

Topology optimization of structures subjected to multiple load cases by introducing the Epsilon constraint method

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1 Abstract

A topology optimization method has been developed for structures subjected to multiple load cases (Example of a bridge pier subjected to wind loads, traffic, superstructure...). We formulate the problem as a multi-criterial optimization problem, where the compliance is computed for each load case. Then, the Epsilon constraint method (method proposed by Chankong and Haimes, 1971) is adapted. The strategy of this method is based on the concept of minimizing the maximum compliance resulting from the critical load case while the other remaining compliances are considered in the constraints. In each iteration, the compliances of all load cases are computed and only the maximum one is minimized. The topology optimization process is switching from one load to another according to the variation of the resulting compliance.

In this work we will motivate and explain the proposed methodology and provide some numerical examples.

Keywords: Topolgy optimization, multiple load cases, Epsilon constraint method

2 Introduction

The objective function of topology optimization problems is to minimize the compliance of the structure due to a given applied force. When the structure is subjected to multiple load cases, the objective function becomes a set of multi-objective functions where the aim is to minimize the compliances due to each load case, seperately.

To deal with the multi-load case, traditional methods [8] propose to minimize the worst case, which means to minimize the maximum compliance corresponding to the load case corresponding to the worst stress state. Another approach consists in combining the different objective functions, so the problem is changed to a scalar optimization problem. A famous example is the weighted sum method where the objective functions are summed with a preference ratio of importance, but the difficulty of this method lies on finding the correct value of the weight ratios prior to optimization, because small perturbations (of these ratios) leads to very different solutions.

3 State of the Art

3.1 Topology optimization

The topology optimization problem aims in finding the optimal layout of a given structure within a specified region, which can be interpreted as finding the adequate design that stores the minimum amount of the strain energy for a given structure. This is equivalent to minimizing the external work done by the applied load. Many methods were used to solve the topology optimization problems. The most famous one is the SIMP method, and compared to other methods, it is based on mathematical theories and permits to convert the optimal topology problem into a sizing problem on a fixed domain [3].

The mathematical formulation of the optimization problem for isotropic material is written as [3]:

$$\begin{aligned} \min_x c(x) = \min_x U(x)^T K(x) U(x) = \min_x \sum_{e=1}^N (x_e)^p u_e^T k_0 u_e \\ \text{subjected to the prescribed volume fraction: } V(x)/V_0 = f \\ \text{subjected to the loads: } KU = F. \end{aligned} \quad (1)$$

Where c is the compliance of the structure. U is the global displacement vector, K is the global stiffness matrix, F is the vector of applied loads, N is the number of the elements, x is the density and p is the penalization factor.

$V(x)$ and V_0 are the volume and the design domain volume, respectively and f is the prescribed volume fraction.

SIMP method is an interpolation scheme that penalizes intermediate densities to realize a black and white design; ($p \geq 3$) is always required. The sensitivity of the objective function with respect to the element densities x_e is formulated as:

$$\frac{\partial c}{\partial x} = -p(x_e)^{p-1} u_e^T k_0 u_e. \quad (2)$$

In order to ensure existence of solutions and to avoid the formation of checker-board patterns, filters are applied to the sensitivities or the densities.

The optimization problem is solved by means of the standard optimality criteria method.

3.2 Robust Topology Optimization

When considering the real engineering world, the solutions obtained from deterministic settings in topology optimization problems may not be practical and do not reflect the real behavior of the structure. This is due to the variabilities in the manufacturing process and the operating conditions. These variabilities lead to uncertainties in geometry and material properties. The material property uncertainty was limited to Young's modulus and perturbations are made on the stiffness matrix to quantify the effect of uncertainties on structural response [2].

Applied loading is often considered as uncertain parameter and uncertainty may include loading direction uncertainty and uncertainty in the magnitudes [9] and [4].

The treatment of uncertainties can be categorized into two types: the robust design optimization (RDO) and the reliability based topology optimization (RBTO). The RDO aims to minimize the influence of the stochastic variabilities. However, the reliability based design optimization figures out the probability of failure and aims to ensure that the reliability of a structure is greater than the required value. The uncertainty is introduced as a constraint on the probability of failure. Kharmanda ([6] and [5]) integrated the reliability analysis into topology optimization problem where the randomness of the applied loads and the geometry have been

taken into account. Also, Zhao and Wang [9] proposed an efficient approach to solve robust topology optimization problem of structures under loading uncertainty and showed the effect of loading magnitude and directional uncertainty on the robust designs.

3.3 Epsilon Constraint method

The Epsilon method is one of the scalarization method [7]. It was proposed first by Chankong and Haimes in 1983, and its concept is to optimize one objective functions among others, while the remaining objectives are constrained to be less than or equal to given target values.

4 Proposed Epsilon Constraint method

4.1 Problem formulation and algorithm

When the structure is subjected to multiple load cases, each load case is considered as an objective function to be optimized and the global optimization problem consists in finding of a single optimal solution that optimizes simultaneously all the objectives functions. Then, the topology optimization problem changes to a problem of minimizing the resulted compliance of the different loads. However, available methods propose to minimize the maximum compliance corresponding to the worst case. These methods seem not to be relevant since they didn't take into consideration the contribution of the other loads. The proposed "adapted" Epsilon method deals with such case, its strategy consists in calculating the compliance corresponding to each load, and in each iteration the maximum compliance should be minimized. The concept of this method can be written mathematically as:

In each iteration:

$$\min_x c_r(x) = \min_x \left\{ \max_{\{r \in 1, \dots, m\}} \left(\sum_{e=1}^N (x_e)^p u_e^T k_0 u_e \right) \right\} \quad (3)$$

subjected to: $c_i(x) \leq \epsilon_i; i=1, \dots, m; i \neq r$

subjected to the the prescribed volume fraction: $V(x)/V_0 = f$

Where m is the numbers of the loads applied in the structure, r is the load case where the compliance reaches its maximum in the iteration. ϵ_i are the upper bounds for the constraints and used to obtain weak Pareto optima, and it is the task of the decision maker to choose the appropriate values.

What characterizes this method is the jumping of the optimization process; while minimizing the first calculated maximum compliance, the optimization is then moved to cover the compliance of the other loads basing on the maximum calculated value of the compliance in each iteration. The optimization stops when the convergence is assured.

This method is described in the following algorithm:

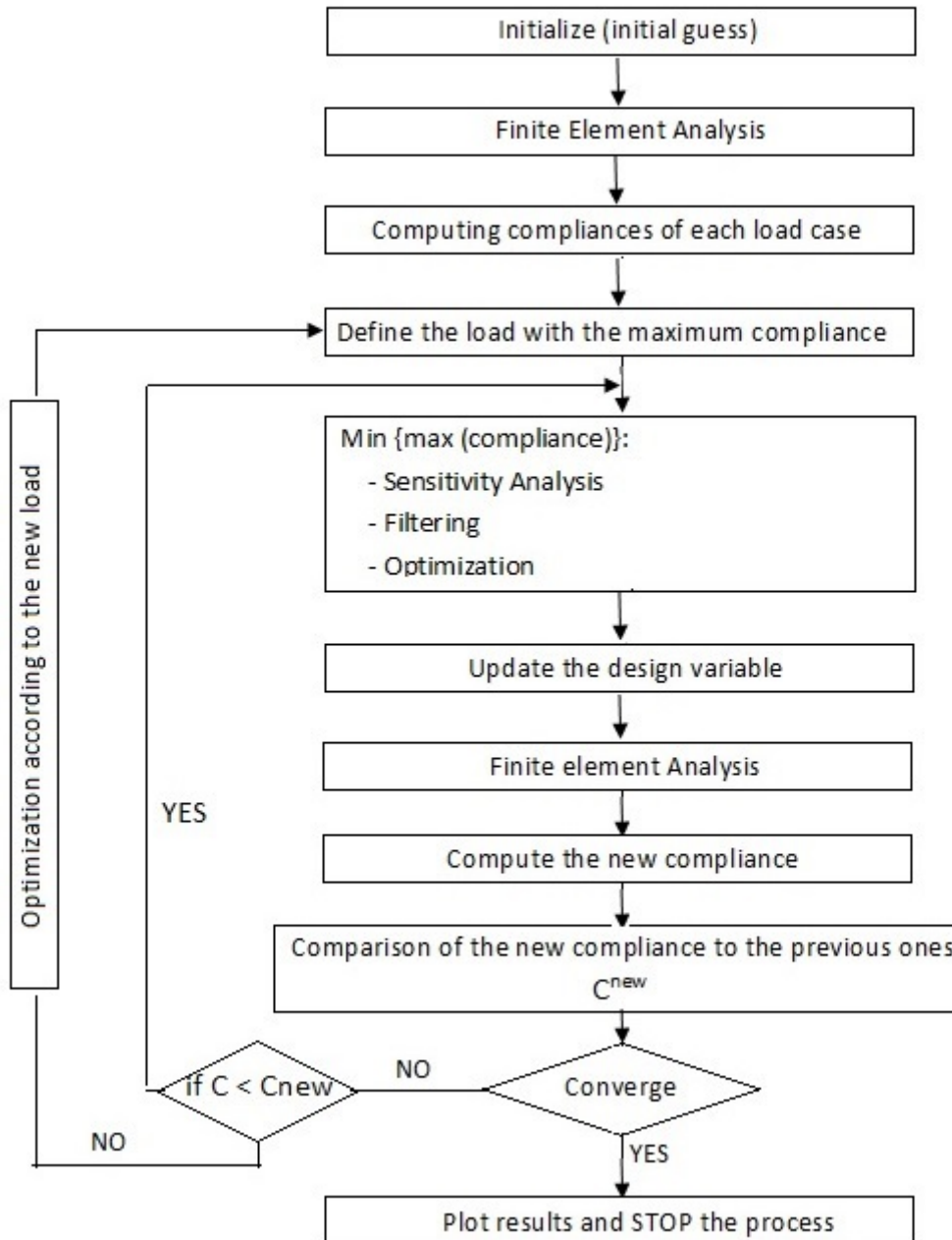


Figure 1: Algorithm of the Epsilon constraint method

4.2 Numerical examples

Consider a cantilever beam where the design domain, the boundary conditions and the external loads are shown in the following table. The aim of the topology optimization is to find the optimal material distribution, with a constraint on the total amount of the material.

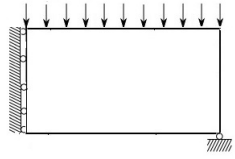
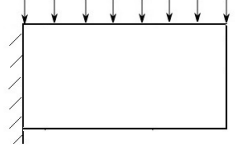


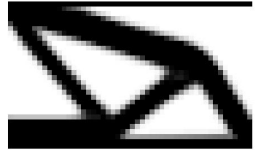

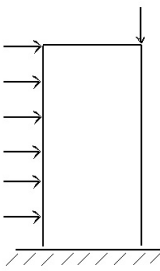
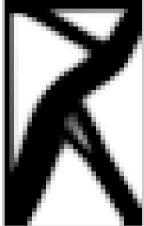

| | Initial design | Optimized design (1) | Optimized design (2) |
|-----------------|--|--|--|
| cantilever beam |   |   |   |
| Column |  |  |  |

Table 1: Optimized structures via Epsilon constraint method
1- without passive elements; 2- with passive elements

From the previous table, in the two examples of the cantilever beam, the distribution of the material depends on two principal factors: the initial boundary conditions and the process of optimization of the Epsilon Constraint method itself. In the first example, and however the upper part is subjected to nodal loads, the final optimized design does not contain that part. This can be explained by the fact that the optimization converged from the first stage and only the important loads were considered (those who have bigger compliance and they are located in the left part: near the boundary condition). Unlike this case, the distribution of material in the second example of the fixed cantilever beam involves the upper part of the structure where the loads were applied. Also, the third example of the fixed column shows the same results. So, the optimal design depends also on the boundary conditions.

In order to ensure the existence of material on the parts where loads are applied and because of the uncertainty of the magnitude and the direction of the loads in the structure, the designer can implement passive elements in those parts with density equal to one (solid part) as it shown in the previous table[1].

The following table shows the effect of considering only 5 loads on the final optimized design while considering the Epsilon constraint method.

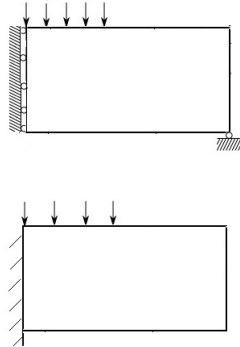

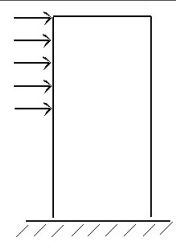

| | Initial design | Optimized design without passive elements |
|-----------------|---|--|
| cantilever beam |  |  |
| Column |  |  |

Table 2: Effect of the loads on the optimization process of the Epsilon constraint method

5 Conclusion

This paper proposed a new method dealing with structures subjected to multiple loads. An "adapted" Epsilon Constraint method have been implemented as a tool for the optimization process, and, unlike the traditional methods that consider only the optimization of the structure according to critical load case, this method consider the contribution of the different applied loads. This method presents as well a faster convergence and the optimal layout depends principally on the initial boundary conditions and the process of optimization of the Epsilon Constraint method itself.

As outlook, the new designs need to be checked for their structural reliability and compared to those obtained by classical topology optimization. This may lead to adaptations of the proposed method in terms of reliability based structural multi-objective optimization.

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