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MODEL OF TRAM LINE OPERATION

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Abstract. *From passenger's perspective punctuality is one of the most important features of trams operations. Unfortunately in most cases this feature is only insufficiently fulfilled. In this paper we present a simulation model for trams operation with special focus on punctuality. The aim is to get a helpful tool for designing time-tables and for analyzing the effects by changing priorities for trams in traffic lights respectively the kind of track separation.*

A realization of trams operations is assumed to be a sequence of running times between successive stops and times spent by tram at the stops.

In this paper the running time is modeled by the sum of its mean value and a zero-mean random variable. With the help of multiple regression we find out that the average running time is a function depending on the length of the sections and the number of intersections. The random component is modeled by a sum of two independent zero-mean random variables. One of these variables describes the disturbance caused by the process of waiting at an intersection and the other the disturbance caused by the process of driving.

The time spent at a stop is assumed to be a random variable, too. Its distribution is estimated from given measurements of these stop times for different tram lines in Kraków.

Finally a special case of the introduced model is considered and numerical results are presented.

1 INTRODUCTION

One of the most important features of trams operations – in passenger‘s opinion – is punctuality, which provides the basis for detailed planning of trips (especially in cases of low frequency of public transport operation). For passengers lack of punctuality is a deceptiveness of service. Punctuality is defined as the fact, that a definite vehicle achieves, leaves or passes a point on line in a specific moment of time, with assumed tolerance. Often this term also means the accordance of the real departure time from stop with time-tables departure moment. So punctuality is essential for passenger and public transport operators. For the operator it is a measure for the level of service, specified in contract with ordering entity (for example: commune).

The most common measure of punctuality is the deviation from time-table, which is described as the difference between time of departure given by timetable and the real time of departures. Positive values of deviations refer to earliness of departures, negative value to lateness of departure. In Kraków, punctually departure is defined to be in the interval from maximum 1-minute early and maximum 3-minutes late.

In this paper we develop a simulation model for tram line operations with special focus to punctuality. The realization of tram line operation consists of running times of successive sections and times spent by tram on following stops. Based on measurements for various tram lines in Kraków the factors which have the widest influence on the running time were selected and the stop time was analyzed.

The results were helpful for designing time-tables and allow forecasts for the effects by changing priorities for trams in traffic lights respectively the kind of track separation.

2 IMPORTANT FACTORS ON TRAM LINE OPERATION

Tram operation in urban traffic conditions causes difficulties with punctuality assurance. There exist many factors that influence the movement of public transport trams. The most important factors are directly connected with basic functions of public transport – the carriage of passengers between following stops and the passenger’s exchange on stops. Other influences are connected with street conditions, human factors, public transport organization, environmental and local conditions. Examples for such factors are

- Non-realistic time-tables,
- Variability of tram's and other vehicle's traffic volume,
- Lack of priorities for trams (separated tram tracks and separated tram or common tram-bus lanes, priorities in traffic lights),
- Insufficient capacity of sections and stops,
- Kind of day, time of day (workday, holiday, rush-hour),
- Accidents and breakdowns,
- Occupancy of vehicles,
- Delay or speeding-up of vehicle, in relation to time-tables at previous stop,
- Types of vehicles (number of doors, capacity, kind of floor),

- Types of stops (single, double, only for trams, for trams and buses),
- Location of tram stop in relation to the nearest intersection,
- Location of tram stop in relation to the zone of the city,
- Number of leaving and boarding passengers,
- Exact information for passengers,
- Dispatching control,
- Transportation policy,
- Seasons, weather, atmospheric conditions,
- Disposition, psychophysical features of drivers and passengers discipline.

Some of the presented factors are immeasurable others are difficult to estimate. So we consider only two kinds of influences. The first kind is defined as disturbances along the line, achieved by tram stop by stop and section by section, and the second one as disturbances on the whole line, because of different running times of following trams on line. Exampled disturbances of tram line operation are presented at the figure 1 (line nr 24 in Kraków).

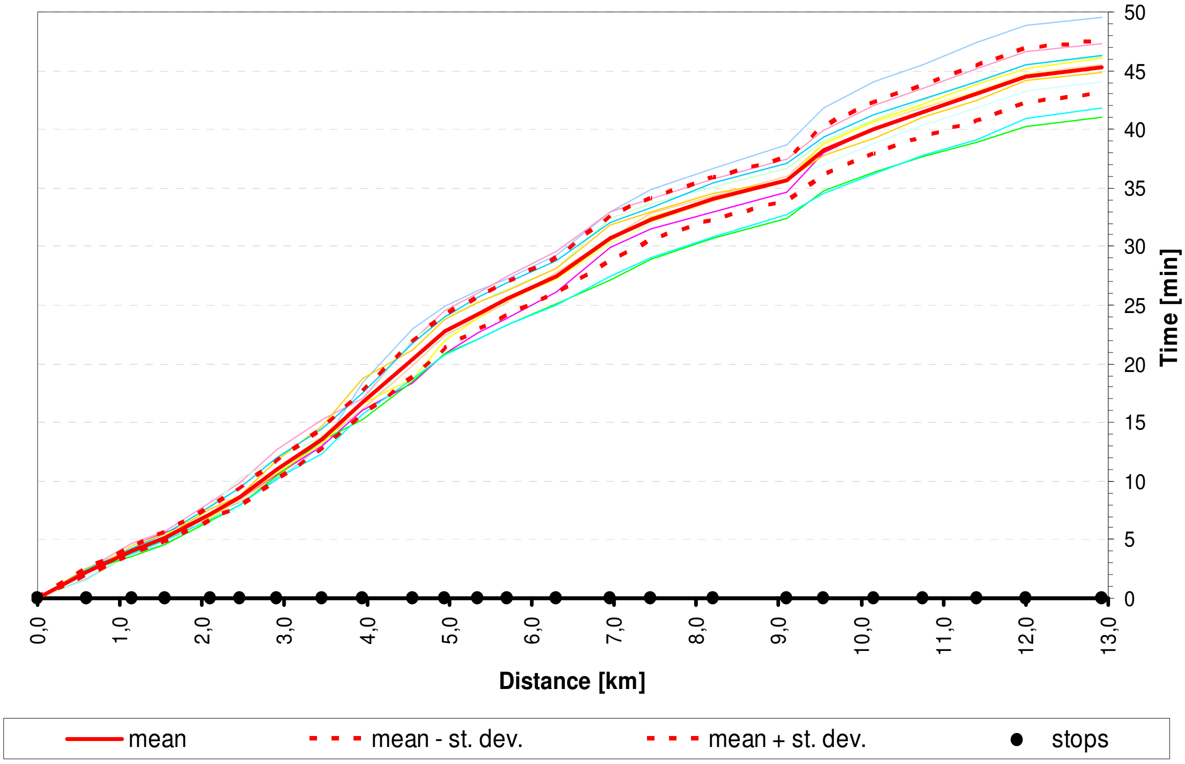

Figure 1: Example of tram line operation's disturbances (line number 24 in Kraków).

One possibility to reduce the disturbances is the privilege of trams, that means especially to separate the track. With the proposed model the effectiveness of privileges for trams can be tested before the real implementing.

3 GENERAL ASSUMPTIONS OF THE MODEL

Tram lines – in mathematical description – have a simple digraph structure, which is presented at the figure 2. There is a adequate way to describe lines as a group of module sequences: section between following stops – stop at the end of the section.

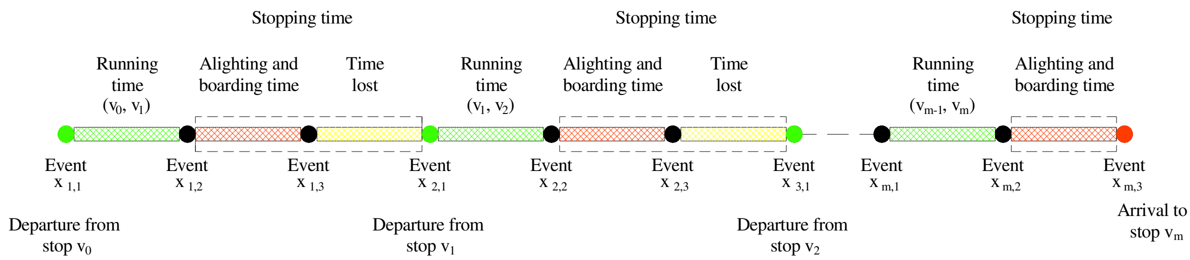

Figure 2: Scheme of tram line model.

In our model the following variables are used. The running time is the time between the departure from first stop on the section and the arrival on the second stop. The stopping time, it defined as the time from the moment of vehicle stopping on stop to the moment of the start of vehicle's movement. According to figure 2 the stopping time consists of alighting and boarding time and time lost on stop before departure, because of impossibility of start its movement. The alighting and boarding time is the time from the moment of starting opening doors to the moment of starting closing the last opened doors. Therefore time lost is the time from the moment of starting closing last opened doors to the moment of beginning running from stop.

A mathematical description of tram line operation is given as a sequence of two modules (section between following stops – stop at the end of section) by the following system of equations

$$\begin{aligned} x_{k+1} &= A_k x_k + B_k u_k + w_k \\ y_k &= C_k x_k + D_k u_k \end{aligned}.$$

The first equation models the state, the second one the output and the index k represents the step. Even value of k means the end of a section and odd value the end of a stop. The state vector x_k contains the important variables for the dynamic of the system like number of passengers or the elapsed time till step k. In contrast the output vector y_k gives the interesting values, for instance the deviation from time table. The matrices A_k and B_k are obtained by statistical analysis as explained below. The vector u_k describes important deterministic influences on the system, which are also determined by statistical analysis. Examples are the number of intersections or the length of a section. The disturbance vector w_k models the random influences that are generated by suitable simulation methods.

4 DESCRIPTION OF SELECTED VARIABLES

Running time and stopping time are assumed to be random variables. For the estimation of their stochastic characteristics we take three tram lines crossing the city centre into consideration. Measurements were done during a typical workday, in afternoon rush hours. They were carried out with radio-controlled clocks. During measurements there were registered moments of opening vehicle's door on stops, moments of finishing alighting and boarding passengers, and moments of starting vehicle's movement.

4.1 Running Time

The running time of the section between two following stops can be estimated by a sum of the mean value and a zero-mean random variable

$$t_r = \bar{t}_r + \hat{t}_r \,.$$

The average running time depends on the section type in relation to the kind of track separating. In the presented model, we take three kinds of tram track separation $p \in \{1,2,3\}$ into consideration:

- $p = 1$, if the tram track is separated by green and the priorities in traffic lights for the trams are assured,
- $p = 2$, if the tram track or the exclusive lane for trams is separated by green or kerb,
- $p = 3$, if the tram track is located in the middle part of the street or the tram lane is separated by painted line.

For all three cases, the average running time is estimated by a function of the length L of the section between two following stops and the number of intersections located on this section s. As a result, there are three multiple regression formulas for the average running time, they are presented in table 1. To determine those formulas, there were taken into consideration section's lengths up to 1,45 km, and total numbers of intersections (only signalized and non-signalized, on which trams have not right of way) – from 0 to 3, per section.

Type of section	Multiple regression formula of running time [min]	Durbin-Watson statistic	Coefficient of Determination [%]
Separated tram track and priorities for trams	$\bar{t}_r$ = 1,16 L + 0,335 s	2,04	91,1
Separated tram track or separated exclusive lane for trams and buses	$\bar{t}_r$ = 2,28 L + 0,158 s	1,77	91,3
Non-separated tram track located in the middle part of the street	$\bar{t}_r$ = 2,75 L + 0,356 s	1,99	80,2

Table 1: Parameters of running time.

The regression formulas of running time are characterized by high values of determination coefficient and values of Durbin-Watson statistic, which provides about good fit of the given models. The smallest influence of section's length onto running time appears for sections with separated track and priorities in traffic lights on intersections, the highest – for sections with non-separated tracks. Analysis of sensitivity of obtained regression formulas is presented in figure 3.

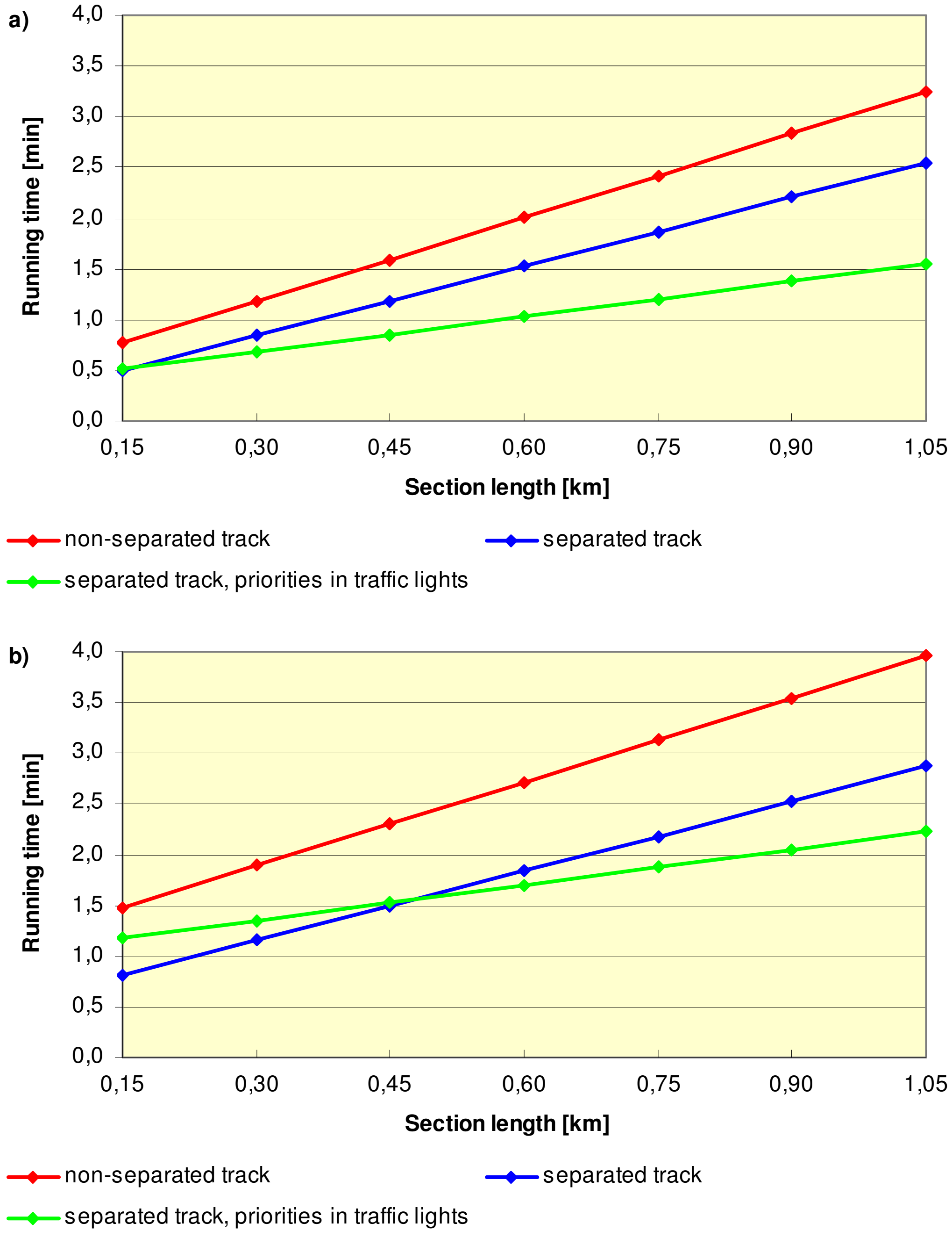

Figure 3: Average running time in dependence on length of section and total number of intersections
a) one intersection b) three intersections on section

Based on the obtained result that the running time depends on the length of the section and the number of intersections, the random variable is also modeled in dependence of these factors. The average running time has the following properties. If the length and the number of intersections are zero then the averaged running time is zero, too. The sum of the averaged running times of following sections equals the averaged running time of the sum of the sections. The random variable $\hat{t}_r$ must fulfill the same properties. For that reason we model the randomness as a sum of two independent zero-mean normal distributed variables for every section. One component describes the disturbance caused by the process of waiting at an intersection and the other the disturbance caused by the process of driving. Because of the assumed independence of the random variables of following sections their variances can be summed up. For a section of length L and a number of intersections s the variance of $\hat{t}_r$ can be calculated by

$$\sigma^2 = \sigma_1^2 L + \sigma_2^2 s,$$

where σ_1^2 is the variance of pure driving time for a section with length one and σ_2^2 the variance of the waiting time at one intersection. Due to the lack of measurements of the waiting time and pure driving time we estimate the variances by the least square approach

$$\hat{t}_r^{\,2} = \sigma_1^2 L + \sigma_2^2 s,$$

given in table 2. The realizations of $\hat{t}_r$ in dependence of the kind of separation are generated by $\hat{t}_r = t_r - \bar{t}_r$ with the regression formulas for $\bar{t}_r$ given in table 1.

Type of section	Least square approach
Separated tram track and priorities for trams in traffic lights on intersections	$\hat{t}_r^{\,2} = 0{,}016L + 0{,}094s$
Separated tram track or separated exclusive lane for trams and buses	$\hat{t}_r^{\,2} = 0{,}54L + 0s$
Non-separated tram track located in the middle part of the street	$\hat{t}_r^{\,2} = 0{,}959L + 0{,}176s$

Table 2: Variance of running time.

4.2 Stopping time

The stopping time consists of alighting and boarding time and time lost on stop before departure. Alighting and boarding time depends mainly on the number of alighting and boarding passengers and the number of passengers inside vehicle, reaching the stop. It depends also on type of vehicle (capacity, number of sits, number of doors), type of stop (number of stopping-spaces, passenger‘s area).

However, the time lost is an effect of certain random events, mainly because of impossibility of departure from stop, caused by lack of priorities of public transport vehicles. A detailed analysis can be found in [1].

In the presented model the stopping time is a random variable with distribution given in figure 4. The characteristics of the distribution are shown in table 3.

Mean value $\bar{t}_s$	Variance	Minimum	Maximum
0,53	0,113	0,07	2,48

Table 3: Characteristics of the distribution of stopping time

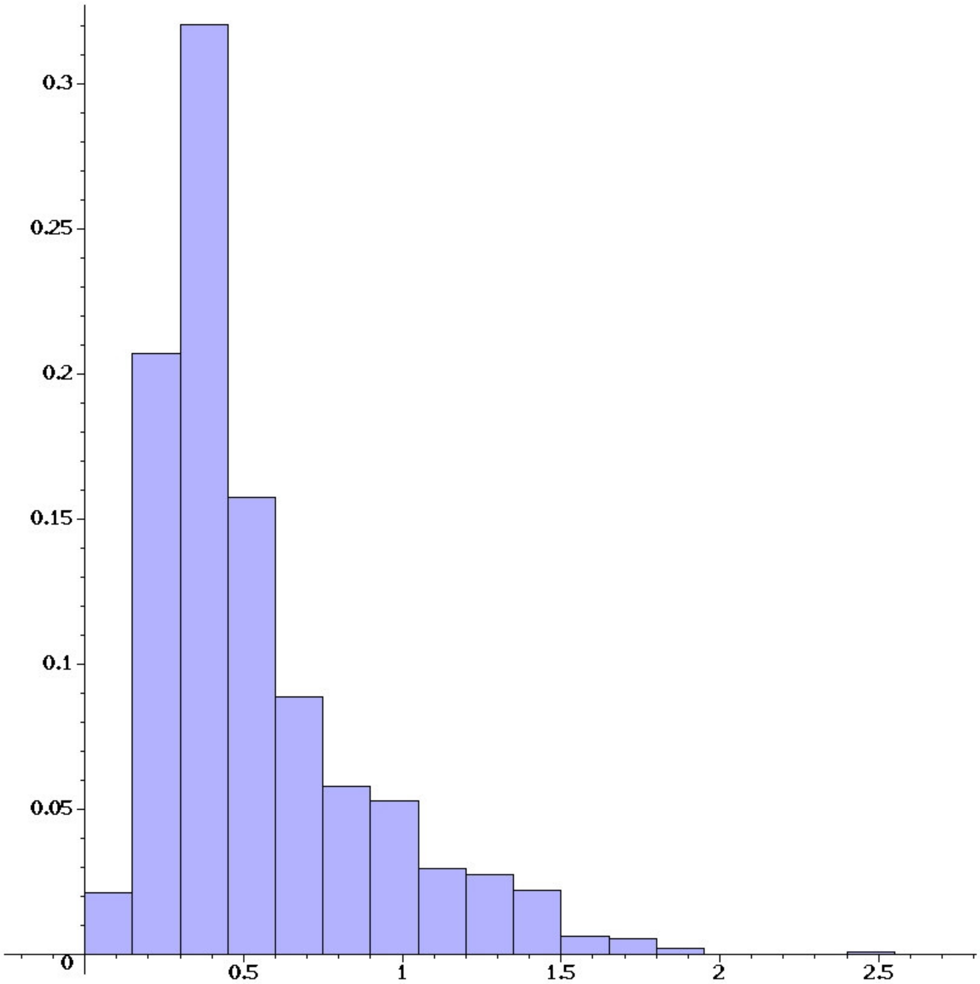

Figure 4: Histogram of stopping time.

5 SIMULATION MODEL

Now we consider a special case of the general model presented in chapter 3. The time t_k from start till the departure from stop k is simulated by

$$t_k = t_{k-1} + \bar{t}_{r,k} + \hat{t}_{r,k} + t_{s,k},$$

where $\bar{t}_{r,k}$ is the average running time of section k, $\hat{t}_{r,k}$ is the random disturbance of running time and $t_{s,k}$ is the stopping time of stop k. The expectation $\bar{t}_k$ of t_k is determined by

$$\bar{t}_k = \sum_{i=1}^{k} \bar{t}_{r,i} + k\bar{t}_s .$$

A measure of punctuality is the deviation from time-table, which is described as the difference between time of departure $t_{t,k}$ from stop k given by time-table and the real time of departure t_k. Using the expectation $\bar{t}_k$ of t_k the mean deviation from time-table shown in figure 5 can be calculated by $t_{t,k} - \bar{t}_k$. Additionally in figure 6 a comparison of seven realizations of t_k with the expectation $\bar{t}_k$ is presented. So we conclude, that $\bar{t}_k$ is a good candidate for an improved time-table.

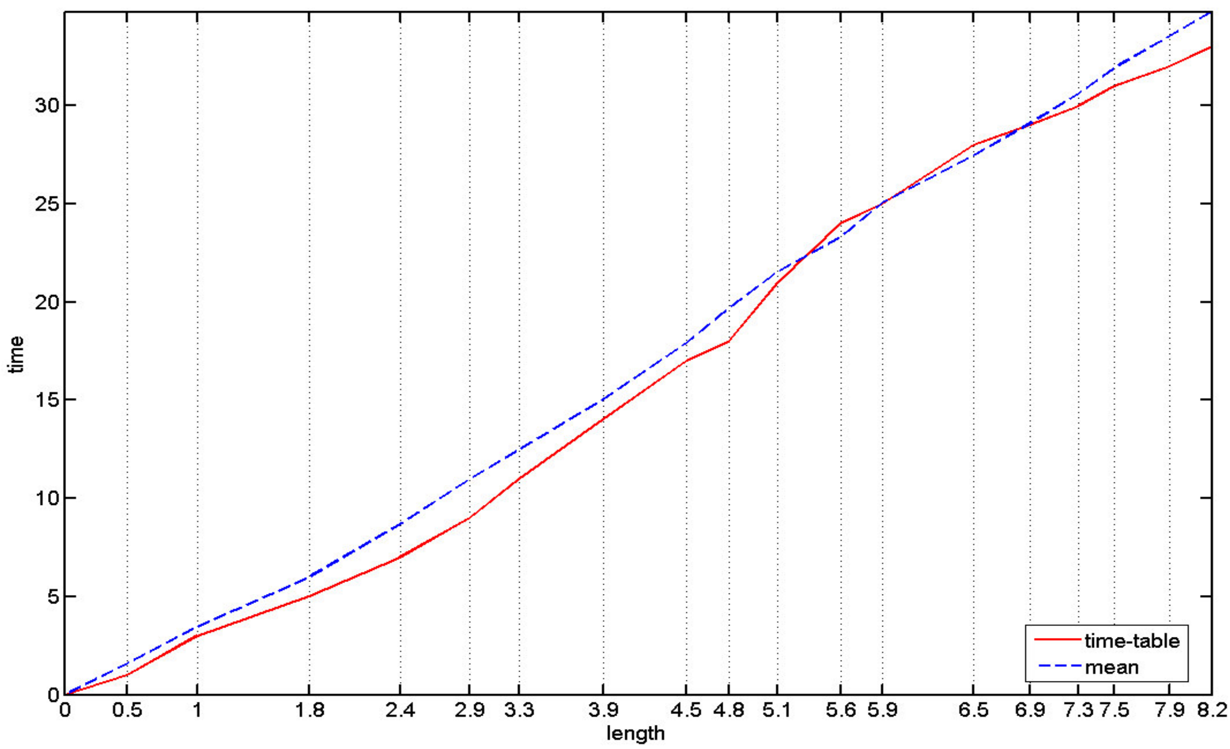

Figure 5: Comparison of mean time and the time-table (lane number 7 in Kraków).

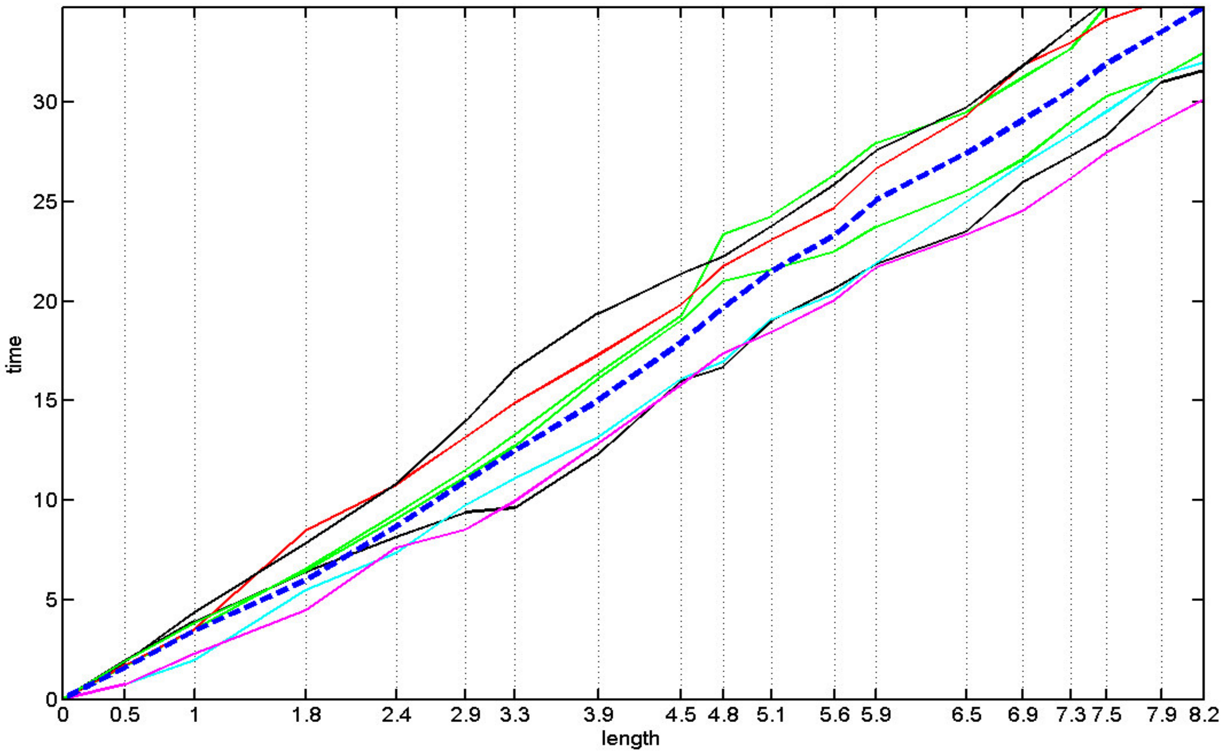

Figure 6: Mean time and seven realizations (lane number 7 in Kraków).

6 MAIN CONCLUSIONS

1. There are many reasons of buses and trams unpunctuality, most of them have stochastic character and it is possible to describe tram line operation as realization of stochastic process.
2. The graph model is the best way to describe the process of tram line operation, as a group two module sequences: section - stop.
3. A suitable Simulation model of tram line operation contains deterministic and probabillistic components, which describe processes appearing in real tram line operation.
4. Average running time could be explained by multiple regression formula includes length of section and total number of intersections on this section with taking into consideration the way of tram track separating.
5. The presented model can be helpful for time-table‘s better designing and for evaluation of effectiveness planned solutions, before their implementation.
6. There is necessity during further investigations for more precisely describtion of the model‘s parameters, especially alighting and boarding time in dependence of numbers of alighting and boarding passengers and number of passengers inside vehicle.

7. It is also needed to examine the influence of period of the day on disturbances on tram line. This leads to taking into consideration the history of process.

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The paper reflects only the author's views and the Community is not liable for any use that may be made of the information contained therein.

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