Doubly-Selective Channel Estimation for Continuous Phase Modulation

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Abstract—In this paper, we present two Data-Aided channel estimators for Continuous Phase Modulation (*CPM*) in the case of transmissions over doubly-selective channels. They both capitalize on the Basis Expansion Model (*BEM*), widely used for OFDM systems and for Single Carrier transmission with linear modulation. However, in the case of CPM signals, we need to work on a over-sampled received signal (fractionally-spaced representation) as the equalization techniques are also working on the over-sampled received signal. The first one is a classical Least Squares (*LS*) estimation of the BEM parameters whereas the second channel estimator introduces first a parametric dependence on the paths delays. Indeed, in the case where those delays are known (by estimation or by geometrical consideration as for the aeronautical channel by satellite), the second LS estimation on the BEM parameters is improved and less computationally demanding. Simulations results are provided and show good performance of our parametric LS estimation.

I. INTRODUCTION

CPM signals are commonly known for their good spectral properties and their constant envelop, which make them robust to the non-linearities such as the ones introduced by embedded amplifiers. They are actually considered for a lot of application such as military communications, 60Ghz communications, Internet of Things and also aeronautical communications.

To our knowledge, only a few papers deal with CPM transmissions over time-varying (*TV*) channels. Indeed, the optimal approach for detection would consist of a Maximum A Posteriori (*MAP*) detection taking into account both channel and CPM memory. However, this is computationally prohibitive as the associated time-varying trellis grows exponentially with the delay spread of the channel and with the CPM memory.

In the case of time-invariant (*TIV*) channel, a viable strategy is to perform separately channel equalization and CPM detection. This approach has been considered in several papers with a Minimum Mean Square Error (*MMSE*) criterion and a Frequency-Domain (*FD*) Equalizer [1]–[4]. Indeed, in the case of a frequency-selective channel, the frequency channel matrix is diagonal which can be used to design low-complexity FD equalizers as for linear modulations. Most of those works have been done under the hypothesis of perfect synchronization and perfect channel knowledge. To our knowledge, only a few papers deal with TIV channel estimation for CPM. [2] presents simulation results with channel estimation errors but no channel estimator. [5] presents a Least Squares (*LS*) channel estimation in the time-domain, based on the polyphase representation of the received signal. It also exploits the *a priori* positioning in order to develop a parametric model on the delays of the paths and so to enhance the performance of the channel estimation. In [4] and [6], the authors perform a FD channel estimation with interpolation (using B-spline functions). [7] performs frequency-domain channel estimation with superimposed pilots. Those Frequency-Domain channel estimators exploit the diagonal structure of the channel matrix in the Frequency-Domain to perform Channel Estimation, which is not the case anymore for TV channels.

In case of TV channels, [8] develops a time-domain MMSE equalizer based on the well-known Basis Expansion Model (*BEM*) [9]. It also supposes that the channel is perfectly known at the receiver and it seems that the study of TV channel estimation is not widely addressed for Continuous Phase Modulation. However, for linear modulation and for OFDM, BEM-based channel estimation has been well studied using different BEMs [10]–[15] and so we propose in this paper to adapt some of those methods for block-based CPM transmissions over TV channels and to evaluate their performance.

In this paper, we will investigate TV channel estimation for CPM signals based on a Basis Expansion Model. We will see that a Least Squares estimation can be performed on the received signal using a fractionally-spaced representation and also that it can be improved by using *a priori* on the delays of the paths (as in the case of aeronautical communication by satellite). We will also provide some performance for single-carrier block-based CPM transmission where we have to extrapolate the estimated channel over the data block.

The paper is organized as follows. In section II, we will present our system model whereas the BEM model for TV channels is presented in section III. Then, in section IV, we will discuss two channels estimators which both capitalize

on the BEM model. We will also show how to exploit those estimators in the case of a block-based structure of the CPM signal. We will provide some simulation results with an emphasis on the aeronautical channel via a satellite link in section V. Finally, conclusions and perspectives are drawn in section VI.

II. System Model

A. Notations

In the following, a vector will be represented by an underlined letter (e.g. $\underline{v}$, $\underline{V}$) and a matrix by a doubly underlined letter (e.g. $\underline{\underline{m}}$, $\underline{\underline{M}}$). The matrix $\underline{\underline{I}}_N$ is the identity matrix of size $N \times N$.
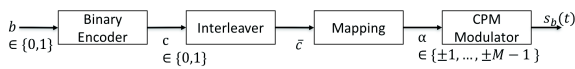
B. Communication system description

Fig. 1. CPM BICM Transmitter

We consider the general Bit Interleaved Coded Modulation (*BICM*) transmission scheme for CPM, as given in Fig 1. Let $\{\alpha_n\}_{0 \leq n \leq N-1} \in \{\pm 1, \pm 3, \ldots, \pm M - 1\}^N$ be a sequence of N symbols taken from the M-ary alphabet . The complex envelope $s_b(t)$ associated with the transmitted CPM signal is written as follows

$$s_b(t) = \sqrt{\frac{2E_s}{T}} \exp\left(j2\pi h \sum_{i=0}^{N-1} \alpha_i q(t - iT)\right) \quad (1)$$

where

$$q(t) = \begin{cases} \int_0^t g(\tau)d\tau, t \leq L_{\text{cpm}} \\ 1/2, t > L_{\text{cpm}} \end{cases}$$

E_s is the symbol energy, T is the symbol period, $g(t)$ is the frequency pulse, h is the modulation index and L_{cpm} is the CPM memory.

Let us now consider a transmission over a TV channel $h_c(t, \tau)$. At the receiver, we assume ideal low-pass filtering using the front-end filter $\Psi(t)$ and ideal synchronization. Denoting $h(t, \tau) = \Psi(t) * h_c(t, \tau)$, where $*$ is the convolution operator, the received signal can be written as:

$$r(t) = \sum_m s(m\frac{T}{k})h\left(t, t - m\frac{T}{k}\right) + w(t), \quad (2)$$

where $w(t)$ is a complex baseband additive white Gaussian noise with power spectral density $2N_0$, and k is the oversampling factor.

As the channel is time-varying, we need to *spread* the pilot symbols within the frame. To do so, we derive a block-based model by using a known Unique Word (UW), also called training sequence. This approach is also useful to perform frequency-domain equalization, as it allows us to *circularize* the channel [5]. Similar model has been considered for linear modulation in [16].

Unlike for linear modulations, due to the CPM memory, we need to add some termination symbols at the end of the data block in order to ensure the phase continuity and the uniqueness of the UW [17], as illustrated in Fig.2. Moreover, the length of a UW must be larger than the time dispersion of the channel to avoid interference between CPM blocks.
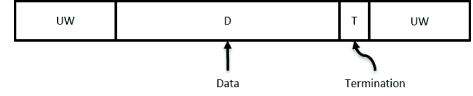

Fig. 2. Block-based structure of the CPM signal

C. Baseband representation

Using a Fractionally-Spaced representation of the received signal, we have the following expression:

$$r[l] = r\left(\frac{lT}{k}\right) = \sum_m s(m\frac{T}{k})h\left(l\frac{T}{k}, (l-m)\frac{T}{k}\right) + w\left(l\frac{T}{k}\right)$$

$$= \sum_m s[m]h[l; l - m] + w[l] \quad (3)$$

By defining the channel matrix $\underline{\underline{h}}$ in Eq.(5) where L is the channel span, and by neglecting the interference coming from the previous data block, the signal has the following matrix-wise representation:

$$\underline{r} = \underline{\underline{h}}\underline{s} + \underline{w} \quad (4)$$
$$\text{with } \underline{r} = [r[0], r[1], \ldots, r[kN - 1]]^T$$
$$\underline{s} = [s[0], s[1], \ldots, s[kN - 1]]^T$$
$$\text{and } \underline{w} = [w[0], w[1], \ldots, w[kN - 1]]^T$$

We point out that the interference from the previous data block can be perfectly removed (and hence, our hypothesis will be valid) by not taking into account the first L received samples.

We then introduce the following representation:

$$\underline{h}_l = \left[h[0, l], h[1, l], \ldots, h[kN - 1, l]\right]^T \quad (6)$$

where $\underline{h}_l$ is a vector of size $kN \times 1$ corresponding to the complex attenuation of the lth path. Then, our channel matrix $\underline{\underline{h}}$ can be written as:

$$\underline{\underline{h}} = \sum_{l=0}^{L-1} \text{diag}(\underline{h}_l)\underline{\underline{Z}}_l \quad (7)$$

where $\underline{\underline{Z}}_l$ is a matrix of size $N \times N$ which represents the delay of the lth path in the *lag* domain, i.e. $[\underline{\underline{Z}}_l]_{n,(n-l)} = 1$ and 0 elsewhere, and $\text{diag}(\underline{h}_l)$ is a diagonal matrix of size $N \times N$ whose diagonal entries are $\underline{h}_l$.

In terms of channel estimation, we can see, by using those notations, that the number of parameters to estimate can be too important (as there is kN complex coefficients per path). The Basis Expansion Model allows us to reduce the number of parameters to estimate as only a few coefficients which are required to model the time-varying channels.

$$\underline{\underline{h}} = \begin{bmatrix} h[0,0] & 0 & \cdots & \cdots & 0 \\ \vdots & \ddots & \ddots & & \vdots \\ h[L-1,L-1] & & \ddots & \ddots & \vdots \\ \vdots & \ddots & & \ddots & 0 \\ 0 & \cdots & h[kN-1,L-1] & \cdots & h[kN-1,0] \end{bmatrix} \tag{5}$$

III. BASIS EXPANSION MODEL

We now introduce the BEM which has been presented in [9]. However, in this section, our notations are inspired by the work of [10] for OFDM systems in case of Doppler Spread Channels.

A. Received signal using BEM

The complex attenuation corresponding to the lth path is described as:

$$\underline{h}_l = \underbrace{[\underline{\zeta}_0, \underline{\zeta}_1, \ldots, \underline{\zeta}_{P-1}]}_{\underline{\underline{\zeta}}} \underbrace{[\eta_{l,0}, \eta_{l,1}, \ldots, \eta_{l,P-1}]^T}_{=\underline{\eta}_l} = \sum_{p=0}^{P-1} \eta_{l,p} \underline{\zeta}_p \tag{8}$$

where P be the number of basis function, $\underline{\zeta}_p$ is the (p+1)th deterministic base of size $kN \times 1$, and $\eta_{l,p}$ is the (p+1)th stochastic parameter for the (l+1)th path.
By introducing Eq.(8) in Eq.(7), we obtain:

$$\underline{\underline{h}} = \sum_l \sum_p \eta_{l,p} \underbrace{\text{diag}(\underline{\zeta}_p)\underline{\underline{Z}}_l}_{=\underline{\underline{\Gamma}}_{l,p}} = \sum_l \sum_p \eta_{l,p}\underline{\underline{\Gamma}}_{l,p} \tag{9}$$

$\underline{\underline{\Gamma}}_{l,p}$ is a deterministic matrix of size $kN \times kN$. We now define the matrix $\underline{\underline{\Gamma}}$ of size $kN \times PLkN$:
$\underline{\underline{\Gamma}} = [\underline{\underline{\Gamma}}_{0,0}, \ldots, \underline{\underline{\Gamma}}_{0,P-1}, \underline{\underline{\Gamma}}_{1,0}, \ldots, \underline{\underline{\Gamma}}_{1,P-1}, \ldots, \underline{\underline{\Gamma}}_{L-1,P-1}]$
and the vector $\underline{\eta} = [\underline{\eta}_0^T, \underline{\eta}_1^T, \ldots, \underline{\eta}_{L-1}^T]^T$ of size $LP \times 1$. Using the Kronecker product $\otimes$, we have:

$$\underline{\underline{h}} = \underline{\underline{\Gamma}}(\underline{\eta} \otimes \underline{\underline{I}}_{kN}) \tag{10}$$

Finally, it can be shown that the received vector is:

$$\underline{r} = \underline{\underline{\Gamma}}(\underline{\eta} \otimes \underline{\underline{I}}_{kN})\underline{s} + \underline{w} = \underline{\underline{\Gamma}}(\underline{\underline{I}}_{LP} \otimes \underline{s})\underline{\eta} + \underline{w} \tag{11}$$

B. Case of a time-invariant channel

In the case of TIV channels, the BEM can be simplified. By taking $P = 0$ and $\underline{\zeta}_0 = [1, 1, \ldots, 1]^T$, we obtain:

$$\underline{h}_l = h[l]\underline{\zeta}_0 \tag{12}$$

We have now:

$$\underline{\eta} = [\underline{\eta}_0, \underline{\eta}_1, \ldots, \underline{\eta}_{L-1}]^T = [h[0], h[1], \ldots, h[L-1]]^T \tag{13}$$
$$\tag{14}$$

and also

$$\underline{\underline{\Gamma}}_{l,0} = \text{diag}(\underline{\zeta}_0)\underline{\underline{Z}}_l = \underline{\underline{I}}_N\underline{\underline{Z}}_l = \underline{\underline{Z}}_l \tag{15}$$
$$\text{and so } \underline{\underline{\Gamma}} = [\underline{\underline{Z}}_0, \underline{\underline{Z}}_1, \ldots, \underline{\underline{Z}}_{L-1}] \tag{16}$$

Our received signal (11) becomes:

$$\underline{r} = \underbrace{\underline{\underline{\Gamma}}(\underline{\underline{I}}_L \otimes \underline{s})}_{=\underline{\underline{s}}}\underline{h} + \underline{w} \tag{17}$$

which corresponds exactly to the equation (33) of [5] but using the fractionally-spaced representation instead of the polyphase representation.

C. BEM Design

In the State of the Art, various BEMs are considered. We discuss here a few models, in a non-exhaustive way, that we will use later.
[11] and [12] use the discrete Karhunen-Loeve expansion (*KL-BEM*). It is assumed that the auto-correlation function of the complex attenuation is the zeroth-order Bessel function (which is the case when the Doppler has a Jakes' Doppler Spectrum). With the knowledge of the maximum Doppler frequency and of the variance of the attenuation, this technique is optimal [12]. We note $\underline{\underline{R}}$ the channel correlation matrix of a tap. By using a Singular Value Decomposition (*SVD*), we obtain $\underline{\underline{R}} = \underline{\underline{V}}\text{diag}(\underline{\Delta})\underline{\underline{V}}^H$. Then, the KL-BEM is designed by taking the P first column of the matrix $\underline{\underline{V}}$ as the basis functions $\{\underline{\zeta}_p\}_p$.
[13] introduces a BEM based on Complex Exponential (*CE-BEM*) functions which has been "extended" to the over-sampled CE-BEM (*OCE-BEM*) in [14], [18]. In this case, no prior knowledge channel statistics is needed, reducing the problem of mismatched model. The basis function are defined by:

$$\zeta_p[n] = e^{j2\pi(p-P/2)n/(KkN)}, \text{ with } K \text{ a positive integer} \tag{18}$$

It is well-known that CE-BEM ($K = 1$) suffers from model error on the edge of the considered block (as it implies that the channel is periodic with a period equal to the duration of the block). OCE-BEM ($K \geq 2$) allows for a more accurate parameterization, but we loose the orthogonality of the base.
Others BEM exist (such as polynomial BEM and discrete prolate spheroidal BEM) but they will not be discussed in this paper. A comparison of some BEMs in terms of modelling performance is given in [19], [20].

IV. CHANNEL ESTIMATION

In this section, we will present the Data-Aided Least Squares channel estimation of the BEM parameters $\underline{\eta}$. Unlike

for linear modulation, due to the use of the CPM, this estimation must be performed on the over-sampled received signal $\underline{r}$. We will consider now the transmitted signal $\underline{s}$ known at the receiver. Hence, the matrix $\underline{\underline{s}} = \underline{\underline{\Gamma}}(\underline{\underline{I}}_{LP} \otimes \underline{s})$ of size $kN \times LP$ is also known.

A. Least Squares Estimation

As for linear modulations, the standard approach is to perform a LS estimation of the BEM parameters. We assume in this case that there is no model error. The Least Square estimation is:

$$\widehat{\underline{\eta}} = (\underline{\underline{s}}^H \underline{\underline{s}})^{-1} \underline{\underline{s}}^H \underline{r} \qquad (19)$$

$$\text{and so } \widehat{\underline{\underline{h}}} = \underline{\underline{\Gamma}}(\widehat{\underline{\eta}} \otimes \underline{\underline{I}}_N) \qquad (20)$$

as the noise in our system model is a white Gaussian noise.

The overall complexity of this estimation is dominated by the multiplication of the matrix $(\underline{\underline{s}}^H \underline{\underline{s}})^{-1} \underline{\underline{s}}^H$ of size $LP \times kN$ by a vector of size kN. Note however that this matrix can be pre-computed and stored at the receiver.

B. Least Squares Estimation with a priori positioning

As in case of TIV Channel estimation for linear modulation [21] and for CPM [5], we can introduce a parametric dependence on the estimation of the BEM parameters and the delays of the L_c paths.

Let define the vector $\underline{\tau} = [\tau_0, ..., \tau_{L_c-1}]^T$ which contains the delay of the different paths and $\underline{a} = [\underline{a}_0^T, ..., \underline{a}_{L_c-1}^T]^T$ the associated BEM parameters such as $h_c(t,\tau) = \sum_{l=0}^{L_c-1} a_l(\tau)\delta(t - \tau_l)$. We now introduce the dependence on the delays $\underline{\tau}$, and by using the BEM on the complex attenuation (we suppose $\underline{\tau}$ constant during the frame transmission) and the same idea as previously (see Eq.(10)), we obtain:

$$\underline{\underline{h}} = \sum_{l=0}^{L_c-1} \sum_{p=0}^{P-1} \eta_{l,p} \text{diag}(\underline{\zeta}_p) \underline{\underline{Z}}_{\tau_l} \qquad (21)$$

$$= \underline{\underline{\Gamma}}_{\underline{\tau}}(\underline{\eta}_P \otimes \underline{\underline{I}}_N) \qquad (22)$$

$$\text{where } \underline{\underline{Z}}_{\tau_l} \doteq \text{Toeplitz}(\underline{\Psi}(\tau_l)) \qquad (23)$$

$$\text{and } [\underline{\Psi}(\tau_l)]_n \doteq \Psi(n\frac{T}{k} - \tau_l) \qquad (24)$$

Let us emphasize the difference between this parametric model and the one presented in Eq.(11). In this model, $\underline{\underline{\Gamma}}_{\underline{\tau}}$ contains only the contribution of the front-end filter $\Psi(t)$ with the delays of the considered paths $\underline{\tau}$. The vector $\underline{\eta}_p$ of size $PL_c \times 1$ contains only the BEM parameters for each considered paths. Our received signal is now:

$$\underline{r} = \underline{\underline{\Gamma}}_{\underline{\tau}}(\underline{\underline{I}}_{PL_c} \otimes \underline{s})\underline{\eta}_P + \underline{w} = \underline{\underline{s}}_P \underline{\eta}_P + \underline{w} \qquad (25)$$

where $\underline{\underline{s}}_p$ is now a matrix of $kN \times PL_c$. This matrix $\underline{\underline{s}}_p$ is known at the receiver as the vector τ is also known by geometrical consideration [5], [21] or previously estimated as in [22]–[24].

As our noise in our system model is still a white Gaussian noise, the Least Squares estimation of $\underline{\eta}_P$ is given by:

$$\widehat{\underline{\eta}}_P = (\underline{\underline{s}}_p^H \underline{\underline{s}}_p)^{-1} \underline{\underline{s}}_p^H \underline{r} \qquad (26)$$

$$\text{and so } \widehat{\underline{\underline{h}}}_P = \underline{\underline{\Gamma}}_{\underline{\tau}}(\widehat{\underline{\eta}}_P \otimes \underline{\underline{I}}_{kN}) \qquad (27)$$

By using this parametric estimator, we reduce the number of parameters to estimate from PL to PL_c parameters. We also reduce the computational complexity of the inverse as the matrix $\underline{\underline{s}}_p^H \underline{\underline{s}}_p$ is of size $L_c \times L_c$ instead of $L \times L$. For instance, in the case of the aeronautical channel, it is commonly admitted that $L_c = 2$

C. Block-based transmission

In case of block-based transmission as presented in Fig. 2, only the UWs (composed of N_{UW} symbols) are known at the receiver, however we need to estimate the channel over both data block and UW in order to perform channel estimation as the one presented in [8]. To do so, we propose to consider the previous UW, the data block and the next UW as proposed in [15]. In this case, we can separate the contribution of the UWs and of the data block. By considering, the vector $\underline{r}_{\text{UW}} = [r[L], r[L+1], \ldots, r[kN_{\text{UW}} - 1], r[k(N - N_{\text{UW}}) + L], \ldots, r[kN - 1]]^T$ of size $2k(N_{\text{UW}} - L) \times 1$ which contains only the received samples corresponding to the contribution of the UWs (we have remove the ISI coming from the data blocks), we can rewrite our system as [15]:

$$\underline{r}_{\text{UW}} = \underline{\underline{h}}_{\text{UW}} \underbrace{[\underline{s}_{\text{UW}}^T, \underline{s}_{\text{UW}}^T]^T}_{\doteq \underline{s}_{\text{UW},2}} + \underline{w}_{\text{UW}} \qquad (28)$$

where the matrix $\underline{\underline{h}}_{\text{UW}}$ of size $k(N_{\text{UW}} - L) \times 2kN_{\text{UW}}$ is a concatenated sub-matrix of $\underline{\underline{h}}$. We point out that this matrix $\underline{\underline{h}}_{\text{UW}}$ depends on the same BEM parameters as the matrix $\underline{\underline{h}}$ as it is only a partitioning of this former matrix. Hence, the LS estimation, based only on the UW, is given by:

$$\widehat{\underline{\eta}}_{\text{UW}} = (\underline{\underline{s}}_{\underline{UW},2}^H \underline{\underline{s}}_{\text{UW},2})^{-1} \underline{\underline{s}}_{\text{UW},2}^H \underline{r}_{\text{UW}} \qquad (29)$$

$$\text{with } \underline{\underline{s}}_{\text{UW},2} \doteq \underline{\underline{\Gamma}}(\underline{\underline{I}}_{PL} \otimes \underline{s}_{\text{UW},2}) \qquad (30)$$

This LS estimation can use *a priori* positioning as described in the previous section. Hence, our estimated channel is obtained by $\widehat{\underline{\underline{h}}} = \underline{\underline{\Gamma}}(\widehat{\underline{\eta}}_{\text{UW}} \otimes \underline{\underline{I}}_{kN})$. The *interpolation* of our estimated channel over the data block is directly perform by the BEM model.

V. RESULTS

For simulation, we consider a binary CPM scheme with $h = 1/2$, a CPM memory $L_{\text{CPM}} = 3$ and a REC pulse shape. We point out that, as our Unique Word can be independent of our CPM parameters, our results can be extended to other CPM schemes. We define the Normalized Mean Square Error *NMSE*, which explicitly takes the BEM modeling error into account, as:

$$\text{NMSE} = \mathbb{E}\left\{ \frac{\sum_{l=0}^{L-1} |\underline{h}_l - \widehat{\underline{h}}_l|^2}{\sum_{l=0}^{L-1} |\underline{h}_l|^2} \right\} \qquad (31)$$
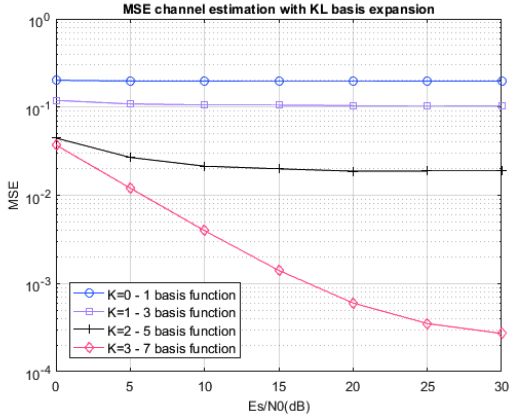
Fig. 3. NMSE over TV channels using KL-BEM

Fig. 4. NMSE over TV channels using OCE-BEM

Fig. 5. NMSE over TV channels using KL-BEM for block-based CPM

We have evaluated our channel estimation in the case of a TIV channel (not shown here) and we obtain results similar to the ones presented in [5].

Now let us consider a time-varying channel. We examine our channel estimators for a Rayleigh channel with a Jakes' Doppler Spectrum and a maximum Doppler frequency of $0.0183R_s$ (where R_s is the symbol rate: $R_s = 1/T$). This channel is composed of 2 paths with a delay of $1.5T$ between them. First, we consider the case where even the data block is known at the receiver in order to validate our methods. In our simulations, we perform the LS estimation with 128 known symbols.

In Fig. 3, we consider the KL-BEM and present simulations results for different numbers of basis functions. We can observe that, by increasing the number of basis functions, the performance are improved, which is explained by the fact that our BEM is closer to the simulated channel. The floor error phenomenon is due to the BEM modeling error. It is acknowledged that a BEM is accurate when the modeling error is on the order of 10^{-4} [15], which is the case when we use 7 basis functions. We do not consider the improved LS channel estimation using the knowledge of the delays due to the fact that the KL-BEM already uses this knowledge, by considering the variance of the attenuation.

Nevertheless, the KL-BEM described in subsection III-C requires an important knowledge on the channel such as the maximum Doppler frequency and the variance of the complex attenuation (which provides the knowledge on the delays). Hence, we investigate more robust BEM.

A common BEM is the (O)CE-BEM only based on complex exponentials. This model does not require any knowledge of the channel statistics. In this case, the BEM channel estimation suffers from the unknown *sparsity* of the channel. Indeed, we try to estimate a *null* path by a sum of weighted complex exponentials. Even the LS estimation with positioning *a priori* exhibits a large BEM modeling error as shown in Fig. 4. We can also observe that for the same number of basis functions, the KL-BEM outperforms the chosen OCE-BEM, which can be explained by the optimally of the KL-BEM. However, we
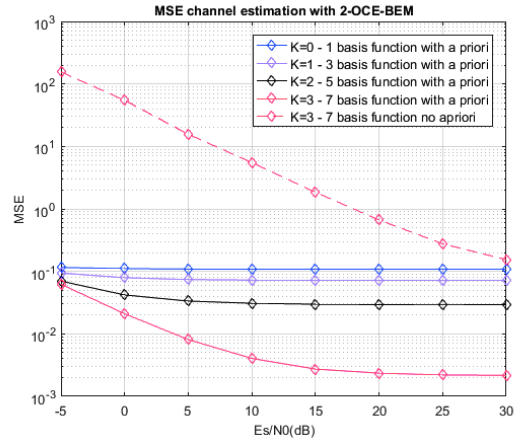
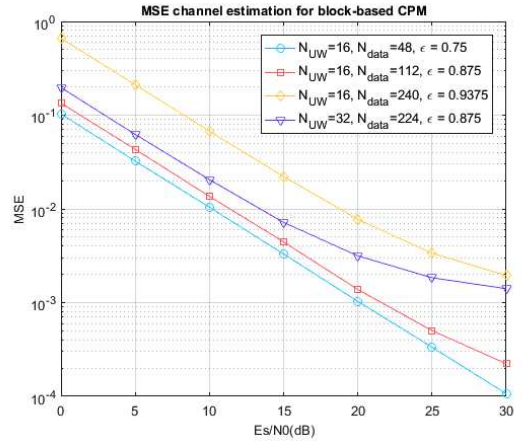can see, in any of our chosen case, that the OCE-BEM can be considered accurate as the error floor is around 10^{-3}, which can lead to further study to find an other suitable model based on the OCE-BEM (by changing the factor or by considering a completely different BEM).

To conclude, we now present the performance of our algorithm in the case of a block-based transmission. As explained in subsection IV-C, only the UWs are known at receiver, and so the channel is *interpolated* over the data block thanks to the BEM.

We will consider different sizes N_{UW} of UW and different sizes of data block of N_{DATA} (which give us a bandwidth efficiency of $1 - N_{\text{UW}}/(N_{\text{DATA}} + N_{\text{UW}})$). We choose for those simulations a normalized Doppler spread of $0.0008R_s$ and the KL-BEM to approximate our channel which suppose a maximum Doppler frequency of $0.002R_s$. Those parameters have been used in [15]. Only 3 basis functions are considered, which gives us 6 BEM parameters to estimate as only 2 paths are considered. Simulations results are presented in Fig.5. We obtain similar performance compared to the ones in [15], which tends to validate our approach. We can observe that
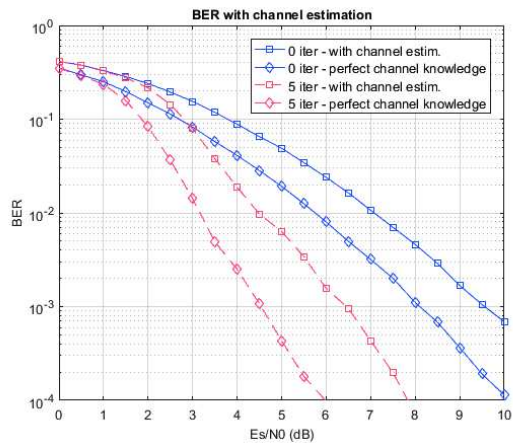
Fig. 6. BER over TV channels for block-based CPM

when we increase the size of the data block, while keeping the same size of UW, the performance are degrading which can be explained easily. Indeed, in this case, the *interpolation* of the channel over the data block is worse due to BEM modelling error. Finally, we evaluate the impact of the channel estimation in terms of Bit Error Rate (*BER*) in Fig.6, considering now a coded transmission (using a convolutional code with polynomial generators $(5,7)_8$) and an iterative concatenated scheme between the CPM MAP detector and the MAP channel decoder. We send 40 data blocks of 112 symbols with Unique Word of 16 symbols, which give us a bandwidth efficiency of 75%. To perform channel equalization, we use the band Frequency-Domain MMSE Equalizer presented in [25] (with $Q = 5$). We use the same simulation parameters as previously. We consider the case of perfect channel knowledge and the case where we use the previous KL-BEM channel estimation. The degradation due to the the channel estimation error is around 2dB at a BER of 10^{-3}. Similar results have been observed in [2] in the case of TIV channels where the MMSE equalizer suffers from a degradation of 3dB.

VI. CONCLUSION

In this paper, we have evaluated the performance of BEM-based channel estimation for CPM over time-varying channels and we have shown how to perform it in case of a block-based transmission. We have shown that the knowledge of the paths delays can improved significantly the performance of such estimation. We also observe that the performance of BEM-based estimation is very depending on the chosen BEM which can have large modeling error.

REFERENCES

[1] F. Pancaldi and G. M. Vitetta, "Equalization algorithms in the frequency domain for continuous phase modulations," *IEEE Trans. on Comm.*, vol. 54, no. 4, pp. 648–658, 2006.

[2] W. Van Thillo, F. Horlin, J. Nsenga, V. Ramon, A. Bourdoux, and R. Lauwereins, "Low-complexity linear frequency domain equalization for continuous phase modulation," *IEEE Trans. on Wireless Comm.*, vol. 8, no. 3, pp. 1435–1445, 2009.

[3] B. Ozgul, M. Koca, and H. Deliç, "Double turbo equalization of continuous phase modulation with frequency domain processing," *IEEE Trans. on Comm.*, vol. 57, no. 2, pp. 423–429, 2009.

[4] C. Brown and P. Vigneron, "Equalisation for continuous phase modulation using basis functions," in *SPIE 2011, Orlando, Florida, April 2011*.

[5] R. Chayot, N. Thomas, C. Poulliat, M. L. Boucheret, N. V. Wambeke, and G. Lesthievent, "Channel estimation and equalization for cpm with application for aeronautical communications via a satellite link," in *IEEE MILCOM*, Oct 2017.

[6] C. Brown and P. Vigneron, "Channel estimation and equalisation for single carrier continuous phase modulation," in *IEEE MILCOM, Baltimore, MD*, pp. 334–340, Nov 2011.

[7] C.-H. Park, R. W. Heath, and T. S. Rappaport, "Frequency-domain channel estimation and equalization for continuous-phase modulations with superimposed pilot sequences," *IEEE Trans. on Vehicular Tech.*, vol. 58, no. 9, p. 4903, 2009.

[8] D. Darsena, G. Gelli, I. Iudice, and F. Verde, "Equalization techniques of control and non-payload communication links for unmanned aerial vehicles," *IEEE Access*, vol. PP, no. 99, pp. 1–1, 2018.

[9] G. B. Giannakis and C. Tepedelenlioglu, "Basis expansion models and diversity techniques for blind identification and equalization of time-varying channels," *Proceedings of the IEEE*, vol. 86, no. 10, pp. 1969–1986, 1998.

[10] L. Rugini, P. Banelli, and G. Leus, "Low-complexity banded equalizers for ofdm systems in doppler spread channels," *EURASIP Journal on Applied Signal Processing*, vol. 2006, pp. 248–248, 2006.

[11] M. Visintin, "Karhunen-loeve expansion of a fast rayleigh fading process," *Electronics Letters*, vol. 32, pp. 1712–, Aug 1996.

[12] K. A. D. Teo and S. Ohno, "Optimal mmse finite parameter model for doubly-selective channels," in *IEEE GLOBECOM*, vol. 6, pp. 5–pp, 2005.

[13] M. K. Tsatsanis and G. B. Giannakis, "Modelling and equalization of rapidly fading channels," *Int. J. on Adapt. Control*, vol. 10, no. 2-3, pp. 159–176, 1996.

[14] G. Leus, "On the estimation of rapidly time-varying channels," in *Signal Processing Conference, 2004 12th European*, pp. 2227–2230, IEEE, 2004.

[15] G. Leus, Z. Tang, and P. Banelli, "Estimation of time-varying channels–a block approach," in *Wireless communications over rapidly time-varying channels*, pp. 155–197, Elsevier, 2011.

[16] H. Witschnig, T. Mayer, A. Springer, A. Koppler, L. Maurer, M. Huemer, and R. Weigel, "A different look on cyclic prefix for sc/fde," in *IEEE PIMRC*, vol. 2, pp. 824–828, IEEE, 2002.

[17] W. V. Thillo, V. Ramon, A. Bourdoux, F. Horlin, K. Sleurs, and R. Lauwereins, "Training sequence versus cyclic prefix for cpm with frequency domain equalization," in *IEEE GLOBECOM, Honolulu, HI*, pp. 1–5, Nov 2009.

[18] T. A. Thomas and F. W. Vook, "Multi-user frequency-domain channel identification, interference suppression, and equalization for time-varying broadband wireless communications," in *Sensor Array and Multichannel Signal Processing Workshop. 2000. Proceedings of the 2000 IEEE*, pp. 444–448, IEEE, 2000.

[19] T. Zemen and C. F. Mecklenbrauker, "Time-variant channel estimation using discrete prolate spheroidal sequences," *IEEE Trans. on Signal Processing*, vol. 53, no. 9, pp. 3597–3607, 2005.

[20] Z. Tang, *Ofdm transmissions over rapidly changing channels*. PhD thesis, TU Delft, Delft University of Technology, 2007.

[21] B. Raddadi, C. Poulliat, N. Thomas, M.-L. Boucheret, and B. Gadat, "Channel estimation with a priori position for aeronautical communications via a satellite link," in *IEEE PIMRC, Hong Kong, China*, pp. 532–537, Sept 2015.

[22] C. Carbonelli, S. Vedantam, and U. Mitra, "Sparse channel estimation with zero tap detection," *IEEE Trans. on Wireless Comm.*, vol. 6, no. 5, 2007.

[23] N. Benvenuto and R. Marchesani, "The viterbi algorithm for sparse channels," *IEEE Trans. on Comm.*, vol. 44, pp. 287–289, Mar 1996.

[24] M. Sharp and A. Scaglione, "Estimation of sparse multipath channels," in *IEEE MILCOM, San Diego, CA*, pp. 1–7, Nov 2008.

[25] R. Chayot, N. Thomas, C. Poulliat, M. L. Boucheret, N. V. Wambeke, and G. Lesthievent, "A frequency-domain band-mmse equalizer for continuous phase modulation over frequency-selective time-varying channels," in *EUSIPCO*, Sept 2018.