STATIC ANALYSIS ON MODELS OF CONTINUOUS ORTHOTROPIC THIN-WALLED PRISMATIC SHELL STRUCTURES

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Abstract. *The paper presents a linear static analysis on continuous orthotropic thin-walled shell structures simply supported at the transverse ends with a random deformable contour of the cross section. The examination of the structure has used with two computation models: a prismatic structure consisting of isotropic strips and a equivalent smooth orthotropic plate. The study is made with the force method and with the analytical finite strip method (AFSM). The basic system is obtained by separating the superstructure (orthotropic) from the understructure at the places of intermediate supports. The solution of the superstructure has been made by the AFSM in displacements, of the support part - by the FEM. The basic unknown forces have been determined from system of canonic equations. The final displacements and forces of a continuous superstructure have been determined. A numerical example is given.*

1. STATE OF THE PROBLEM AND PROBLEM SETTING

Due to their high bearing ability, technological features and economical nature the thin-walled prismatic structures have been applied more and more widely in building industry. This class of structures involves many bridge upper structures, scaffold bridges, channels, some floor and roof structures etc. Their static analysis as thin-walled spatial systems is most often done automatically with appropriate software based on the contemporary discrete computation methods.

The aim of the paper is to do spatial static analysis on a multi-span thin-walled prismatic structure.

2. NATURE OF THE STUDY

Theoretical model of structure

A computation model corresponding to a straight thin-walled prismatic structure of a random length, random deformable contour of the cross-section and a constant longitudinal

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stiffness has been assumed. The structure is simply supported in their transverse ends by diaphragms that are assumed to be endlessly stiff in their own plane and ideally flexible out of the plane. The support along its longitudinal ends is random. The external loads and influences can be various. The study has been carried out in a linear setting.

Method of solving the structure

The study of the structure has been done using a force method in combination with the analytical finite strips method (AFSM) in displacements [1], [2], [4].

Basic system and basic unknowns

The basic system is obtained separating the superstructure of the lower structure in the places of intermediate supports and consists of two parts. The first part presents a one-opening thin-walled prismatic structure; the second one presents intermediate supports (columns, bending supports, spatial frames). The connection between the superstructure and the intermediate supports is accomplished under random conditions of support.

Forces at the points of support in the direction of connections removed $\{X_i\} = \{X_i^1\ X_i^2\ X_i^3\ X_i^4\ X_i^5\ X_i^6\}^T$ $(i = 1,...,nb)$ (Fig. 1) are assumed to be the basic unknowns. The number of the unknown components $\{X_i^p\}$ for each point of support can be distinguished and depends on the connection between the superstructure and supports.

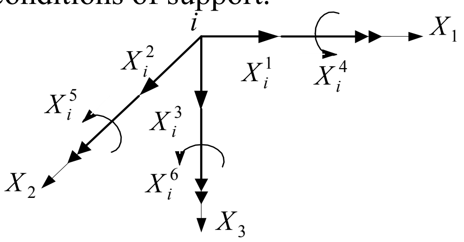


Fig. 1

A system of canonic equations

To obtained the basic system, the conditions of the continuity of deformations are written in the places of removed support connections

(1) $$[D]\{X\} + \{D_f\} = 0\,,$$ where

$[D]$ - matrix of the structure pliability;

$\{X\}$ - vector of the unknown forces in all points of support of the structure;

$\{D_f\}$ - vector of the external load on the structure.

The matrix of pliability $[D]$ presents the sum of two matrixes

(2) $$[D] = [D^I] + [D^{II}],$$ where

$[D^I]$ is the matrix of the superstructure;

$[D^{II}]$ is the matrix of the supports.

The vector of the external loads and actions is analogously presented $\{D_f\}$

(3) $$\{D_f\} = \{D_f^I\} + \{D_f^{II}\}.$$

Single States. Obtaining the matrix of pliability and the vector of the external load on the structure

The two parts of the basic system are solved in sequence from the action of single values of each basic unknown and by the external load acting on the structure.

The solution of the superstructure is done by the Analytical Finite Strip Method (AFSM) in displacements [4]. The structure is dissembled in only one (transverse) direction into a finite number of plane strips connected among themselves in longitudinal linear nodes. The three displacements of the points of the nodal lines and the rotation around these lines are assumed to be the basic unknown quantities in each node. The boundary conditions of each strip of the basic system correspond to the symply support along the transverse ends and the fixed support along the longitudinal ends. The separate strip of the basic system is solved by the method of the single trigonometric lines. The method is reduced to a solution a discrete structure in displacements and restoration of its continuity at the places of the sections made in respect to both the displacements and forces.

The external loads, the basic unknowns summarized nodal forces develop in a trigonometric order (cosinus order of loads acting in the longitudinal direction of the structure and sinus order for all the rest) with Fourier's coefficients:

(4) $$X_i^1(x) = X_{i,0}^1 + \sum_{n=1}^{\infty} X_{i,n}^1 \cos\alpha_n x, \qquad X_i^{2,3,4,5,6}(x) = \sum_{n=1}^{\infty} X_{i,n}^{2,3,4,5,6} \sin\alpha_n x, \qquad (i = 1,\ldots,nb).$$

To obtain the matrix of pliability $\left[D^I\right]$, the solution of one-opening structures is done by AFSM by action of each basic unknown $\left\{X_i^P\right\} = 1\ ((p = 1,2,3,4,5,6),(i = 1,\ldots,nb))$. To obtain the vector of load members $\left\{D_f^I\right\}$ the solution of one-opening structures is done by AFSM by external actions.

The solution of the support part is done by ready-made software of analysis of structures according to the finite element method. Matrix $\left[D^{II}\right]$ and vector $\left\{D_f^{II}\right\}$ are obtained.

Solution of the set of linear algebraic equations

The basic unknown equations are determined from their solution.

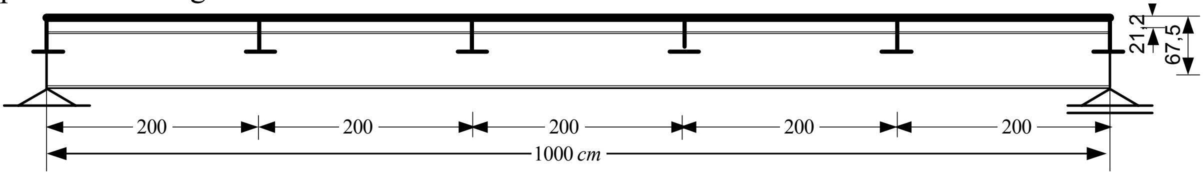
Forces and displacements in the continuous structure

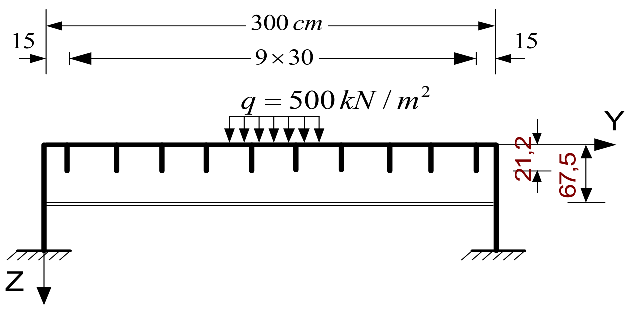
The final displacements and forces at a random point of the multi-opening superstructure are determined using the principle of superposition $S = S_F^0 + S_F^I + S_X^I$ where S_F^0 are the displacements (forces) in an one-opening system caused by loads acting in the field of strips; S_F^I are the displacements (forces) in an one-opening system caused by external nodal loads; S_X^I are the displacements (forces) in an one-opening system caused by the basic unknown taken in their real dimensions and directions (nodal loads).

The computations are done by software developed in Visual Fortran v. 5.0 for a PC [4].

3. NUMERICAL IMPLEMENTATION. ANALYSIS AND COMPARISON OF RESULTS

A steel continuous bridge five-opening structure has dimensions, support and load presented in Fig. 2.





The physical features of the structure material are:

module of the linear elastic deformation:

$E = 2{,}1.10^8\,kN/m^2$;

Poisson's coefficient :

$\nu = 0{,}3$.

Fig. 2

The longitudinal ribs and crossbeams, which are supported on the main longitudinal beams, are with an open profile of the cross-section. It is assumed that the plate is included in the open section of the longitudinal ribs and the crossbeams with an aiding width equal to the half distance between the longitudinal ribs and between the crossbeams respectively. The sections of the longitudinal ribs and crossbeams together with the plate width aiding to them are presented in Fig. 3 and Fig. 4 respectively.

Longitudinal ribs

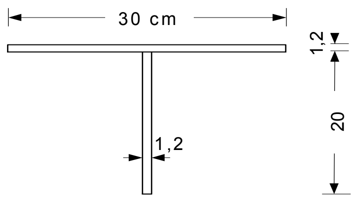


Fig. 3

The inertia moment of the longitudinal rib section in respect to its horizontal main central inertia axis with bucking is $I = 2422{,}304.10^{-8}\, m^4$.

Crossbeams

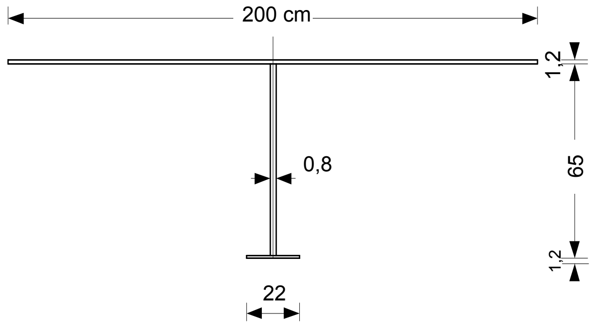


Fig. 4

The inertia moment of the crossbeam section in respect to its main horizontal central inertia axis with bending is $I = 0{,}00153216\, m^4$.

Due to the big bucking stiffness and the small twisting stiffness of the main beams, it is accepted that the crossbeam is simply supported on them.

The basic system is obtained by removal of the connections between the longitudinal ribs and crossbeams (intermediate supports). The unknown forces X_i acting in the direction of the removed connections are introduced: in this case there are two straight opposite vertical and two straight opposite horizontal forces.

To solve the two parts of the basic system for each basic unknown $X_i = 1$ together, the matrix of pliability of each crossbeam has been obtained. The matrix of pliability is $\lfloor D \rfloor = \lfloor d_{ij} \rfloor$ where $\lfloor d_{ij} \rfloor$ is the displacement of section i of the crossbeam caused by force $F = 1$ ($X_i = 1$) applied at point j. To determine $\lfloor d_{ij} \rfloor$, vertical force $F = 1$ has been successively put on the crossbeam in the place of their connections at each longitudinal rib (Fig. 5). The vertical displacements at a random point of the crossbeam are determined by expressions

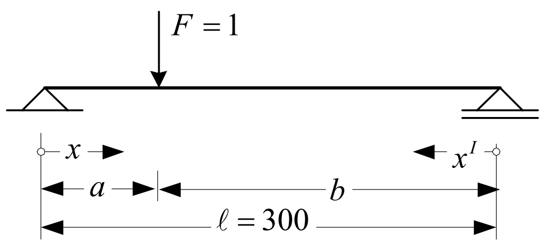


Fig. 5

$$w(x) = \frac{F\ell^2 b}{6EI}\xi\left(1 - \beta^2 - \xi^2\right) \text{ за } x \le a,$$

$$w(x) = \frac{F\ell^2 a}{6EI}\xi^I\left(1 - \alpha^2 - \xi^{I2}\right) \text{ за } x^I \le b,$$

$$\text{където } \xi = \frac{x}{\ell},\ \alpha = \frac{a}{\ell},\ \beta = \frac{b}{\ell}.$$

For the position of force $F = 1$, the displacements of all points of the connection of the longitudinal rib with the crossbeam have been determined (Table 1).

Table 1. Numerical kind of the matrix of crossbeam pliability

6,311E-08	1,635E-07	2,281E-07	2,614E-07	2,673E-07	2,502E-07	2,142E-07	1,635E-07	1,023E-07	3,479E-08
	4,547E-07	6,530E-07	7,568E-07	7,788E-07	7,316E-07	6,278E-07	4,799E-07	3,005E-07	1,023E-07
		9,834E-07	1,170E-06	1,221E-06	1,156E-06	9,974E-07	7,649E-07	4,799E-07	1,635E-07
			1,448E-06	1,548E-06	1,487E-06	1,294E-06	9,974E-07	6,278E-07	2,142E-07
				1,713E-06	1,685E-06	1,487E-06	1,156E-06	7,316E-07	2,502E-07
	symmetrical				1,713E-06	1,548E-06	1,221E-06	7,788E-07	2,673E-07
						1,448E-06	1,170E-06	7,568E-07	2,614E-07
							9,834E-07	6,530E-07	2,281E-07
								4,547E-07	1,635E-07
									6,311E-08

The solution of the structure has been done with the following computation models.

Solution of the structure as an isotropy one according to AFSM

The computation model of the structure – descretisized scheme with the numbers of nodes and strips, the common coordinate system, the structure dimensions and loading are shown in Fig. 6.

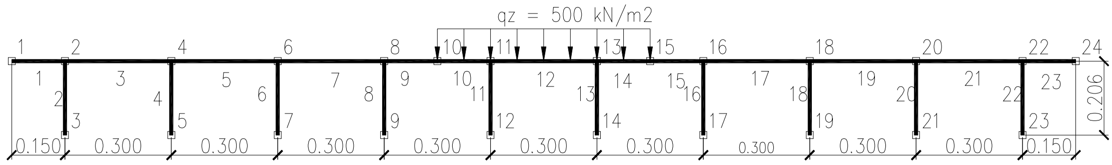


Fig. 6. The discretisized system, loading and dimensions

Twenty-three finite strips have been used. The number of the members retained in Fourier's series are 49. The network of output results for displacements and forces is of a step of $\frac{1}{20}\ell$ in longitudinal direction and $\frac{1}{4}$ of the width of each strip in crosswise direction of the structure. The displacements and forces at all structure nodes and the points marked between them have been determined (Fig. 7, Table 2).

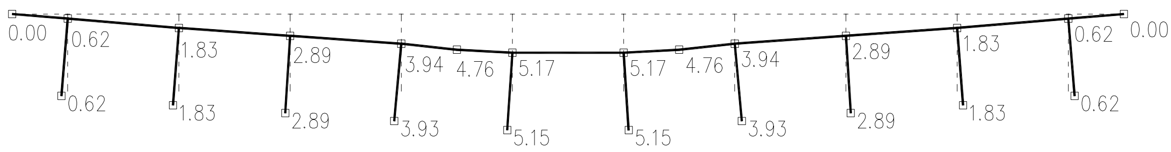


Fig. 7. Deformed diagram in section $(X = \ell/2)$ with marked displacements $w\left(\frac{\ell}{2}, Y\right).10^{-3}\ [m]$

Table 2. Forces in some characteristic sections of the structure

Strip No	$N_x\left(\frac{\ell}{2}, y\right)$ $[kN/m]$	$N_y\left(\frac{\ell}{2}, y\right)$ $[kN/m]$	$M_x\left(\frac{\ell}{2}, y\right)$ $[kNm/m]$	$M_y\left(\frac{\ell}{2}, y\right)$ $[kNm/m]$	$Q_x(0, y)$ $[kN/m]$	$Q_y(0, y)$ $[kN/m]$
1	-0,142E+02	-0,472E+02	0,292E-15	0,997E-15	-0,115E-15	0,000E+00
	-0,152E+02	-0,469E+02	0,490E-02	0,107E-01	-0,105E-01	0,000E+00
	-0,171E+02	-0,464E+02	0,987E-02	0,215E-01	-0,175E-01	0,000E+00
2	-0,323E+01	-0,363E-01	0,484E-02	0,155E-01	-0,346E-01	0,000E+00
	0,258E+02	0,378E-01	0,460E-02	0,957E-02	0,126E+00	0,000E+00
	0,551E+02	0,149E-01	0,339E-02	-0,110E-13	0,540E+00	0,000E+00

3	-0,172E+02	-0,465E+02	0,524E-02	0,609E-02	0,175E-01	0,000E+00
	-0,230E+02	-0,457E+02	0,167E-01	0,332E-01	0,325E-03	0,000E+00
	-0,323E+02	-0,450E+02	0,282E-01	0,589E-01	-0,161E-01	0,000E+00
4	-0,188E+02	-0,115E+00	0,589E-02	0,171E-01	-0,316E-01	0,000E+00
	0,715E+02	0,593E-01	0,592E-02	0,992E-02	0,114E+00	0,000E+00
	0,162E+03	0,445E-01	0,505E-02	-0,112E-13	0,492E+00	0,000E+00
5	-0,323E+02	-0,450E+02	0,231E-01	0,418E-01	0,159E-01	0,000E+00
	-0,444E+02	-0,447E+02	0,406E-01	0,820E-01	-0,165E-02	0,000E+00
	-0,620E+02	-0,440E+02	0,564E-01	0,134E+00	-0,176E-01	0,000E+00
6	0,267E-03	-0,403E-02	0,267E-03	-0,403E-02	-0,254E-01	0,000E+00
	-0,104E-02	0,211E-02	-0,104E-02	0,211E-02	0,930E-01	0,000E+00
	-0,291E-02	-0,736E-14	-0,291E-02	-0,736E-14	0,399E+00	0,000E+00
7	-0,620E+02	-0,440E+02	0,576E-01	0,138E+00	0,816E-02	0,000E+00
	-0,945E+02	-0,389E+02	-0,506E-01	-0,178E+00	-0,463E-02	0,000E+00
	-0,145E+03	-0,279E+02	-0,142E+00	-0,622E+00	-0,147E-01	0,000E+00
8	-0,142E+03	-0,178E+02	0,678E-01	0,219E+00	-0,167E-01	0,000E+00
	0,211E+03	-0,664E+01	0,841E-01	0,791E-01	0,637E-01	0,000E+00
	0,571E+03	-0,917E-02	0,989E-01	-0,691E-14	0,271E+00	0,000E+00
9	-0,145E+03	-0,285E+02	-0,208E+00	-0,841E+00	0,230E-02	0,000E+00
	-0,168E+03	-0,248E+02	0,184E+00	0,121E+00	0,232E-02	0,000E+00
	-0,201E+03	-0,163E+02	0,588E+00	0,113E+01	0,216E-02	0,000E+00
10	-0,201E+03	-0,163E+02	0,588E+00	0,113E+01	0,227E-02	0,000E+00
	-0,250E+03	-0,934E+00	0,462E+00	0,804E+00	0,431E-02	0,000E+00
	-0,323E+03	0,262E+02	-0,660E+00	-0,269E+01	0,980E-02	0,000E+00
11	-0,378E+03	-0,156E+03	0,232E+00	0,352E-17	-0,594E-02	0,000E+00
	0,635E+03	-0,566E+02	0,822E-01	-0,469E-16	0,239E-01	0,000E+00
	0,171E+04	-0,678E+00	-0,322E-14	-0,150E-15	0,101E+00	0,000E+00
12	-0,323E+03	0,255E+02	-0,730E+00	-0,292E+01	0,166E-01	0,000E+00
	-0,271E+03	0,815E+00	0,133E+01	0,273E+01	0,942E-02	0,000E+00
	-0,323E+03	0,255E+02	-0,730E+00	-0,292E+01	0,166E-01	0,000E+00

Solution of the structure as orthotropic plate according AFSM

The structure consisting of a plate and longitudinal ribs have been reduced to a smooth plate, equal to it.

Determining the features of the equivalent plate [3].

The stiffness of bending of the plate in longitudinal direction per a unit of the plate is determined by the expression

$$D_x = \frac{EI_x}{b_1}, \qquad D_x = \frac{2{,}1.10^8.2422{,}304.10^{-8}}{0{,}3} = 16956{,}128\ kN\,m^2/m \qquad \text{where}$$

E is the module of the steel elasticity;

I_x is the inertia moment of the structure longitudinal rib with respect to its zero line;

b_1 is the axial distance between the longitudinal ribs;

The stiffness of bending of an equivalent orthotropic plate per a unit of length of the plate is determined by the expression:

$$D_y = \frac{E.t^3}{12}, \qquad D_y = \frac{2{,}1.10^8.0{,}012^3}{12} = 30{,}24\,kN.m^2/m\,.$$

The stiffness of torsion of an equivalent orthotropic plate is a sum of the stiffness of the plate torsion and the stiffness of the rib torsion of the longitudinal beam in the structure.

The stiffness of the plate torsion на плочата in the structure is determined by the expression:

$$D_{k,pl} = \frac{Gt^3}{12}, \qquad D_{k,pl} = \frac{0,8077.10^8.0,012^3}{12} = 11,631\, kN\, m^2/m,$$ where

G is the shear module of deformations;

t is the plate thickness in the structure.

From the condition for equality of the potential energies of deformation with pure torsion of the rib and an orthotropic plate equivalent to it for the equivalent stiffness of the longitudinal beam rib torsion it is obtained that $D_{k,r} = \dfrac{GI_t}{4b_1}$. Here I_t is the inertia moment of the torsion of longitudinal rib $D_{k,r} = \dfrac{0,8077.10^8.0,302.0,012^3.0,2}{4.0,3} = 7,025\, kN\, m^2/m$.

Finally, for the stiffness of the torsion of the equivalent orthotropic plate it is obtained that $D_k = D_{k,pl} + D_{k,r} = 18,656\, kN\, m^2/m$.

Thus, the obtained stiffness of bucking and bending D_x, D_y and the stiffness of torsion D_k of the equivalent orthotropic plate refer to the case of constructional orthotropy as the structure being examined is.

Determination of the constants of the equivalent materially smooth orhotropic plate - E_x, E_y, ν_x *and* ν_y. The inertia moments of bucking and bending of per a unit of length of the plate orthotropic in construction are $I_x = 8074,347.10^{-8}\, m^4/m$, $I_y = 1,44.10^{-7}\, m^4/m$ respectively. Their ratio is $\dfrac{I_x}{I_y} = 560,718$. It is assumed that materially equivalent orhotropic plate is of width of $t = 0,012\, m$. To keep stiffness $E_x I_x$, same $E_x I_x = (560,718 E_x)\left(\dfrac{I_x}{560,718}\right)$. Hence $E_x = 560,718.2,1.10^8 = 1,1775.10^{11}\, kN/m^2$, $E_y = 2,1.10^8\, kN/m^2$. It is assumed that $\nu_x = 0,3$, from condition $\nu_x.E_y = \nu_y.E_x$ it is obtained that $\nu_y = 0,000535$.

The final features of the materially equivalent orhotropic plate are

$$E_x = 1,1775.10^{11}\, kN/m^2,\ E_y = 2,1.10^8\, kN/m^2,\ \nu_x = 0,3 \text{ и } \nu_y = 0,000535.$$

The computation model and the results obtained from the solution of the structure are shown in Fig. 8, Fig. 9, Tabl. 3.

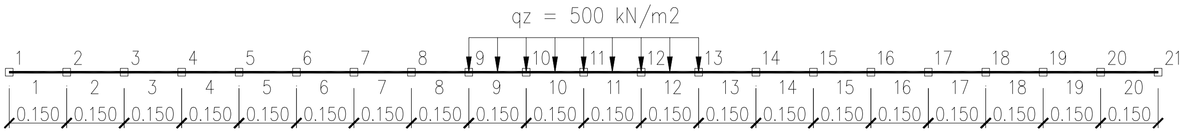


Fig. 8. Discretisized system, load and dimensions of equivalent orthotropy plate

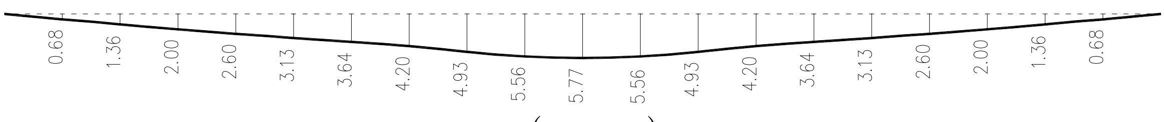


Fig. 9. Deformed scheme in section $(X = \ell/2)$ of the equivalent orthotropy plate

Table 3. Forces in some characteristic sections of the structure

Точка №	$M_x\left(\frac{\ell}{2}, y\right)$ $[kNm/m]$	$M_y\left(\frac{\ell}{2}, y\right)$ $[kNm/m]$	$Q_x^*(0, y)$ $[kN/m]$	$Q_y^*(0, y)$ $[kN/m]$
1	0,469E-15	0,109E-14	0,383E-15	0,000E+00
	0,111E+01	0,916E-02	-0,423E+00	0,000E+00
2	0,231E+01	0,174E-01	-0,666E+00	0,000E+00
	0,356E+01	0,274E-01	-0,123E+01	0,000E+00
3	0,471E+01	0,396E-01	-0,165E+01	0,000E+00
	0,570E+01	0,501E-01	-0,213E+01	0,000E+00
4	0,672E+01	0,622E-01	-0,203E+01	0,000E+00
	0,775E+01	0,799E-01	-0,297E+01	0,000E+00
5	0,849E+01	0,959E-01	-0,333E+01	0,000E+00
	0,879E+01	0,901E-01	-0,375E+01	0,000E+00
6	0,935E+01	0,589E-01	-0,345E+01	0,000E+00
	0,107E+02	0,369E-02	-0,428E+01	0,000E+00
7	0,128E+02	-0,763E-01	-0,434E+01	0,000E+00
	0,157E+02	-0,165E+00	-0,440E+01	0,000E+00
8	0,199E+02	-0,233E+00	-0,264E+01	0,000E+00
	0,281E+02	-0,229E+00	-0,416E+01	0,000E+00
9	0,450E+02	0,141E+00	-0,391E+01	0,000E+00
	0,636E+02	0,566E+00	-0,388E+01	0,000E+00
10	0,729E+02	0,643E+00	0,285E+01	0,000E+00
	0,750E+02	0,609E+00	-0,352E+01	0,000E+00
11	0,743E+02	0,566E+00	-0,330E+01	0,000E+00

5. DEDUCTIONS AND CONCLUSION

The comparison of the results of the displacements and forces obtained by the solution of continuous structure such as isotropic and orthotropic plates has shown the following: the normal displacements $w(0{,}5\ell;Y)$ approximately coincide. The bending moments and the shear forces in the crossbeam in the both solutions are very close. The comparability of the integrated forces in the longitudinal ribs and the plate in the solution of the structure as an isotropic one and the forces in the orthotropic plate is good.

In the general analysis of the structure it can be solves as an equivalent smooth orthtropic plate. In the detailed analysis the structure is solved as an isotropic one. Each of the components (the plate between closed profiles, walls, bottom, the plate of ribs on closed profiles) is examined and dimensioned by the action of the reactive forces and displacements of the general solution and by the local action of external loads.

REFERENCES

[1]. Dragolov, A., Analytical Finite Strip Method for Examination on Prismatic Shells, Sofia, Construction No 1-2 and No 3-4, 1996 (in Bulgarian).

[2]. Christov Ch. T., L. B. Petrova, Computer-Aided Static Analysis of Complex Prismatic Orthotropic Shell Structures by the Analytical Finite Strip Method, Proceedings of the IKM, Weimar, 1997.

[3]. Cheung M., W., Li, S. Chidiac, Finite Strip Analysis of Bridges, London, 1996.

[4]. Petrova, L. B., Static Analysis on Thin-walled Prismatic Orthotropic Structures by the Analytical Finite Strip Method, PhD paper, Sofia, 2004 (in Bulgarian).