Iteration dynamical systems of discrete Laplacians on the plane lattice (I) (Basic properties and computer simulations)

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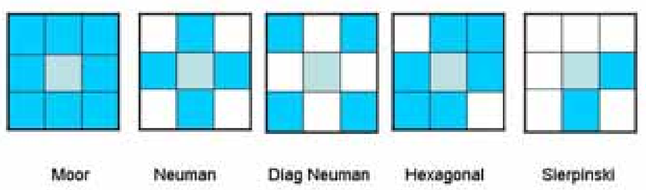
Recently several authors have interests in the discretization of differential operators, for example, the Dirac operator and the Cauchy-Riemann operator ([2]). Also we know that the iteration dynamical system of a quadratic polynomial can describe many fluctuations ([4]). In this study we introduce a concept of a dynamical system defined by an iteration of a discrete Laplacian on the plane lattice and study some basic properties and give several computer simulations. The several kinds of discrete Laplacians and their iteration dynamical systems can not be found in the literatures. Hence we may say that the introduction is quite new and original.
This is the first part of the study on the dynamical systems. In the second part we give an application of our study to psychology.
We choose the lattice **L** on the real plane and consider {0,1}-valued functions on **L**. We calculate sums and products in mod 2 calculation rule. A set of cells which attach the referenced cell is called a neighborhood Up of p. We list several examples



(1) (**Discrete Laplacian)**Choosing a neighborhood Up, we define the discrete Laplacian:

$$\Delta_{Up}f(p)=\sum_{q\varepsilon\, Up-p} (f(q) - f(p)).$$

(2)(**Iteration dyamical system of discrete Laplacian**)Choosing an initial function $f_0 \varepsilon F$, we define the dynamical system defined by the iteration of the Laplacian:

$$\{f_n\}, \quad f_n = \Delta_U f_{n-1} \ (n \geq 1).$$

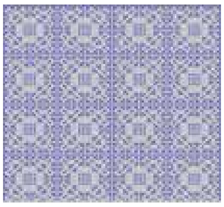
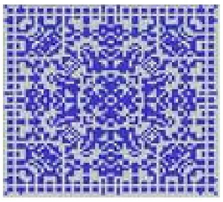
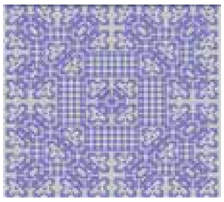
(3)(**Source**) We call p a source (or seed) of the dynamical system when $f_n(p)=1$ for any $n \varepsilon \mathbf{N}$. We regard the sources as boundary conditions.

In this talk we discuss the following two topics:

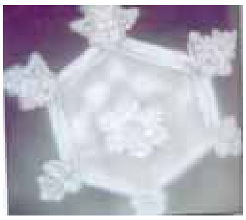
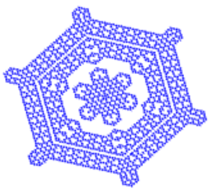
(1) We propose some problems on the basic properties of periodicity and stability for the dynamical systems.

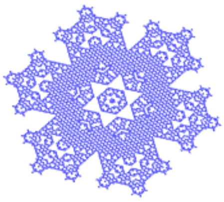
(2) We give several computer simulations by suitable choices of sources and neighborhoods. We may expect to realize many phenomena on evolutions and organizations by these simulators. We notice that we choose the lattice of a suitable size M with the periodicity condition:

(1) **Designs:** We can produce designs of carpets, laces and embroideries systematically choosing suitable neighborhoods. We have a software Designer KENTAURUS(2005).

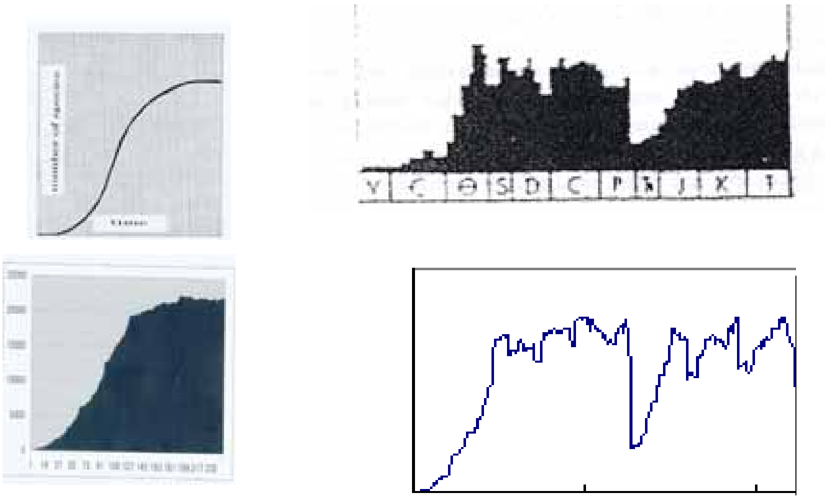
(2) **Crystals of water:** Hexagonal neigh. with a source can realize crystals of waters quite well. This may expect us to make a theory of the crystallization by the iteration dynamical system..

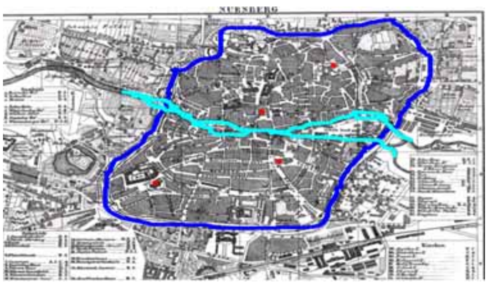


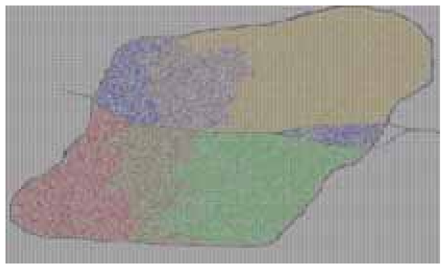
(3) **Evolutions of extinct animals:** The Cambrian explosion of species (The right side: The upper is the Sigmund curve of the evolution and the lower is the simulation)and the Permian mass extinction can be realized by our dynamical systems quite well(The left side: the number of the family of echinoderms, the upper is the data of the time change

and the lower is the simulation([3])



(4) **Growth of cities:** We can try to make simulations of the growth of cites and ecological systems. Here we give an example of the growth of city Nurnberg(the right side is the city in 1882 and the left side is the computer simulation)





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