$

```
    \(m \leftarrow|C| / n\)
```

    \(m \leftarrow|C| / n\)
    \(h \leftarrow|H| / n\)
    \(h \leftarrow|H| / n\)
    \(p \leftarrow \operatorname{LLCP}\left(\mathfrak{Q}_{\mid H, C, T},(H, C, T)\right)\)
    \(p \leftarrow \operatorname{LLCP}\left(\mathfrak{Q}_{\mid H, C, T},(H, C, T)\right)\)
    \(U \leftarrow 0^{n}\)
    \(U \leftarrow 0^{n}\)
    for \(i=1, \ldots, h\)
    for \(i=1, \ldots, h\)
        \(\tau \leftarrow \widetilde{E}_{K}\left(U, H_{i}\right)\)
        \(\tau \leftarrow \widetilde{E}_{K}\left(U, H_{i}\right)\)
        \(U \leftarrow H_{i} \oplus \tau\)
        \(U \leftarrow H_{i} \oplus \tau\)
        if \(\left(U \in \mathfrak{B}_{1}\right.\) and \(\left.p<i\right)\) then
        if \(\left(U \in \mathfrak{B}_{1}\right.\) and \(\left.p<i\right)\) then
            bad \(\leftarrow\) true \(; \quad U \stackrel{\$}{\leftarrow}\{0,1\}^{n} \backslash \mathfrak{B}_{1}\)
            bad \(\leftarrow\) true \(; \quad U \stackrel{\$}{\leftarrow}\{0,1\}^{n} \backslash \mathfrak{B}_{1}\)
        \(\mathfrak{B}_{1} \leftarrow \mathfrak{B}_{1} \cup U\)
        \(\mathfrak{B}_{1} \leftarrow \mathfrak{B}_{1} \cup U\)
    for $i=1, \ldots, m-1$
for $i=1, \ldots, m-1$
$M_{i} \leftarrow \widetilde{E}_{K}^{-1}\left(U, C_{i}\right)$
$M_{i} \leftarrow \widetilde{E}_{K}^{-1}\left(U, C_{i}\right)$
$U \leftarrow M_{i} \oplus C_{i}$
$U \leftarrow M_{i} \oplus C_{i}$
if $\left((p<h+i)\right.$ and $\left.\left(U \in \mathfrak{B}_{1}\right)\right)$
if $\left((p<h+i)\right.$ and $\left.\left(U \in \mathfrak{B}_{1}\right)\right)$
bad $\leftarrow$ true $; U \stackrel{\$}{\leftarrow}\{0,1\}^{n} \backslash \mathfrak{B}_{1}$
bad $\leftarrow$ true $; U \stackrel{\$}{\leftarrow}\{0,1\}^{n} \backslash \mathfrak{B}_{1}$
$\mathfrak{B}_{1} \leftarrow \mathfrak{B}_{1} \cup\{U\}$
$\mathfrak{B}_{1} \leftarrow \mathfrak{B}_{1} \cup\{U\}$
$S \leftarrow E_{K}\left(1^{n},\left|M_{m}\right|\right)$
$S \leftarrow E_{K}\left(1^{n},\left|M_{m}\right|\right)$
$Y \leftarrow\left(C_{m} \| T^{\alpha}\right) \oplus S$
$Y \leftarrow\left(C_{m} \| T^{\alpha}\right) \oplus S$
$X \leftarrow \widetilde{E}_{K}^{-1}(U, Y)$
$X \leftarrow \widetilde{E}_{K}^{-1}(U, Y)$
if $\left((p<h+m)\right.$ and $\left.\left((U, Y) \in \mathfrak{B}_{3}\right)\right)$
if $\left((p<h+m)\right.$ and $\left.\left((U, Y) \in \mathfrak{B}_{3}\right)\right)$
bad $\leftarrow$ true $; X \stackrel{\$}{\leftarrow}\{0,1\}^{n} \backslash \mathfrak{B}_{1} \oplus Y$
bad $\leftarrow$ true $; X \stackrel{\$}{\leftarrow}\{0,1\}^{n} \backslash \mathfrak{B}_{1} \oplus Y$
$\mathfrak{B}_{3} \leftarrow \mathfrak{B}_{3} \cup\{(U, Y)\}$
$\mathfrak{B}_{3} \leftarrow \mathfrak{B}_{3} \cup\{(U, Y)\}$
$\left(M_{m} \| \tau^{\prime}\right) \leftarrow X \oplus S$
$\left(M_{m} \| \tau^{\prime}\right) \leftarrow X \oplus S$
$U \leftarrow X \oplus Y$
$U \leftarrow X \oplus Y$
if $\left((p<h+m)\right.$ and $\left.\left(U \in \mathfrak{B}_{1}\right)\right)$
if $\left((p<h+m)\right.$ and $\left.\left(U \in \mathfrak{B}_{1}\right)\right)$
bad $\leftarrow$ true $; U \stackrel{\$}{\leftarrow}\{0,1\}^{n} \backslash \mathfrak{B}_{1}$
bad $\leftarrow$ true $; U \stackrel{\$}{\leftarrow}\{0,1\}^{n} \backslash \mathfrak{B}_{1}$
$\mathfrak{B}_{1} \leftarrow \mathfrak{B}_{1} \cup\{U\}$
$\mathfrak{B}_{1} \leftarrow \mathfrak{B}_{1} \cup\{U\}$
$\left(T^{\prime} \| Z\right) \leftarrow \widetilde{E}_{K}(U, \tau)$
$\left(T^{\prime} \| Z\right) \leftarrow \widetilde{E}_{K}(U, \tau)$
if $\left(\tau^{\prime}=\tau^{\alpha}\right.$ and $T^{\beta}=T^{\prime}$ and
if $\left(\tau^{\prime}=\tau^{\alpha}\right.$ and $T^{\beta}=T^{\prime}$ and
$\left.(H, C, T) \notin \mathfrak{Q}_{\mid H, C, T}\right)$
$\left.(H, C, T) \notin \mathfrak{Q}_{\mid H, C, T}\right)$
win $\leftarrow$ true
win $\leftarrow$ true
$\mathcal{Q} \leftarrow \mathcal{Q} \cup\{(H, \perp, C, \perp)\}$
$\mathcal{Q} \leftarrow \mathcal{Q} \cup\{(H, \perp, C, \perp)\}$
return win

```
    return win
```

Figure 8.4.: Games $G_{2}$ and $G_{3}$ for the proof of Lemma 8.3, Game $G_{3}$ contains the code in the box while $G_{3}$ does not.

Scenario 1: ( $\left|M_{m}\right|=n$ ). For our analysis, we distinguish between two mutually exclusive cases.

Case $1\left(U_{m+1} \in \mathfrak{B}_{1}\right)$ :
In this case we first consider that $U_{m+1}$ is not fresh. This implies that the ciphertext $\left(C_{1}, \ldots, C_{m}\right)$ must be part of a common prefix of a previous query. The adversary can win only if $T$ is a fresh value, i.e., not a part of a previously occurred prefix. Since $\mathfrak{P}$ is a set of random permutations, the upper bound is then given by

$$
\operatorname{Pr}\left[\mathfrak{P}^{-1}\left(U_{m+1}, T\right)=\tau\right]=0
$$

Case $2\left(U_{m+1} \notin \mathfrak{B}_{1}\right)$ :
If $U_{m+1}$ is fresh, we can upper bound the success probability for one query by $1 /\left(2^{n}-q\right)$. Hence, for $q$ queries, we can upper bound the success probability by

$$
\frac{q}{2^{n}-q}
$$

Due to the fact that Case 1 and Case 2 are mutually exclusive, we can upper bound the success probability for $q$ queries by

$$
\max \left\{0, \frac{q}{2^{n}-q}\right\} \leq \frac{q}{2^{n}-q}
$$

Scenario 2: ( $\left|M_{m}\right| \neq n$ ). As in Scenario 1 we analyze two mutually exclusive cases.
Case $1\left(U_{m+1} \in \mathfrak{B}_{1}\right)$ :
This case implies that the ciphertext-tag tuple $\left(C_{1}, \ldots, C_{m}, T^{\alpha}\right)$ must be part of a prefix previously occured query. Hence, the adversary can win only if $T^{\beta}$ is new. Though, it is impossible, i.e., for all $\forall Z \in\{0,1\}^{\alpha}$ it holds that

$$
\operatorname{Pr}\left[\mathfrak{P}^{-1}\left(U_{m+1}, T^{\beta} \| Z\right)=\tau\right]=0
$$

since $\mathfrak{P}\left(U_{m+1}, \cdot\right)$ is a random permutation.
Case $2\left(U_{m+1} \notin \mathfrak{B}_{1}\right)$ :
This case implies that $C_{m} \| T^{\alpha}$ must be fresh. The probability that the condition $\tau^{\prime}=\tau^{\alpha}$ from Line 469 holds, can be upper bounded by

$$
\underset{\alpha}{\operatorname{Pr}}=\max _{M_{m}}\left\{\operatorname{Pr}\left[\mathfrak{P}^{-1}\left(U_{m}, C_{m} \| T^{\alpha}\right)=\left(M_{m} \| \tau^{\alpha}\right)\right]\right\} \leq \frac{1}{2^{\left(n-\left|C_{m}\right|\right)}-q}
$$

Hence, the probability for $q$ queries can be upper bounded by $\frac{q}{2^{\left(n-\left|C_{m}\right|\right)}-q}$. From the assumption $U_{m+1} \notin B_{1}$ follows that $U_{m+1}$ is new. Since $\mathfrak{P}$ is a set of random permutations, we can upper bound the probability that the condition $T^{\beta}=T^{\prime}$ from Line 469 holds by

$$
\operatorname{Pr}_{\beta}=\max _{Z}\left\{\operatorname{Pr}\left[\mathfrak{P}\left(U_{m+1}, \tau\right)=T^{\beta} \| Z\right]\right\} \leq \frac{1}{2^{\left|C_{m}\right|}-q}
$$

Then, the probability for $q$ queries can be upper bound by $q /\left(2^{\left|C_{m}\right|}-q\right)$.
The (total) success probability of this case depends on the length of $\left|C_{m}\right|$. So we can distinguish between the following three subcases:

Subcase 2.1 ( $\left|C_{m}\right|<n / 2$ ):
In this case we can upper bound $\operatorname{Pr}_{\alpha}$ by $\frac{1}{2^{n / 2}-q}$ and $\operatorname{Pr}_{\beta}$ by 1 . Hence, the total success probability for $q$ queries is at most $\frac{q}{2^{n / 2}-q}$.

## Subcase 2.2 ( $\left|C_{m}\right|=n / 2$ ):

In this case we can upper bound $\operatorname{Pr}_{\alpha}$ by $\frac{2}{2^{n / 2}-2 q}$ and $\operatorname{Pr}_{\beta}$ by $\frac{1}{2^{n / 2}-q}$. Hence, the total success probability for $q$ queries is at most $\frac{2 q^{2}}{2^{n-1}-q^{2}}$.
Subcase 2.3: ( $\left|C_{m}\right|>n / 2$ ):
In this case we can upper bound $\operatorname{Pr}_{\alpha}$ by 1 and $\operatorname{Pr}_{\beta}$ by $\frac{1}{2^{n / 2+1}-q}$. Hence, the total success probability for $q$ queries is at most $\frac{q}{2^{n / 2+1}-q}$.
Since all three subcases are mutually exclusive, we can upper bound the success probability for $q \leq 2^{n / 2-2}$ queries by

$$
\max \left\{\frac{q}{2^{n / 2}-q}, \frac{2 q^{2}}{2^{n-1}-q^{2}}, \frac{q}{2^{n / 2+1}-q}\right\} \leq \frac{q}{2^{n / 2}-q}
$$

Due to the fact that Case 1 and Case 2 are mutually exclusive, we can upper bound the success probability for $q$ queries by

$$
\max \left\{0, \frac{q}{2^{n / 2}-q}\right\} \leq \frac{q}{2^{n / 2}-q}
$$

Since both scenarios are mutually exclusive, we can upper bound the success probability for $q$ queries by

$$
\frac{q}{2^{n / 2}-q} .
$$

Our claim follows by adding up the individual bounds.

For simplification, we provide an upper bound that is easier to grasp than the original bound, but not as tight as the original bound given above.

Corollary 8.4 (Simplified ONDMA Bound). Lets assume that $16 \leq q \leq \ell$ and the IND-PRP advantage is at most $\epsilon$ for an adversary which amount of queries is at most $2^{n / 2-2}$. Then the following bound holds:

$$
\mathbf{A d v} \frac{O N D M A}{\Pi}(q, \ell, t) \leq \frac{6 \ell^{2}+9}{2^{n-1}-q}+\frac{q}{2^{n / 2}-q}+2 \epsilon
$$

Discussion. The proofs in this chapter show that any IND-CCA-secure on-line encryption scheme can be easily transformed into a full-fleged robust OAE scheme by simply (a) prepending the associated data and (b) appending the tag generation procedure to the message.

### 8.3. Practical Instances

The generic $\operatorname{McOE}$ scheme is based on a tweakable block cipher $\widetilde{E} \in \operatorname{Block}(k, n, n)$. Usually, an (efficient) tweakable block cipher is constructed out of a standard $n$-bit block cipher [15, 63, 152, 205]. Threefish 90] is the only native tweakable block cipher, published so far. Since the tweak size (128-bit) of Threefish is smaller than its block size $(256,512$ or 1024 bit) it does not match our requirements. In the following we introduce two block cipher based instances of McOE: McOE-X and McOE-G.

### 8.3.1. McOE-X

The $\operatorname{McOE}-\mathrm{X}$ scheme uses a regular block cipher $E \in \operatorname{Block}(k, n)$ (e.g., AES [71]) which is converted to a tweakable block cipher, namely TX, by mixing the tweak (i.e., the chaining value) into the key $K$ by using the XOR operation (cf. Figure 8.5). A formal definition of TX follows.

Definition 8.5 (TX). Let $E \in \operatorname{Block}(k, n)$ be a block cipher, $M, C, T \in\{0,1\}^{n}$, and $K \in\{0,1\}^{k}$. Then, the tweakable block cipher TX- $E \in B l o c k(k, n, n)$ is defined by

$$
\mathrm{TX}-E_{K}(U, M):=E_{K \oplus U}(M)
$$



Figure 8.5.: Constructed tweakable block cipher TX.
and its inverse is defined by

$$
\mathrm{TX}-E_{K}^{-1}(U, C):=E_{K \oplus U}^{-1}(C)
$$

Note that we can generalize the XOR operation between the key and the tweak by a function $\varphi:\{0,1\}^{k} \times\{0,1\}^{n} \rightarrow\{0,1\}^{n}$. For any fixed key $K, \varphi(K, \cdot)$ and the XOR operation are injective. Therefore, we can replace the XOR operation by the function $\varphi$. It is easy to see that a secure instance of TX requires related-key resistance for the block cipher $E$ since the adversary can partially control some relations among the keys used in the computation. Thus, we have

$$
\mathbf{A d} \sqrt{\frac{P R P}{\left(\mathrm{TX}-E, \mathrm{TX}-E^{-1}\right)}}(q, t)=\mathbf{A d v} \frac{P R P-R K A}{E, E^{-1}}(q, O(t))
$$

Therefore, we can deduce the OCCA3 security of MCOE-X from Theorem 8.1 and thus, from Corollary 8.4.

Corollary 8.6 (McOE-X Security). For $16 \leq q \leq \ell$ and $q \leq 2^{n / 2-2}$ we have

$$
\mathbf{A d v} \frac{O N D M A}{\operatorname{McOE}-\mathrm{X}}(q, \ell, t) \leq \frac{6 \ell^{2}+9}{2^{n-1}-q}+\frac{q}{2^{n / 2}-q}+2 \mathbf{A d v} \frac{\sqrt{P R P-R K A}}{E, E^{-1}}(2 q+\ell, O(t))
$$

Key-Recovery Attack. In [165], Mendel et al. showed a key-recovery attack on McOE-X with a birthday-bound complexity. The adversary is allowed to query the

McOE-X encryption oracle $\mathcal{E}$ and to access the block cipher $E$ itself. In general, this is a reasonable assumption since we can assume $E$ to be a common block cipher like AES. The attack works as follows:

1. Choose an arbitrary value $a$, compute $b_{i}=E_{K_{i}}(a)$ for $i=1, \ldots, q$ and store all pairs $\left(b_{i}, k_{i}\right)$ in a list $\mathfrak{L}$.
2. Choose an arbitrary constant value $M_{1}$, and set $M_{2}=a$. Then, choose an arbitrary nonce value $N_{i}$. Thus, set $M=M_{1} \| M_{2}$ and ask for $C=\mathcal{E}\left(N_{i}, M\right)$ with $C=C_{1} \| C_{2}$.
3. If $C_{2} \in \mathfrak{L}_{\mid b}$, compute the secret key by $K=k_{i} \oplus M_{1} \oplus C_{1}$, otherwise, go back to Step 2.

Let us denote $q^{\prime}$ as the number of iterations for the loop described in Steps 2 and 3. The total number of block cipher invocations $\ell$ is restricted to $\ell \leq q+4 q^{\prime}$ since for one query to $\mathcal{E}$, four block cipher calls are necessary to compute the pair $(C, T)$. If we choose $\ell \approx 2^{n / 2}$, there exists an adversary with a success probability of at most $1 / 2$, which is able to recover the secret key $K$ using the presented attack. Note that this attack does only confirm with our security claim, but also requires more queries then our security bound. Nevertheless, this attack shows that it is very crucial to change the cipher key after $\ll 2^{n / 2}$ invocations of the block cipher $\mathcal{E}$. The proposed attack can be avoided by increasing the key size to, e.g., $2 n$.

### 8.3.2. McOE-G

The McOE instance McOE-G updates the chaining value by applying an almost-XOR-universal ( $\epsilon$-AXU) hash function $\mathcal{H}$ to the XOR result of the previous message block and ciphertext block (see Figure 8.6). In our practical implementation, we use the Galois-Field multiplication for $\mathcal{H}$, i.e., the key $K_{2}$ is multiplied with the chaining value over $\operatorname{GF}\left(2^{128}\right)$ defined by the low-weight irreducible polynomial $g(x)=$ $x^{128}+x^{7}+x^{2}+x+1$ as used in OCB [208] and GCM [164].

Definition 8.7 (TG). Let $E \in \operatorname{Block}(k, n)$ be a block cipher, and $\mathcal{H}:\{0,1\}^{n} \times$ $\{0,1\}^{n} \rightarrow\{0,1\}^{n}$ be an $\epsilon$-AXU hash function. Suppose $M, C, T \in\{0,1\}^{n}$ and $\left(K_{1}, K_{2}\right) \in\{0,1\}^{k} \times\{0,1\}^{n}$ with $K=K_{1} \| K_{2}$. Then, the tweakable block cipher TG- $E \in \operatorname{Block}(k+n, n, n)$ is defined by

$$
\mathrm{TG}-E_{K}(U, M)=E_{K_{1}}\left(M \oplus \mathcal{H}_{K_{2}}(U)\right) \oplus \mathcal{H}_{K_{2}}(U),
$$



Figure 8.6.: Constructed tweakable block cipher TG.
and its inverse is defined by

$$
\mathrm{TG}-E_{K}^{-1}(U, C)=E^{-1}\left(C \oplus \mathcal{H}_{K_{2}}(U)\right) \oplus \mathcal{H}_{K_{2}}(U)
$$

Liskov et al. showed in Theorem 2 of [153] that TG is a secure tweakable block cipher with

$$
\mathbf{A d v} \frac{\sqrt{P R P}}{\mathrm{TG}-E, \mathrm{TG}-E^{-1}}(q, t) \leq 3 q^{2} \epsilon \cdot \mathbf{A d v} \frac{\sqrt{P R P}}{E, E^{-1}}(q, O(t))
$$

Therefore, we can deduce the ONDMA security of McOE-X from Theorem 8.1 and from Corollary 8.4.

Corollary 8.8 (McOE-G Security). For $16 \leq q \leq \ell$ and $q \leq 2^{n / 2-2}$, we have

Remark. McOE-G is not secure if an adversary has oracle access to the internal building blocks (i.e., the block cipher $E$ and the $\epsilon$-AXU hash function $\mathcal{H}$ ) [45]. Hence, it is crucial that an adversary has only black-box oracle access to the tweakable block cipher $\widetilde{E}$.

| Block cipher | Impl. | Message length in bytes |  |  |  |  |  |  |
| :--- | :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  |  | 64 | 256 | 512 | 1024 | 4096 | 16384 | 32768 |
| MCOE-X-AES | software | 27.0 | 20.6 | 19.5 | 19.0 | 18.6 | 18.6 | 18.6 |
| MCOE-X-AES | AES-NI | 13.7 | 10.9 | 10.5 | 10.2 | 10.0 | 10.0 | 10.0 |
| MCOE-X-Threefish | software | 15.6 | 7.9 | 6.7 | 6.1 | 5.6 | 5.5 | 5.5 |
| MCOE-G-AES |  |  |  |  |  |  |  |  |
| MCOE-G-AES | GF-NI/AES-NI | 12.5 | 9.9 | 9.4 | 9.2 | 9.0 | 9.0 | 9.0 |
| AES-CBC encryption | software | 18.6 | 18.0 | 16.4 | 15.5 | 14.2 | 11.4 | 11.4 |
| AES-CBC encryption | AES-NI | 5.1 | 5.9 | 5.7 | 5.4 | 4.1 | 4.1 | 4.1 |

Table 8.2.: Performance values cpb, single core), measured on a Core i5 540M for AES-128 and Threefish-512.

### 8.4. Benchmarks

This section provides software-performance benchmarks of the two presented members of the McOE family, i.e., McOE-X and McOE-G. All measurement results are based on the real-time clock (RTC) and obtained by the median of 5,000 measurements of the target function. The performance values are given in Cycles per Byte (cpb). For the sake of comparison, we also provide performance benchmarks for AES-CBC, a common encryption scheme without authentication, standardized by the National Institute of Standards and Technology (NIST) [84]. The AES software implementation is based on Gladman [111], whereas the hardware implementation is based on the Intel AES-NI Sample Library [66]. The Threefish implementation is based on the NIST/SHA-3 reference source as provided by the Skein authors [178]. Finally, the implementation of Galois-Field multiplication uses Intels carry-less multiplication instruction PCLMULQDQ that allows to compute the carry-less product of two 64 -bit operands in about 3.54 cpb (119]. Note that all performance benchmarks are based on naive implementations based on reference code. Therefore, it is most likely that the benchmarks can be further improved by the usage of sophisticated optimization methods.

Target Platform. The benchmarks were performed on a single core of an Intel Core i5-3210M CPU 2.50 GHz computer. All software benchmarks were written in C or ASM and compiled with the GNU C compiler (gcc) version 4.8.2 using the optimization flag -O3.

Results. The results of the 64-bit performance benchmarks are summarized in Table 8.2 .

Further Implementation Results. Bogdanov et al. recently published a high-speed implementation of McOE-G optimized for Haswell CPUs [50]. Their implementation runs at about 6.24 cpb on a single core of an Intel Core i5-4300U CPU ( 1900 MHz ) processor. In addition, Bogdanov et al. also performed benchmarks in the multimessage scenario. Here, McOE-G matches the performance of GCM at about 1.45 cpb . Their results show that scenarios exist where a conventional OAE scheme can be replaced by a robust one without noticeable performance loss.

### 8.5. Results Summary

Originally, our research was inspired by the search for an authenticated encryption scheme that can be used in a general-purpose cryptographic library. It should offer by default a huge failure tolerance for practical software developers and still allow being used in an on-line manner.

Since the well-known schemes (such as OCB and SIV) did not fit our requirements, we developed MCOE- the first robust OAE scheme. Furthermore, it is provably secure in the standard model and fast enough for most common applications like (full) hard-disk encryption or secure network communication. This renders McOE a well-suited candidate for any general-purpose cryptographic library.

## Part III

## Design and Usage of <br> Cryptographic Hash Functions

## Twister $_{\pi}$ - A Framework for Fast and Secure Hash Functions

A person who never made a mistake never tried anything new.

Albert Einstein

In this chapter we present Twister $_{\pi}$, a framework for hash functions. It is an improved version of Twister, one of the 51 accepted fist-round participants of the SHA-3 Competition. TwISTER $\pi_{\pi}$ is built upon the ideas of wide-pipe and sponge functions. The core of this framework is a - very easy to analyze - Mini-Round, providing both fast diffusion as well as collision-freeness. The total security level is claimed to be not below $2^{n / 2}$ for collision attacks and $2^{n}$ for ( 2 nd-) preimage attacks. TWISTER $_{\pi}$ instantiations are secure against all known generic attacks. We also propose two instances Twister $_{\pi}-n$ for hash output sizes $n=256$ and $n=512$. These instantiations are highly optimized for 64 -bit architectures and run fast in hardware and software. Furthermore, Twister $\pi_{\pi}$ scales very well on low-end platforms.

Related Work. In the last decade, design flaws in popular hash functions such as MD5 200] and SHA-0 185 were exposed, leading to a huge amount of attacks for SHA-1 183] [39, 40, 62, 80, 199, 234 236]. But, also newer hash functions, e.g., [12, 121, 140, 141] - which try to take care for weaknesses in the Merkle-Damgård construction itself - were broken soon after their publications [118, 162, 168, 192, 193].

Back then, some cryptographers were worried that the standardized SHA-2 [183, 184] family could also be vulnerable to state-of-the-art techniques in cryptanalysis.

Therefore, in 2008, the NIST started the SHA-3 Competition [181] with the goal to find a successor for the SHA-2 family. On October 2, in 2012, the NIST announced Keccack [34], a sponge-function based software, as the winner of the SHA-3 Competition. The concept of sponge functions [36] is given for example by a big internal state that absorbs a message of infinite length and that later squeezes out a hash value of variable size. In 2014, it will become the official SHA-3 standard.

Our Contribution. The design of secure and practical hash functions is of great interest. Due to the SHA-3 Competition, many new proposals for hash functions have been published during this process. In this chapter we present a new hash function framework called Twister ${ }_{\pi}$. Our proposal is based on a sponge construction [37] as well as on the wide-pipe approach [155]. The main goal is to present a fast and secure hash function which is flexible to use and easy to analyze.

More precise, it uses XOR-sponges with a big internal state as proposed in [155]. The Grindahl design [141], which is the closest to our approach, but contains some flaws which cannot be exploited in Twister $_{\pi}$. We take advantage of the well studied basic operations of AES [71] and adopt several of them, including some optimization techniques.

Due to recent breakdowns of many proposed hash functions, we analyze the resistance of $\mathrm{TWISTER}_{\pi}$ against all known generic attacks on hash functions. We show that the $\mathrm{TWISTER}_{\pi}$ framework resists all of them if the size of the internal chaining value is at least double the size of the hash output.

In 2009, Mendel et al. discovered a serious design flaw of Twister 166 , the predecessor of Twister $\pi$, leading to several attacks against Twister-512 [166]. Those attacks are not transferable to TwISTER $_{\pi}-512$ since it was especially designed to resist them.

Outline. Section 9.1 points out the principles of $\mathrm{TwISTER}_{\pi}$. Section 9.2 specifies $\operatorname{Twister}_{\pi}$ and Section 9.3 briefly discusses its resistance against generic attacks. Section 9.4 shows some optimization techniques related to the implementation of Twister $_{\pi}$ on different platforms, and Section 9.5 provides software-performance benchmarks. Section 9.6 introduces the differences between Twister $\pi_{\pi}$ and its predecessor Twister [93]. Finally, Section 9.8 summarizes our contribution.

### 9.1. Design Principles of Twister $_{\pi}$

In this section we explain our design purpose and point out the principles of the Twister $_{\pi}$ design.

Security. The concept of Maximum Distance Separable (MDS) matrices allows us to obtain a maximum of diffusion inside each column of the state matrix. Since the message input is orthogonal with the diffusion of the state, i.e., a message word is always injected into the last row of our state matrix, we allow a minimum of control on the state for an adversary. Introducing a local feed-forward as well as the BlankRound (Twister-Round with no message input), furthe reduces the influence of an adversary on the state.

Evolutionary. Throughout the last decade, a lot of hash functions using many different concepts have been broken. And often we had to learn that using newly developed techniques lead to stronger attacks, which render pretended strong hash functions weak.
The well-studied and analyzed block ciphers that were in the final round of the AES Competition ${ }^{1}$ lead to some well established design principles offering a high level of cryptographic knowledge. Therefore, Rijndael [70] can be seen as one of the most studied block ciphers during this process and also in the time after. Its concepts of simple byte-wise operations SubBytes, ShiftRows, and MixColumns, are well-analyzed and it turns out that their combination can offer a high level of speed and some form of provable security. We adopt a few of these concepts for Twister ${ }_{\pi}$ and we also learn from recent hash function breakouts.

Simplicity. A strong hash function should not be hard to analyze since not finding an attack due to the algorithmic complexity does not mean that there is no simple attack which breaks the whole function in an easy way. We therefore only use simple and few components as building blocks for the Twister-Round. These components have been studied before and are well known - but combining these components to obtain a good hash function is new. Our clear design and straightforward structure of Twister $_{\pi}$ makes cryptanalysis easy and serves the purpose that there are no simple attacks which cannot be found due to a complex and unreadable algorithm.

[^2]Portability and Scalability. A main design criteria of $\operatorname{Twister}_{\pi}$ is its application to a wide range of applications. Due to its byte-wise operations, it scales very neat on 8 -, 16 -, 32 -, and 64 -bit platforms. Twister ${ }_{\pi}$ can be very efficiently applied on smart cards with small 8 -bit processors. We also offer an optimized version for 32bit and 64 -bit environments. The portability will be enhanced by its low-memory requirements, which makes Twister $_{\pi}$ even valuable for low-end platforms.

Analyzability. Twister $_{\pi}$ consists of well-known and well-analyzed components. The security level of $\mathrm{TwISTER}_{\pi}$ can be proven for the inner components, which is more worth than just a security claim. Using the concept of an MDS matrix, lead to a very fast diffusion. After only two Twister-Round invocations, a full diffusion is guaranteed (if no feed-forward has taken place). This high level of diffusion makes $\mathrm{TwISTER}_{\pi}$ very close to a randomized hash function offering a high level of security.

### 9.2. Specification of the Twister $_{\pi}$ Hash-Function Family

```
Algorithm 11 TWISTER \(_{\pi}\)
Input: \(M\) \{Message to Hash\}, \(n\) \{Output Length\}
Output: \(Y\) \{Hash Value\}
    \(T \leftarrow 0\)
    \(S \leftarrow \operatorname{Init}(\pi)\)
    \(S_{(1 \rightarrow)} \leftarrow S_{(1 \rightarrow)} \oplus n\)
    if \(n \leq 256\) then
        \(T \leftarrow\) null
    end if
    \(M^{\prime} \leftarrow M_{\|} \mid 10^{*}\)
    for \(i=1, \ldots,\left|M^{\prime}\right| / 512\) do
        \((S, T) \leftarrow\) Twister-Round \(\left(S, T, M_{i}^{\prime}\right)\)
    end for
    \(S \leftarrow\) State-Finalization \((S, T,|M|, n)\)
    return \(Y \leftarrow \operatorname{Output-Round}(S, n)\)
```

In this section we specify the overall structure and the individual building blocks of TWISTER $_{\pi}$, a byte-oriented hash function family, operating on a square state matrix $S$. The core primitive of each individual TwISTER $_{\pi}$ hash function is the underlying compression function, Twister-Round, processing 512-bit message blocks together
with a 512 -bit state (i.e., the chaining value) and outputs a 512 -bit value. Messages are padded using the $10^{*}$-Padding-Rule (cf. Definition (2.7) to become a multiple of 512 bits. The compression function is based on an AES-like round function MiniRound, processing a 64 -bit message word. The 512 -bit checksum $T$ can be seen as an optional parameter, only needed for computing a message digest greater than 256 bits, otherwise it is set to null. It serves as an additional state to preserve our wide-pipe approach where the state is at least twice as large as the message digest. In the following, Twister-Round ( $S, M_{i}$ ) denotes the invocation of Twister-Round where the parameter $T$ is either null or omitted. Thus,

$$
\text { Twister-Round }\left(S, M_{i}\right)=\text { Twister-Round }\left(S, \text { null, } M_{i}\right) \text {. }
$$

The finalization phase starts after the padded message was fully processed, i.e., the message is completely absorbed by the state $S$. At first, the message length and the checksum - if present - is absorbed into the state by the means of the StateFinalization. Finally, the message digest is computed by following the design ideas of the sponge function [36]; instead of presenting the complete internal state to the attacker, the Output-Round computes as many 64 bit output slices as needed. A description of $\mathrm{TWISTER}_{\pi}$ in pseudo-code notation is shown in Algorithm (11. Next, we give an in-depth description of the individual components of TWISTER $\pi$.

### 9.2.1. Context

The State $S . \quad \operatorname{TwISTER}_{\pi}$ operates on a square state matrix $S=\left(S_{i, j}\right), 1 \leq i, j \leq 8$, consisting of eight rows and columns, where each cell $S_{i, j}$ represents one byte.

| $S_{1,1}$ | $S_{1,2}$ | $\ldots$ | $S_{1,8}$ |
| :---: | :---: | :---: | :---: |
| $S_{2,1}$ | $S_{2,2}$ | $\ldots$ | $S_{2,8}$ |
| $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| $S_{8,1}$ | $S_{8,2}$ | $\ldots$ | $S_{8,8}$ |

Note, $S_{(i \rightarrow)}:=\left(S_{i, 1}, \ldots, S_{i, 8}\right)$ denotes the $i$-th row vector and $S_{(j \downarrow)}:=\left(S_{1, j}, \ldots, S_{8, j}\right)$ the $j$-th column vector. similar to Blowfish [220], the initial state of Twister $\pi_{\pi}$ is given by the first 64 hex digits of the fractional portion of $\pi$. After the initialization
the internal state $S$ is given by the following matrix:

| 24 | 3 F | 6 A | 88 | 85 | A3 | 08 | D3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 13 | 19 | 8 A | 2 E | 03 | 70 | 73 | 44 |
| A4 | 09 | 38 | 22 | 29 | 9 F | 31 | D0 |
| 08 | 2 E | FA | 98 | EC | 4 E | 6 C | 89 |
| 45 | 28 | 21 | E6 | 38 | D0 | 13 | 77 |
| BE | 54 | 66 | CF | 34 | E9 | 0 C | 6 C |
| C0 | AC | 29 | B7 | C9 | 7 C | 50 | DD |
| 3 F | 84 | D5 | B5 | B5 | 47 | 09 | 17 |

Checksum $T$. The checksum enlarges the state of Twister $_{\pi^{-}}-384$ and Twister $_{\pi^{-}}$ 512 to stick to our wide-pipe design [155] decision. In other words: using the checksum, we can double size of the internal state.
similar to the state $S$, the checksum $T$ is represented by a square matrix $T=\left(T_{i, j}\right)$, $1 \leq i, j \leq 8$, consisting out of eight rows and columns, where each cell $T_{i, j}$ represents one byte.

| $T_{1,1}$ | $T_{1,2}$ | $\ldots$ | $T_{1,8}$ |
| :---: | :---: | :---: | :---: |
| $T_{2,1}$ | $T_{2,2}$ | $\ldots$ | $T_{2,8}$ |
| $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| $T_{8,1}$ | $T_{8,2}$ | $\ldots$ | $T_{8,8}$ |

We define a checksum-update operation as

$$
T_{(i \downarrow)}=\left(3 T_{(i \downarrow)}\right) \oplus\left(T_{(i+1 \downarrow)} \boxplus S_{(i \downarrow)}\right),
$$

where $\boxplus$ denotes an addition modulo $2^{64}$. The initial checksum state is given by the all zero state, i.e., $T_{i, j}=0$ with $1 \leq i, j \leq 8$

### 9.2.2. Mini-Round

The Mini-Round (cf. Algorithm (12) is the basic building block of any Twister ${ }_{\pi}$ hash function. The design of this primitive follows the lead of the AES round transformation and thus, prefers simple components over complex ones. The main purpose of this non-linear permutation is to inject a message word $W$, (InjectMessage) and to take care of the diffusion of the state matrix $S$. The core of the Mini-Round is the MixColumns operation where $S$ is multiplied with an MDS matrix to achieve a

```
Algorithm 12 Mini-Round
Input: S {State}, W {Message word}
Output: S {Updated state}
    S\leftarrow InjectMessage(S,W)
    S\leftarrow AddTwistCounter (S)
    S\leftarrowSubBytes(S)
    S\leftarrow ShiftRows}(S
    S\leftarrowMixColumns(S)
    return S
```

proper diffusion. The Mini-Round is visualized in Figure 9.1. TwISTER . $_{\pi}$ can handle at most $2^{64}$ Mini-Rounds. This limitation is caused by the AddTwistCounter operation where a 64 -bit counter is added. Each Mini-Round can process 64 bits of message data. Therefore, with a native usage of a Mini-Round it is possible to process up to $2^{64} \cdot 64$ message bits. If this limitation becomes a real world issue in the future, it is possible to increase the size of the TwistCounter to 128 s bit with almost no performance loss.

InjectMessage (IM). A 64-bit message word $W$ is XOR-injected into the last row. Let $W=W[1], \ldots, W[8]$ where $W[i]$ denotes the $i$-th significant byte of $W$, e.g., $W[8]$ denotes the most significant byte of $W$. Then, we define the message-injection process by

$$
S_{(8 \rightarrow)} \oplus W:=S_{(8,1)} \oplus W[1]\|\cdots\| S_{(8,8)} \oplus W[8]
$$

AddTwistCounter (AC). The TwistCounter ctr is an unsigned 64-bit integer initialized by the maximum value, i.e., OxFFFF_FFFF_FFFF_FFFF. It is XORed byte by byte into the first row of the state $S$.

$$
S_{(1 \rightarrow)} \oplus c t r:=S_{(1,1)} \oplus \operatorname{ctr}[1]\|\ldots\| S_{(1,8)} \oplus \operatorname{ctr}[8]
$$

After successful addition, ctr is decreased by 1.

SubBytes (SB). This function is defined as a bijection

```
SubBytes: {0,1}}\mp@subsup{}{}{8}->{0,1\mp@subsup{}}{}{8
```



Figure 9.1.: Illustration of a Mini-Round.
and is used as an S-box for each byte. It should, among other properties, be highly non-linear. A discussion on how to obtain such cryptographically strong S-boxes (for $8 x 8$ S-boxes) can be found in [241]. TWISTER $\pi$ uses the well-known and intensively studied AES S-Box which can be found in [71].

We define the SubBytes operation by

$$
S_{(i, j)}:=S B\left(S_{(i, j)}\right) \quad \text { with } \quad 1 \leq i, j \leq 8
$$

ShiftRows (SR). ShiftRows is a cyclic left shift similar to the ShiftRows operation of AES. It rotates Row $j$ by $(j-1)$ mod 8 bytes to the left. Suppose $S_{\leftarrow_{j}(i \rightarrow)}$ denotes the i-th row rotated by j bytes to the left. Then, the ShiftRows operation is defined
by

$$
S_{(i \rightarrow)}:=S_{\leftarrow_{i-1}(i \rightarrow)} \quad \text { with } \quad 1 \leq i \leq 8
$$

MixColumns (MC). The MixColumns step is a permutation operation on the state. It applies a $N \times N-$ MDS $A$ (a maximum distance separable matrix as defined below) to each column, i.e., $A \cdot S_{(j \downarrow)}$ for $1 \leq j \leq 8$.

Definition 9.1 (MDS Matrix). An $[n, k, d]$-code with a generator matrix

$$
G=\left[I_{k \times k} A_{k \times(n-k)}\right]
$$

is an MDS code if every square submatrix of $A$ is non-singular, i.e., $d \neq 0$, where $d$ denotes the determinant of $A$. The matrix $A$ is called an MDS matrix.

The MDS matrix chosen for TwISTER ${ }_{\pi}$ is cyclic, i.e., its $i$-th row can be obtained by a cyclic right rotation of (0201010507080601) by $i$ entries. It has a branch number of 9 meaning that if two 8-byte input vectors differ in $1 \leq k \leq 8$ bytes, the output of the MixColumns operation differs in at least $9-k$ bytes. More precisely, the approximate probability that two 8-byte input words with $D_{I}$ different bytes on predefined positions maps to two 8 -byte output words with $D_{O}$ different bytes on predefined positions by the MixColumns operation is given in Table 9.1. The $8 \times 8-\mathrm{MDS}$ matrix used for all proposed instances of $\mathrm{TWISTER}_{\pi}$ is:

$$
\overline{\mathrm{MDS}}=\left(\begin{array}{llllllll}
02 & 01 & 01 & 05 & 07 & 08 & 06 & 01 \\
01 & 02 & 01 & 01 & 05 & 07 & 08 & 06 \\
06 & 01 & 02 & 01 & 01 & 05 & 07 & 08 \\
08 & 06 & 01 & 02 & 01 & 01 & 05 & 07 \\
07 & 08 & 06 & 01 & 02 & 01 & 01 & 05 \\
05 & 07 & 08 & 06 & 01 & 02 & 01 & 01 \\
01 & 05 & 07 & 08 & 06 & 01 & 02 & 01 \\
01 & 01 & 05 & 07 & 08 & 06 & 01 & 02
\end{array}\right)
$$

All of the byte-entries are considered to be elements of $G F\left(2^{8}\right)$. An element of $G F\left(2^{8}\right)$ is represented by $\sum_{i=0}^{7} a_{i} 2^{i}$. The reduction polynomial $R(x)$ of $G F\left(2^{8}\right)$ is defined by

$$
R(x)=x^{8}+x^{6}+x^{3}+x^{2}+1
$$

A detailed discussion about the properties of MDS matrices/codes is given in [158].

| $D_{I} / D_{O}$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 1 | $-\infty$ | $-\infty$ | $-\infty$ | $-\infty$ | $-\infty$ | $-\infty$ | $-\infty$ | $-\infty$ |
| 1 | $-\infty$ | $-\infty$ | $-\infty$ | $-\infty$ | $-\infty$ | $-\infty$ | $-\infty$ | $-\infty$ | 1 |
| 2 | $-\infty$ | $-\infty$ | $-\infty$ | $-\infty$ | $-\infty$ | $-\infty$ | $-\infty$ | $2^{-8}$ | 0.99 |
| 3 | $-\infty$ | $-\infty$ | $-\infty$ | $-\infty$ | $-\infty$ | $-\infty$ | $2^{16}$ | $2^{-8}$ | 0.99 |
| 4 | $-\infty$ | $-\infty$ | $-\infty$ | $-\infty$ | $-\infty$ | $2^{-24}$ | $2^{16}$ | $2^{-8}$ | 0.99 |
| 5 | $-\infty$ | $-\infty$ | $-\infty$ | $-\infty$ | $2^{32}$ | $2^{-24}$ | $2^{16}$ | $2^{-8}$ | 0.99 |
| 6 | $-\infty$ | $-\infty$ | $-\infty$ | $2^{-40}$ | $2^{32}$ | $2^{-24}$ | $2^{16}$ | $2^{-8}$ | 0.99 |
| 7 | $-\infty$ | $-\infty$ | $2^{-48}$ | $2^{-40}$ | $2^{32}$ | $2^{-24}$ | $2^{16}$ | $2^{-8}$ | 0.99 |
| 7 | $-\infty$ | $2^{-56}$ | $2^{-48}$ | $2^{-40}$ | $2^{32}$ | $2^{-24}$ | $2^{16}$ | $2^{-8}$ | 0.99 |

Table 9.1.: Column properties of the state matrix after multiplication with an MDS matrix.

### 9.2.3. Compression Function



Figure 9.2.: Illustration of a Twister-Round.

The compression function

$$
\text { Twister-Round : }\{0,1\}^{512} \times\left[\{0,1\}^{512}\right] \times\{0,1\}^{512} \rightarrow\{0,1\}^{512}
$$

maps three 512-bit input values (i.e., the state, an optional checksum and a message block) to a 512-bit output value. It consists of nine Mini-Rounds. Each of the first eight rounds absorbs a 64 -bit word (i.e., $W_{1}, \ldots, W_{8}$ ) of the message block into the state. The last Mini-Round is invoked with a virtual all-zero message word to limit an adversaries control over the internal state of the hash function. Such a BlankRound is a common building component in the design of cryptographic hash functions [35, 141, 180]. The individual Mini-Rounds are separated by a feed-forward operation with the state before its invocation (cf. Figure 9.2) to guarantee the one-wayness of Twister-Round since the remaining Mini-Round operations are invertible. The feedforward operation is defined by

$$
S:=S \oplus \operatorname{Mini}-\operatorname{Round}(S, X),
$$

```
Algorithm 13 Twister-Round
Input: \(S\) \{State\}, \(T\) \{Checksum\}, \(W\) \{Message Block \}
Output: \(S\) \{Updated State\}, \(T\) \{Updated Checksum \(\}\)
    for \(i=1, \ldots, 8\) do
        if \(T \neq\) null then
            \(T_{(i \downarrow)} \leftarrow 3 \cdot T_{(i \downarrow)} \oplus\left(T_{(i+1) \downarrow} \boxplus S_{(i \downarrow)}\right)\)
        end if
        \(S \leftarrow \operatorname{Mini}-\operatorname{Round}\left(S, W_{i}\right) \oplus S\)
    end for
    \(S \leftarrow \operatorname{Mini}-\operatorname{Round}(S, 0) \oplus S\)
    return \((S, T)\)
```

where $X$ is either a message word or a sequence of 64 zero-bits. The optional checksum for computing a message digest longer than 256 bits is updated before processing a message word. A description of Twister-Round in pseudo-code notation is given in Algorithm 13

### 9.2.4. Post-Processing

This section describes the Twister ${ }_{\pi}$ finalization process. It starts after the message is completely processed by iterating the compression function over all message blocks. The post-processing consists of the following two steps:

State Finalization. At first, to prevent length-extension attacks (see Section 2.3), the state is updated by processing the bit-length $|M|$ of the unpadded message $M$, encoded as a 64 -bit value, together with the current state by means of a Mini-Round (i.e., $S \leftarrow \operatorname{Mini}-\operatorname{Round}(S,|M|))$. Afterwards, the state finalization ends with either a Blank-Round or the processing of the checksum with the state by means of a Twister-Round, depending on the length of the message digest. In the latter case, the checksum $T$ is transformed in a 64 -byte message block $M=M[1], \ldots, M[64]$ column by column where

$$
(M[(i \cdot 8)-7], \ldots, M[i \cdot 8]) \leftarrow T_{(i \downarrow)} \text { with } 1 \leq i \leq 8 .
$$

A formal definition of State-Finalization is given in Algorithm 14

Message Digest Computation. The task of Output-Round is the computation of the actual message digest from a given state $S$. A description in pseudo-code notation

```
Algorithm 14 State-Finalization
    \(S \leftarrow \operatorname{Mini-Round}(S,|M|)\)
    if \(n \leq 256\) then
        \(S \leftarrow \operatorname{Mini-Round}(S, 0)\)
    else
        \(S \leftarrow\) Twister-Round \((S\), null,\(T)\)
    end if
```

is shown in Algorithm 15. For every 64 bits of the message digest, the current state is first saved, and then updated by a Blank-Round followed by a feed-forward operation with the saved state, and finally, an additional invocation of a Blank-Round (see Lines $3-6)$. A 64 -bit output value is then obtained by XOR-ing the first column of $S$ with the first column of the saved state (see Line 7). This procedure is repeated until the needed amount of message digest bits is obtained. The last output stream can be varied between 32 bits and 64 bits by taking only the first half of the output value. This allows to vary the output size for a huge amount of applications. Note that due to Output-Round, TWISTER $\pi$ can theoretically produce hash values up to $2^{64}$ bits. This limitation results from the initial step where the output length is written into to first row of the state (cf. Algorithm 11). In some particular scenarios, long hash values can become handy, e.g., full domain hashing [26]. Anyway, the security of TWISTER $_{\pi}$ is limited by the state size.

```
Algorithm 15 Output-Round
Input: \(S\) \{State\}, \(n\) \{Output Length\}
Output: \(Y\) \{Hash Value \(\}\)
    \(Y \leftarrow \emptyset\)
    for \(i=1, \ldots,\lceil n / 64\rceil\) do
        \(X \leftarrow S\)
        \(S \leftarrow \operatorname{Mini}-\operatorname{Round}(S, 0) \oplus S\)
        \(S \leftarrow \operatorname{Mini}-\operatorname{Round}(S, 0)\)
        \(Y \leftarrow Y \|\left(S_{(1 \downarrow)} \oplus X_{(1 \downarrow)}\right)\)
    end for
```


### 9.3. Security against Generic Attacks

In this section we give a brief discussion why $\operatorname{TWISTER}_{\pi}$ is resistant to existing generic attacks.

Length-Extension Attacks. The combination of the Twister $\pi_{\pi}$ padding rule and the processing of the message length in the post-processing phase avoids such type of attacks. Another possible attack can be as follows:

For a known hash value $\mathcal{H}(M)$, one can compute the hash value $\mathcal{H}(M\|Y\| Z)$ for any suffix $Z$ if the length of an unknown message $M$ is known as well as the padding $Y$ of $M$. Twister ${ }_{\pi}$ is secure against such attacks due to two countermeasures: (1) By knowing only the hash value, an attacker cannot easily determine the state $S$ after the last compression function call as it has only access to the hash value generated by the Output-Round, which squeezes out some bits of the state by applying the output transformation. The bits of the squeezing process do not leave enough information to recover the internal state; (2) The multiple feed-forward does also prevent any attacker to successfully gain any knowledge about prior state information. In each squeezing process, one feed-forward is applied.

Multi-Collision Attacks. An instance of $\mathrm{TWISTER}_{\pi}$ fully resists a multi-collision attack if $8 \cdot 8^{2}=512 \geq n$ since the complexity is determined by $k \cdot 2^{512 / 2}$. All instances of $\mathrm{TWISTER}_{\pi}$ have this feature, although the state of $\mathrm{TWISTER}_{\pi}-384$ and TWISTER $_{\pi}-512$ is not big enough to prevent this attack by itself, but including the checksum can be viewed as an enlargement of the state which then provides resistance against this kinds of attacks.

Herding Attacks. For Twister $\pi_{\pi}-256$ (Twister $_{\pi}-512$ ), we have an internal state of 512 bits $(|S|+|T|=1024)$ and with $512>\frac{3 \cdot 256-5}{2}=381.5$ bits $\left(1024>\frac{3 \cdot 512-5}{2}=\right.$ 765.5). The attack has the same complexity as for a (2nd-) preimage attack on a random oracle. The complexity of this attack decreases with increased size of the message. If the message is of size about $2^{\ell}$, the complexity of the attack is $2^{(2 n-5) / 3-\ell}$. It is easy to see that all of our proposed instances of TWISTER ${ }_{\pi}$ provides resistance $^{\text {Pr }}$ against this kind of attacks.

Long 2nd-preimage Attacks. Long 2nd-preimage attacks cannot be applied to the Twister $_{\pi}$ framework for three reasons. First, in each Mini-Round, the TwistCounter $c t r$ is added to the second column of the state $S$ which does not allow to
find expandable messages. Second, Twister ${ }_{\pi}$ uses multiple feed-forwards and third, the internal chaining value is in general much larger than $n$. This makes it harder to find collisions and fix points since we essentially have constructions similar to the wide-pipe design [155].

Slide Attacks. The TwistCounter ctr prevents slide attacks since in each iteration of the Mini-Round, a fresh value is injected into the state matrix which does not allow an adversary to find slid pairs of messages. Furthermore, the last inserted message block cannot be the all-zero block due to the padding rules. Thus, slide attacks are not possible for $\mathrm{TwISTER}_{\pi}$.

### 9.4. Implementation Details

In this section we discuss issues related to the implementation of TWISTER ${ }_{\pi}$ on different platforms. Test vectors to verify a specific implementaion are given in Appendix A.

In essence, our techniques for implementing this cryptographic hash function rely on the following key sources of information:

- Optimization techniques as given in [71] and
- some of the new techniques on how to reduce the number of instructions for an AES implementation as given in 33].

Most of the discussed issues are relevant for more than one platform.
Note that there are no multiplications of two arbitrary values of $G F\left(2^{8}\right)$, but only multiplications of a variable with some fixed constants. The latter is easier to implement than the former - especially in the context of hardware and high-speed software implementations.

### 9.4.1. 64-Bit Platforms

All steps of the round transformation, (i.e.,SubBytes, ShiftRows, and MixColumns) can be combined in a single set of lookup tables, allowing for very fast implementations on processors with word length greater equal 64 bits. The following notations
will be used for the elements at matrix position $(i, j)$ and for (for $1 \leq i, j<8$ ):

$$
\begin{array}{ll}
a_{i, j} & \text { input state matrix element, } \\
b_{i, j} & \text { state matrix element after SubBytes, } \\
c_{i, j} & \text { state matrix element after ShiftRows, } \\
d_{i, j} & \text { state matrix element after MixColumns. }
\end{array}
$$

After finishing the MixColumns operation, we have for $1 \leq j \leq 8$ :

$$
\left[\begin{array}{c}
d_{1, j} \\
d_{2, j} \\
d_{3, j} \\
d_{4, j} \\
d_{5, j} \\
d_{6, j} \\
d_{7, j} \\
d_{8, j}
\end{array}\right]=\left[\begin{array}{llllllll}
02 & 01 & 01 & 05 & 07 & 08 & 06 & 01 \\
01 & 02 & 01 & 01 & 05 & 07 & 08 & 06 \\
06 & 01 & 02 & 01 & 01 & 05 & 07 & 08 \\
08 & 06 & 01 & 02 & 01 & 01 & 05 & 07 \\
07 & 08 & 06 & 01 & 02 & 01 & 01 & 05 \\
05 & 07 & 08 & 06 & 01 & 02 & 01 & 01 \\
01 & 05 & 07 & 08 & 06 & 01 & 02 & 01 \\
01 & 01 & 05 & 07 & 08 & 06 & 01 & 02
\end{array}\right] \times\left[\begin{array}{c}
S B\left[a_{1, j}\right] \\
S B\left[a_{2, j+1}\right] \\
S B\left[a_{3, j+2}\right] \\
S B\left[a_{4, j+3}\right] \\
S B\left[a_{5, j+4}\right] \\
S B\left[a_{6, j+5}\right] \\
S B\left[a_{7, j+6}\right] \\
S B\left[a_{8, j+7}\right]
\end{array}\right]
$$

where $S B:\{0,1\}^{8} \rightarrow\{0,1\}^{8}$ denotes the S-Box operation, and where ' + ' denotes a wraparound addition, e.g., $j+6 \equiv 2$ for $j=4$. This matrix multiplication can be interpreted as a linear combination of all eight column vectors:

$$
\left[\begin{array}{l}
d_{1, j} \\
d_{2, j} \\
d_{3, j} \\
d_{4, j} \\
d_{5, j} \\
d_{6, j} \\
d_{7, j} \\
d_{8, j}
\end{array}\right]=\left[\begin{array}{c}
02 \\
01 \\
06 \\
08 \\
07 \\
05 \\
01 \\
01
\end{array}\right] S B\left[a_{1, j}\right] \oplus\left[\begin{array}{c}
01 \\
02 \\
01 \\
06 \\
08 \\
07 \\
05 \\
01
\end{array}\right] S B\left[a_{1, j+1}\right] \oplus \ldots \oplus\left[\begin{array}{c}
01 \\
06 \\
08 \\
07 \\
05 \\
01 \\
01 \\
02
\end{array}\right] S B\left[a_{1, j+7}\right] .
$$

We now define eight $V$-tables: $V_{1}, V_{2}, \ldots, V_{8}$ :

$$
V_{1}[\alpha]=\left[\begin{array}{c}
02 \\
01 \\
06 \\
08 \\
07 \\
05 \\
01 \\
01
\end{array}\right] S B[\alpha], \quad V_{2}[\alpha]=\left[\begin{array}{c}
01 \\
02 \\
01 \\
06 \\
08 \\
07 \\
05 \\
01
\end{array}\right] S B[\alpha], \quad \ldots \quad V_{8}[\alpha]=\left[\begin{array}{c}
01 \\
06 \\
08 \\
07 \\
05 \\
01 \\
01 \\
02
\end{array}\right] S B[\alpha] .
$$

It follows that we can write the combined operation of SubBytes, ShiftRows, and MixColumns as

$$
\left[\begin{array}{l}
d_{1, j} \\
d_{2, j} \\
d_{3, j} \\
d_{4, j} \\
d_{5, j} \\
d_{6, j} \\
d_{7, j} \\
d_{8, j}
\end{array}\right]=V_{1}\left[a_{1, j}\right] \oplus V_{2}\left[a_{1, j+1}\right] \oplus \quad \ldots \quad \oplus V_{1}\left[a_{1, j+7}\right] .
$$

So, there are only 64-bit XOR operations involved in the computation of a TwisterRound that can be implemented quite efficiently on most platforms.

### 9.4.2. 32-Bit Platforms

By splitting the 64-bit lookup tables $V_{1}, \ldots, V_{8}$ into 32-bit chunks, it takes twice as much operations as compared to the 64 -bit variant. More general, this TwisterRound implementation has the desirable feature of scaling down linearly in terms of speed depending on the available word size of the platform.

### 9.4.3. Specific Remarks for 8 -Bit Platforms

The performance on 8-bit processors is an important issue since most smart cards with cryptographic applications are restricted to their usage. There are several options for implementing $\mathrm{TWISTER}_{\pi}$, depending on whether the requirements demand for minimum space (i.e., low memory for storing lookup tables) or maximum speed. If minimum space is requested, the multiplication of two elements in $G F\left(2^{8}\right)$ has to be performed in software and should not be stored as a lookup table. Specific details for such issues can be found in [71, Chapter 4.1.1]. If space limitations are not an issue, the technique for implementing TWISTER ${ }_{\pi}$ via lookup tables should be chosen as discussed in Section 9.4.1 or by splitting them up into single operations as discussed in Section 9.4.2, As all operations linearly scale down in terms of speed, i.e., a 64 -bit XOR can be easily implemented via eight times an 8 -bit XOR, the running time is eight times the running time on a 64 -bit platform.

### 9.4.4. Dedicated Hardware

$\mathrm{TwISTER}_{\pi}$ is suited to be implemented in dedicated hardware. There are several tradeoffs between chip area and speed possible. Since the implementation in software on general-purpose processors is already very fast, the need for hardware implementation will probably be limited to very specific cases like:

1. Extreme high-speed chips with no area restrictions: The tables $V_{1}, \ldots, V_{8}$ can be hard-wired and the XOR operations can be conducted in parallel.
2. Compact coprocessors on smart cards: There can either be only the S-Box hardwired or, additionally (and if enough memory is available), the tables $V_{1}, \ldots, V_{8}$ be generated at runtime.
3. If there is essentially no space to hard-wire anything, even the S-Box can be generated at runtime. Since Twister ${ }_{\pi}$ uses the Rijndael S-Box, one can assemble it using two transformations:

$$
S B[\alpha]=f(g(\alpha)),
$$

where $g(\alpha)$ is defined as

$$
\alpha \rightarrow \alpha^{-1} \text { in } G F\left(2^{8}\right)
$$

and $f(\alpha)$ is an affine transformation.
Note that there are finite-field multiplier over $\operatorname{GF}\left(2^{n}\right)$ available in hardware that execute in a single clock cycle. More information is available in , e.g., [188] or, for a short summary, in [71].

### 9.5. Benchmarks

This section provides software-performance benchmarks for the Twister $_{\pi}$ reference implementation. All measurement results are based on the real-time clock (RTC) and obtained by the median of 5,000 measurements of the target function. The performance values are given in cpb . For the sake of comparison, we also provide performance benchmarks for SHA-256 and SHA-512, where we used the implementation from OpenSSI ${ }^{2}$ version 1.0.1e.

[^3]| Bytes | SHA-256 | TWISTER $_{\boldsymbol{\pi}} \mathbf{- 2 5 6}$ | SHA-512 | TWISTER $_{\boldsymbol{\pi}} \mathbf{- 5 1 2}$ |
| :---: | :---: | :---: | :---: | :---: |
| 20 | 40.2 | 86.6 | 49.8 | 160.2 |
| 64 | 22.8 | 41.0 | 15.6 | 64.1 |
| 256 | 13.5 | 20.5 | 10.4 | 26.4 |
| 512 | 12.0 | 17.1 | 8.4 | 20.1 |
| 576 | 11.8 | 16.7 | 7.6 | 19.4 |
| 1024 | 11.5 | 15.3 | 7.5 | 16.9 |
| 1500 | 10.7 | 14.5 | 6.8 | 15.6 |
| 4096 | 10.6 | 14.0 | 6.8 | 14.4 |
| 10000 | 10.5 | 13.8 | 6.6 | 14.0 |
| 16384 | 10.3 | 13.7 | 6.6 | 13.9 |
| 32768 | 10.3 | 13.7 | 6.6 | 13.8 |

Table 9.2.: The benchmark results for 64-bit platforms in cpb on an Intel Core i53210M CPU 2.50GHz; OS: Linux 3.9-1-amd64; Compiler: gcc 4.7.3.

| Bytes | SHA-256 | TwISTER $_{\boldsymbol{\pi}} \mathbf{- 2 5 6}$ | SHA-512 | TWISTER $_{\boldsymbol{\pi}} \mathbf{- 5 1 2}$ |
| :---: | :---: | :---: | :---: | :---: |
| 20 | 137.3 | 223.9 | 456.0 | 415.4 |
| 64 | 71.7 | 105.3 | 142.7 | 165.8 |
| 256 | 39.4 | 53.0 | 92.6 | 68.6 |
| 512 | 34.0 | 44.3 | 74.9 | 52.5 |
| 576 | 33.4 | 43.3 | 66.6 | 50.7 |
| 1024 | 31.4 | 39.9 | 66.0 | 44.3 |
| 1500 | 30.0 | 37.9 | 59.7 | 41.1 |
| 4096 | 29.5 | 36.7 | 59.3 | 38.3 |
| 10000 | 29.1 | 36.0 | 57.9 | 37.0 |
| 16384 | 29.0 | 35.9 | 57.7 | 36.8 |
| 32768 | 29.0 | 35.8 | 57.4 | 36.5 |

Table 9.3.: The benchmark results for 32-bit platforms in cpb on an Intel Core i53210M CPU 2.50GHz; OS: Linux 3.9-1-amd64; Compiler: gcc 4.7.3.


Figure 9.3.: The compression function Twister-Round-256.

Target Platform. The benchmarking took place on a single core of an Intel Core i5-3210M CPU 2.50 GHz processor. All of the software benchmarking was written in C or ASM and compiled with the GNU C compiler ( $g c c$ ) version 4.7.3 using the optimization flag -03.

Implementation Remarks. TWISTER $_{\pi}$ was especially designed with 64 -bit platforms in mind by making it possible to aggregate eight times an 8-bit table lookup into one single 64 -bit table lookup.

Results. The results of the 64 -bit and 32 -bit performance benchmarks are summarized in Tables 9.2 and 9.3 , respectively.

### 9.6. From Twister to $\mathrm{Twister}_{\pi}$

The task of this section is to introduce the differences between Twister $\boldsymbol{\pi}_{\pi}$ and its predecessor Twister [93]. All modifications have been taken into account as a results of the external TwISTER ${ }_{\pi}$ cryptanalysis from Mendel et al. [166]. A detailed discussion on their findings is given in Section 9.7. The major change between the two hash function families lies in the structure of the compression function. All members of the $\mathrm{TwISTER}_{\pi}$ family use the same compression function, independent of the length of the message digest. On the other hand, the Twister family uses two similar compression functions, (1) Twister-Round-256, for the computation of message digest up to 256 bits and (2) Twister-Round- 512 to compute message digests longer than 256 bits. Both hash functions follow the Min-Max approach, i.e., either three or four Mini-Rounds are pooled to a Maxi-Round. The feed-forward operation (as visualized in Figure 9.3 and 9.4) is only performed after each Maxi-Round and not after each Mini-Round since two consecutive Mini-Rounds without a feed-forward in between guarante full diffusion - a nice property which unfortunately does not imply either collision or preimage security. In Twister ${ }_{\pi}$, we perform a feed-forward after each Mini-Round to thwart rebound attacks. Furthermore, in Twister ${ }_{\pi}$, we


Figure 9.4.: The compression function Twister-Round-512.

| Type | Compression Function Calls | Memory Requirement |
| :--- | :---: | :---: |
| collision attack | $2^{251}$ | $2^{9}$ |
| 2nd-preimage attack | $2^{384}$ | $2^{64}$ |
| preimage attack | $2^{456}$ | $2^{10}$ |

Table 9.4.: Cryptanalytic results for Twister-512 166 .
improved the checksum algorithm by a non-linear operation, namely multiplication by 3. Finally, Twister injects the TwistCounter into the second column. Twister $\pi_{\pi}$ injects the TwistCounter into first row to circumvent a cancellation of a non-zero difference between two message words $W$ and $W^{\prime}$.

### 9.7. Untwisting the Myth - Cryptanalysis of Twister

This section consists of three parts. The first part introduces the preliminaries needed for understanding differential cryptanalysis. The second part presents the cryptanalytic results for Twister-512 from Mendel et al. [166] - summarized in Table 9.4 - and the third part discusses why those attacks are not applicable to $\mathrm{TwISTER}_{\pi}$ anymore.

### 9.7.1. Preliminaries of Differential Cryptanalysis

Differential cryptanalysis, introduced by Biham and Shamir at Crypto'90 [42], turned out to be one of the most powerful techniques to attack cryptographic primitives like hash functions and block ciphers. It follows differential trails that occur with a significant probability, instead of looking at specific values. Next, we give a brief introduction about the basic definitions that are needed in the following cryptanalysis of TWISTER. The notions are borrowed from 219].

Definition 9.2 (XOR Difference). Suppose $x_{1}$ and $x_{2}$ are two $n$-bit values. Then, the ( $n$-bit) XOR difference is defined by

$$
\Delta x=x_{1} \oplus x_{2}
$$

Definition 9.3 (Differential Probability). Let $F:\{0,1\}^{n} \rightarrow\{0,1\}^{m}$ be a function. Suppose $\Delta x$ is an n-bit input difference and $\Delta y$ is an m-bit output difference. Then, we define the differential probability by

$$
\operatorname{Pr}[\Delta x \xrightarrow{F} \Delta y]=2^{-m} \sum_{i=0}^{2^{n}-1}(F(i) \oplus F(i \oplus \Delta x)=\Delta y)
$$

In this thesis we name $\Delta x$ input difference, $\Delta y$ output difference and $\Delta x \xrightarrow{F} \Delta y$ differential. Note that differentials with a differential probability of zero are called impossible differentials.

Difference-Distribution Table (DDT). The number of right pairs of a differential $\Delta x \xrightarrow{F} \Delta y$ denoted as $N_{F}(\Delta x \xrightarrow{F} \Delta y)$ is the number of pairs which satisfy that an input difference $\Delta x$ leads to an output difference $\Delta y$. Usually, cryptanalysts are interested in the number of right pairs for all possible input and output values of a non-linear function, e.g., an S-box, which can be encoded as DDT.

Definition 9.4 (Difference-Distribution Table (DDT)). Let $F$ be an $n \times m S$ Box. Then, the DDT of $F$ is a $2^{n} \times 2^{m}$ table whose entries are the number of right pairs $N_{F}(\Delta x \xrightarrow{F} \Delta y)$ for all differentials $\Delta x \xrightarrow{F} \Delta y$. The rows and columns of the $D D T$ are indexed by the input differences $\Delta x$ and output differences $\Delta y$, respectively.

Due to the fact that the XOR operation is commutative, all entries of a DDT are even values, and the sum of each row must be $2^{n}$. A toy example is given in Figure 9.5 .

$$
F(x)= \begin{cases}1, & x=0 \\ 3, & x=1 \\ 2, & x=2 \\ 0, & x=3\end{cases}
$$

| $\Delta x / \Delta y$ | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 4 | 0 | 0 | 0 |
| 1 | 0 | 2 | 2 | 0 |
| 2 | 0 | 2 | 0 | 2 |
| 3 | 0 | 4 | 0 | 0 |

Figure 9.5.: Example computation of a DDT for a $2^{2} \times 2^{2}$ S-box.

AES S-box. The $2^{8} \times 2^{8}$ AES S-box $\mathrm{SB}-$ which is also used in TwISTER $_{\pi}$ and Twister - provides a nearly uniform distribution of XOR differentials. More precisely, the 65,536 entries of the $\mathrm{SB}-\mathrm{DDT}$ are given by 33,150 entries $0,32,130$ entries 2,255 entries 4 , and one entry of 256 [219]. So, the probability that an entry chosen uniformly at random contains a value greater than zero is about $1 / 2$. A more formal description of this observation is given in the following proposition:

Proposition 9.5 (Non-Zero Probability of the AES S-Box). Suppose $S B$ is the AES S-box, and let $(\Delta x, \Delta y)$ a fixed tuple of input-output differences. Then, it holds that

$$
\operatorname{Pr}\left[N_{S B}(\Delta x \xrightarrow{S B} \Delta y)>0\right]=\frac{32386}{65536} \approx 1 / 2
$$

### 9.7.2. Collision Attack on $\mathrm{TWISTER}_{\pi}-512$

The core of the collision attack is a semi-free-start collision attack for the compression function based on a rebound attack presented at FSE'09 by Mendel et al. [167] - about four months after the SHA-3 submission deadline. This attack was then developed into a collision attack for the hash function using Wagner's generalized birthday attack 233].

Semi-Free-Start Collision Attack. This attack only considers the first Maxi-Round of the Twister-Round-512 compression function. This operation updates the state $S^{0}$ by processing the first message words $W_{1}, W_{2}$, and $W_{3}$ of a 512-bit message block $M_{i}$ by three consecutive invocations of the Mini-Round, i.e., $S^{\prime}=\left(\right.$ Mini-Round $W_{3} \circ$ Mini-Round $\left.W_{2} \circ \operatorname{Mini-Round} W_{1}\right)\left(S^{0}\right)=\operatorname{Maxi-Round}\left(S^{0}, W_{1}, W_{2}, W_{3}\right)$. Note that we can further decompose a Mini-Round into its individual basic operations:

$$
\operatorname{Mini}-\operatorname{Round}\left(S^{0}, W_{i}\right)=\left(M C \circ S R \circ S B \circ A C \circ I M_{W_{i}}\right)\left(S^{0}\right)
$$



Figure 9.6.: A schematic view of the semi-free-start collision attack of TwISTER-512. Black state bytes are active.

Then, we have

$$
\begin{aligned}
S^{\prime}=\operatorname{Maxi}-\operatorname{Round}\left(S^{0}, W_{1}, W_{2}, W_{3}\right)= & \left(M C \circ S R \circ S B \circ A C \circ I M_{W_{3}} \circ\right. \\
& M C \circ S R \circ S B \circ A C \circ I M_{W_{2}} \circ \\
& \left.M C \circ S R \circ S B \circ A C \circ I M_{W_{1}}\right)\left(S^{0}\right) .
\end{aligned}
$$

The basic idea of the attack is to inject a message-word difference $\Delta W_{1}$ into the first Mini-Round, which can be canceled by the message-word difference $\Delta W_{3}$ in the third Mini-Round. The rebound attack can then be described by an inbound step and an outbound step (cf. Figure 9.6). The inbound step propagates differences in $W_{1}$ and $W_{3}$ forwards and backwards through the MixColumns operation with a probability of 1. The goal of the outbound step is to find matches for the resulting differences of the SubBytes operation of the second round and propagate them outwards.

Inbound Step. Let $S^{0}, \ldots, S^{6}$, and $S^{\prime}$ be the internal states of Twister as shown in Figure 9.6. First, a message word of eight active bytes, i.e., an XOR difference unequal zero, is injected into the last row of the State $S^{1}$ by means of the InjectMessage operation followed by another message injection into the state $S^{5}$ to cancel out the remaining active bytes in the last row. Then, the two active States $S^{2}$ and $S^{4}$ are computed by forward and backward computation, respectively. The column property of the MixColumns operation and its inverse (see Section 9.2.2) ensures that all 64 bytes of $S^{3}$ and $S^{4}$ are active.

Outbound Step. In this step, the adversary has to find a match for the input and output differences of the SubBytes operation of the second Mini-Round. Note that all 64 bytes of $S^{3}$ and $S^{4}$ are active. So, we have to estimate the effort for the adversary in finding 64 matches. For a single S-box call, the probability that a fixed $\Delta x \xrightarrow{S B} \Delta y$ exists is about $2^{-1}$ (see Proposition 9.5). For such a differential, it is possible to assign at least two possible values to the S-box, due
to the symmetry of XOR differences. Next, the adversary chooses a random difference for the active Byte $S_{(8,1)}^{2}$ and then, computes the first column of the interim state $S^{3}$. The probability of finding non-zero differentials for all entries of this column is $2^{-8}$. After finding a match, the adversary continues with the next column, until all columns are successfully processed. The complexity of this step is less than $2^{8}$ compression function calls.

After executing the outbound step, the adversary has found a differential match for the SubBytes operation and can choose from at least $2^{8}$ possible states for $S^{3}$. Each of those states can be computed forwards or backwards and produces a semi-freestart collision for a Maxi-Round. Each computation determines as well the State $S^{0}$ as the values and differences of $W_{1}$ and $W_{3}$, where the value of the message word $W_{2}$ can be freely chosen.

Next, we show how this semi-free-start collision attack can be extended to a collision attack on Twister-512.

Collision Attack on the Compression Function. At first, the adversary $\mathcal{A}$ computes $2^{224}$ semi-free-start collisions for the last Maxi-Round of Twister-Round-512 and stores them in a list $\mathfrak{L}$. This has a time complexity of about $2^{224}$ compression function calls. By varying the values for the massage word $W_{7}, \mathcal{A}$ can gain additional $2^{64}$ degree of freedom. After this pre-computations step, $\mathcal{A}$ randomly chooses some values for the message words $W_{1}-W_{5}$ and computes the input of the last Maxi-Round. Statistically, the adversary finds a match in $\mathfrak{L}$ after $2^{224}$ tries. The complexity for this step of the attack is about $2 / 3 \cdot 2^{224}$ compression function calls. In total, this collision attack has a time complexity of at most $2^{225}$ compression function calls and a memory complexity of $2^{224}$. The adversary can get rid of the memory complexity by applying the memory-less variant of the meet-in-the-middle attack introduced by Quisquater and Delescaille [195].

In the next paragraph we discuss how $\mathcal{A}$ can extend a collision for the compression function into a genuine collision for Twister-512.

Collision Attack on the Hash Function. In addition to the $8 \times 8$ state matrix $S$, Twister also has an $8 \times 8$ checksum matrix $T$ which is updated immediately before the processing of a message word by a Mini-Round. In the post-processing, $T$ is absorbed into the state via the compression function. Therefore, to construct a collision for Twister-512, an adversary has to implement a collision for both the chaining value represented by the state $S$ and the checksum $T$. This can be
done by applying the multi-collision attack introduced by Joux [129]. The effort to construct $2^{t}$ multi-collisions is $t \cdot \alpha$ where $\alpha$ denotes the complexity for constructing a single collision. A collision for the compression function of Twister- 512 can be constructed with a time complexity of about $2^{225}$. Thus, an adversary can construct $2^{256}$ collision with a time complexity of about $256 \cdot 2^{225}=2^{233}$ evaluations of the compression function. The memory complexity for this attack is about $2^{9}$ to store the $2^{256}$ multi-collisions. Due to the birthday paradox, the probability that two out of $2^{256}$ corresponding checksums are equal is greater than $1 / 2$. Thus, we can assume that the set of multi-collisions also contains a collision for Twister-512. The time complexity to find a colliding checksum pair is about $2^{256}$ checksum computations, i.e., 16 integer operations consisting of eight XOR operations and eight modular additions per checksum computation. In contrast, a Twister-Round-512 evaluation, omitting the checksum computation, can be done in 684 integer operations and 64 load/store operations. The individual numbers can be computed by adding up the cost of the individual operations which are

- $3 \cdot 8$ XOR operations for the feed-forward operations,
- 10•1 XOR operations for the InjectMessage step,
- 10•64 XOR operations for a combined operation that consists of the SubBytes, ShiftRows, and MixColumns step, and
- $10 \cdot 64$ table lookups for such a combined operation.

Thus, we can assume that the cost of evaluating the compression function is at least 32 times higher than the computation of a checksum, so, $2^{256}$ checksum computations has an effort of at most $2^{251}$ compression function evaluations. The memory complexity for this step is negligible when applying the memory-less variant of the birthday attack [195].

### 9.7.3. 2nd-Preimage Attack on Twister-512

Let $S^{i}$ denote the state $S$ after the invocation of the $i$-th Mini-Round within a Twister-Round-512. The following 2nd-preimage attack has a minor limitation; it only works for a given message-hash tuple ( $M$,Twister-512( $M$ ) ), where the message consists of at least 513 message blocks, i.e., $|M|>512 \cdot 513$. Without loss of generality, we assume that the message $M$ consists of 513 message blocks. The process of the 2nd-preimage adversary $\mathcal{A}$ can be described by the following six consecutive steps:

1. $\mathcal{A}$ applies the multi-collision attack to construct $2^{512}$ multi-collisions with a time complexity of about $512 \cdot 2^{225}=2^{234}$ evaluations of the compression function and a memory complexity of $2^{9}$. After this step, $\mathcal{A}$ gets $2^{512}$ messages leading to the same chaining value $V_{512}$, i.e., the state after 512 iterations of the compression function.
2. $\mathcal{A}$ chooses arbitrary values for the last message block $M_{513}^{\prime}$ with correct padding and computes the chaining value $V_{513}$, i.e., the state which is used as input for the post-processing step.
3. $\mathcal{A}$ chooses arbitrary values for the first five columns of the checksum $T^{\prime}$, i.e., $T_{(1 \downarrow)}^{\prime}, \ldots, T_{(5 \downarrow)}^{\prime}$. Then, it computes the interim state $S_{F}^{6}=V_{513}^{\prime} \oplus S^{3} \oplus S^{6}$ of the last compression function call, Twister-Round-256( $\left.V_{513}^{\prime}, T^{\prime}\right)$. From the first preimage, $\mathcal{A}$ can compute $S^{\prime}=$ Twister-Round-256 $\left(S^{0}, T\right)$, and then the value $S^{10}=S^{\prime} \oplus S_{F}^{6}$.
4. For each of the $2^{64}$ possible values of $T_{(8 \downarrow)}^{\prime}, \mathcal{A}$ computes backwards the values $S^{\prime 7}=\operatorname{InjectMessage}\left(S^{\prime 7}, T_{(7 \downarrow)}^{\prime}\right)$ and stores them in the list $\mathfrak{L}$.
5. For each of the $2^{64}$ possible values of $T_{(6 \downarrow)}^{\prime}, \mathcal{A}$ computes forwards from $S^{6}$ to the injection of the checksum word $T_{(7 \downarrow)}^{\prime}$, and then checks if the result matches any element of the list $\mathfrak{L}$. Since $\mathcal{A}$ is still able to choose an arbitrary value for $T_{(7 \downarrow)}^{\prime}$, it is sufficient to match all rows except the last. The probability that all of those 448 bits - from the remaining seven rows - match, is about $2^{448-128}$ since $\mathcal{A}$ has $2^{128}$ pairs to check. Statistically, $\mathcal{A}$ finds a match after $2^{320}$ iterations of the Steps 3-5. So, we can upper bound the costs of finding such a pair by about $2^{320+64}=2^{384}$ compression function evaluations.
6. Once $\mathcal{A}$ found a 2 nd-preimage for the iterative part of Twister-512, it has to ensure that the checksum $T^{\prime}$ is valid. From the computation of Step $1, \mathcal{A}$ has access to $2^{512}$ checksums leading to the same chaining values $V_{512}, V_{513}$, and $V_{514}$. By applying a memory-less meet-in-the-middle-attack 195], the adversary can construct the needed checksum value. The complexity for this attack is about $2^{257}$ checksum operations.

The introduced 2nd-preimage attack for TwISTER-512 has a time complexity of about $2^{384}$ and a memory complexity of $2^{9+64}$. Due to the output transformation OutputRound, the attack cannot be extended to a preimage attack on Twister-512 in a straightforward way. But, in the next section we will present a sophisticated way to use this attack as a key element of a preimage attack.


Figure 9.7.: Inversion of the first 64 -bit word of a Twister- 512 hash value.

### 9.7.4. Preimage Attack on Twister-512

To construct a preimage for Twister-512, an adversary $\mathcal{A}$ has to invert the output transformation OutputRound (see Section 9.2.4). Afterwards, it can apply the 2ndpreimage attack former presented to construct a preimage. Suppose the adversary has to find a preimage for the value $Y=Y_{1}\|\ldots\| Y_{8}$ where $Y_{i}$ denotes the $i$-th hash word which was generated by the OutputRound function. Furthermore, we assume without loss of generality that $\mathcal{A}$ produces a preimage $M$ of 513 message blocks with correct padding, such that $Y=$ Twister-512 $(M)$. Let $S^{1}-S^{6}$ be the internal states of Twister as shown in Figure 9.7, where $S^{1}$ denotes the state after the invocation of the function State-Finalization, i.e., $V_{514}$. The OutputRound inversion attack can be described by the following six consecutive steps:

1. $\mathcal{A}$ chooses an arbitrary value for the first column of $S^{1}$ and sets the first column of $S^{6}$ using the first 64 -bit hash word, i.e., $S_{(1 \downarrow)}^{6}=S_{(1 \downarrow)}^{1} \oplus Y_{1}$.
2. $\mathcal{A}$ computes forwards 64 bits from the state $S^{1}$, the Byte $S_{(1,1)}^{2}$, and the seven Bytes $S_{(i, j)}^{2}$ for $2 \leq i \leq 8$ where $j=10-i$.
3. $\mathcal{A}$ computes backwards from State $S^{6}$ the first column of $S^{5}$ and the diagonal bytes of $S^{4}$.
4. $\mathcal{A}$ chooses arbitrary values for the seven remaining diagonal bytes of State $S^{1}$, i.e., $S_{(2,2)}^{1}, \ldots, S_{(8,8)}^{1}$. This determines the first column of $S^{2}$. Then, $\mathcal{A}$ computes backwards the eight diagonal bytes $S_{(i, i)}^{3}=S_{(i, i)}^{1} \oplus S_{(i, i)}^{4}$ for $1 \leq i \leq 8$.
5. Next, $\mathcal{A}$ has to find a match of $S^{2}$ and $S^{3}$ through the MixColumns operation
(cf. Figure (9.7). Note that the first column of $S^{2}$ is already determined by the previous step. First off all, $\mathcal{A}$ tests if the first Byte $S_{(1,1)}^{3}$ matches, if not, $\mathcal{A}$ starts over again from Step 1 . Statistically, $\mathcal{A}$ finds a match after $2^{8}$ iterations of Steps 1-4. Then, it find matches for the remaining columns of $S^{2}$ by executing the following two steps:
a) $\mathcal{A}$ chooses for each column $2 \leq i \leq 8$ arbitrary values for the remaining seven Bytes $S_{(j, i)}^{2}$ with $1 \leq j \leq 8$ and $j \neq 10-i$ since $S_{(10-i, i)}^{2}$ is already fixed due to Step 2.
b) Then, $\mathcal{A}$ computes the MixColumns operation and checks if Byte $S_{(i, i)}^{3}$ matches, and repeates the previous step if not. This step has a time complexity of $2^{8}$ Mini-Round operations.

Note that each column can be modified independently. Therefore, this step has a time complexity of about $8 \times 2^{8}=2^{11}$ Mini-Round operations. This corresponds to about $2^{8}$ compression function evaluations.
6. After $\mathcal{A}$ has found a match for all columns of $S^{2}$, it computes $S^{1}$ backwards from $S^{2}$. Note that the values fixed in Step 1 and Step 4 do not change anymore.
$\mathcal{A}$ can invert the OutputRound of Twister-512 by repeating Steps 1-6 about $2^{448}$ time. Thus, this inversion attack has a time complexity of about $2^{448+8}=2^{456}$ compression function evaluations and a total memory complexity of $2^{10}$. Now, $\mathcal{A}$ can apply the 2nd-preimage attack of the previous section to the State $S^{1}=V_{514}$ to construct a preimage for Twister- 512 which consists of 513 message blocks. This preimage attack has a total time complexity of $2^{448}+2^{456} \approx 2^{456}$ compression function operations and a total memory complexity of $2^{10}$.

Remarks. None of the introduced collision, preimage, and 2nd-preimage attacks on Twister-512 are practical due to the time complexity of at least $2^{384}$ compression function calls. Nevertheless, they reveal non-random properties that are not present in SHA-512. Due to the publication of those attacks, the committee of the SHA-3 Competition did not pass Twister-512 to the second round; a rightful choice.

### 9.7.5. Twister $_{\pi}$ Security Discussion

In this section we argue why the presented attacks on Twister-512 are not applicable to Twister $_{\pi}-512$.

Collision Attacks. The semi-free-start collision attack exploits the invertibility of a Maxi-Round. Twister $\pi_{\pi}$ abandoned the concept of Maxi-Rounds and applies a feed-forward after each Mini-Round. Thus, following the notations of Figure 9.6, we have $S^{6}=(A C \circ M I)\left(S^{5} \oplus S^{2}\right)$ instead of $S^{6}=(A C \circ M I)\left(S^{5}\right)$. Since $S^{2}$ is a full active state and the message word is injected in the last row, the interim state $S^{6}$ has at least 56 active bytes. Therefore, it is no longer possible to achieve a state containing an all-zero difference after three invocations of a Mini-Round. So, the additional feed-forward operations thwart the proposed rebound attack. Furthermore, this modification also thwarts the presented collision attack on Twister-512 since it is just a sophisticated extension of the semi-free-start collision attack.
(2nd-) Preimage Attack. The preimage attack on Twister-512 presented in Section 9.7.4 invokes the 2nd-preimage attack from Section 9.7.3. Thus, it is sufficient to show why the 2nd-preimage attack is no longer applicable to TwISTER $\pi_{\pi}-512$.

Steps 3 and 4 of the 2nd-preimage attack on Twister-512, the adversary also exploits the invertibility of a Maxi-Round to compute the interim states $S^{10}$ and $S^{\prime 7}$. The extra feed-forward operation would not allow to compute $S^{10}$ without determining $T_{(6 \downarrow)}, T_{(7 \downarrow)}$, and $T_{(8 \downarrow)}$. But this would take away the freedom from an adversary to choose an arbitrary value for $T_{(7 \downarrow)}$ and $T_{(8 \downarrow)}$, increasing the time complexity by $2^{128}$ from $2^{384}$ to $2^{512}$, which renders this attack not better than exhausting search.

### 9.8. Results Summary

We proposed a family of hash functions which overcomes several identified weaknesses of the commonly used MD4/5 family of hash functions (MD4, MD5, SHA-0/1). By using some of the well-analyzed building blocks and ideas of Rijndael, we obtained a design for which we claim that no efficient differential collision structure exists. In addition, we limit access to the internal structure and take care that any possible difference quickly diffuses into the internal state. Furthermore, it is highly scalable as there are - as proposed in, e.g., TWISTER $\pi_{\pi}-256$ and TWISTER $_{\pi}-512$ - many possible ways to adopt our main building block, the Twister-Round.

Furthemore, we proposed two specific instantiations of the Twister ${ }_{\pi}$ framework, , Twister $_{\pi}-256$ and Twister $_{\pi}-512$. The claimed security level for Twister ${ }_{\pi}-256$ with respect to collision resistance is $2^{128}$ and with respect to (2nd-) preimage resistance $2^{256}$. For TWISTER $_{\pi}-512$, the claimed security level for collision resistance is $2^{256}$ and for (2nd-) preimage resistance $2^{512}$. The TWISTER $_{\pi}$ family of hash functions exploits mathematical structures (i.e., MDS matrices) and, at the same time,
has comparable speed to the SHA-2 family. Thus, instances of the $\operatorname{Twister}_{\pi}$ family are suitable for a huge range of applications from low-end 8-bit microcontroller platforms up to high-end 64-bit software architectures.

## Catena: A Memory-Consuming Password Scrambler

Talent hits a target no one else can hit; Genius hits a target no one else can see.

Arthur Schopenhauer

From the early 1960s [218] till now, the concept of textural passwords are dominant in terms of human-computer authentication. In the context of this thesis we define a password as a user-chosen secret, and thus, we also consider both a passphrase and a Personal Identification Number (PIN) as a password. As observed by Wilkes in the late 1960s [237], storing plain authentication passwords is insecure. Everybody that is granted access to the password storage of a specific multi-user system, immediately learns all user passwords, and can just impersonate any user by a legitimate login. About a decade later, the UNIX system integrated some of Wilkes ideas [174] by deploying a DESłbased [175] one-way encryption function, called crypt. This function is limited to passwords up to eight characters since the seven least significant bits of each of the first eight characters of the password represents the 56-bit key of the 64 -bit block cipher DES, which is used to encrypt iteratively - 25 times - a string of 64 zero-bits.

Under the assumption that crypt is preimage-secure, there is no efficient way to recover the original password from its output, i.e., the password hash. Nevertheless, this scheme can nowadays not longer be considered secure, due to its very small key space of 56 bits. By the means of modern Graphical Processing Units (GPUs) with hundreds of cores 67] - as embedded in all state-of-the art graphics cards - it is
possible to recover the key in feasible time. For example, the advanced passwordrecovery tool hashcat can process about $2^{26}$ password candidates per second (c/s) on a single $A M D$ hd6990 graphics card [226]. An adversary with access to a GPU cluster with 128 nodes can compute a preimage in about four months. Thus, the question of how to slow down such adversaries becomes a pressing one.

Memory is expensive; so, a typical GPU or other cheap massively-parallel hardware with lots of cores can only have a limited amount of memory for each single core. More importantly, each core will have only a very limited amount of fast memory (cache). So, the way to prevent $c$-core adversaries from gaining some close-to- $c$ times speed-up is by making a password scrambler not only intentionally slow on standard sequential computers, but also intentionally memory consuming. Under the preimage-security assumption, any adversary using $c$ cores in parallel with less than about $c$ times the memory of a sequential implementation must experience a strong slow-down. A formal definition of this property called sequential memoryhardness is given in Section 10.3 (cf. Definition 10.3). The first password scrambler that took this condition into account was scrypt [191].

In the light of the current situation, the designer of a modern password scrambler is caught between Scylla and Charybdis. On the one hand, the acceptance of a password scrambler depends on its time and memory usage. Usually, user want to $\log$ in without noticeable delay [231], and especially on embedded devices, such as routers or switches, it is unlikely that the developers choose to implement a login process that consumes a significant amount of expensive memory. On the other hand, the more time and memory a password scrambler needs to compute a hash from a password, the less efficient are guessing attacks such as exhaustive search. This is the reason why the password processing takes some time for both kinds of users legitimate ones and attackers. Thus, a good password scrambler $\mathcal{P}$ has to satisfy at least the following three basic conditions:
(1) Given a password $p w d$, computing $\mathcal{P}(p w d)$ should be "fast enough" for the user.
(2) Computing $\mathcal{P}(p w d)$ should be "as slow as possible", without contradicting the previous condition.
(3) Given $h=\mathcal{P}(p w d)$, there must be no significantly faster way to test $q$ password candidates $x_{1}, x_{2}, \ldots, x_{q}$ for $\mathcal{P}\left(X_{i}\right)=Y$ than by actually computing $\mathcal{P}\left(x_{i}\right)$ for each $x_{i}$.

Memory-Access Pattern and Outline. Note that a memory-consuming password scrambler may suffer from a new problem. If the memory-access pattern depends on the password, and the adversary can observe that pattern, this may open the way to another kind of shortcut attack. For example, a spy process running on the same machine as the password scrambler $\mathcal{P}$ (without access to the internal memory of $\mathcal{P}$ ) may gather information about the memory-access pattern by measuring cache timings. This information can be used to greatly speed-up massively-parallel attacks with low memory for each core. In Section 10.4 we show that this is actually an issue for scrypt, and then, in Section 10.5 we present a fix by introducing Catena, a new password scrambler framework which consumes lots of memory (like scrypt), but does not have a password-dependend memory-access pattern. In Section 10.6 we formally analyze the security of Catena framework and its memory consumption. Section 10.7 presents a secure Key Derivation Function (KDF) based on Catena. In Section 10.8 we introduce an instantiation of Catena, namely Catena-DBG, and in Section 10.9 we analyze its security and memory-hardness. Finally, Section 10.10 summarizes our contribution.

### 10.1. Background

Since the introduction of crypt, storing the hash of a password and avoiding to store the plain password itself has become the minimum standard for secure passwordbased user authentication. But, even as late as 2012, major players like Yahoo and CSDN (China Software Developer Network) seem to store plain user passwords 157].
Two important innovations from crypt were key stretching and salts. Key stretching is the answer to the typically low entropy of user-chosen passwords: The password scrambler is intentionally slow, but not too slow for the regular operation, e.g., a password-based login. This makes exhaustively searching through all likely passwords more expensive.

Salt. A salt refers to an additional random input for the password scrambler and is stored together with the password hash. It enables a password scrambler to derive lots of different password hashes from a single password as an initialization vector enables an encryption scheme to derive lots of different ciphertexts from a single plaintext. Since the salt must be chosen uniformly at random, it is most likely that different users have different salts. Thus, it defends against attacks where password hashes from many different users are known to the attacker, e.g., against the usage of rainbow tables [182].

Pepper. There are different ways to perform key stretching. One is to keep $p$ bits of the salt secret, turning them into pepper [159]. Both attackers and legitimate users have to try out all $2^{p}$ values the pepper can have (or $2^{p-1}$ on the average). Note that a careless implementation of this approach could leak a few bits of the pepper via timing information when trying out all possible values in a specific order. Thus, a recommended approach would be to start at a random value and wrap around at $2^{p}$. Kelsey et al. [136] analyzed another key stretching approach where a cryptographic operation is iterated $n$ times. Boyen proposed in [52] a user-defined implicit choice of $n$ by iterating until the user presses a "halt" button.

### 10.2. Related Work

Table 10.1 provides an overview of password scramblers that are or have been in frequent use, compared to Catena. It indicates whether the password scrambler supports salt, server relief, and client-independent updates. Furthermore, the table lists all possible values of the cost factor (security parameter) including the default values, the memory usage, and issues from which the considered password scrambler may suffer from.

Hash Function Based Password Scramblers. Not long ago, md5crypt [132] has been used in nearly all Free-BSD and Linux-based systems to scramble user passwords. It is based on the well-known MD5 hash function with a fixed number of 1,000 iterations. Due to the fact that CPUs and GPU; become more and more powerful, md5crypt can now be computed too fast, e.g., over 5 million times per second on an $A M D H D 6990$ graphics card [226]. Additionally, its own author does not consider md5crypt secure anymore [132]. Common Linux distributions nowadays employ sha512crypt [82], e.g., Debian, Ubuntu, and Fedora. It provides similar features as md5crypt, but uses SHA-512 instead of MD5. Furthermore, the number of iterations can be chosen by the user; default is 5,000 iterations. NTLMv1 [112] is a fast password scrambler which is deployed to generate hash values for several versions of Microsoft Windows passwords. It is very efficient to compute: One can check over nine billion password candidates per second on a single Commercial Off-The-Shelf (COTS) graphics card [226]. For this and other reasons, we recommend that NTLMv1 should not be used anymore, if possible.

The Password-Based Key Derivation Function 2 (PBKDF2) has been specified by the NIST 231]. It is widely used either as a KDF (e.g., in Wi-Fi Protected Access (WPA), WPAp, OpenOffice, or WinZip) or as a password scrambler (e.g., in

| Password Scrambler | Cost Factor | Memory | Server <br> Relief | Client-Indep. <br> Updates | Issues |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| crypt | $[174]$ | 25 | small | - | - | "too fast" |
| md5crypt | $[132]$ | 1,000 | small | - | - | "too fast" |
| sha512crypt | $[82]$ | $1,000-999,999$ | small | - | - | small memory |
| NTLMv1 | $[112]$ | 1 | small | - | - | "too fast" |
| PBKDF2 | $[231]$ | $1-\infty$ | small | - | - | small memory |
| bcrypt | $[194]$ | $2^{4}-2^{99}$ | 4,168 bytes | - | - | constant memory |
| scrypt | $[191]$ | $1-\infty$ | flexible, big | - | - | cache-timing attacks |
| CATENA | (this work) | $2^{1}-2^{\infty}$ | flexible, big | $\checkmark$ | $\checkmark$ | new and untested |

Table 10.1.: Comparison of contemporary password scramblers.

Mac OS X, and LastPass). The security of PBKDF2 is based on $c$ iterations of HashBased Message Authentication Code (HMAC) 17] instantiated with SHA-1, where $c$ is a user-chosen value which is given by default with $c=1,000$.
bcrypt. The bcrypt algorithm [194] is built upon the Blowfish block cipher [220]. Internally, Blowfish uses a slow key scheduler to generate an internal state of 4,168 bytes for the key-dependent S-boxes ( $4 \times 1,024$ bytes) and the round keys ( 72 bytes). Thus, while bcrypt has not been designed with the intention to thwart parallelized attackers by exhaustive memory usage, the state is sufficiently large to slow down bcrypt significantly on current GPUs, e.g., it can only be computed about 4,000 times per second on an $A M D H D 7970$ graphic card [226]. However, the state size is fixed - so if future GPUs have a larger cache, it may actually run much faster. There is no tunable parameter to increase the memory requirement. For key stretching, bcrypt invokes the Blowfish key scheduler $2^{c}$ times, e.g., OpenBSD uses $c=6$ for users and $c=8$ for the superuser.
scrypt. Occupying a lot of memory hinders attacks using special-purpose hardware (storage is expensive) and GPUs. We are aware of one single password-scrambler that has been designed to fulfill this requirement: scrypt [191]. (There was HEKS [197], but it has been broken by the author of scrypt [191].) As its core, scrypt uses the sequentially memory-hard function ROMix, which can take $G$ units of memory and performs $2 G$ operations. With only $G / K$ units of memory, the number of operations goes up to $2 G \cdot K$. In [191], Percival recommends $G=2^{14}$ and $G=2^{20}$ for password hashing and key derivation, respectively. We will describe and analyze scrypt and ROMix in Section 10.4.

### 10.3. Memory-Related Properties

In this section we introduce a listing of desired properties a modern password scrambler should have - beyond salt and pepper. We start, by introducing the security parameter $g$, called garlic. The notion of garlic reflects the property that incrementing this parameter by ' 1 ' doubles the memory usage and at least doubles the computational time.

Memory-Hardness. To describe memory requirements, we adopt and slightly change the notion from [191]. The intuition is that for any parallelized attack, using $c$ cores, the required memory per core is decreased by a factor of $1 / c$, and vice versa.

Definition 10.1 (Memory-Hard Function). For a memory-hard function $\mathcal{F}$ which is computed on a Random Access Machine using $S(g)$ space and $T(g)$ operations, it holds that

$$
T(g)=\Omega\left(\frac{G^{2}}{S(g)}\right)
$$

where $G=2^{g}$.

Thus, for $S(g) \cdot T(g)=G^{2}$ with $G=2^{g}$, using $c$ cores, it holds that

$$
(1 / c \cdot S(g)) \cdot(c \cdot T(g))=G^{2} .
$$

A formal generalization of this notion is given in the following.

Definition 10.2 ( $\lambda$-Memory-Hard Function). For a $\lambda$-memory-hard function $\mathcal{F}$ which is computed on a Random Access Machine using $S(g)$ space and $T(g)$ operations, it holds that

$$
T(g)=\Omega\left(\frac{G^{\lambda+1}}{S(g)^{\lambda}}\right)
$$

where $G=2^{g}$.

Thus, if one has only $1 / c$ of the memory available, one needs $c^{\lambda}$ processor units to gain the same time-memory tradeoff, i.e.,

$$
\left(1 / c \cdot S(g)^{\lambda}\right) \cdot\left(c^{\lambda} \cdot T(g)\right)=G^{\lambda+1}
$$

In the following we use $S(g)$ and $S$ as synonyms.

Definition 10.3 (Sequential Memory-Hard Function). A sequential memoryhard function is a function $\mathcal{F}$ with the following properties:
(a) $\mathcal{F}$ is memory-hard and
(b) there is no $\beta>0$ such that $\mathcal{F}$ can be computed on a Parallel Random Access machine with $S^{*}(g)$ processors and $S^{*}(g)$ space in expected time $T^{*}(g)$, where $S^{*}(g) T^{*}(g)=O\left(T(g)^{2-\beta}\right)$.

Password Recovery (Preimage Security). For a modern password scrambler it must hold that the advantage of an adversary (modelled as a computationally unbounded but always-halting algorithm) for guessing a valid password should be reasonable small, i.e., not higher than for trying out all possible candidates. Therefore, given a password scrambler $\mathcal{P}$, we define the password-recovery advantage of an adversary $\mathcal{A}$ as follows:

Definition 10.4 (Password Recovery Advantage). Let $s$ denote a randomly chosen salt value and pwd a password randomly chosen from a source $\mathcal{Q}$ with $m$ bits of min-entropy. Then, given a hash value $h \leftarrow \mathcal{P}(s, p w d)$, it holds that

$$
\operatorname{Adv}_{\mathcal{P}, \mathcal{Q}}^{R E C}(\mathcal{A})=\operatorname{Pr}_{s}\left[p w d \leftarrow \mathcal{Q}, h \leftarrow \mathcal{P}(s, p w d): x \leftarrow \mathcal{A}^{\mathcal{P}, s, h}: \mathcal{P}(s, x)=h\right] .
$$

Furthermore, by $\mathbf{A d v}_{\mathcal{P}}^{R E C}(q)$ we denote the maximum advantage taken over all adversaries asking at most $q$ queries to $\mathcal{P}$.

Client-Independent Update. According to Moore's Law [173], the available resources of an adversary increase continually over time - and so do those of the legitimate user. Hence, a security parameter chosen once may be too weak after some time and needs to be updated. This can easily be accomplished immediately after the user has entered its password the next time. However, in many cases, a significant amount of user accounts is inactive or rarely used, e.g., $70.1 \%$ of all Facebook accounts experience zero updates per month [177], and $73 \%$ of all Twitter accounts do not have at least one tweet per month [215]. Therefore, it is desirable to be able to compute a new password hash (with some higher security parameter)
from the old one (with the old and weaker security parameter), without having to involve user interaction or otherwise having to know the password. We call this feature a client-independent update of the password hash. When key stretching is done by iterating an operation, client-independent updates may or may not be possible, depending on the details of the inner workings of a password scrambler. For example, when the original password is one of the inputs for the final operation (see [191]), client-independent updates are impossible.

Server Relief. A slow and - even worse - memory-demanding password-based login process may be too much of a burden for many service providers. Server relief splits the password-scrambling process into two parts: (1) a slow (and possibly memorydemanding) one-way function $\mathcal{F}$ and (2) an efficient one-way function $\mathcal{H}$. By default, the server computes the password hash $\mathcal{H}(\mathcal{F}(p w d, s))$ from the password $p w d$ and a salt $s$. Alternatively, the server sends $s$ to the client who responds $x=\mathcal{F}(p w d, s)$. Finally, the server just computes $\mathcal{H}(x)$. While it is probably easy to write a generic server-relief protocol using any password scrambler, none of the existing password scramblers has been designed to naturally support this property.

Key Derivation Function (KDF). Beyond authentication, passwords are also used to derive symmetric keys. Obviously, one can just use the output of the password scrambler as a symmetric key - perhaps after truncating it to the required key size. This is a disadvantage if one either needs a key that is longer than the password hash or has to derive more than one key. Thus, it is prudent to consider a KDF as a tool of its own right - with the option to derive more than one key and with the security requirement that compromising some of the keys does not endanger the other ones. Note that it is required for a KDF that the input and output behaviour cannot be distinguished from that of a set of random functions.

Resistance against Cache-Timing Attacks. Password scramblers with a passworddependent memory-access pattern risk to be vulnerable against cache-timing attacks. Depending on the implementation and under certain circumstances, timing information related to a given machine's cache behavior may enable the adversary to observe which addresses have been accessed. This can be exploited to implement a very efficient password-candidate sieve. Therefore, any password scrambler whose memory-access pattern is independent from the password is not vulnerable against cache-timing attacks.

### 10.4. The scrypt Password Scrambler

Algorithm 16 describes the scrypt password scrambler and its core operation ROMix. For pre- and post-processing, scrypt invokes the one-way function PBKDF2 to support inputs and outputs of arbitrary length. ROMix uses a hash function $\mathcal{H}$ with an $n$-bit output where $n$ is the size of a cache line (at current machines, often 512 bits). To support hash functions with smaller output sizes, [191] proposes to instantiate $\mathcal{H}$ by a function called BlockMix, which we will not elaborate on. For our security analysis of ROMix, we modelled $\mathcal{H}$ as a random oracle.
ROMix takes two inputs: An initial state $x$ which depends on both salt and password, and the array size $G$ that defines the required storage. One can interpret $\log _{2}(G)$ as the garlic factor of scrypt. In the first phase (Lines 20-23), ROMix initializes an array $v$, i.e., the array variables $v_{0}, \ldots, v_{G-1}$ are set to $x, \mathcal{H}(x), \mathcal{H}(\mathcal{H}(x)), \ldots$, $\mathcal{H}(\ldots(\mathcal{H}(x)))$, respectively. In the second phase (Lines 24-27), ROMix updates $x$ depending on $v_{j}$. The sequential-memory hardness comes from the way how the index $j$ is computed, depending on the current value of $x$, i.e., $j \leftarrow x \bmod G$. After $G$ updates, the final value of $x$ is returned and undergoes the post-processing.
A minor issue is that scrypt uses the password $p w d$ as one of the inputs for post-processing (Line 12). Thus, it has to be in storage during the entire passwordscrambling process. This is risky if there is any chance that the memory can be

```
Algorithm 16 The scrypt Algorithm and its Core Operation ROMix [191].
scrypt ROMix
    pwd \{Password\}
    \(s\) \{Salt \(\}\)
    \(G\) \{Cost Parameter\}
Output: \(x\) \{Password Hash\}
10: \(x \leftarrow\) PBKDF2 \((p w d, s, 1,1)\)
11: \(x \leftarrow \operatorname{ROMix}(x, G)\)
12: \(x \leftarrow \overline{\text { PBKDF2 }}(p w d, x, 1,1)\)
13: return \(x\)
```


## Input:

## Input:

$x$ \{Initial State\}
$G$ \{Cost Parameter\}
Output: $x$ \{Hash Value $\}$
for $i=0, \ldots, G-1$ do $v_{i} \leftarrow x$ $x \leftarrow \mathcal{H}(x)$
end for
for $i=0, \ldots, G-1$ do $j \leftarrow x \bmod G$ $x \leftarrow \mathcal{H}\left(x \oplus v_{j}\right)$
end for
return $x$

```
Algorithm 17 ROMixMC
Input:
    x {Initial State},
    G {1st Cost Parameter},
    K {2nd Cost Parameter}
Output: x {Hash Value}
    for i=0,\ldots,G-1 do
        if imod}K=0 then
            vi}\leftarrow
        end if
        x\leftarrow\mathcal{H}(x)
    end for
```

```
for \(i=0, \ldots, G-1\) do
```

for $i=0, \ldots, G-1$ do
$j \leftarrow x \bmod G$
$j \leftarrow x \bmod G$
$\ell \leftarrow K(j / K)$
$\ell \leftarrow K(j / K)$
$y \leftarrow v_{\ell}$
$y \leftarrow v_{\ell}$
for $m=\ell+1, \ldots, j$ do
for $m=\ell+1, \ldots, j$ do
$y \leftarrow \mathcal{H}(y)\left\{\right.$ invariant: $\left.y \leftarrow v_{m}\right\}$
$y \leftarrow \mathcal{H}(y)\left\{\right.$ invariant: $\left.y \leftarrow v_{m}\right\}$
end for
end for
$x \leftarrow \mathcal{H}(x \oplus y)$
$x \leftarrow \mathcal{H}(x \oplus y)$
end for
end for
return $x$

```
    return \(x\)
```

compromised during the time scrypt is running. Compromising the memory should not happen anyway, but this issue could easily be fixed without any bad effect on the security of scrypt, e.g., one could replace Line 12 of Algorithm 16 by $x \leftarrow$ PBKDF2 $(s, x, 1,1)$.

### 10.4.1. Brief Analysis of ROMix

In the following we introduce a way to run ROMix with less than $G$ units of storage. Suppose we only have $S<G$ units of storage for the values in $v$. For convenience, we assume $G$ is a multiple of $S$ and set $K \leftarrow G / S$. As it will turn out, the memoryconstrained Algorithm ROMixMC (cf. Algorithm 17) generates the same result as ROMix with less than $G$ storage units and is $\Theta(K)$ times slower than ROMix. From the array $v$, we will only store the values $v_{0}, v_{K}, v_{2 k}, \ldots, v_{(S-1) K}$ - using all the $S$ memory units available.

At Line 9 , the variable $\ell$ is assigned the biggest multiple of $K$ less or equal $j$. By verifying the invariant at Line 12, one can easily see that ROMixMC computes the same hash value as the original ROMix, except that $v_{j}$ is computed on the fly, beginning with $v_{\ell}$. These computations call the random oracle on average $(K-1) / 2$ times. Thus, the second phase of ROMixMC is about $\Theta(K)$ times slower than the second phase of ROMix, and this dominates the workload for ROMixMC.

Next, we briefly discuss why ROMix is sequentially memory-hard (for the full proof see [191]). The intuition is as follows: The indices $j$ are determined by the output of the random oracle $\mathcal{H}$ and thus, essentially, uniformly distributed random values over $\{0, \ldots, G-1\}$. With no way to anticipate the next $j$, the best approach is to
minimize the size of the "gaps", i.e., the number of consecutively unknown $v_{j}$. This is indeed what ROMixMC does, by storing one $v_{i}$ every $K^{\prime}$ 'th step.

### 10.4.2. Cache-Timing Attacks

Algorithm 16 (scrypt/ROMix) revisited. What could possibly go wrong?

The Spy Process. As it turns out, the idea to compute an unpredictable index $j$ and then ask for the value $v_{j}$, which is useful for sequential memory-hardness, is also an issue. Consider a spy process running on the same machine as scrypt. This spy process cannot read the internal memory of scrypt. But, as it is running on the same machine, it shares its cache memory with ROMix. The spy process interrupts the execution of ROMix twice:

1. When ROMix enters the second phase (Line 24 of Algorithm 17), the spy process reads from a bunch of addresses, to force out all the $v_{i}$ that are still in the cache. Thereupon, ROMix is allowed to run for another very short time.
2. Now, the spy process interrupts ROMix again. By measuring access times when reading from different addresses, the spy process can figure out which of the $v_{i}$ have been read by ROMix, in between.

So, the spy process can tell us the indices $j$ for which $v_{j}$ has been read, and with this information we can mount the following cache-timing attack.

The Preliminary Cache-Timing Attack. Let $p w d^{\prime}$ denote the current password candidate. Suppose $x$ is the output of PBKDF2 $\left(p w d^{\prime}, s a l t, 1,1\right)$. Then, we can apply the following password candidate sieve:

1. Execute the first phase of ROMix, without storing the $v_{i}$, i.e., skip Line 21 of Algorithm 16.
2. Compute the index $j \leftarrow x \bmod G$.
3. If $v_{j}$ is one of the values that have been read by ROMix, then store $p w d^{\prime}$ in a list.
4. Else, conclude that $p w d^{\prime}$ is a wrong password.

This sieve can run in parallel on any number of cores, each core trying out another password candidate $p w d^{\prime}$. Note that each core needs only a small and constant amount of memory - the data structure to decide if $j$ is one of the indices of the value $v_{j}$ which has been read. Further, this can be shared between all the cores. Hence, we can use exactly the kind of hardware that scrypt has been designed to hinder.

Next, we discuss the gain of this attack. Let $r$ denote the number of iterations the loop in Lines 24-27 of ROMix has performed before the second interrupt by the spy process. So, there are at most $r$ indices $j$ with $v_{j}$ being read. That means, we expect this approach to sort out all but $r / G$ candidates. If our spy process manages to interrupt very soon after allowing it to run again, we have $r \ll G$. This may enable us to use conventional hardware to run full ROMix to search for the correct password among the candidates on the list.

The Final Cache-Timing Attack. In this attack we allow the second interrupt to arrive very late - maybe even as late as the termination time of ROMix. So, the loop in Lines 24-27 of ROMix has been run $r=G$ times. As it seems, each $v_{i}$ has been read once. But actually, this is only true on the average; some $v_{i}$ have been read more than once, and we expect about $(1 / e) G \approx 0.37 G$ array elements $v_{i}$ not being read at all. So, applying the basic attack allows us to eliminate about $37 \%$ of all password candidates - a rather small gain for such hard work.

In the following we introduce a way to push the attack further, inspired by Algorithm 17, the memory-constrained ROMixMC. Our final cache-timing attack on scrypt does only need the smallest possible amount of memory: $S=1, K=G / S=G$, and thus, we only have to store the single value $v_{0}$. Like the second phase of ROMixMC, we will compute the values $v_{j}$ on the fly when needed. Unlike ROMixMC, we will stop execution whenever one of our values $j$ is such that $v_{j}$ has not been read by ROMix (according to the information from our spy process).

Thus, if the first $v_{j}$ has not been read, we immediately stop the execution without any on-the-fly computation; if the first $v_{j}$ has been read, but not the second, we need one on-the-fly computation of $v_{j}$, and so forth.

Since a fraction, i.e., $1 / e$, of all values $v_{i}$ has not been read, we will need about $1 /(1-1 / e) \approx 1.58$ on the fly computations of some $v_{j}$, each at the average price of $(G-1) / 2$ times calling $\mathcal{H}$. Additionally, each iteration needs one call to $\mathcal{H}$ for computing $x \leftarrow \mathcal{H}\left(x \oplus v_{j}\right)$. Including the work for the first phase, with $G$ calls to $\mathcal{H}$,
the expected number of calls to reject a wrong password is about

$$
G+1.58 \cdot\left(1+\frac{G-1}{2}\right) \approx 1.79 G
$$

As it turns out, rejecting a wrong password with constant memory is faster than computing ordinary ROMix with all the required storage, which actually makes $2 G$ calls to $\mathcal{H}$, without computing any $v_{i}$ on the fly. We stress that the ability to abort the computation, thanks to the information gathered by the spy process, is crucial.

### 10.4.3. Discussion

At the current point of time, our cache-timing attacks are theoretical. Even if one manages to run a spy process on a machine using scrypt, the requirement to interrupt ROMix twice at the right points of time is demanding. Nevertheless, even the theoretical ability of mounting such attacks should be seriously taken into account.
The idea of attacking cryptographic algorithms from hardware side (side-channel attacks) is not new [145], neither is the usage of a spy process for theoretical cachetiming attacks [190]. In [31], Bernstein demonstrated practically how to recover AES keys by using cache-timing information:

The problem lies in AES itself: it is extremely difficult to write constanttime high-speed AES software [...]. Constant time low-speed AES software is fairly easy to write but is also unacceptable for many applications.

Similarly, we argue that there is a problem in scrypt itself. One can certainly implement scrypt such that cache-timings do not leak information about the password. But, we believe this would drastically reduce the performance of scrypt. As a compensation - recall that password scramblers are intentionally slow, but must be "fast enough" for the user - one would have to set the cost parameter $G$ to some smallish value. But, this would only make regular attacks more efficient since attackers can use faster implementations. At the end of the day, this may defeat the entire point of using scrypt at all.

Note that this cache-timing attack has even more severe consequences. It does not only speed-up regular password-guessing attacks where the password hash is already in possession of the adversary. It also enables an adversary $\mathcal{A}$ to recover a password without knowing the password hash at all by just verifying the memoryaccess pattern.

The core of the problem is the fact that ROMix reads a value $v_{j}$, where the index $j \leftarrow x \bmod G$ depends on $x$ and thus, on the password. It would be very convenient
to have a password scrambler which is sequentially memory-hard and computes $j$ in some password-independent way, i.e., only depending on the loop index $i$. In the next section we actually present such a $\lambda$-memory-hard password scrambler, Catena.

### 10.4.4. The Garbage-Collector Attack

Here we introduce another memory-based issue of the ROMix algorithm. Typical attackers try plenty of password candidates in parallel, and this gets a lot more costly if they need a huge amount of memory for each candidate. The defender, on the other hand, will only compute a single hash, and the parameters (especially the "garlic") should be chosen such that the required amount of memory is easily available to the defender.

But, memory-demanding password scrambling may also provide completely new attack opportunities for the adversary. If we allocate a huge block of memory for password scrambling, holding $v_{0}, v_{1}, \ldots, v_{G-1}$, this memory becomes "garbage" after the password scrambler has terminated, and will be collected for reuse, eventually. One usually assumes that the adversary learns the hash of the secret. The garbagecollector attack assumes that the adversary additionally learns the memory content, i.e., the values $v_{i}$, after the termination of the password scrambler.

For ROMix, the value $v_{0}=\mathcal{H}(x)$ is a plain hash of the original secret $x$. Hence, the garbage-collector adversary can bypass ROMix completely and search directly for $x$ with $\mathcal{H}(x)=v_{0}$, implying that each password candidate can be checked in time and memory $O(1)$. Thus, ROMix does not provide much defense against garbage-collector attacks. As a possible countermeasure, one can simply overwrite $v_{0}, \ldots, v_{G-1}$ after running ROMix. But, this step might be removed by a compiler due to optimization, since it is algorithmically ineffective.

### 10.5. Specification of Catena

In this section we introduce our password scrambler Catena. More detailed, we first specify Catena and explain its properties regarding to password hashing, i.e., client-independent update and server relief. Afterwards, we present a instantiations of Catena, called Catena-DBG.

A formal definition is shown in Algorithm 18, based on two building blocks: (1) the cryptographic hash function $\mathcal{H}$ (see Lines 1 and 4) and (2) the memory-consuming $n$-bit hash function $\mathcal{F}_{\lambda}$ (see Line 3). Note that we require that the function $\mathcal{F}_{\lambda}$ is

1. $\lambda$-memory hard,
```
Algorithm 18 Catena
Input: \(\lambda\) \{Depth\}, \(g_{0}\{\) Initial Garlic\}, pwd \{Password\},
    u \{Tweak \(\}, s\) \{Salt \(\}, g\) \{Garlic \(\}\)
Output: \(x\) \{hash of the password\}
    \(x \leftarrow \mathcal{H}(u\|p w d\| s)\)
    for \(c=g_{0}, \ldots, g\) do
        \(x \leftarrow \mathcal{F}_{\lambda}(c, x)\)
        \(x \leftarrow \mathcal{H}(c \| x)\)
    end for
    return \(x\)
```

2. collision resistant and,
3. its memory-access pattern is independent of the password derived input $x$.

Note that the for loop (Line 2-5) is required to provide client-independent updates. For the initial deployment of Catena, we recommend to set the initial garlic value $g_{0}$ to $g$ to achieve the best ratio between running time and memory usage. For the sake compatibility $\lambda$ and $g_{0}$ should never be updated.
Note that a secure password scrambler must satisfy preimage security. It is easy to see that Catena inherits the preimage security from the underlying hash function $\mathcal{H}$.

Next, we discuss the tweak and two further novel features of Catena.
Tweak. The tweak $u$ is an additional multi-byte value which is given by:

$$
u \leftarrow d\|\lambda\| n\||s|\| \mathcal{H}(H),
$$

where the first byte $d$ denotes the mode (domain) for which Catena is used: $d=0$ when used as a password scrambler, and $d=1$ when used as a KDF (see Section 10.7). All remaining possible values for $d$ are reserved for future applications. The second byte $\lambda$ (depth) defines together with the memory cost parameter $g$ (garlic) the security parameters for Catena. The next 16 -bit value $n$ denotes the output length of the underlying hash function $\mathcal{H}$ in bits. The 32 -bit value $|s|$ denotes the total length of the salt in bits. The last $n$-bit value $\mathcal{H}(H)$ is the hash of the associated data $H$, which can contain additional information like hostname, user-ID, name of the company, or the IP address of the host, with the goal to customize the password hashes. The tweak is processed together with the secret password and the salt (see

Algorithm 18, Line 1). Thus, the tweak $u$ can be seen as a weaker version of a salt, increasing the additional computational effort for an adversary when using different values. Furthermore, it allows to differentiate between password hashing and key derivation.

Client-Independent Update. Its sequential structure does enable Catena to provide client-independent updates. Let $h=\operatorname{CatenA}_{\lambda}(p w d, u, s, g)$ be the hash of a specific password $p w d$, where $\mathrm{u}, s$, and $g$ denote the tweak, the salt, and the garlic. After increasing the security parameter from $g$ to $g^{\prime}=g+1$, we can update the hash value $h$ without user interaction by computing:

$$
h^{\prime}=\mathcal{H}\left(g^{\prime} \| \mathcal{F}_{\lambda}\left(g^{\prime}, h\right)\right)
$$

It is easy to see that the equation $h^{\prime}=\operatorname{CatenA}_{\lambda}\left(p w d, u, s, g^{\prime}\right)$ holds.

Server Relief. In the last iteration of the for-loop in Algorithm 18, the client has to omit the last invocation of the hash function $\mathcal{H}$ (see Line 4) and then transmits the output of CATENA to the server. Afterwards, the server computes the password hash by applying the hash function $\mathcal{H}$. Thus, the vast majority of the effort (memory usage and computational time) for computing the password hash is handed over to the client exonerating the server. This enables someone to deploy Catena even under restricted environments or when using constrained devices - or when a single server has to handle a huge amount of authentication requests.

Keyed Password Hashing. To further thwart off-line attacks, we introduce a technique to use Catena for keyed password hashing, where the password hash depends on both a password and a secret key $K$. Note that $K$ is the same for all users, and thus, it has to be stored on the server. To preserve the server-relief property (see above), we encrypt the output of CATENA using the XOR operation with $\mathcal{H}(K \|$ userID $\|g\| K)$, which, under the reasonable assumption that the value (userID \| $\|$ ) is a nonce, was proven to be CPA-secure in [205]. Then, the keyed password hash $y$ is given by

$$
y:=\operatorname{Catena}_{\lambda}(p w d, u, s, g) \oplus \mathcal{H}(K \| \text { userID }\|g\| K)
$$

where the userID is a unique and user-specific identification number which is assigned by the server. Now, we show what happens during the client-independent update, i.e., when $g=g+r$ for arbitrary integer $r>0$. The process takes the following four steps:

1. Given $K$ and userID, compute $w=\mathcal{H}(K \|$ userID $\|g\| K)$.
2. Compute $x=y \oplus w$, where $y$ denotes the current keyed hash value.
3. Update $x$, i.e., $x=\mathcal{H}\left(c \| \mathcal{F}_{\lambda}(c, x)\right)$ for $c \in\{g+1, \ldots, g+r\}$.
4. Compute the new hash value $y=y \oplus \mathcal{H}(K \|$ userID \| $g+r \| K)$.

Remark. Obviously, it is a bad idea to store the secret key $K$ on the same place as the password hashes since it can be leaked in the same way as the password-hash database. One possibility to separate the key from the hashes is to securely store the secret key by making use of hardware security modules (HSM), which provide a tamper-proof memory environment with verifiable security. Then, the protection of the secret key depends on the level provided by the HSM (see FIPS140-2 [57] for details). Another possibility is to derive $K$ from a password during the bootstrapping phase. Afterwards, $K$ will be kept in the RAM and will never be on the hard disk drive. Thus, the key and the password-hash database should never be part of the same backup file.

### 10.6. Security Analysis of Catena

We denote a password scrambler to be secure if it provides at least 1-memory-hardness and preimage security. Furthermore, it should be resistant against cache-timing attacks. It is easy to see that Catena inherits its $\lambda$-memory-hardness from $\mathcal{F}_{\lambda}$. Since the memory-access pattern of Catena is static and therefore, independent from the password, it provides resistance against cache-timing attacks. Finally, we show that Catena is a secure passsword scrambler that behaves like a good random function, which is useful for using CATENA as a secure KDF, Before we present our claims, we introduce some essential knowledge, which ease the understanding of our proofs.

Password-Recovery Resistance. In this section we show that Catena is a good password scrambler, i.e., given the hash value $h$ it is infeasible for an adversary to do better than trying out password candidates in likelihood order to obtain the correct password.

Theorem 10.5 (Catena is Password-Recovery Resistant). Let $m$ denote the min-entropy of a passwords source $\mathcal{Q}$. Then, it holds that

$$
\operatorname{Adv}_{\mathrm{CATENA}, \mathcal{Q}}^{R E C}(q) \leq \frac{q}{2^{m}}+\operatorname{Adv}_{\mathcal{H}}^{p r e}(q, t)
$$

Proof. Note that an adversary $\mathcal{A}$ can always guess a (weak) password by trying out about $2^{m}$ password candidates. For a maximum of $q$ queries, it holds that the success probability is given by $q / 2^{m}$. Instead of guessing $2^{m}$ password candidates, an adversary can also try to find a preimage for a given hash value $h$. It is easy to see from Algorithm 18 that an adversary thus has to find a preimage for $\mathcal{H}$ in Line 4. More detailed, for a given value $h$ with $h \leftarrow \mathcal{H}(g, x)$, $\mathcal{A}$ has to find a valid value for $x$. The success probability for this can be upper bounded by $\mathbf{A d v}_{\mathcal{H}}^{p r e}(q, t)$. Our claim follows by adding up the individual terms.

Pseudorandomness. In the following we analyze the advantage of an adversary $\mathcal{A}$ in distinguishing the output of CATENA from a random bitstring of the same length as the output of Catena. Therefore, we model the internally used hash function $\mathcal{H}:\{0,1\}^{*} \rightarrow\{0,1\}^{n}$ as a random oracle.

Theorem 10.6 (PRF Security of Catena). Let $q$ denote the number of queries made by an adversary and s a randomly chosen salt value. Furthermore, let $\mathcal{H}$ be modelled as a random oracle and $g \geq g_{0} \geq 1$. Then, it holds that

$$
\operatorname{Adv}^{\frac{P R F}{\text { CATENA }_{\lambda}}}(q, t) \leq(q \cdot g+q)^{2} / 2^{n}+\mathbf{A d v}_{F_{\lambda}}^{c o l l}(g \cdot q)
$$

Proof. Suppose that $a^{i}=\left(p w d^{i}\left\|u^{i}\right\| s^{i} \| g\right)$ represents the $i$-th query, where $p w d^{i}$ denotes the password, $u^{i}$ denotes the tweak, $s^{i}$ the salt, and $g$ the garlic. For this proof, we impose the reasonable condition that all queries of an adversary are distinct, i.e., $a^{i} \neq a^{j}$ for $i \neq j$.

Suppose that $y^{j}$ denotes the output of $F_{\lambda}\left(g, a^{j}\right)$ of the $j$-th query (cf. Algorithm 18, Line 3). Then, $\mathcal{H}\left(g \| y^{j}\right)$ is the output of $\operatorname{CatenA}_{\lambda}\left(a^{j}\right)$. In the case that $y^{1}, \ldots, y^{q}$ are pairwise distinct, an adversary $\mathcal{A}$ cannot distinguish $\mathcal{H}(g \| \cdot)$ from a random function $\$(\cdot)$ since in the random-oracle model, both functions return a value chosen uniformly at random from $\{0,1\}^{n}$.

Therefore, we have to upper bound the probability of the event $y^{i}=y^{j}$ with $i \neq j$. Due to the assumption that $\mathcal{A}^{\prime} s$ queries are pairwise distinct, there must be at least one collision for $\mathcal{H}$ or $\mathcal{F}_{\lambda}$. For $q$ queries, we have at most $q(g+1)$ invocations of $\mathcal{H}$. Thus, we can upper bound the collision probability by

$$
(q \cdot g+q)^{2} / 2^{n}
$$

Furthermore, we have $q \cdot g$ invocations of the memory-consuming function $\mathcal{F}_{\lambda}$. We can upper bound the probability of a collision by $\operatorname{Adv}_{F_{\lambda}}^{\text {coll }}(g \cdot q)$. Our claim follows from the union bound.

### 10.7. The Catena-KG Key-Derivation Function

In this section, we introduce Catena-KG - a mode of operation based on Catena, which can be used to generate keys of different sizes (even larger than the natural output size of Catena (cf. Algorithm (19). To provide uniqueness of the inputs, the domain value $d$ of the tweak is set to 1 , i.e., the tweak $u^{\prime}$ is given by

$$
u^{\prime} \leftarrow 0 \times 01\|\lambda\| n\||s|\| \mathcal{H}(H) .
$$

Then, the call of Catena is followed by an output transformation that takes the output $x$ of Catena, a key identifier $\mathcal{I}$, and a parameter $\ell_{K}$ for the key length as input, and generates key material of the desired output size. CATENA-KG is even able to handle the generation of extra-long keys (longer than the output size of $\mathcal{H}$ ) by applying $\mathcal{H}$ in CTR Mode [84]. Note that longer keys do not imply improved security, in that context. The key identifier $\mathcal{I}$ is supposed to be used when different keys are

```
Algorithm 19 Catena-KG
Input: \(p w d\) \{Password \(\}, u^{\prime}\{\) Tweak \(\}, s\{\) Salt \(\}, g\) \{Garlic \(\}, \mathcal{I}\) \{Key Identifier \(\}\)
Output: \(K\left\{\ell_{K}\right.\)-Bit Key Derived from the Password \(\}\)
    \(x \leftarrow\) Catena \(_{\lambda}\left(p w d, u^{\prime}, s, g\right)\)
    \(K \leftarrow \emptyset\)
    for \(i=1, \ldots,\left\lceil\ell_{K} / n\right\rceil\) do
        \(K \leftarrow K \| \mathcal{H}\left(i\|\mathcal{I}\| \ell_{K} \| x\right)\)
    end for
    return Truncate \(\left(K, \ell_{K}\right)\) \{truncate \(k\) to the first \(\ell_{K}\) bits \(\}\)
```

generated from the same password. For example, when Alice and Bob set up a secure connection, they may need four keys: An encryption and a message-authentication key for messages from Alice to Bob, and another two keys for the opposite direction. One could argue that $\mathcal{I}$ should also become part of the associated data. Actually, this would be a bad move. Setting up the connection would require legitimate users to run Catena several times. But, the adversary can search for the password for one key, and just derive the other keys, once that password has been found. Instead, one should rather employ a single call to Catena with larger security parameters and
then run the output transformation for each key. In contrast to the password-hashing scenario, where a user want to log in without noticeable delay, users may tolerate a delay of several seconds to derive an encryption key from a password [231], e.g., when setting up a secure connection, or when mounting a cryptographic file system. Thus, we recommend to use higher values for $g$ for key-derivation.

Security Analysis. It is easy to see that Catena-KG inherits its $\lambda$-memory-hardness from Catena since it invokes Catena (Line 1 of Algorithm 19). Next, we show the PRF security of CATENA-KG in the random-oracle model.

Theorem 10.7 (Catena-KG Security). Let $\mathcal{H}:\{0,1\}^{*} \rightarrow\{0,1\}^{n}$ be a random function. Then, for $g \geq g_{0} \geq 1$, it holds that

$$
\mathbf{A d v}^{\frac{P R F}{\mathrm{CATENA}}} \mathrm{KG}_{\lambda}(q, t) \leq(q \cdot g+q)^{2} / 2^{n}+\mathbf{A d v}_{F_{\lambda}}^{\text {coll }}(g \cdot q)
$$

Proof. For the sake of simplification, we omit the truncation step and let the adversary always get access to the untruncated key $K$. Since $\mathcal{H}$ is a random function, the only chance for an adversary to distinguish CATENA- $\mathrm{KG}_{(\mathcal{H}, \lambda, g)}(\cdot)$ from a random $n$-bit function is an input collision in Line 4 of Algorithm 19. Thus we have to upper bound the probability that two outputs of CATENA $\lambda_{\lambda}$ collide. Let $a^{i}=\left(p w d^{i}\left\|u^{i}\right\| s^{i} \| g^{i}\right)$ denote the $i$-the query, where $p w d^{i}$ denotes the password, $u^{i}$ denotes the tweak, $s^{i}$ the salt, and $g^{i}$ the garlic. A collision between two distinct queries $a^{i}$ and $a^{j}$, i.e., $\operatorname{Catena}_{\lambda}\left(a^{i}\right)=\operatorname{CatenA}_{\lambda}\left(a^{j}\right)$ with $a^{i} \neq a^{j}$, implies a collision in $\mathcal{H}$. The probability for this event can be upper bounded by

$$
(q \cdot g+q)^{2} / 2^{n}+\mathbf{A d v}_{F_{\lambda}}^{\text {coll }}(g \cdot q)
$$

using similar arguments as in the proof of Theorem 10.10 .

### 10.8. Catena-DBG

In this section we introduce CATENA-DBG, a concrete instantiation of Catena where $\mathcal{F}_{\lambda}$ is instantiated with the Double Butterfly Hashing ( (DBH) operation that is based on a stack of $\lambda G$-superconcentrators. The following definition of a $G$-superconcentrator is a slightly adapted version of that introduced in 151].


Figure 10.1.: A Cooley-Tukey FFT graph with eight input and output vertices.

Definition 10.8 ( $G$-Superconcentrator). A Directed Acyclic Graph (DAG) with a set of vertices $\mathfrak{V}$ and a set of edges $\mathfrak{E}$, a bounded indegree, $G$ inputs, and $G$ outputs is called a $G$-superconcentrator if for every $k$ such that $1 \leq k \leq G$ and for every pair of subsets $\mathfrak{V}_{1} \subset \mathfrak{V}$ of $k$ inputs and $\mathfrak{V}_{2} \subset \mathfrak{V}$ of $k$ outputs, there are $k$ vertex-disjoint paths connecting the vertices in $\mathfrak{V}_{1}$ to the vertices in $\mathfrak{V}_{2}$.

Double Butterfly Graph ( $\overline{\mathrm{DBG}}$ ). A $\overline{\mathrm{DBG}}$ is a $G$-superconcentrator which is defined by the graph representation of two back-to-back placed Fast Fourier Transform (FFT) 53]. More detailed, it is a representation of twice the Cooley-Tukey [FFT] algorithm 65] omitting one row in the middle (see Figure 10.1 for an example where $g=3$ ). Therefore, a DBG consists of $2 g$ rows.

Based on the DBG we define the sequential and stacked (DBGR) where the security parameters $\lambda$ and $g$ determine the depth ( number of stacked superconcentrators) and the width ( number of nodes per row, i.e., $2^{g}$ ), respectively. In the following, we denote $v_{i, j}^{k}$ as the $j$-th vertex in the $i$-th row of the $k$-th superconcentrator. Note that in this thesis we use the vertices $v_{0, j}^{k}$ and $v_{2_{-1, j}^{k-1}}^{k-1}$ as synonyms since due to the stacking of $\lambda$ DBG the last row of the $k-1$-th DBG is identical to the first row of the $k$-th DBG.


Figure 10.2.: Types of edges of an (3,1)-Double Butterfly Graph.

Definition $10.9\left(\right.$ DBG $\left._{\lambda}^{f}\right)$. Fix two integers $g, \lambda \geq 1$, then the $(g, \lambda)$-Double Butterfly Graph $\left(\overline{D B G}_{\lambda}\right) \Pi(\mathcal{V}, \mathcal{E})$ consists of $2^{g}(\lambda(2 g-1)+1)$ vertices

$$
\left(\bigcup_{i=0}^{2 g-2} \bigcup_{j=0}^{2^{g}-1} \bigcup_{k=1}^{\lambda}\left\{v_{i . j}^{k}\right\}\right) \cup\left(\bigcup_{j=0}^{2^{g}-1}\left\{v_{2 g-1, j}^{\lambda}\right\}\right)
$$

and $\lambda \cdot(2 g-1) \cdot\left(3 \cdot 2^{g}\right)+2^{g}-1$ edges

- vertical: $2^{g} \cdot(\lambda \cdot(2 g-1))$ edges

$$
\bigcup_{i=0}^{2 g-2} \bigcup_{j=0}^{2^{g}-1} \bigcup_{k=1}^{\lambda}\left\{v_{i, j}^{k}, v_{i+1, j}^{k}\right\}
$$

- diagonal: $2^{g} \cdot \lambda \cdot g+2^{g} \cdot \lambda \cdot(g-1)$ edges

$$
\bigcup_{k=1}^{\lambda} \bigcup_{j=0}^{2^{g}-1}\left(\left(\bigcup_{i=0}^{g-1}\left\{v_{i, j}^{k}, v_{i+1, j \oplus 2^{g-1-i}}^{k}\right\}\right) \cup\left(\bigcup_{i=g}^{2 g-2}\left\{v_{i, j}^{k}, v_{i+1, j \oplus 2^{i-(g-1)}}^{k}\right\}\right)\right)
$$

- sequential: $\left(2^{g}-1\right) \cdot(\lambda \cdot(2 g-1)+1)$ edges

$$
\left(\bigcup_{i=1}^{2 g-1} \bigcup_{j=0}^{2^{g}-2} \bigcup_{k=1}^{\lambda}\left\{v_{i, j}^{k}, v_{i, j+1}^{k}\right\}\right) \cup\left(\bigcup_{j=0}^{2^{g}-2}\left\{v_{2 g-1, j}^{\lambda}, v_{2 g-1, j+1}^{\lambda}\right\}\right)
$$

- connecting layer: $\lambda \cdot(2 g-1)$ edges

$$
\bigcup_{i=1}^{2 g-2} \bigcup_{k=1}^{\lambda}\left\{v_{i, 2^{g}-1}^{k}, v_{i+1,0}^{k}\right\}
$$



Figure 10.3.: An (3, 1)-Double Butterfly Graph.

For the parameter set $g=3$ and $\lambda=1$ Figure 10.2 illustrates the individual types of edges we use in our Definition above. Moreover, an example for an (3,1)-Double Butterfly Graph (DBG) Figure 10.3.

Double Butterfly Hashing (DBH). The DBHR , operation is defined in Algorithm 20 , The structure is based of a DBGe. Note that the function $\sigma$ (see Lines 7 and 9) is given by

$$
\sigma(g, i, j)= \begin{cases}j \oplus 2^{g-1-i} & \text { if } 0 \leq i \leq g-1, \\ j \oplus 2^{i-(g-1)} & \text { otherwise. }\end{cases}
$$

Thus, $\sigma$ determines the indices of the vertices of the diagonal edges.
Since the security of Catena in terms of password hashing is based on a timememory tradeoff, it is desired to implement it in an efficient way, making it possible to increase the required memory. We recommend BLAKE2b [13] as the underlying hash function, implying a block size of 1024 bits with 512 bits of output. Thus, it can process two input blocks within one compression function call. For Catena-DBG, we cannot simply concatenate the inputs to the hash function $\mathcal{H}$ while keeping the same performance per hash function call, i.e., three inputs to $\mathcal{H}$ require two compression function calls. Therefore, we compute $\mathcal{H}(X, Y, Z)=\mathcal{H}(X, \oplus Y \| Z)$ instead of $\mathcal{H}(X, Y, Z)=\mathcal{H}(X\|Y\| Z)$. Obviously, this doubles the probability of input collisions. Nevertheless, for a 512-bit hash function, the advantage for an adversary is still negligible.

```
Algorithm 20 Double Butterfly Hashing (DBH)
Input: \(g\) \{Garlic \(\}, x\{\) Value to hash\}, \(\lambda\) \{Depth\}, \(\mathcal{H}\{\) Hash Function \(\}\)
Output: \(x\) \{Password Hash\}
    \(v_{0} \leftarrow \mathcal{H}(x)\)
    for \(i=1, \ldots, 2^{g}-1\) do
        \(v_{i} \leftarrow \mathcal{H}\left(v_{i-1}\right)\)
    end for
    for \(k=1, \ldots, \lambda\) do
        for \(i=1, \ldots, 2 g-1\) do
            \(r_{0} \leftarrow \mathcal{H}\left(v_{2^{g}-1} \oplus v_{0} \| v_{\sigma(g, 0, j)}\right)\)
            for \(j=1, \ldots, 2^{g}-1\) do
            \(r_{i} \leftarrow \mathcal{H}\left(r_{i-1} \oplus v_{i} \| v_{\sigma(g, i, j)}\right)\)
            end for
            \(\vec{v} \leftarrow \vec{r}\)
        end for
    end for
    return \(x \leftarrow v_{2^{g}-1}\)
```


### 10.9. Analysis of Catena-DBG

Next, we discuss the security of Catena-DBG against side-channel attacks. Furthermore, we discuss the memory-hardness and collision resistance of the DBH ${ }_{\lambda}^{\beta}$ operation.

### 10.9.1. Side-Channel Attacks and Collision Resistance

Straightforward implementations of Catena-DBG have neither password-dependent memory-access pattern nor have they password-dependent branches. Therefore, our proposed instantiation of CATENA is resistant against cache-timing attacks.

Considering a malicious garbage collector, Algorithm 20 exposes two arrays, namely $v$ and $r$. Both are overwritten multiple times. Therefore, Catena-DBG is resistant against garbage-collector attacks. Note that Catena-DBG with some $\lambda \geq 2$ is at least as resistant to garbage-collector attacks as the same variant with $\lambda-1$ without a malicious garbage collector.

Next, we analyze the collsion resistance of $\mathrm{DBH}_{\lambda}$. Therefore, we model the internally used hash function $\mathcal{H}:\{0,1\}^{*} \rightarrow\{0,1\}^{n}$ as a random oracle.

Theorem 10.10 (Collision Security of $\mathbf{D B H}_{\lambda}$ ). Let $q$ denote the number of queries. Furthermore, let $\mathcal{H}$ be modelled as a random oracle for some fixed integers $g, g_{0}, \lambda \geq 1$ with $g \geq g_{0}$ and $G=2^{g}$. Then, it holds that

$$
\operatorname{Adv}_{\underset{D B H}{D B / l}}^{\text {coll }}(q, t) \leq \frac{(q \cdot \lambda \cdot g)^{2}}{2^{n-2 g-3}} .
$$

Proof. From Algorithm 20 it is easy to see that collision DBH $\boldsymbol{N}_{\lambda}(x)=$ DBH $_{\lambda}^{A}\left(x^{\prime}\right)$ for $x \neq x^{\prime}$ implies either a input or output collision for $\mathcal{H}$.

For our analysis, we replace the random oracle $\mathcal{H}$ by $\mathcal{H}^{\prime}(x):=\mathcal{H}\left(\operatorname{truncate}_{n}(x)\right)$ that truncates any input to $n$ bits before hashing. Thus, any collision in the first $n$ bits $\mathcal{H}$ in Line 7 and 9 of Algorithm 20 leads to a collision, regardless of the remaining inputs.

Output Collision. In this case, we can upper bound the collision probability of $\mathcal{H}$ by deducing the total amount of invocations of $\mathcal{H}^{\prime}$ per query. There are $G$ invocations of $\mathcal{H}^{\prime}$ in Lines 1-4. of Algorithm 20, In addition, there are $\lambda(2 g-1) G$ invocations in Lines 5-14 of Algorithm 20. In total, we have $\lambda 2 g G$ invocations. Since $\mathcal{H}$ is modelled as a random oracle, we can upper bound the collision probability for $q$ queries by

$$
\frac{(q \cdot \lambda \cdot 2 g \cdot G)^{2}}{2^{n}} \leq \frac{q^{2} \lambda^{2} g^{2}}{2^{n-2 g-2}} .
$$

Input Collision. In this case, we have to take into account that a input collision for distinct queries $a$ and $b$ in Line 7 and 9 can occur:

$$
v_{2^{g}-1}^{a} \oplus v_{0}^{a}=v_{2^{g}-1}^{b} \oplus v_{0}^{b} \quad \text { (Algorithm [20, Line 7) }
$$

or

$$
\left.r_{i-1}^{a} \oplus v_{i}^{a}=r_{i-1}^{b} \oplus v_{i}^{b} \quad \text { (Algorithm 20, Line } 9\right) .
$$

For each query this can happen $\lambda \cdot(2 g-1) \cdot 2^{g}$ times. Note that all values $v_{i}, r_{i}$ are outputs from the random oracle $\mathcal{H}^{\prime}$, except the initial value for $v_{0}$. Hence, we can upper bound the collision probability for this event by

$$
\frac{\left(q \lambda \cdot(2 g-1) \cdot 2^{g}\right)^{2}}{2^{n}} \leq \frac{q^{2} \lambda^{2} g^{2}}{2^{n-2 g-2}} .
$$

Our claim follows from the union bound.

### 10.9.2. Memory Hardness.

In 1970, Hewitt and Paterson introduced a method for analyzing Time-Memory Tradeoffs (TMTO;) on directed acyclic graphs [189], called pebble game. While their method has been known for decades, it was recently used in a cryptographic context, see e.g., [86]. In general, a pebble game is a common model to derive and analyze TMTOs as shown in [216, 217, 222, 227, 229].

The pebble-game model is restricted to DAGs with bounded in-degree and can be seen as a single-player game. Let $\Pi(\mathcal{V}, \mathcal{E})$ be a DAG and let $G=|\mathcal{V}|$ be the number of vertices within $\Pi(\mathcal{V}, \mathcal{E})$. In the setup phase of the game, the player gets $S$ pebbles (tokens) with $S \leq G$. A pebble can be placed (pebble) or be removed (unpebble) from a vertex $v \in \mathcal{V}$ under certain requirements:

1. A pebble may be removed from a vertex $v$ at any time.
2. A pebble can be placed on a vertex $v$ if all predecessors of the vertex $v$ are marked.
3. If all immediate predecessors of an unpebbled vertex $v$ are marked, a pebble may be moved from a predecessor of $v$ to $v$.

A move is the application of either the second or the third action stated above. The goal of the game is to pebble $\Pi$, i.e., to mark all vertices of the graph $\Pi$ at least once. The total amount of moves represent the computational costs.

In [151], Lengauer and Tarjan have already analyzed the TMTO for a stack of $\lambda G$ superconcentrators. Since the double-butterfly is a special form of a $G$-superconcentrators there bound also holds for DBGR.

Theorem 10.11 (TMTO for a stack of $\lambda G$-Superconcentrators [151]). For pebbling a stack of $\lambda G$-Superconcentrators using $S \leq G / 20$ pebbles it holds that

$$
T \geq G\left(\frac{\lambda G}{64 S}\right)^{\lambda}
$$

Note that the DBH operation computes a special variations of the DBG where each vertex represents the the hash values of its direct predecessors. Thus, DBHR and therefore CATENA-DBG inherits the $\lambda$-memory hardness from DBG $\lambda_{\lambda}^{9}$.

Discussion. We have to point out that the computational effort for DBH, with reasonable values for $G$, e.g., $G \in\left[2{ }^{17}, 2^{21}\right]$, may stress the patience of many users since the number of vertices and edges grows logarithmic with $G$. Thus, it remains an open research problem to find a $G$-superconcentrator - or any other $\lambda$-memory-hard function - that can be computed more efficiently than a DBHH.

### 10.10. Results Summary

We introduced a new class of side-channel attacks, called garbage-collector attack, which bases on a malicious garbage collector. We showed that the common password scrambler scrypt is vulnerable to this kind of attacks. Furthermore, we presented a (theoretical) cache-timing attack on scrypt that exploits its password-dependent memory-access pattern. Both attacks allows an adversary to construct a memoryless password filter that enables massively-parallel password-guessing attacks. Moreover, we show that our attacks work even without knowledge of the password hash. All regular implementations, i.e., implementations that are not hardened against sidechannel attacks, of password scramblers with a password-dependent memory-access pattern appear to be vulnerable to these attacks.
As a remedy, we introduced a novel password-scrambler framework Catena, which is based on a $\lambda$-memory-hard function. It is the first framework which naturally supports client-independent updates and server relief. It consists of two security parameters $\lambda$ (depth) and $g$ (garlic), where $\lambda$ reflects the memory hardness and $g$ the memory consumption. In addition, we have shown that Catena is provably secure in the random oracle model.

Furthermore, we presented a $\overline{\text { DBH }}$ based instantiatation of Catena, Catena-DBG. Note that DBH basically computes a stack of several Double Butterfly Graph where each vertex of the graph is the hash value of its direct predecessors.

Finally, we want to stress out that the limited practicality of this implementation. There is a good chance that the runtime of Catena-DBG might exceed the patience of many users.

## Part IV

## Epilog



## Conclusion

Science never solves a problem without creating ten more.

George Bernard Shaw

In this section we conclude this thesis by giving a brief summary of the main contributions and emphasize some further work as well as open research topics.

### 11.1. Summary

Robustness. One of the main topics of this thesis is the analysis and design of misuse-resistant authenticated encryption schemes. The field of robust authenticated encryption schemes was pioneered by Rogaway and Shrimpton [210] who introduced the notion of (nonce-) misuse resistance in 2006. During our research we came up with a generalized definition of robustness as well as the security notion of decryption misuse.

Moreover, we introduced two novel on-line authenticated encryption schemes: MCOE and COFFE. The latter one is the first provably secure OAE scheme that has been designed for the usage of a hash function rather than a block cipher as the underlying primitive. In contrast to conventional AE schemes, COFFE also provides ciphertext integrity in the nonce-misuse scenario. The former one, MCOE, was presented at FSE 2012 [98]. It was the first robust OAE scheme published by then. This academic work inspired fellow researchers to introduce new nonce-misuse resistant OAE schemes [5, 7, 73]. Our work seem to have influenced the submission requirements of the
upcoming Competition for Authenticated Encryption: Security, Applicability, and Robustness (CAESAR):
... that the cipher is designed to provide the maximum possible robustness against message-number reuse 1 .

We expect that our contributions in terms of nonce- and decryption-misuse resistance will foster misuse awareness in future (O)AE scheme designs. Furthermore, we do believe that providing a second line of defence - by applying robust authenticated encryption - helps to make the IT-world a little bit more secure.

Hash Function Design. Another main topic of this work is the presentation of Twister $_{\pi}$, a family of cryptographic hash functions. It is a revised version of the SHA-3 submission Twister [88] that has some vulnerabilities against certain rebound attacks [166]. In the revision process we applied effective countermeasures to overcome those weaknesses. Until now, no attacks are known.

Password Scrambler. Inspired by the discovery of cache-timing attacks on scrypt, we designed Catena, the first provably secure and memory-consuming password scrambler that does not only thwart GPU-based attacks, but also provide a password independent memory-access pattern to render cache-timing attacks infeasible. Furthermore, Catena naturally supports client-independent updates and server relief, and it is provable secure in the random oracle model. The program chair of the Password Hashing Competition ( $(\mathrm{PHC})$ has serendipitously added support for clientindependent update as a functional requirement and cache-timing resistance as a security requirement $\sqrt[2]{2}$. Finally, we hope that our contribution lay the groundwork for all subsequent password-hashing schemes.

### 11.2. Further Research

Due to the CAESAR contest, the design and analysis of authenticated encryption schemes is a hot topic in the field of symmetric cryptography. Inside the cryptographic community, there is a clear consensus about the fact that the notion of secure AE (CCA3)security) is the de facto gold standard for the vast majority of secure channels. Nevertheless, there are still some open research topics.

[^4]- Is it possible to design an integrated tweakable block cipher $\widetilde{E} \in \operatorname{Block}(k, u, n)$ which is more efficient than a constructed one? Instances of MCOE or TC1 [211] would greatly benefit from such a primitive.
- What is about the software and hardware efficiency of such integrated primitives?
- McOE is highly sequential. Is it possible to construct a provably secure on-line authenticated encryption scheme which is both parallel and robust?
- Are there, apart from nonce and decryption misuse, any other misuse scenarios that should be taken into account by the cryptographic community?
- Shall possible security issues in the case of robustness also be discussed in publickey cryptography, e.g., digital signatures or fully homomorphic encryption?

Cryptographers have almost orphaned the field of password-hashing schemes in spite of the medial omnipresence of leaked password databases. Therefore, designing a good password-hashing scheme is more an art than a science. The PHC tries to raise the awareness of this research topic. Imperatively, a solid theoretic foundation is needed, i.e., rigorous analysis, formal definitions, and security notions.

- Is it possible to design a fast and parameterizable cryptographic hash function which can be turned - by an appropriate parameter choice - into a memoryhard KDF or password scrambler?
- How efficient would such a construction be in soft- or hardware?
- Are there any other relevant properties research should take a look at?
- Which reasonable security notions should become the gold standard for passwordhashing schemes?
- Is it possible to construct a $\lambda$-memory hard function that is more efficient than our proposed DBG operation? For example, does a scalable $G$-superconentrator exist, that can be computed more efficently, i.e., that has a linear number of edges and vertices?


## List of Publications

The lists are ordered by the date of publication.

## Lecture Notes in Computer Science

1. Christian Forler, Stefan Lucks, and Jakob Wenzel. Memory-Demanding Password Scrambling. In Tatsuaki Okamoto, editor, ASIACRYPT, volume 8874 of Lecture Notes in Computer Science, pages 289-305. Springer, 2014. [104]
2. Farzaneh Abed, Christian Forler, Eik List, Stefan Lucks, and Jakob Wenzel. Counter-bDM: A Provably Secure Family of Multi-Block-Length Compression Functions. In David Pointcheval and Damien Vergnaud, editors, AFRICACRYPT, volume 8469 of Lecture Notes in Computer Science, pages 440-458. Springer, 2014. [4]
3. Farzaneh Abed, Scott Fluhrer, Christian Forler, Eik List, Stefan Lucks, David McGrew, and Jakob Wenzel. Pipelineable On-Line Encryption. In Carlos Cid and Christian Rechberger, editor, Fast Software Encryption Lecture Notes in Computer Science, Springer, 2014. (Note: to appear.) [1]
4. Farzaneh Abed, Christian Forler, Eik List, Stefan Lucks, and Jakob Wenzel. A Framework for Automated Independent-Biclique Cryptanalysis. In Shiho Moriai, editor, FSE, volume 8424 of Lecture Notes in Computer Science, pages 561-581. Springer, 2013. (3)
5. Ewan Fleischmann, Christian Forler, and Stefan Lucks. McOE: A Family of Almost Foolproof On-Line Authenticated Encryption Schemes. In Anne Canteaut, editor, FSE, volume 7549 of Lecture Notes in Computer Science, pages 196-215. Springer, 2012. [98]
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1. Ewan Fleischmann, Christian Forler, Michael Gorski, and Stefan Lucks. TWISTER $_{\pi}$ - A Framework for Secure and Fast Hash Functions. International Journal of Applied Cryptography (IJACT), Volume 2 Number 1, pages 68-81, Inderscience, 2010. [96]

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1. Christian Forler, David A. McGrew, Stefan Lucks, and Jakob Wenzel. COFFE: Ciphertext Output Feedback Faithful Encryption. IACR Cryptology ePrint Archive, 2014:1003, 2014.
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3. Farzaneh Abed, Christian Forler, and Stefan Lucks. Classification of the CAESAR Candidates. IACR Cryptology ePrint Archive, 2014:792, 2014.
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5. Farzaneh Abed, Scott Fluhrer, Christian Forler, Eik List,Stefan Lucks, David McGrew, and Jakob Wenzel. The POET Family of On-Line Authenticated Encryption Schemes. Submission to the CAESAR competition, 2014.
6. Christian Forler, Stefan Lucks, and Jakob Wenzel. Catena: A MemoryConsuming Password Scrambler. Submission to the PHC, 2014.
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## Twister $_{\pi}$ : Test Vectors

## A.1. $\mathrm{TWISTER}_{\pi}-256$



## A.2. $\mathrm{TwISTER}_{\pi}-512$

Input: 616263
Output: d9 2e 69 c1 86 9e 0c c1 170677 fc fb 79 b4 33 ea b9 2393 b6 5907 bb d1 69 0e f2 1f 69 d8 3a 72 ae 44308456 f0 49 e6 ec 3864 bc 37 7a 477602 ee 9 e $9867485009666 f 6080$ 1d 16 2a

Input: 61626364656667686263646566 67686963646566676869 6a 6465 66676869 6a 6b 6566676869 6a 6b 6c 66676869 6a 6b 6c 6d 676869 6a 6b 6c 6d 6e 6869 6a 6b 6c 6d 6e 6f 69 6a 6b 6c 6d 6e 6f 70 6a 6b 6c 6d 6e 6f 7071 6b 6c 6d 6e 6f 707172 6c 6d 6e 6f 70717273 6d 6e 6f 7071727374 6e 6f 707172737475
Output: b0 e3 4b aa 3d a1 5487 Of 1f 7a c4 ef a1 5e 33 d6 d3 23 f0 74 c6 $2 f$ e1 40 ea 37579 e ee 1 a 2 e 4 b ce 3 e be 6 c 0 e 40 56 bb 8357 e8 41 b0 05 0e 3d df ea e3 5a 0249 0c ac $0 f$ e0 1b dd 4a 7f f4

Input: $616161616161 \ldots 616161616161$ (1,000,000 octets)
Output: 8b 9935 5a 36 c6 $295362024 a$ de 91
(64 octets)

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[^0]:    ${ }^{1}$ Ciphertext indistinguishability where a secure encryption scheme must not leak any information about the plaintext, except its length.

[^1]:    ${ }^{2}$ Ode on a Distant Prospect of Eton College

[^2]:    ${ }^{1}$ http://csrc.nist.gov/archive/aes/

[^3]:    ${ }^{2}$ http://www.openssl.org, last access: July 2013

[^4]:    ${ }^{1}$ http://competitions.cr.yp.to/caesar-call.html
    ${ }^{2}$ https://password-hashing.net/call.html

