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APPLICATION OF MULTI-CRITERIA NUMERICAL OPTIMISATION IN GEOTECHNICAL ENGINEERING

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Abstract. *Geotechnical constructions are sophisticated structures due to the non-linear soil behaviour and the complex soil-structure interaction, which entails great exigencies on the liable engineer during the design process. The process can be schematised as a difficult and, depending on the opportunities and skills of the processor more or less innovative, creative and heuristic search for one or a multiple of defined objectives under given boundary conditions.*
Wholistic approaches including numerical optimisation which support the constructing engineer in this task do not currently exist. Abstract problem formulation is not state of the art; commonly parameter studies are bounded by computational effort. Thereby potential regarding cost effectiveness, construction time, load capacity and/or serviceability are often used insufficiently.
This paper describes systematic approaches for comprehensive optimisation of selected geotechnical constructions like combined pile raft foundations and quay wall structures. Several optimisation paradigms like the mono- and the multi-objective optimisation are demonstrated and their use for a more efficient design concerning various intentions is shown in example.
The optimisation is implemented by using Evolutionary Algorithms. The applicability to geotechnical real world problems including nonlinearities, discontinuities and multi-modalities is shown. The routines are adapted to common problems and coupled with conventional analysis procedures as well as with numerical calculation software based on the finite element method.
Numerical optimisation of geotechnical design using efficient algorithms is able to deliver highly effective solutions after investing more effort into the parameterization of the problem. Obtained results can be used for realizing different constructions near the stability limit, visualizing the sensitivity regarding the construction parameters or simply procuring more effective solutions.

1 INTRODUCTION

Design processes in engineering represent an innovative, creative and heuristic search to ideally accomplish one or more defined objectives within given constraints. System-related problems in geotechnical concepts can be ascribed to the complex soil-structure interaction as well as the non-linear behaviour of the soil.

A priori, the quantitative impact of adjusting certain design parameters to reach target values is unknown. Therefore, the design engineer rarely succeeds in fully meeting all requirements [11].

Wholistic approaches which involve several aspects of geotechnical concepts in a unified view do not currently exist. Design concepts and related numerical models are either not at all or just marginally evaluated by sensitivity analyses of free construction parameters. Usual objective values for constructions are often inherently static. Processes concerning the construction or cost effectiveness only have inferior influence on the design process.

Modern methods of mathematical optimisation already gained access to other fields, for example mechanical engineering. There, methods of numerical optimisation are successfully employed offering solutions to complex problems. Comparable approaches in geotechnical engineering have only played a minor role to date [3].

This article will illustrate the advantages in the application of optimisation methods supporting the design of geotechnical structural systems. The potential of these methods for design engineers is elucidated in an example. Approaches and strategies for wholistic mono- and multi-criteria optimisation are presented for selected geotechnical structures together with the results.

The concept is extended to versatile requirements [10]. In addition to safety principles further aspects like construction and maintenance costs, time of construction, construction method, geometry etc. are incorporated.

2 CONCEPTS OF OPIMIZATION

Optimisation procedures are important instruments for manifold decision processes. The aim of optimisation is to determine the optimal solution for a problem under several constraints with passable efforts.

Because of the different requirements regarding definition and solution a distinction between mono- and multi-objective optimisation concepts is made. While mono-criteria optimisation tasks involve the minimisation of one single objective, multi-objective optimisations deal with several objectives simultaneously. Mono-criteria optimisations ideally have a global solution, whereas solutions to multi-objective optimisations consist of sets of ideal solutions.

The mathematical descriptions of mono- and multi-objective problems as well as their solutions are specified below.

2.1 Mono-criteria optimisation

The elementary form of the mono-criteria optimisation problem consists of a unique objective function $f\left(\underline{\mathbf{x}}\right)$ with constraints for the equations $g_i\left(\underline{\mathbf{x}}\right)$ and the inequalities $h_j\left(\underline{\mathbf{x}}\right)$, shown in Eq. (1).

$$\min \left\{ f\left(\underline{\mathbf{x}}\right) | \underline{\mathbf{x}} \in \mathbb{M} | \, g_i\left(\underline{\mathbf{x}}\right) = 0; h_j\left(\underline{\mathbf{x}}\right) \leq 0 \right\} \tag{1}$$

A valid point $\underline{\mathbf{x}}^*$ which fulfills

$$f\left(\underline{\mathbf{x}}^*\right) \leq f\left(\underline{\mathbf{x}}\right) \quad \forall \quad \underline{\mathbf{x}} \in \mathbb{M} \tag{2}$$

is called global minimum of the in Eq. (1) defined problem.

2.2 Multi-objective optimisation

Analogous to Eq. (1) the constrained multi-objective optimisation problem is defined as

$$\min \left\{ \underline{\mathbf{f}}\left(\underline{\mathbf{x}}\right) | \underline{\mathbf{x}} \in \mathbb{M} | \, g_i\left(\underline{\mathbf{x}}\right) = 0; h_j\left(\underline{\mathbf{x}}\right) \leq 0 \right\} \ . \tag{3}$$

In contrast to Eq. (1) rather a set of objective functions is considered. The optimality definition of the mono-criteria problem is replaced by the concept of dominance. A solution $\underline{\mathbf{x}}_1$ dominates a solution $\underline{\mathbf{x}}_2$ if in $f_k\left(\underline{\mathbf{x}}_1\right)$ for all $k = 1, \ldots, l$ no objective is worse and at least one is superior to $f_k\left(\underline{\mathbf{x}}_2\right)$.

If a set $\mathbb{P}$ of solutions $\underline{\mathbf{x}}^*$ is not dominated by other solutions, Eq. 4 is fulfilled and $\mathbb{P}$ is denoted as PARETO-optimal set.

$$\underline{\mathbf{x}}^* \prec \underline{\mathbf{x}} \quad \forall \quad \underline{\mathbf{x}} \in \mathbb{M} \tag{4}$$

To apply the concept of dominance to the optimisation task a mapping from decision space into objective space is required, see Fig. 1. The set $\mathbb{P}$ of non-dominated solutions is defined as the PARETO-optimal set.

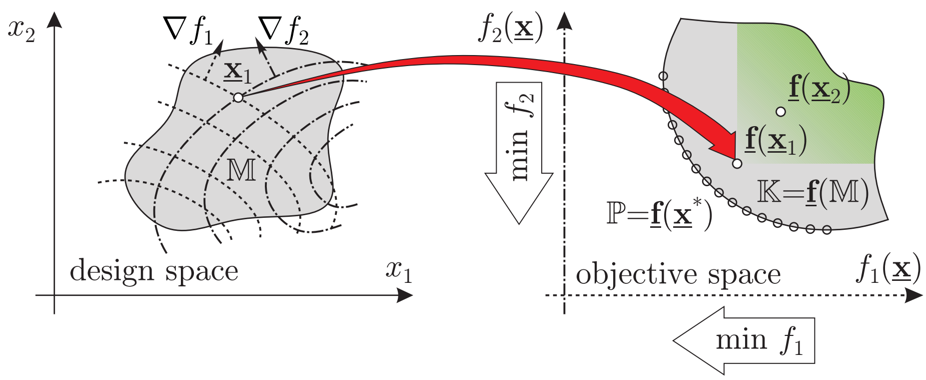

Figure 1: Mapping from decision space into objective space [1]

3 NUMERICAL OPTIMIZATION

For the practical solution of optimisation problems numerical methods are used. The history of numerical optimisation goes back to Leibnitz and Newton. Conventional procedures are used to determine the extreme values of real-valued functions with one or more variables with or without constraints based on gradient information. There is a multitude of optimisation algorithms that efficiently solve special optimisation tasks.

Practical application problems require capable and robust procedures. Especially objective functions with a large number of local extreme values as well as complicated problems with non-real parameters or discontinuities in the objective function cannot be satisfyingly solved with the classical methods. In this case stochastic methods could be successfully applied.

Characteristic for stochastic methods is the combination of specific search and randomness to obviate premature convergence. The structure is relatively simple and often inspired by natural adaption processes. The essential advantage is the independence of objective functions and constraints from the structure. Detrimental is the high calculation effort caused by a large number of evaluations of the objective function.

By using Evolutionary Algorithms for the design optimisation a stochastic optimisation method is adopted. Evolutionary Algorithms are able to solve so called black-box problems, their application for solving a wide range of problems has been shown in a large number of studies [13].

Evolutionary Algorithms are inspired by the biological evolution. An Evolutionary Algorithm is a population-based iteration scheme, which examines parallel multiple parameter sets in each iteration step. A benchmark principle is adopted on the sets in terms of one or more objective functions to accomplish an optimal solution by truncation of the algorithm [5].

After an initial phase at the beginning of the optimisation a specific search over multiple generations is enforced. In every generation the objective function is evaluated for all sets, the highest fitness is assigned to the best solutions. Executing evolutionary operators like recombination and mutation on fitness-based selected individuals leads to new parameter sets for the next generation. This process ideally leads to even better solutions. Fig. 2 shows the basic scheme of an Evolutionary Algorithm.

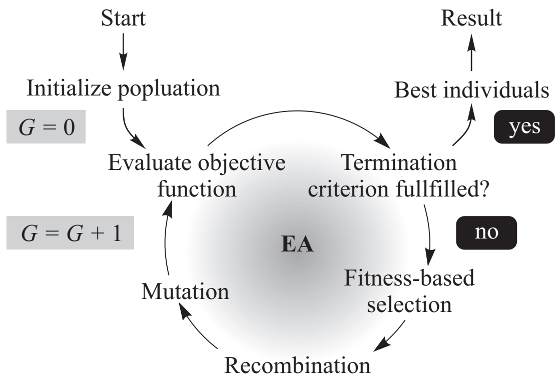

Figure 2: Basic scheme of a generalised evolutionary algorithm

In contrast to classical deterministic optimisation schemes Evolutionary Algorithms are able to carry out parallel searches. Moreover, gradient information is not required and the solution of a problem is independent from the representation of its parameters. Thus even highly non-linear objective functions as well as discontinuities in objective functions or constraints can be optimised with sufficient accuracy [4]. Due to parallel search multi-objective problems could also be solved in a single optimisation run.

4 APPLICATION TO GEOTECHNICAL STRUCTURES

4.1 Mono-criterial problem: cost-optimisation of a quay wall structure

The requirements of quay wall structures have increased over the past years. Realisation of large water depths, substantial load increases of the gantry cranes as well as changes in calculation standards result in the need for even more sophisticated constructions.

The high number of requirements precludes a cost-effective solution by a classical static calculation. Linear constructions like quay walls qualify for optimisation because the effort for optimisation is overlaid by its use with increasing length of the calculation profile. This effect is shown for the optimisation of a quay wall in consideration of the building costs. In addition the dependencies of costs on specific parameters are shown explicitly.

The optimisation of a quay wall structure is carried out based on an economic performance calculation established on the estimated costs for the different positions under statical constraints. The structure consists of a combined sheet pile wall behind a row of friction type piles. The concrete superstructure is founded on three vertical piles; the inclined pile carries horizontal loads. The section of the construction is shown in Fig. 3. Water depth, soil profile and dimensions of the structure are typical for a container terminal at the port of Hamburg.

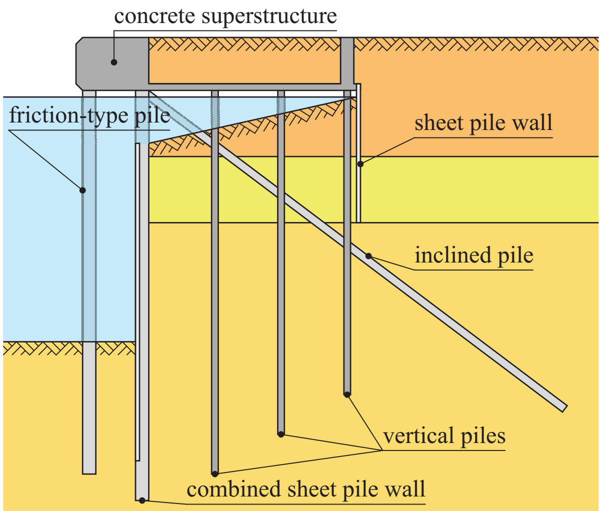

Figure 3: Examined quay wall section

To show the dependencies of construction costs on the geometry and the position of selected components parameterised variations are calculated. In the following two examples are presented. In Fig. 4 the cost-dependencies for the free construction parameters of the friction-type pile are shown, Fig. 5 shows the influence on the costs of the inclination and the distance of the inclined pile.

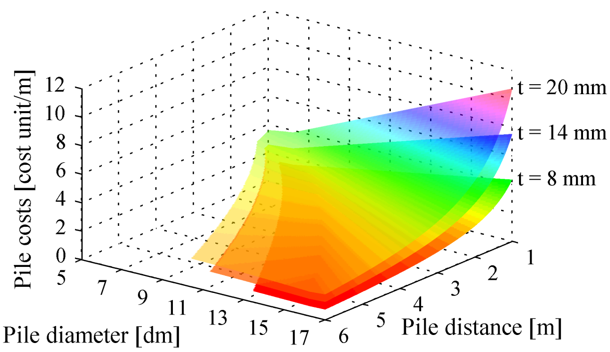

Figure 4: Costs of friction-type pile depending on pile diameter and pile distance for three different wall thicknesses

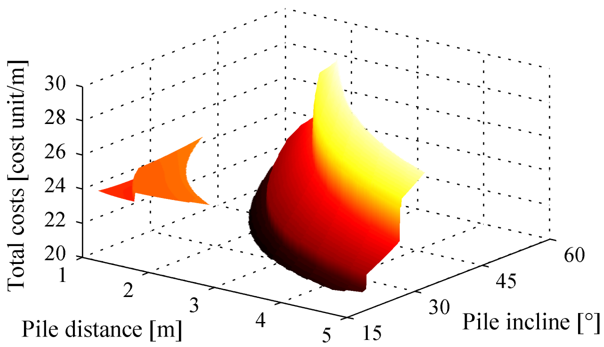

Figure 5: Total cost of the quay wall structure depending on anchor pile distance and anchor pile inclination

Through inspection of the shown plots of the objective functions the character of the objective functions could be estimated. The cost-function of the friction-type pile is obviously a non-linear multi-modal function on a restricted domain. The cost-function of the inclined pile is discontinuous. The conducted parameter study demands a high calculation effort; the estimation of graphical contexts is limited to three dimensions.

By using Evolutionary Algorithms to optimise the total costs of the shown quay wall structure the minimum can be found efficiently. For the wall section shown in Fig. 3 a cost saving of about 11 % based on the estimated partial cost could be realized compared to average construction costs.

4.2 Multi-criterial problem: Cost- and settlement optimisation of a pile foundation

Pile foundations are characterized by complex load/settlement behaviour. For the realistic determination of the non-linear load-bearing behaviour of a single pile, interactions within a pile group and between adjacent piles have to be considered as well as the linkage between the piles by the superstructure. A comprehensive static calculation approach for pile groups is presented in [9].

Construction costs of a pile foundation encompass costs of the substructure as well as costs for the piles themselves. In [7], a detailed and parameterized cost calculation for the presented foundation type is elucidated. It is subdivided into a standard price calculation for the superstructure as well as a performance-related calculation for the piles. Determination of the crucial performance data for the piling rig is based on an approach according to [12]. Material costs were determined utilizing the producer cost index and machinery costs were set according to the BGL. For simplicity fixed costs for the construction site as well as set-up and transportation costs were neglected.

The resulting static cost planning model shows a high degree of complexity and it is not feasible to solve it by hand with regard to parallel minimisation of the dual objective. Furthermore the intuitively assumed correlation between an increase in costs and a reduction in settlement can be confirmed. A parametric optimization using a multi-criterial Evolutionary Algorithm was solved to obtain the set of possible solutions.

The outcome of the optimization is displayed in Fig. 6 in the criteria space. Fig. 7 depicts the results subdivided for the individual parameters.

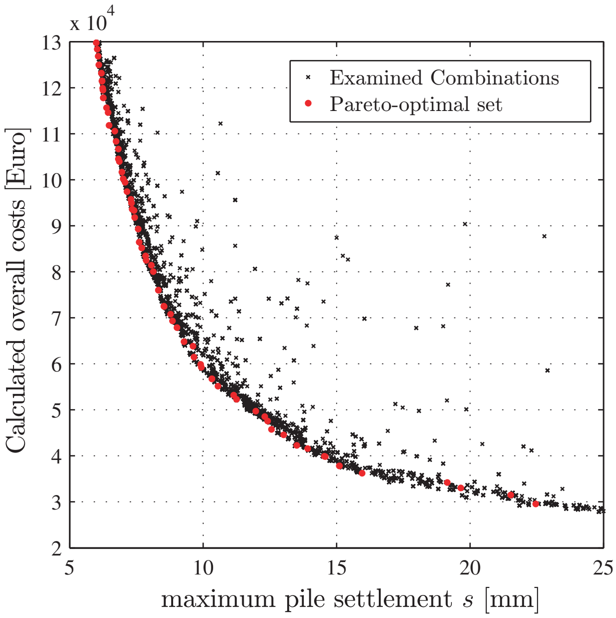

Figure 6: Calculated overall costs and displacements of the foundation for all examined combinations and for the ingenious solution

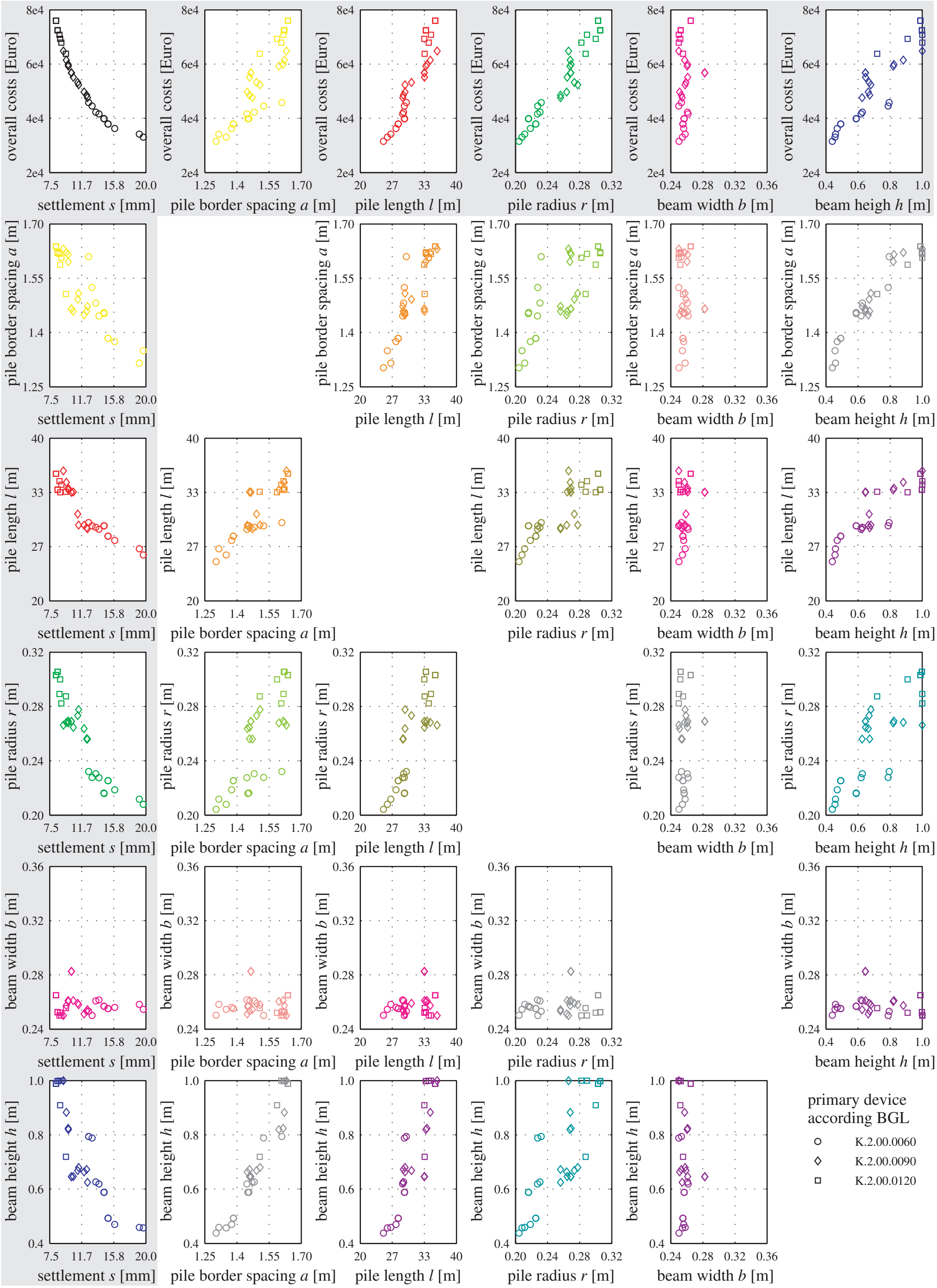

Figure 7: Optimisation results concerning overall costs and displacements depending on the optimisation parameters pile border spacing, pile length, pile radius as well as beam width and beam height divided into applied primary device

In Fig. 6 the afore mentioned correlation between an increase in costs and a reduction in settlement is apparent. Also an accurate and fast convergence of the method is evident since only few points deviate from the emerged PARETO- optimal set. The red points designate the best solutions after the last iteration step and are located at the lower left-most edge of the possible solution set.

Details of the solutions obtained are illustrated in Fig. 7. The graph in the top left corner shows the PARETO- optimal points of the criteria space. In this case the cost-dominating parameter, which for the present example is the output of the primarily employed machinery, is varied. This diagram evidently demonstrates an over-linear increase of the engine power of the drilling rig with decreasing settlement. The cause of this increase becomes obvious if one examines the underlying parameters. Reduction of settlement is accompanied by an elongation of the piles as well as an increase in pile radius which would explain the higher moment of inertia of the primary machinery. For a change in machinery a jump in the cost function is observed.

The diagrams shown in Fig. 7 have to be viewed in context; only the depiction in criteria space is meaningful by itself. Diagrams highlighted in grey show the dependency of the target function values from the starting parameters, all other diagrams illustrate the correlations between two parameters.

Based on these results one can arrive at a substantiated decision for an optimal solution for any subjective considerations. With the demonstration in Fig. 6 the dependence between deformation and production costs of the chosen foundation becomes explicitly apparent. In correlation with the depiction in Fig. 7 one can verify the feasibility of a construction. Based on the optimisation results the decision maker is able to quantify the causes and effects.

5 CONCLUSIONS

Many practical optimisation problems cannot be rearranged for the desired target values, as a result of non-linearity, multi-modality and discontinuity. The correlation between design parameters and design targets is too complicated already for simple cases so that a manual search does not lead to the desired results.

Numerical optimization provides methods to support the geotechnical design process. With greater focus on the parameterized problem formulation efficient algorithms are able to deliver highly effective solutions.

Further research is needed to analyse the optimisation strategies as well as the increase in efficiency for the respective problem under investigation. More efficient linkage of the optimisation and calculation software is crucial to establish approaches and facilitate practicability. Potential with regard to more efficient constructions can be determined and taken advantage of where applicable.

Especially modern calculation methods such as the finite element method enable an integrative linkage of calculation process and optimisation algorithms. First studies reveal promising results[8].

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