Harvesting selectivity and stochastic recruitment in economic models of age-structured fisheries

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A R T I C L E   I N F O

A B S T R A C T

We develop the age-structured fishery model by including endogenous harvesting selectivity and stochastic recruitment, as a growing body of fishery ecological evidence suggests these factors to be critical for fisheries management. Optimal harvesting selectivity aims to direct fishing towards age classes that are preferable to catch given information on fish growth, natural mortality, and recruitment in addition to implications on harvesting cost. We analytically show that maximum sustainable yield (MSY) leads to potentially serious and previously unrecognized deviation from economic optimality, as it neglects the dependence of harvesting costs on gear selectivity. We further show that the steady-state population level may fall below the MSY population even with zero discounting and stock-dependent harvesting costs. We quantify our results using empirical data for Baltic cod. Applying the age-structured model with endogenous harvesting selectivity, we find large differences between maximum sustainable yield and the economic optimum, although the classic biomass model suggests that these differences should be unimportant. Stochastic recruitment implies a threefold increase in young age classes, but the stochastic solution can be accurately approximated by the certainty equivalence principle.

Introduction

Optimization models in fishery economics originate from two different traditions, i.e. the Schaefer (1954) and Gordon (1954) lumped-parameter (or biomass) approach and the Beverton and Holt (1957) cohort analysis. After the contributions of Clark and Munro (1975) and Clark (1990), the Schaefer-Gordon approach has dominated fishery economics. This model enables analytical simplicity compared to a more detailed age-structured description of harvestable populations and the shortage of data additionally forced earlier fishery economics to apply models with a minimum set of parameters. However, due to advances in fishery ecology, data concerning the most important fish populations are presently provided in the age-structured setup (e.g. ICES, 2016). In spite of this development, the fishery economic applications of such models remain poorly understood. Moreover, the generality of the theoretical results obtained within the biomass model can be questioned, and the setup seriously restricts the scope of the fishery economic analysis (cf. Clark and Munro, 2017).

Fishery ecologists have found that fishing acts as a powerful selective force causing age-class truncation effects that destabilize fished stocks and increase the risk of population collapse (e.g. Hixon et al., 2014; Hsieh et al., 2006). Regulation of

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gear selectivity, mesh sizes in particular, is an important management instrument for many fisheries globally. The inclusion of these factors into economic analyses is impossible within the biomass framework, and calls for age-structured (cohort or life history) optimization models, which pose new and open questions for fisheries economics. Our study shows how the inclusion of optimized harvesting gear selectivity changes certain established concepts in fishery economics, and how stochastic environmental fluctuations favoring the spawning of old age classes can be incorporated into age-structured fishery models.

Growing biological evidence shows that harvesting tends to remove the oldest age classes, which form the most productive fraction of the spawning stock: the fraction that is able to maintain successful reproduction over unfavorable periods (Green, 2008). This leads to the question of how to control the catch pressure over the population age classes. Choice of fishing gear is the prime method for achieving this, especially the mesh size in gillnet and trawl fisheries. Mesh sizes are regulated all over the globe and they have a central role in the Beverton and Holt cohort model, but endogenous gear selectivity is seldom included in fishery economic analysis.[1] The most natural framework for this purpose is a dynamic age-structured model. This line of reasoning is followed by Diekert et al. (2010a, 2010b), who writes that northeast Arctic cod suffers from "growth overfishing", i.e. fish are caught from age classes whose value is still rapidly increasing. This can be separated from "recruitment overfishing", which endangers egg production and recruitment.

Our study specifies an economically sound and novel age-structured model that covers both schooling fisheries and fisheries where a larger population size decreases the harvesting cost. Instead of effort, typically used in age-structured models (but difficult to measure empirically), we use total annual harvest as an optimized variable together with mesh size. This leads to a concept we label "efficient biomass", which weights fish biomasses by age-specific and optimized catchabilities. This concept is new in fishery economics and it is essential in interpreting the features of age-structured models that sharply contrast with those based on the classic biomass model. We show analytically that optimizing gear selectivity creates a new type of discrepancy between the economically optimal steady state and steady states aiming to maximize sustainable yield (MSY).[2] When old age classes are essential for MSY, optimal mesh size is large. This decreases the efficient biomass leading to prohibitively high harvesting costs (and implicitly high effort). In contrast, economically optimal mesh size is smaller, implying that the economic outcomes and population structures between these steady states become remarkably different. In his review of existing fishery economics (mainly based on the biomass setup), Wilen (2000) concludes that the difference between the MSY and economic steady state is practically unimportant. Our first contribution is the result that this picture becomes very different within the age-structured framework with optimized fishing gear.

Studying the properties of the age-structured model reveals that the admissible and sustainable yield, and MSY in particular, become dependent on fishing gear and technology. This can be compared to the Schaefer-Gordon lumped-parameter approach, where MSY is presented to follow from biological factors only (Clark, 1990, p. 13). The dependence of MSY on harvesting technology has generally not been recognized in fishery economics. Our second contribution is a new proposal for defining three different MSY concepts, depending on fishing technology and the occurrence of pulse fishing, and to contrast these three MSY equilibria with the economically optimal outcome. This sheds new light on fisheries policy analysis.

In a recent debate Grafton et al. (2007, 2010) represent the view that economically optimal steady-state population levels in actual fisheries tend to exceed MSY levels, and are thus preferable from both conservation and economic viewpoints. This is criticized by Clark et al. (2010a, 2010b), who emphasize that the economic optimum with discounting may well imply stock sizes below MSY and even extinction. This discussion relies directly on results from the Schaefer-Gordon biomass model, where the economically optimal steady state always exceeds the MSY steady state given stock-dependent harvesting costs (and no discounting). As our third contribution, we show that the age-structured model with optimized gear selectivity may lead to a very different conclusion. In the age-structured model, economically optimal *efficient biomass* exceeds the MSY *efficient biomass*, while, depending e.g. on the harvesting technology, the difference for the physical stock biomass may be the reverse. This result is a novel example showcasing that classic results based on the biomass model do not carry over to the age-structured setup.

Our fourth contribution is a test of our analytical findings and a comparison of the biomass and age-structured models using a detailed empirical model for Baltic cod. This fishery is an example of the vast amount of detailed age-structured data available for many fisheries of economic importance around the globe. Additionally, it serves as a particularly relevant case for contrasting biological recommendations for fishery management with results from economic optimization in the stochastic setting. For Baltic cod, the older age classes form the most efficient fraction of the spawning stock. According to substantial biological literature (e.g. Hinrichsen et al., 2016) environmental variability affects Baltic cod egg survivability via ambient oxygen conditions determined by stochastic inflow events of North Sea water. The magnitude of this effect depends on the age of the spawning females, and adverse effects are strongest among the eggs of young females. Thus ecologists conjecture that maintaining old age classes is important for this fishery (Cardinale and Arrhenius, 2000).[3] Besides detailed ecological

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[1] Turvey (1964) is an early discussion of these questions along the lines of the Beverton and Holt model.

[2] MSY still forms the main policy guide in fishery management. The UN Convention on the Law of the Sea is based on restoring and maintaining fish stocks for producing MSY (UNCLOS, 1982, Article 61, 3). The European Union has committed to the MSY policy (EU, 2013).

[3] The ecologists' conjecture is more or less based on aiming for MSY or similar objectives without including economic features Köster et al. (2009).

parameterization, our model for Baltic cod is based on estimated nonlinear demand, stock-dependent harvesting costs, and experimental data on gear selectivity.

We first study the potential role of our theoretical findings assuming a deterministic setup. Comparison of the economic and various MSY equilibria shows major differences in economic performance and population levels when analyzed within the age-structured model, albeit the difference is minor if analyzed using the classic biomass model (cf. Wilen, 2000). Additionally, the economically optimal steady-state biomass is found to be lower than the MSY biomass even with zero interest rate and stock-dependent harvesting costs. These results are novel, and are explained by our analytical investigation, but cannot be understood in the biomass framework. They additionally challenge the generality of the discussion by Grafton et al. (2007, 2010a) and Clark et al. (2010a, 2010b).

We finally investigate whether including the stochasticity of age class-specific egg survivability will change the economically optimal harvesting results. Recruitment stochasticity is an inherent feature of fish populations (Voss et al., 2012; Kuparinen et al., 2014), but earlier stochastic optimization models with population structure are scarce.[4] Costello et al. (1998) study the stochastic effects of El Niño events on a three-age-class Coho salmon fishery assuming perfect gear selectivity. Holden and Conrad (2015) add stochasticity to a model with perfect selectivity and with juvenile, immature, and adult stage classes.

It is difficult or impossible to compute stochastic solutions for a model with nine state variables using dynamic programming. Given this difficulty, we apply stochastic programming that transforms the problem into a high-dimensional nonlinear programming problem. Stochastic programming has previously been applied in e.g. finance and game theory, but we are unaware of earlier applications in fisheries. Compared to dynamic programming, the strength of this method is avoidance of the "curse of dimensionality". Its downside − the necessity of applying rather short planning horizons − turns out to be of minor importance for the problem at hand. Given the actual 2013 population size as the initial state and discount rates of 10−15%, the optimal 2013 harvest based on stochastic optimization is higher than harvest based on expected environmental conditions, but the difference is only 2.3% or smaller. With a higher initial population level the stochastically optimized harvest becomes conservative compared to the deterministic solution, but once again the difference is small. Computing stochastic solutions over time shows that while a high egg survivability event triples recruitment, the variation in harvest is only 25%, and the stochastic solution can be approximated by the certainty equivalence solution with surprisingly high accuracy.

The paper is organized as follows. We next specify the age-structured model for two age classes and study the model analytically. In the next step the model is specified to include any number of age classes, stochastic recruitment, and Baltic cod parameter values. We first present the deterministic results for Baltic cod and finally the stochastic computation runs.

A deterministic two-age classes fishery with optimized gear selectivity: theoretical results

Let the total harvest be denoted by H_t at period t. Assume that a concave function U ($U \in C^2$) denotes the single-period gross surplus of total harvest. Let the harvesting cost depend linearly on effort, implying that per period net surplus is given as $U(H_t) - cE_t$, where E_t is effort and $c > 0$ is a constant. The total per period harvest is given as

$$H_t = B_t^{\chi} E_t, \quad t = 0, 1, ..., \tag{1}$$

where $0 < \chi \leq 1$ is a constant and B_t, $t = 0, 1, ...$, is "efficient biomass". It is given as

$$B_t = \sum_{s=1}^{n} w_s q_s(\gamma_t) x_{st}, \quad t = 0, 1, ..., \ n = 2, \tag{2}$$

where w_s is the weight of fish in age class s, $q_s(\gamma_t)$ ($q_s \in C^2$, $s = 1, 2$) is harvesting selectivity as a function of mesh size γ_t and x_{st} are the number of fish in age glass $s = 1, 2$. Imperfectly selective fishing gear is described by a logistic function, as common in fisheries science (Madsen, 2007)

$$q_s(\gamma_t) = \left[1 + e^{(\gamma_t - \vartheta_{1s})\vartheta_{2s}^{-1}}\right]^{-1}, \quad s = 1, 2, \ t = 0, 1, ..., \tag{3}$$

where ϑ_{is}, $i = 1, 2$, $s = 1, 2$, are positive constants. Catcability decreases in mesh size γ and equals $\frac{1}{2}$ when $\gamma = \vartheta_{1s}$. Higher ϑ_{2s} implies that catcability depends more smoothly on mesh size and vice versa. The larger the difference between the ϑ_{21} and ϑ_{22}, the more effectively can larger age classes be excluded from the catch by increasing mesh size, i.e. the more selective the fishing gear.

Note that if $\chi = 1$ in (1), the harvest per age class is given as a linear function of effort and age-specific biomass (cf. Schaefer, 1957). Denoting the discount factor by b we write the present value of net surplus from harvesting as

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$$J = \sum_{t=0}^{\infty} [U(H_t) - cE_t]\, b^t. \tag{4}$$

The number of fish in age class 1 depends on the population structure $\mathbf{x}_t = (x_{1t}, x_{2t})$, and is given as

$$x_{1,t+1} = \phi(\mathbf{x}_t),\ t = 0, 1, \ldots, \tag{5}$$

where ϕ ($\phi \in C^2$) is a recruitment function. We assume Beverton-Holt (1957), Ricker (1954) or constant recruitment (Clark, 1990, p. 204–216).

We have used effort as one optimized variable. However, data on total harvest are more reliable than data on effort. We thus eliminate effort and assume that the fraction of biomass harvested from each age class $\hat{h}_{st}$ equals the fraction of age class-specific efficient biomass on the total efficient biomass:

$$\frac{\hat{h}_{st}}{H_t} = \frac{w_s q_s(\gamma_t) x_{st}}{B_t},\ \ s = 1, 2,\ \ t = 0, 1, \ldots. \tag{6}$$

This implies that the composition of the total catch in terms of the numbers of fish harvested from different age classes are given as $h_{st} = \hat{h}_{st} w_s^{-1} = q_s(\gamma_t) x_{st} H_t (B_t)^{-\chi}$, $s = 1, 2$, $t = 0, 1, \ldots$. Development of age class 2 takes the form

$$x_{2,t+1} = \alpha_1 x_{1t} + \alpha_2 x_{2t} - H_t G(\gamma_t, \mathbf{x}_t) \tag{7}$$

for $t = 0, 1, \ldots,$ where $\alpha_s,\ s = 1, 2$ are age class-specific survivability coefficients and

$$G(\gamma_t, \mathbf{x}_t) \equiv [\alpha_1 q_1(\gamma_t) x_{1t} + \alpha_2 q_2(\gamma_t) x_{2t}] B_t(\gamma_t, \mathbf{x}_t)^{-1} \tag{8}$$

allocates total harvest to age classes 1 and 2 respectively. Finally, we include the conditions

$$x_{s0},\ \ s = 1, 2,\ \ given, \tag{9}$$

$$x_{st} \geq 0, \tag{10}$$

$$H_t \geq 0, \tag{11}$$

$$0 \leq \underline{\gamma} \leq \gamma_t \leq \overline{\gamma}, \tag{12}$$

for $s = 1, 2,\ t = 0, 1, \ldots$ and where (12) specifies lower and upper constraints for the mesh size.

After eliminating effort from (4) using (1), the optimization problem is

$$\max_{\{H_t,\ \gamma_t,\ t=0,1,\ldots\}} J = \sum_{t=0}^{\infty} \left[U(H_t) - cB(\gamma_t, \mathbf{x}_t)^{-\chi} H_t \right] b^t \tag{13}$$

subject to (5), (7) and (9)–(12).

We shall use this model for comparing the economically optimal solutions and the MSY solution. However, the MSY concept must be sharpened in the age-structured setting. First, we define "ultimate MSY" (uMSY) (Getz, 1980) that solves

$$\max_{\{h_{st},\ t=0,1,\ldots,\ s=1,\ldots,n\}} J_{uMSY} = \sum_{t=0}^{\infty} \sum_{s=1}^{n} w_s h_{st}, \tag{14}$$

subject to (5), (7), (9), (10) where age specific harvests $h_{st} \geq 0$, $s = 1, \ldots, n$, $t = 0, 1, \ldots$ are optimized directly. The uMSY is a purely theoretical concept that reveals the biologically feasible maximum catch size if perfectly selective harvesting gear was available.

Taken the available harvesting gear into account we may maximize the objective function (13) after setting $U(H_t) = H_t$, $c = 0$ and $b = 1$. However, this setup may yield pulse fishing or limit cycles (Hannesson, 1975; Tahvonen, 2009a, 2009b), i.e. solutions that do not represent strict sustainability. Thus, we label this outcome as "gear constrained maximum yield" (gcMY). Finally, it is possible to define an equilibrium that closest resembles the traditional MSY, i.e. a steady state that maximizes constant annual yield given the existing gear. We label this as "gear constrained MSY" (gcMSY).

We contrast the age-structured model with the solution for the biomass model that describes the same fish stock. The classic biomass model (cf. Clark, 1990, p. 236) is specified as

$$\max_{\{h_t,\ t=0,\dots\}} \sum_{t=0}^{\infty}\left[U(H_t) - H_t c(\tau x_t)^{-\chi}\right] b^t, \ s.\ t.\ x_{t+1} = x_t + F(x_t) - H_t, \ x_0 = given, \tag{15}$$

where x_t is total biomass, F is a biomass growth function, c and τ are parameters, and τx_t approximates the efficient biomass used in the cost function of the age-structured model.

Properties of the deterministic age-structured problem

Writing $cB_t^{-\chi}H_t = C[B(\gamma_t, \mathbf{x}_t), H_t,]$ the Lagrangian and the first order necessary conditions read as

$$L = \sum_{t=0}^{\infty} b^t \{U(H_t) - C[H_t, B(\gamma_t, \mathbf{x}_t)] + \lambda_{1t}[\phi(\mathbf{x}_t) - x_{1,t+1}] + \lambda_{2t}[\alpha_1 x_{1t} + \alpha_2 x_2 - H_t G(\gamma_t, \mathbf{x}_t) - x_{2,t+1}]\}$$

$$\frac{\partial L}{\partial H_t}b^{-t} = U\prime - C_H - \lambda_{2t}G = 0, \tag{16}$$

$$\frac{\partial L}{\partial \gamma_t}b^{-t} = -C_B B_\gamma - \lambda_{2t}H_t G_\gamma = 0, \ if \ \underline{\gamma} < \gamma_t < \overline{\gamma}, \ \leq 0 \ if \ \underline{\gamma} = \gamma_t \ and \ \geq 0 \ if \ \gamma_t = \overline{\gamma}, \tag{17}$$

$$\frac{\partial L}{\partial x_{s+1,t+1}}b^{-t} = -bC_B B_{x_{s+1,t+1}} + b\lambda_{1,t+1}\phi_{x_{s+1,t+1}} - \lambda_{s+1,t} + b\lambda_{s+2,t+1}\alpha_{s+1} - bH_{t+1}\lambda_{2,t+1}G_{x_{s+1,t+1}} = 0, \tag{18}$$

including (5), (7), (9)–(12), where in (18) $s = 0, 1$, $t = 0, 1,\dots$, and $\lambda_3 \equiv \lambda_2$. In (16), (17) we assume interior solutions excluding the mesh size choice. At steady state the time symbols can be dropped and the conditions (16)–(18) take the form

$$U\prime - C_H - \lambda_2 G = 0, \tag{19}$$

$$-C_B B_\gamma - \lambda_2 H G_\gamma = 0, \ if \ \underline{\gamma} < \gamma < \overline{\gamma}, \ \leq 0 \ if \ \gamma = \underline{\gamma} \ and \ \geq 0 \ if \ \gamma = \overline{\gamma}, \tag{20}$$

$$-bC_B B_{x_1} + b\lambda_1 \phi_{x_1} - \lambda_1 + b\lambda_2(\alpha_1 - HG_{x_1}) = 0, \tag{21}$$

$$-bC_B B_{x_2} + b\lambda_1 \phi_{x_2} + \lambda_2 b\left(\alpha_2 - HG_{x_2} - b^{-1}\right) = 0. \tag{22}$$

Eliminating λ_1 by (21) and λ_2 by (19) condition (22) can be written as

$$-C_B\left[\frac{B_{x_2} + bB_{x_1}\phi_{x_2}(1 - b\phi_{x_1})^{-1}}{(U\prime - C_H)G^{-1}}\right] + \frac{b\alpha_1\phi_{x_2}}{1 - b\phi_{x_1}} - H\left(\frac{b\phi_{x_2}G_{x_1}}{1 - b\phi_{x_1}} + G_{x_2}\right) - (1 - \alpha_2) = r. \tag{23}$$

Eq. (23) is an age-structured counterpart of the Clark and Munro (1975) fundamental equation of renewable resources. To understand the various factors that determine the optimal steady state in the age-structured framework, we consider a couple of special cases. First, if efficient biomass does not decrease harvesting cost, i.e. if $C_B = 0$ Eq. (23) describes schooling fishery and if discount rate is zero, it describes the gcMSY equilibrium.

Assume first knife edge selectivity, $q_1(\gamma) = 0$. This implies $G = \alpha_2/w_2$, $G_{x_1} = G_{x_2} = G_\gamma = 0$. The left hand side (LHS) of (20) reduces to $-C_B B_\gamma \leq 0$ implying that $\gamma = \underline{\gamma}$ if harvesting cost decrease with population size ($C_B < 0$) and $\gamma \in \left[\underline{\gamma}, \overline{\gamma}\right]$ otherwise. Condition (23) reduces to

$$-C_B B_{x_2} + \frac{b\alpha_1\phi_{x_2}}{1 - b\phi_{x_1}} - (1 - \alpha_2) = r. \tag{24}$$

When ignoring harvesting costs (as for the gcMSY objective), or for a schooling fishery ($C_B = 0$), (24) states that the interest rate equals the marginal productivity of the biological capital net of marginal natural mortality of age class 2. Further, assume that a young age class does not produce eggs, i.e. $\phi_{x_1} = 0$. Saving one unit of age class 2 fish increases recruitment, but the increase is realized net of the natural mortality of the "invested" age class 2 individual and after a delay of one period. Note that the increased recruitment is discounted by the biological and economic discount factors α_1 and b. When the young age class contributes to recruitment, the present value marginal recruitment effect of age class 2 individuals must be normalized by the term $1 - b\phi_{x_1}$. For a Beverton-Holt recruitment function, the quotient term in (24) is decreasing in x_2. This implies that if harvesting cost depends negatively on efficient biomass ($C_B < 0$), the optimal steady state population level will increase. Additionally, differentiation shows that higher interest rate implies a smaller steady state population. Excluding the fact that the internal delays in the age structure complicate the determination of population (present value) marginal productivity, the properties of optimal steady state still resembles those of the biomass framework (Clark and Munro, 1975).
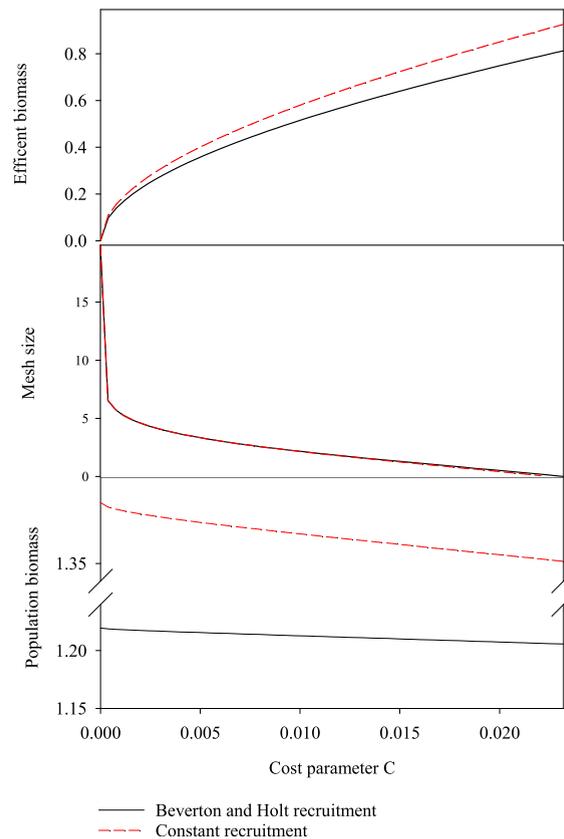
Fig. 1. Optimal steady state as a function of harvesting cost parameter c. Parameter values, see Appendix A.

Given imperfectly selective fishing gear and $q_s(\gamma) > 0$, $s = 1,2$ condition (20) for optimal mesh size can be written as

$$-C_B B_\gamma - \lambda_2 H x_1 x_2 (q'_1 q_2 - q_1 q'_2)(\alpha_1 w_2 - \alpha_2 w_1) B^{-2} = 0 \text{ if } \underline{\gamma} < \gamma < \overline{\gamma}, \ \leq 0 \text{ if } \gamma = \underline{\gamma} \text{ and } \geq 0 \text{ if } \gamma = \overline{\gamma}, \quad (25)$$

where $x_1 x_2 (q'_1 q_2 - q_1 q'_2)(\alpha_1 w_2 - \alpha_2 w_1) B^{-2} = G_\gamma$. When mesh size is increased, harvesting cost increase as efficient biomass decreases and $C_B B_\gamma > 0$. Additionally, increasing mesh size changes the allocation of harvesting pressure among the age classes. This is reflected in term G_γ and its sign depends on harvesting technology and population biology. Condition (25) can be considered in particular under the gcMSY objective, in which case $C_B = 0$, and only the latter factor determines optimal mesh size. The term $q'_s q_{s+1} - q_s q'_{s+1}$, $s = 1$ follows from fishing technology and according to the parameters estimated in Quaas et al. (2013) it is negative for Baltic cod for all $s = 1,...,n-1$ and any mesh size $\underline{\gamma} \leq \gamma \leq \overline{\gamma}$ (Table B1, Appendix B). Survivability parameters are typically independent of age classes in fisheries data implying that by $w_2 > w_1$ the empirically most relevant case is $\alpha_1 w_2 - \alpha_2 w_1 > 0$. For gcMSY this implies that $-\lambda_2 H G_\gamma > 0$, i.e. the upper bound mesh size is optimal for the gcMSY objective, as this minimizes the catch of small individuals from age class 1. However, a large mesh size increases harvesting cost and $-C_B B_\gamma < 0$ for all $\gamma \in [\underline{\gamma}, \overline{\gamma}]$, implying that the optimal mesh size in the economically optimal steady state

equals the lower bound if the dependence of harvesting cost on efficient biomass is strong enough (c is high enough Eq. (5)). This discrepancy in choosing the optimal mesh size may imply major differences between the outcomes of the gcMSY and economic objectives. This feature is not captured within the biomass framework.

To facilitate the further analysis of this model property assume that recruitment is constant[5] and equal to ϕ, interest rate is zero, $U(H) = H$ and $\chi = 1$ in (2). Eqs. (23) and (25) become quadratic and linear in x_2 (Appendix A). Given exogenous mesh size the steady state biomass increases with the cost parameter c (Eq. (A1)). However, mesh size also depends on cost parameter c. In general, population biomass may increase or decrease with c. To show that the latter outcome is possible we apply the parameter values specified in Appendix A. As shown in Fig. 1, it is optimal to decrease the mesh size as the harvesting cost parameter is increased from zero. This increases the efficient biomass, which compensates the increased harvesting cost. However, a smaller mesh size decreases the number of fish in the older age class, implying that population

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[5] Constant recruitment is frequently assumed in fishery ecology as well as a "hockey stick" recruitment, i.e. recruitment that is increasing and linear with low spawning stock level and constant thereafter (Beverton and Holt, 1957; Clark, 1990, p. 276; Barrowman and Myers, 2000).

biomass decreases. Thus, the traditional result in fishery economics, in which economically optimal population biomass exceeds the MSY biomass when interest rate is zero and harvesting cost depend on stock level (Clark, 1990, p. 231), may not hold in age-structured models given the optimized but imperfect selectivity. We turn to study how this finding and the potential discrepancy between MSY and economic equilibria are realized in the case of Baltic cod.

Empirical application of the age-class model for Baltic cod

Baltic cod has been the most valuable fish species in the Baltic Sea, and the fishery has offered livelihood to a large proportion of fishermen in the region. During the 1980s the cod stock reached its historical record, and 22% of global cod catches were obtained from the Baltic Sea. One consequence of this economic contribution is the availability of exceptionally abundant scientific knowledge on various ecological, economic and technical features concerning the Baltic cod. This fishery is characterized by strong variability in recruitment which is caused by stochastic inflow events of North Sea water. During adverse conditions old females exhibit highest egg survivability (Hinrichsen et al., 2016).

We include these effects by an age class-specific egg survivability vector that depends on environmental conditions. The nature of stochasticity and available data allow us to specify stochasticity as a two-stage Bernoulli process. This assumption renders the computational problem manageable, and still captures the main effect that there is a clear distinction between "good" and "bad" environmental conditions. Let the stochastic state of environmental conditions in the sea (temperature, salinity, oxygen concentration) follow the difference equation

$$z_{t+1} = \hat{z} + \varepsilon_t, \quad t = 0, \dots, T, \tag{26}$$

where $\hat{z}$ denotes average conditions and ε_t are i.i.d. random variables. We assume a Bernoulli process where ε_t can take a "high" or "low" value with probabilities p and $1 - p$. The length of the time horizon is $T + 1$, implying that the stochasticity can be described with a binomial tree that contains $2^{T+1} \equiv Q$ outcomes with the probabilities π^i, $i = 1, \dots, Q$.

In the stochastic programming formulation all variables are specified separately for each node, i.e. for each date and for each binomial tree outcome i. Thus, for example total harvest is given as H_t^i, $t = 0, \dots, T + 1$, $i = 1, \dots, Q$. Taken this into account and extending the model to include any number of age classes $n \geq 2$ the problem is to

$$\max_{\{H_t^i, \gamma_t^i, t=0,\dots,T, i=1,\dots,Q\}} J = \sum_{i=1}^{Q} \pi^i \sum_{t=0}^{T} \left[U\left(H_t^i\right) - c\left(B_t^i\right)^{-\chi} H_t^i \right] b^t \tag{27}$$

s.t.

$$x_{1,t+1}^i = \phi\left(z_t^i, \mathbf{x}_t^i\right), \tag{28}$$

$$x_{s+1,t+1}^i = \alpha_s x_{st}^i - H_t^i q_s\left(\gamma_t^i\right) x_{st}^i B^{-1}\left(\gamma_t^i, \mathbf{x}_t^i\right), \quad s = 1, \dots, n-2, \tag{29}$$

$$x_{n,t+1}^i = \alpha_{n-1} x_{n-1,t}^i + \alpha_n x_{nt}^i - H_t^i \left[\alpha_{n-1} q_{n-1}\left(\gamma_t^i\right) x_{n-1,t}^i + \alpha_n q_n\left(\gamma_t^i\right) x_{nt}^i \right] B^{-1}\left(\gamma_t^i, \mathbf{x}_t^i\right), \tag{30}$$

for $t = 0, \dots, T$, $i = 1, \dots, Q$ and including the analogous nonnegativity and initial state restrictions as in (9)–(12). In (28) the number of fish in age class one depends on stochastically evolving environmental conditions and on the population structure $\mathbf{x}_t^i = [x_{1t}^i, \dots, x_{nt}^i]$, $t = 0, 1, \dots, T+1$, $i = 1, 2, \dots, Q$. The environmental condition affecting node it recruitment is observable at the same period t and uncertainty is present at future nodes. When this model is applied in a deterministic setup the value of the stochastic variable z_t is constant and equal its expected value. In this case $Q = 1$ and the superscript i is neglected.

Stochastic programming has its roots in stochastic linear programming (Dantzig, 1955; Infanger, 2011) and in the generalization to nonlinear models (Rockafellar and Wets, 1975). The method is used e.g. in financial portfolio problems (Gülpınar et al., 2004), dynamic games (Genc et al., 2007), land conversion problems (Messina and Bosetti, 2006), and forestry (Tahvonen and Kallio, 2006). The decision process is taken as non-anticipative, i.e. the decisions at any given period and state of the world become independent of the future realization of the random variable. Thus, the decision variables become functions of the system state (including z_t) as in stochastic dynamic programming. A strength of stochastic programming is that state variables can be kept continuous. In addition, while the "curse of dimensionality" is a problem of dynamic programming, the number of state variables is not similarly limited. However, the optimization problem becomes overwhelmingly large with long horizons. Yet, when discount rate is positive the effect of a finite horizon on the first-period decision becomes smaller. To approximate some stochastic realization over longer periods, we apply an iterative procedure (Getz and Haight, 1989, p. 281), where the system state after the first-period decision is taken as a new state and the optimization is repeated over some desirable horizon.

Ecological parameters

We use ICES (2013) estimates for age-specific survival rates and weights (Appendix B, Table B1). The age structure of the spawning stock plays an important role on egg survivability, as older and larger female cod produce more buoyant, larger eggs (Vallin and Nissling, 2000). Stochastic inflow events improve oxygen conditions in the deeper water layers of the Bornholm Basin, and hence increase cod egg survival. While the general effects of a major Baltic inflow may be traced for a longer period, the data in Hinrichsen et al. (2007) shows that the impact on cod egg survival is restricted to one year. Furthermore, egg survival time-series for 1951–2010 show a bimodal distribution (Hinrichsen et al., 2016), and hence support the assumption of a two-stage Bernoulli process in recruitment. We incorporate these effects in an extended Ricker (1954) stock-recruitment function:

$$x^i_{1,t+1} = \phi_0 \left\{ \sum_{s=1}^{n} \left[\left(1 - z^i_t\right)\beta^-_s + z^i_t\beta^+_s \right] \mu_s \, w_s \, x^i_{st} \right\} \exp\left(-\phi_1 x^{ci}_{0t}\right), \ t = 0, ..., T, \ \ i = 1, ..., Q,$$

where parameters $\phi_0 > 0$ and $\phi_1 > 0$ and $\mu_s, \ s = 1, ..., n$ denote age-specific fecundity. The term $(1 - z_{it})\beta^-_s + z_{it}\beta^+_s$ captures stochastic age-specific egg survivability that is low at a value β^-_s in the absence of an inflow event, $z_{ti} = 0$, or high at a value β^+_s when an inflow event occurs, $z_{ti} = 1$.

We construct a time series (1951–2012) of stochastic egg survivability for each female spawning age class in May (Hinrichsen et al., 2016). Survivability was 'low' during 44 years and 'high' during 16 years. Thus, $p = 16/60 = 0.27$. The resulting age-specific egg survivabilities β^+_s for good and β^-_s for bad oxygen conditions are shown in Appendix B Table B1. Together this specifies all the parameter values related to the stochastic model features in Eqs. (26) and (28). Finally, Appendix B, and Table B1 show the estimation of the stock-recruitment function that includes density-dependence and cannibalistic behavior of cod.

For the biomass model we use the growth function from Froese and Proelss (2010): $F = rx_t(1 - x_t/K)$, $r = 0.48$ and $K = 2283$ thousand tons. Using the 2013 initial population, the efficient biomass is 113 thousand tons, while the total stock biomass is 175 thousand tons. This yields $\tau = 0.6$ in (14). However, the 2013 population age structure is far from equilibrium solutions, where the value of τ is between 0.79 and 0.81. As apply $\tau = 0.8$, as we concentrate on steady states in the model comparisons, but our main conclusions are not sensitive with respect to this parameter value.

Economic parameters

To estimate the parameters for the gross surplus and cost functions, we construct the time series of efficient biomasses B_t using estimated age class-specific stock numbers, weights, and fishing mortalities from ICES (2013). Further, we utilize the fact that the Baltic cod fishery has been *de-facto* open access in the past (Kronbak, 2005; Quaas et al., 2012). Under open access, harvest is determined by the market price P_t equaling the marginal harvesting cost, i.e. $P_t = cB^{-\chi}_t$.

We specify the marginal gross benefits as an iso-elastic inverse demand function $U\prime(H_t) = . \, \overline{P}Y^{-\eta}_t H^{-\nu}_t$. Here, H_t is the overall catch quantities of Baltic cod (ICES, 2013), Y_t is the catch of the largest North Atlantic cod stock, the Northeast Arctic cod, as a substitute for Baltic cod, and $\overline{P}$, η, and ν are parameters to be estimated. Further details are given in the Appendix B, Table B2.

To estimate catchabilities we focus on the trawling fleet, as trawlers are the most common type of gear in the Baltic cod fishery (Kronbak, 2005). Madsen (2007) estimates how the shape of the parameters depends on mesh size for the Bacoma escape window, which is the most common trawl net used for Eastern Baltic cod. Parameters ϑ_{1s} and ϑ_{2s} as estimated in Quaas et al. (2013), and all other age-specific parameters, are given in the Appendix B, Table B1. Fig. 2 shows that the catchability is higher the smaller the mesh size and the older the age class.

Results and discussion

Deterministic solutions: contrasting economic optimum and MSY

According to the biomass model (Fig. 3), transitions to steady states are monotonic, and the interest rate varying between 0 and 15% has only minor effects on the optimal steady-state harvest. Additionally, the difference between economically optimal steady-state harvest and MSY harvest is small. This exactly reflects the argument in Wilen (2000), which states that the discrepancy between MSY and economic solutions appears unimportant. Secondly, given interest rates below 11%, the steady-state population level is higher than the MSY population, which is similar to the results found by Grafton et al. (2007) for many fisheries.

In a deterministic age-structured model egg survival is given by an expected value, which is $p = 0.27$ times the high survivability plus 0.73 times the low survivability. In Fig. 4a–d the initial population state is as estimated for 2013 (Table B1), and interest rate varies between 0 and 15%. Optimal harvesting is smooth over time and pulse fishing does not occur. This is a consequence of nonlinearity due to decreasing demand. Results additionally suggest that the 2013 initial biomass is far below
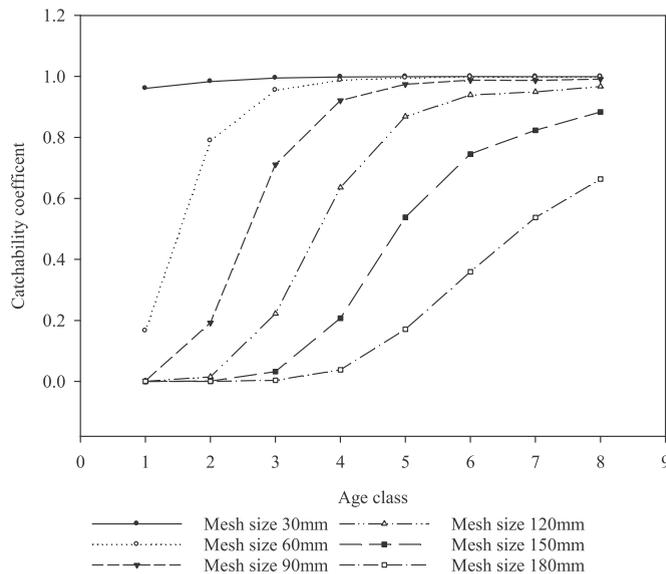
Fig. 2. Bacoma trawl net mesh size and catchability.

the long-run optimal steady state of 850–960 thousand tons (Fig. 4c). Along the transition the harvest level increases eightfold while the profit level only doubles, as market price decreases from €3.1 to €0.8 per kg of fish. The higher the interest rate, the lower are the steady-state population and profit levels. The mesh size and number of older fish decrease with interest rate. However, the effects of the interest rate on steady-state harvest are minor.

Given an interest rate of 10% the steady-state biomass and harvest in the biomass model are 1158 and 274 thousand tons, while they are considerably lower in the age-structured model, equaling 894 and 220 thousand tons. Biomass has varied between 100 and 1100 thousand tons and harvest between 31 and 391 thousand tons (Eero et al., 2007; ICES, 2013). Mesh size has varied between 110 mm and 120 mm in the last 15 years. The optimal solutions of the age-structured model are thus within the observations excluding the mesh size that is slightly lower compared to present practices.

Fishing mortality is highest for age classes 4–8, increases with interest rate (Fig. 5a), and reaches 25–30% for age classes 4–8. Accordingly, the number of fish decreases with age and a higher interest rate yields a younger population (Fig. 5b). The main yield in biomass units is obtained from age classes 4 and 8 (Fig. 5c), while the yield from age classes 1–3 is nearly zero.

However, optimizing mesh size leads to a harvest structure that may still be rather far from the structure that would be optimal if a harvest from any age class could be freely chosen, i.e. if perfect selectivity and uMSY was possible. The uMSY solution is to harvest fish only from age class 8, which is harvested completely at the end of each year (Fig. 5b,c). This implies the beginning of period age structure in Fig. 5b, where, compared to economic solutions, the number of fish is lower in young age classes, but much higher in older classes.

The uMSY produces a high annual catch of 310 thousand tons, while the economic steady-state catch without discounting is 218 thousand tons. These results are in line with ecologists' arguments, which note that Baltic cod it is vital to maintain older and larger fish (Hinrichsen et al., 2016).
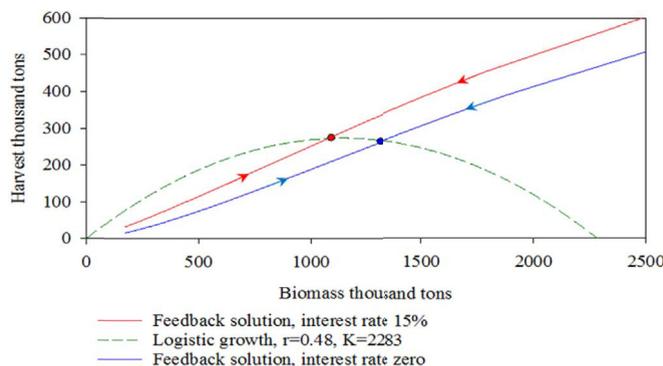
Fig. 3. Numerically computed solution to the biomass model.
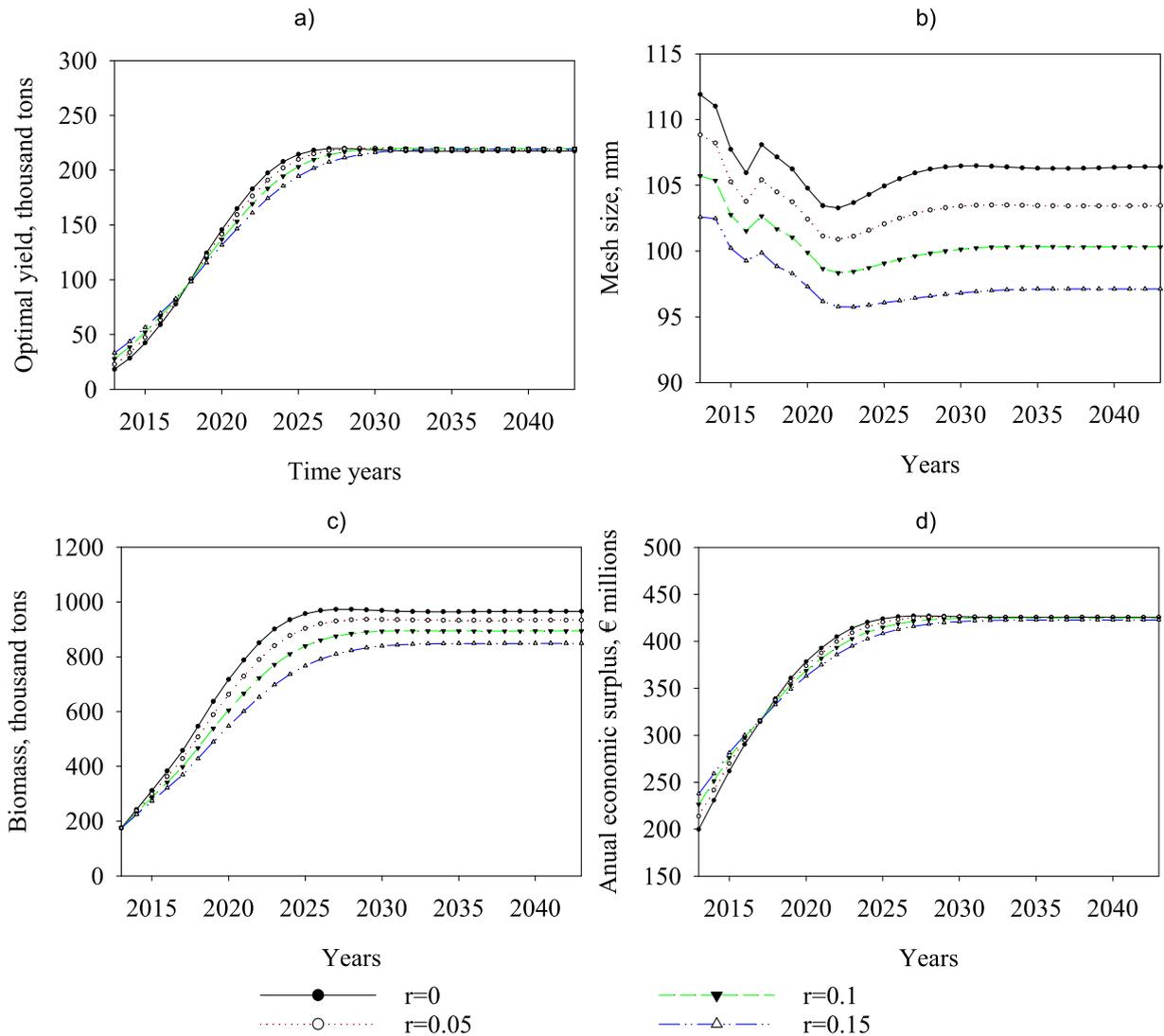
Fig. 4. a–d. Optimal harvesting and population development from the 2013 initial state.

To compute the maximum yield obtainable by the Bacoma trawl net we maximize the sum of annual catches over 100 years and optimize annual harvest and mesh size. This gcMY leads to biannual pulse fishing (not shown), an average annual catch equal to 270 thousand tons, and an average annual economic loss equal to $-€667$ million. This is a major loss, as the maximum annual profit is positive (€425 million). The loss occurs from the high effort level and mesh size that are optimal, as this yields an outcome closest to the uMSY solution with catch from age class 8 only.

The gcMSY can be computed by removing time and the initial state restrictions from Eqs. (28)–(30), and solving the resulting static problem that maximizes annual yield. In Fig. 6a the aim is to reach gcMSY, which is attained at the level of 264 thousand tons when biomass is 1094 thousand tons. Reaching this solution requires a maximum mesh size (300 mm) and an effort level ten times higher than in the economic equilibrium (Fig. 6b and d). Note that in Fig. 6a and b effort and economic loss begin declining when mesh size reaches its maximum.

In Fig. 6c the annual economic surplus (i.e. interest rate is zero) is maximized when biomass and catch equal 965 and 218 thousand tons, respectively. The mesh size is 120 mm, and the index for effort remains much lower compared to gcMSY equilibrium. The gcMSY equilibrium (Fig. 6a) leads to an annual loss equal of $-€692$ million, while the maximized annual economic surplus is €425 million. Again, the major loss is a consequence of excessive effort and large mesh size that are employed to direct catch towards the oldest age class to the extent possible with actual gear available.

Fig. 6a and c also show that the population size in the economically optimal steady state (965 thousand tons) is smaller than the gcMSY biomass (1094 thousand tons). This is a consequence of endogenous mesh sizes. If mesh size is exogenously determined or if analyzing the fishery using the biomass model (Fig. 3), annual economic surplus is maximized with higher
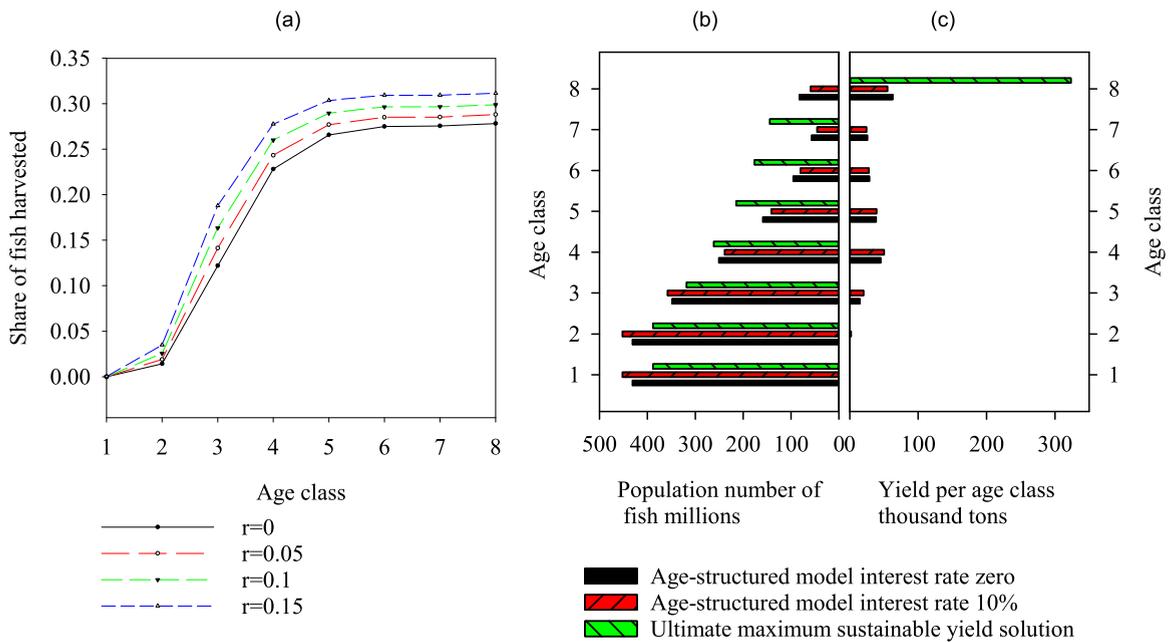
Fig. 5. a,b,c. Steady-state mortalities, population age structure, and yield per age class.

biomass compared to MSY (zero interest rate and stock-dependent cost), as familiar from the biomass setup (Clark, 1990; Grafton et al., 2007). For the more realistic age-structured setting, with optimal mesh size, the dispute between Grafton et al. (2007, 2010a, 2010b) and Clark et al. (2010a, 2010b) may not be valid as such. The economic equilibrium advocated by Grafton et al. (2007, 2010a, 2010b) is lower for Baltic cod than gcMSY given any interest rate. Second, Wilen (2000) concludes that the difference between economic equilibrium and MSY is typically minor and unimportant. This is not at all true in an age-structured model for the Baltic cod fishery, albeit it appears to be true if the fishery is analyzed within the biomass framework.

These outcomes are in line with the analytical understanding we obtained using the two-age-class model version. Additionally, the age-structured model shows that it is wrong to view MSY as determined by biological factors only. The three definitions for MSY all include serious problems in the case of Baltic cod: one is not admissible with existing gear and the other two lead to major economic losses. This can be contrasted with the UN Convention on the Law of the Sea and the European Common Fisheries Policy that consider MSY a central aspect in fisheries management.

The results are so far based on deterministic recruitment. However, the variation in recruitment is large and if the interest rate is 5%, the optimal steady-state harvest varies between 166 and 360 thousand tons depending on whether low or high survivability is assumed. We finally investigate whether our results on economically optimal harvesting are robust and whether stochasticity could further emphasize saving the old age classes, as adverse conditions disproportionately decrease the egg survivability of young females.

Stochastic solutions

Stochastic programming expands the number of optimized variables and requires the application of short planning horizons. A 15-year horizon leads to a nonlinear programming problem with 589,804 variables.[6] The deterministic solutions in Fig. 7a–d show that discount rates equal to 10–15% and a time horizon of 12–15 years produce a rather close approximation for the first year optimal harvest, as extending the time horizon only leads to a minor change in the first-year harvest level.

The first-year deterministic and stochastic solutions are compared in Fig. 7a,b with 2013 initial population level. On average, the stochastically optimal solution is 1.2–2.3% higher compared to the deterministic solution. When the initial population equals the deterministic steady state (Fig. 7c,d), stochasticity leads to slightly lower harvest levels but the decrease is only 0.5–0.8%.

Fig. 8 shows a possible realization of the stochastic fishery over time computed by the iterative computation, where the optimal harvest for year 2013 is obtained by applying a 12-year time horizon. This solution produces a new population state

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[6] A time horizon of 15 years requires a computing time of over 20 h (Intel (R) Xeon (R) E5-2643 v3 @3.40GHZ, 24 processors and AMPL/Knitro algorithm version 10.1). With longer horizons the number of variables > 10^6, which is beyond the capacity of the available methods. The optimization codes are available upon request.
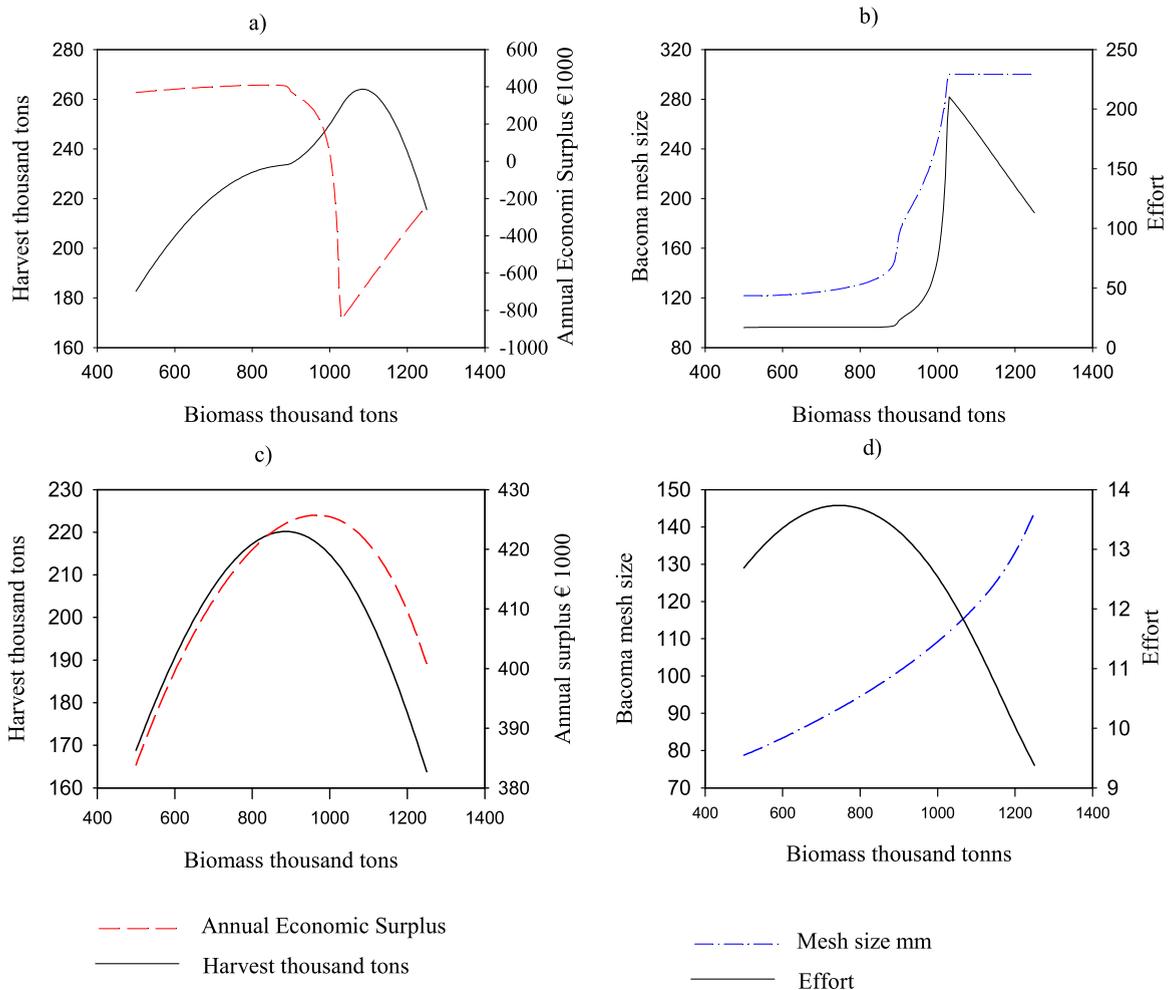
Fig. 6. a–d. Comparison of economic steady-state equilibria and gear-constrained MSY. (a,b) Maximization of gear-constrained MSY. (c,d) Maximization of constant annual economic surplus (zero interest rate).

for year 2014, which is used as the new initial state. The procedure is next repeated over 70 years. Observe that while high egg survivability approximately triples recruitment compared to low survivability, the corresponding increase in annual harvest is only approximately 25%. The increase in spawning stock and harvest is realized four years after the occurrence of high egg survivability. This is the delay until the large cohort enters age classes 4–8, which form the main fraction of yield. Because of the age-class structure the stochastic effects of high egg survivability on yield and profit can be anticipated beforehand. As the problem is nonlinear in harvest, it is optimal to spread the increase in harvest over several years. It is optimal to slightly increase the mesh size two years after high egg survivability to prevent catch from the large cohort before it has reached age classes 4–8. Compared to the deterministic solution shown in Fig. 3b, stochasticity increases the average mesh size, but only by 3–5 mm.

As stochastic optimization in our case yields a small effect on harvest compared to a deterministic solution (Fig. 7), the deterministic feedback solution may be expected to potentially be used to approximate the stochastic (feedback) solution. This is tested in Fig. 9a–c, where Figure a shows a stochastic realization of harvest and spawning stock over a 70-year time period. Indeed, the difference in the outcomes is only minor, and hardly visible. We obtain a similar outcome for 10 different stochastic realizations, and Fig. 9b and c show the average difference between the two solution types and the related standard deviations in terms of harvest and spawning stock biomass. The outcome does not change even when the solution is computed 1000 years forward. These computations suggest that in the specified setup the true stochastic solution can be rather accurately approximated applying the deterministic feedback solution, i.e. by repeating the optimization at the beginning of each period using the updated information on the system state.

Earlier stochastic age-structured optimization studies are scarce. Costello et al. (1998) study the value of information and stochastic effects of El Niño events on a three-age-class Coho salmon fishery assuming perfectly selective fishing gear. The El Niño events are discretized to three phases (normal, weak, and strong). The length of the planning horizons in the
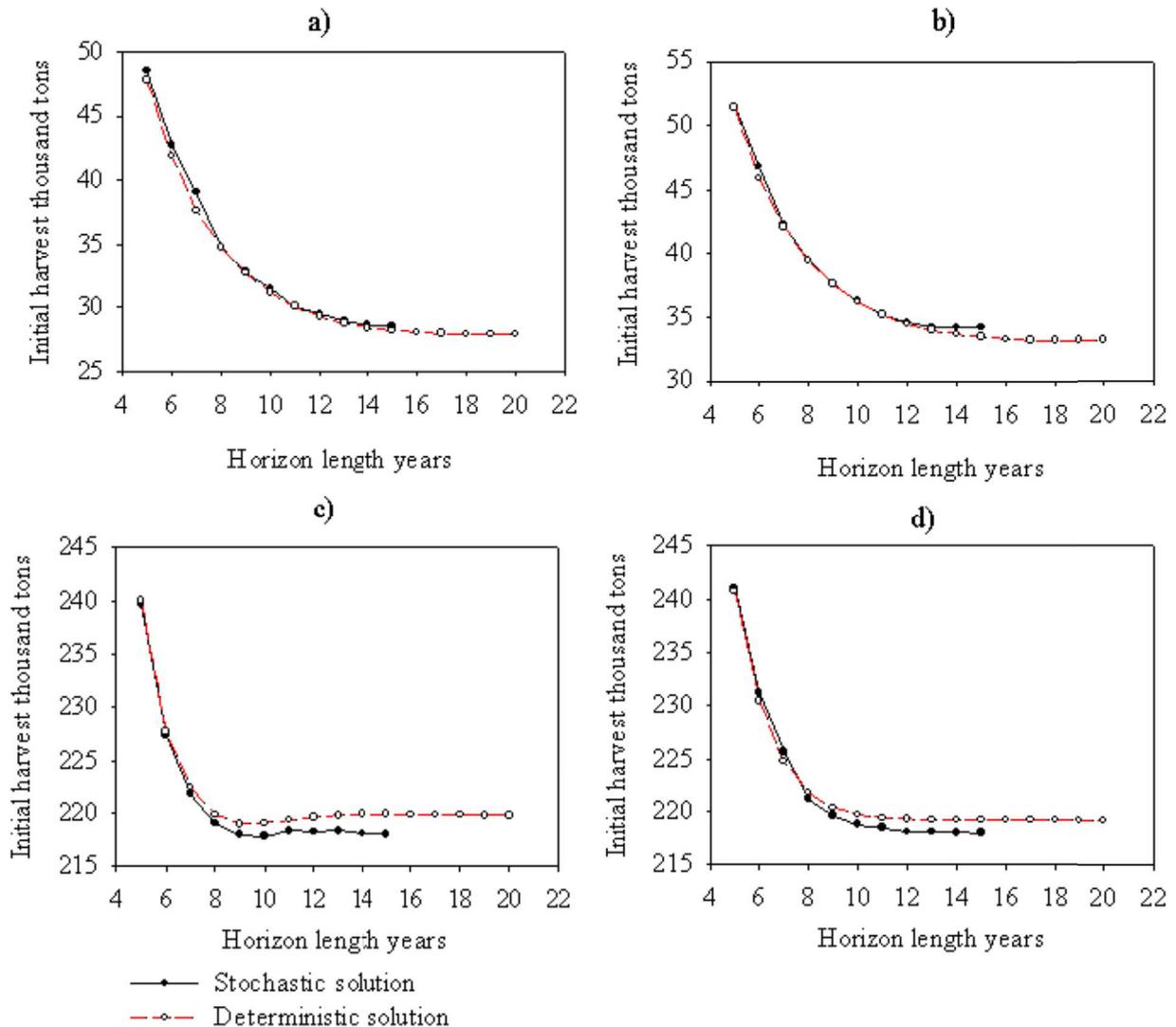
Fig. 7. a–d. Optimal initial harvests at as a function of horizon length a) Initial state: 2013 population, interest rate 10%, b) Interest rate 15% c) Initial state: deterministic steady-state interest rate 10%, d) Interest rate 15%.

computations is four and eight years. According to their results, stochasticity leads to lower harvest compared to solutions without uncertainty, but details are not shown.

Holden and Conrad (2015) assume perfect selectivity in the sense that fishing can either target immatures or adults. If only adults are harvested, the optimal escapement is the same as in the deterministic model. When harvesting immatures only, stochastic harvesting can be more or less conservative compared to deterministic harvesting. Their numerical analysis does not reveal the magnitude of the difference between the stochastic and deterministic solutions. In addition to different selectivity specifications, our model differs from Holden and Conrad (2015), as our objective function is nonlinear and harvesting cost depends on population abundance.

Conclusions

Ecological data on commercially important fish populations is nowadays typically provided in age-structured format. This enables the study of important questions such as the role of fishing technology and age truncation effects, which are beyond reach within the lumped-parameter biomass approach. According to a vast literature in fishery ecology, harvesting activity causes age-class truncation effects with potentially adverse consequences to population recruitment, growth, and stability. Such effects have direct economic relevance and should be taken into account in harvesting decisions. We have developed the economics of age-structured models by the inclusion of optimized harvesting selectivity and stochastic recruitment.
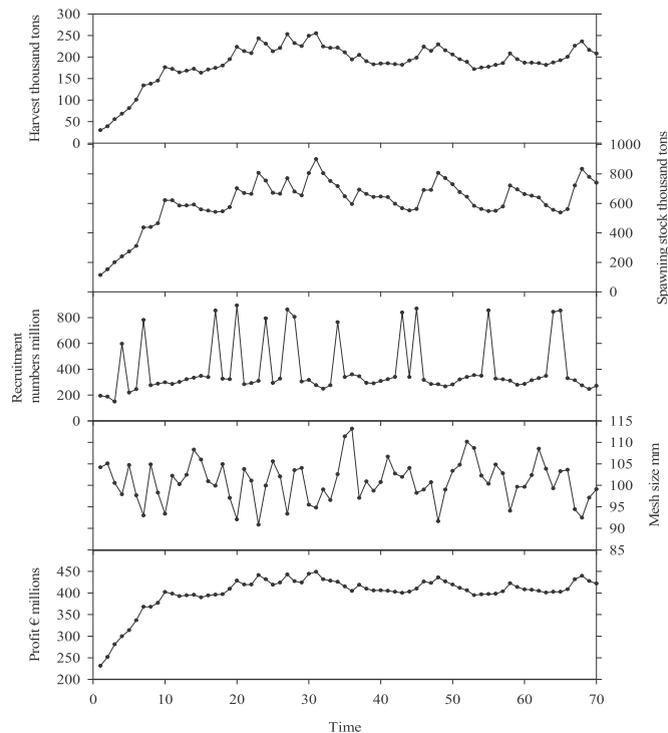
Fig. 8. A stochastic solution example. Note: Initial state: 2013 population, interest rate 10%.

Optimized harvesting selectivity aims to direct fishing activity toward the age classes that would be optimal to harvest if perfectly selective fishing gear were available. Simultaneously, adjusting selectivity has direct consequences on the level of efficient biomass (or effort) that determines harvesting cost. Economically optimal harvesting selectivity recognizes both of these effects, while MSY neglects the harvesting cost effect. This creates a discrepancy between economically optimal and MSY solutions that has been unknown in fishery economics (cf. Wilen, 2000), albeit the classic result in Beverton and Holt (1957, p. 373) of reaching MSY by utilizing a specific mesh size and infinite effort includes a hint of this.[7] A second consequence of optimizing harvesting selectivity was that sustainable harvesting, and MSY in particular, is dependent on fishing gear and technology. This led us to define three different MSY concepts that should be observed when analyzing actual fishery policy that still heavily relies on MSY. A third consequence of endogenous harvesting selectivity is that the level of economically optimal steady-state fish stock may be lower than the MSY stock level even with zero discounting and stock-dependent harvesting cost. This possibility cannot be understood in the traditional biomass model, and it casts a shadow on the debate of whether economic objectives in fisheries promote both conservation and higher economic surplus (cf. Grafton et al., 2007; Clark et al., 2010a, 2010b). We showed that both fishing costs and discounting may tend to reduce optimal stock sizes when taking imperfect and optimized harvesting selectivity into account.

We tested these analytical findings by studying the Baltic cod fishery. Ecologists conjecture that in the Baltic cod fishery case maintaining a large stock of older and larger fish is especially important, as their eggs have higher survivability during adverse environmental conditions. We showed that this conjecture obtains direct support if Baltic cod could be harvested with perfectly selective fishing gear and the aim is ultimate MSY. In contrast, MSY concepts based on existing fishing technology lead to the largest possible mesh size and prohibitively high harvesting cost. Thus, the MSY concept faces serious problems with respect to practicability. We conjecture that similar outcomes may follow in any fishery where catching the ultimate MSY age class is possible by applying costly gear (e.g., fish traps), but some other gear type is preferable when taking cost into account (e.g., trawling gear). Examples for such fisheries include tuna (using purse seines, perhaps even with fish aggregating devices, or more costly but also more selective gears such as hand lines; Groeneveld and Quaas, 2016) and crab fisheries; while our results may be less relevant for anchovy fisheries where age structure plays a less important role.

These findings challenge the UN and European Union policy recommendations that rest on MSY (e.g. Council of the European Union, 2007). Another policy-relevant question is whether aiming for economic objectives leads to a win–win outcome serving both conservation and economic surplus. According to our model for Baltic cod this outcome does not

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[7] However, the result of Beverton and Holt (1957) on the eumetric yield follows a somewhat different logic. Beverton and Holt assume gear with 'knife-edge' selectivity, and the infinite effort in their continuous age model is employed to catch all of the fish exactly at the age where yield is maximized.
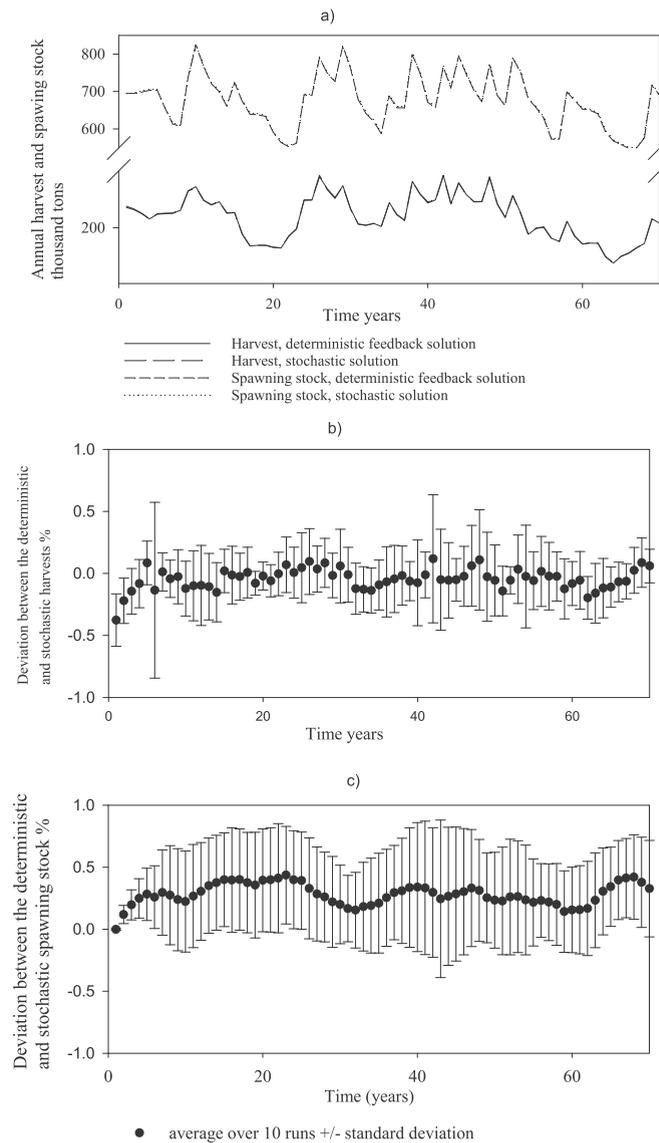
Fig. 9. a–c. Comparison of the stochastic programming and deterministic feedback solutions. Note: Interest rate 10%, horizon length 12 years (in repeated optimization).

hold true: economically optimal steady-state population biomass is lower than MSY biomass even with zero discounting and stock-dependent harvesting cost.

Environmental stochasticity is especially high for Baltic cod and is directly related to the high recruitment value of old age classes during adverse environmental consequences. We study whether our economic results are robust with respect to these factors by solving a nine-state-variable stochastic optimization problem applying stochastic programming. Seasonal variation in recruitment and population levels was high, but the variability in optimal harvest is much lower and approximately ± 25%. Additionally, the stochastically occurring large cohort in an age-structured setup is observed several years before its effects are realized in harvestable age classes. This may partly explain our result in which an optimal stochastic solution can be accurately approximated by the deterministic feedback solution. This is not *a priori* evident, as the model does not satisfy the preconditions for certainty equivalence, i.e. quadratic objective function, linear dynamics, and random walk stochasticity. In spite of these findings it should be noticed that from the management point of view the data requirements are much more demanding under stochasticity and the optimal feedback solution compared to more deterministic setups.

Acknowledgements

Appendix A

Given constant recruitment denoted as ϕ, zero interest rate, $U(H) = H$ and $\chi = 1$ Eq. (23) is quadratic in x_2 with the solution:

$$x_2^{\star}(\gamma) = \varphi \, \frac{\alpha_1 \, q_1}{\alpha_2 \, q_2} \left(\sqrt{\left(1 + \frac{\alpha_2 \, q_2}{(1 - \alpha_2) \, q_1}\right) \left(1 - \frac{w_1}{w_2} \frac{\alpha_2}{\alpha_1} \left(1 - \frac{c}{q_1 \, w_1 \, \varphi}\right)\right)} - 1 \right). \tag{A1}$$

To determine mesh size apply Eq. (25) which by the given assumptions and for interior solutions reads as

$$c \left(\alpha_1 \, q_1' \, \phi + \alpha_2 q_2' \, x_2 \right) = \phi \, x_2 \, \theta, \tag{A2}$$

where $\theta = (q_1' q_2 - q_2' q_1)(\alpha_1 w_2 - \alpha_2 w_1)$. After applying the specification (4) for q_s, $s = 1, 2$, x_2 can be eliminated from (A2) which yields unique solution for the optimal steady state mesh size γ. Applying the parameter values

$$w_1 = 1, \; w_2 = 1.6, \; \alpha_1 = 0.8, \; \alpha_2 = 0.45, \; \vartheta_{11} = 1.5, \; \vartheta_{21} = \vartheta_{12} = \vartheta_{22} = 2, \; \phi = 1, \; \underline{\gamma} = 0, \; \overline{\gamma} = 20,$$

and the definition (3) for the efficient biomass and $B_m = w_1 x_1 + w_2 x_2$ for population biomass yields the solutions shown in Fig. 1. For the solutions with the Beverton-Holt recruitment parameter values are the same except that $\phi(\mathbf{x}) = 5(x_1 + x_2)[1 + 5(x_1 + x_2)]^{-1}$.

Appendix B. Estimation of ecological and economic parameters

Cod exhibits cannibalistic behavior. We capture this age- and density-dependent process based on the approach of Lewy and Vinther (2004). Let δ_s denote an indicator of instantaneous cannibalistic predation mortality that age class s exerts on juveniles. Density dependence of recruitment thus depends on 'cannibalistic' spawning stock biomass $x_{0t}^c = \sum_{s=1}^{n} \delta_s \mu_s w_s x_{st}$. Using estimates of stock numbers, stock weights, and maturity from ICES (2013), and assuming log-normal auto-correlated[8] errors (Cook et al., 1997), we estimate

$$\ln\left\{ \frac{x_{1,t+1}}{\sum_{s=1}^{n} \left[(1 - z_{it})\beta_s^- + z_{it}\beta_s^+\right] \mu_s w_s x_{st}} \right\} = \ln(\phi_0) - \phi_1\left(\sum_{s=1}^{n} \delta_s \mu_s w_s x_{st}\right) + \sigma_t \quad \text{with} \quad \sigma_t = \nu \, \sigma_{t-1} + \xi_{t-1},$$

where ξ_t is IIDN$(0, \sigma^2)$. We obtain $\ln(\varphi_0) = 1.744$ with a 95% confidence interval [1.277,2.211] and $\phi_1 = 0.00157$ [1/1000 t], with a 95% confidence interval [0.0001,0.0031]. For the computations, we use $\phi_0 = \exp(1.744) = 5.720$.

To estimate the parameters of the harvesting cost function, we include an exponential time trend ($t = 2013-$ year of observation), to capture effects of inflation on prices and exogenous technical progress in fishing technology in the open-access condition that price P_t equals the marginal harvesting cost, $P_t = cB_t^{-\chi}$. We estimate the log-linear model

$$\ln(P_t) = c_0 + c_t \, t - \chi \ln B_t + \xi_t,$$

where ξ_t is an IID error term. Applying OLS, we obtain the estimates $c_0 = 1.888$ with 95% confidence interval [1.041,2.734], $c_t = -0.0066$ with 95% confidence interval [$-0.0213,0.0081$], and $\chi = 0.426$ with 95% confidence interval [0.128,0.724], with $R^2 = 0.45$. The confidence interval for χ includes the value $\chi = 0.644$ that Kronbak (2005) reports for this parameter.

Using the specification $U\prime(H_t) = \overline{P} Y_t^{-\eta} H_t^{-\nu}$ for the marginal utility function in the open-access condition $U\prime(H_t) = cB_t^{-\chi}$, and taking logs, yields the following model,

$$\ln(H_t) = a_1 + a_2 Y_t + a_3 t + a_4 \ln(B_t) + \xi_t,$$

where ξ_t is an the IID error term, and again a time trend is included. We use data on catch quantities and efficient biomass, and estimate the coefficients by means of OLS. We obtain the estimates $a_1 = 3.557$ with 95% confidence interval [2.456,4.658], $a_2 = 0.199$ with 95% confidence interval [0.0048,0.350], $a_3 = 0.0098$ with 95% confidence interval [0.0031,0.0164], and $a_4 =$

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[8] We allow for auto-correlation, because there is no direct data on cod of age one. In the stock assessment data the age of recruitment is two years. Point estimates of the coefficients of the stock-recruitment function are very similar with and without allowing for autocorrelation.

0.652 with 95% confidence interval [0.541,0.763], with $R^2 = 0.93$. In the computations, we use $\nu = \chi/a_4 = 0.426/0.652 = 0.653$. Further, we use

$\overline{P} = \exp(c_0 + \nu a_1) = \exp(1.888 + 0.643 \times 3.557) = 67.4$. For the demand function, we use a reference value of Northeast Arctic cod supply of a million tons, and thus use

$$P_t = \overline{P} \ 1000^{0.199 \times 0.654} \ H_t^{-0.654} = 27.434 \ H_t^{-0.654}$$

euros per kg of fish, where $\overline{P} = \overline{P}_0 1000^{0.199 \times .654} = 27.434$. Parameters of the stock-recruitment function and economic parameters are summarized in Table B2.

Table B1
Age class-specific parameter values for Eastern Baltic cod fishery.

Parameter		Age class							
		1	2	3	4	5	6	7	8
Natural survival	α_s	1.00	0.82	0.82	0.82	0.82	0.82	0.82	0.82
High egg survival	β_s^+	0.00	0.607	0.760	0.730	0.710	0.732	0.763	0.861
Low egg survival	β_s^-	0.00	0.017	0.084	0.156	0.232	0.301	0.359	0.473
Maturities	μ_s	0.00	0.13	0.36	0.83	0.94	0.96	0.96	0.98
Cannibalism	δ_s	0.00	0.04	0.60	1.04	1.35	1.30	1.25	1.38
Weights	w_s	0.00	0.177	0.347	0.794	0.912	1.100	1.662	2.740
Catchability	ϑ_{1s}	49.92	74.36	102.52	128.80	152.65	169.52	183.24	195.12
Parameters	ϑ_{2s}	6.24	10.90	13.92	15.82	17.03	18.14	21.61	22.26
Individuals 2013	x_{s0}	194.853	173.859	105.768	63.768	28.198	14.333	5.447	2.298

Table B2
Parameter values for an Eastern Baltic cod fishery.

Parameter	φ_0	φ_1	p	χ	c	ν	$\overline{P}$
Unit	Year^{-1}	1000^{-1} t			10^6 €		€kg^{-1}
Value	5.720	0.00157	0.267	0.426	6.604	0.654	27.434

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