CORE
Metadata, citation and similar papers at core.ac.uk
Provided by Digitale Bibliothek Thüringen

18th International Conference on the Application of Computer
Science and Mathematics in Architecture and Civil Engineering
K. Gürlebeck and C. Könke (eds.)
Weimar, Germany, 07–09 July 2009

SPECIAL FUNCTIONS VERSUS ELEMENTARY FUNCTIONS IN HYPERCOMPLEX FUNCTION THEORY

H. R. Malonek

Department of Mathematics
University of Aveiro
3810-193 Aveiro, PORTUGAL
E-mail: hrmalon@ua.pt

Keywords: Special Functions, Elementary Functions, hypercomplex derivative, series expansion.

Abstract. *In recent years special hypercomplex Appell polynomials have been introduced by several authors and their main properties have been studied by different methods and with different objectives. Like in the classical theory of Appell polynomials, their generating function is a hypercomplex exponential function. The observation that this generalized exponential function has, for example, a close relationship with Bessel functions confirmed the practical significance of such an approach to special classes of hypercomplex differentiable functions. Its usefulness for combinatorial studies has also been investigated. Moreover, an extension of those ideas led to the construction of complete sets of hypercomplex Appell polynomial sequences. Here we show how this opens the way for a more systematic study of the relation between some classes of Special Functions and Elementary Functions in Hypercomplex Function Theory.*

1 INTRODUCTION

2 SPECIAL POLYNOMIALS AND GENERATING FUNCTIONS

3 SPECIAL FUNCTIONS VERSUS ELEMENTARY FUNCTIONS

4 APPLICATIONS: BINOMIAL IDENTITIES

5 ACKNOWLEDGEMENTS

The work was partially supported by the R&D unit *Matemática e Aplicações* (UIMA) of the University of Aveiro, through the Portuguese Foundation for Science and Technology (FCT).

REFERENCES

[1] M. Abul-ez, and D. Constales, Basic sets of polynomials in Clifford analysis, *Complex Variables, Theory Appl.* **14**, 177–185 (1990).

[2] P. Appell, Sur une class de polynomes, *Ann. Sci. École Norm. Sup.* **9**, 119–144 (1880).

[3] R. P. Boas, and R. C. Buck, *Polynomial Expansions of Analytic Functions*, Springer, Berlin, 1958.

[4] F. Brackx, R. Delanghe, and F. Sommen, *Clifford Analysis*, Pitman, Boston-London-Melbourne, 1982.

[5] I. Cação, and H. R. Malonek, On Complete Sets of Hypercomplex Appell Polynomials", *AIP-Proceedings*, Vol. 1048, 647-650 (2008)

[6] B. C. Carlson, Polynomials Satisfying a Binomial Theorem *J. Math. Anal. Appl.* **32**, 543–558 (1970).

[7] M. I. Falcão, and H. R. Malonek, Generalized Exponentials through Appell sets in $\mathbb{R}^{n+1}$ and Bessel functions, *AIP-Proceedings,* Vol. 936, 738–741 (2007).

[8] R. L. Graham, D. E. Knuth, and O. Patashnik, *Concrete mathematics: a foundation for computer science*, Addison-Wesley, 1990.

[9] K. Gürlebeck, and H. Malonek, A hypercomplex derivative of monogenic functions in $\mathbb{R}^{n+1}$ and its applications *Complex Variables Theory Appl.* **39**, 199–228 (1999).

[10] D. E. Knuth, *The Art of Computer Programming*, vol. 1, Addison Wesley, Reading MA, 1968.

[11] G. Laville, and I. Ramadanoff, An integral transform generating elementary functions in Clifford analysis. *Math. Methods Appl. Sci*. 29, No. 6, 637-654 (2006)

[12] H. Malonek, A new hypercomplex structure of the euclidean space $\mathbb{R}^{m+1}$ and the concept of hypercomplex differentiability, *Complex Variables* **14**, 25–33 (1990).

[13] H. Malonek, Power series representation for monogenic functions in $\mathbb{R}^{m+1}$ based on a permutational product *Complex Variables, Theory Appl.* **15**, 181–191 (1990).

[14] H. R. Malonek, and M. I. Falcão, Special Monogenic polynomials- Properties and Aplications, *AIP-Proceedings*, Vol. 936, 764–767 (2007).

[15] H. R. Malonek, and M. I. Falcão, Clifford Analysis between Continuous and Discrete", *AIP-Proceedings* Proceedings, Vol. 1048, 682-685 (2008).

[16] H. Malonek, G. Tomaz, Bernoulli polynomials and matrices in the context of Clifford Analysis, *Discrete Appl. Math.* 157 838-847, (2009)

[17] M. Petkovšek, H. S. Wilf, and D. Zeilberger,*A=B*, A. K.Peters, Wellesley, 1996.

[18] H. Wilf, *generatingfunctionology*, Academic Press, San Diego,, 1994, 2 edn.