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The Higgs Boson and the Cosmos

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Considerate la vostra semenza:
fatti voi non foste a viver come bruti,
ma per seguir virtute e canoscenza
–*Dante Alighieri*

Abstract

The Higgs boson is a cornerstone of the standard model (SM) of particle physics, thus it comes as no surprise that the announcement of its discovery on July 2012 by ATLAS and CMS marked a very important date for particle physics. All the pieces of the SM have finally been observed and the parameters of the theory measured. However, we know that the SM is far from a complete theory and the fact that the Higgs boson has been, up to date, the only discovery of the LHC may be seen as unfortunate by many. In fact, the LHC is just confirming with exceptional accuracy the predictions of the SM, pushing the scale of new physics to larger and larger values, giving us no hints about its correct extension.

Having measured all the parameters of the SM we can assume its validity to an arbitrarily high energy scale and extrapolate its behavior using the renormalization group equations. It turns out that the value of the Higgs mass is low enough to allow this extrapolation and the SM remains consistent up to the Planck scale. However, this computation reveals yet another puzzle: our universe does not lie in the global minimum of the Higgs potential; instead a much deeper vacuum exists at large field values. In principle, quantum tunneling into the true vacuum is possible but fortunately the decay time is about 10^{130} times the age of our universe. For all practical purposes our vacuum is not in danger and the decay will not happen any time soon. This peculiar situation is called *metastability*.

Since the decay time is very long, new physics modifying the Higgs potential at high energies is not needed. The situation, however, changes dramatically if we want to understand why the Higgs ended up in such an energetically disfavored state in the framework of big bang cosmology. It is clear that some sort of fine-tuning is required in order to put the Higgs in the false vacuum. Not only that: the evolution of the universe goes through violent periods, such as inflation and reheating, where the Higgs may experience large fluctuations, making it difficult to justify why it did not decay into the true vacuum without assuming the existence of physics beyond the SM (BSM).

The Higgs is a natural window into particles which are not part of the SM. In fact, it is the only particle with spin-0 and the only field which can form a dimension-2 gauge and Lorentz invariant operator: $H^\dagger H$. Within the SM this property is used to write a mass term for the Higgs which generates spontaneous breaking of the electroweak symmetry, while in BSM models it allows to write interaction terms at the renormalizable level with gauge singlets and with gravity. In this thesis and in the papers attached we explore the effects that these renormalizable BSM operators have on the Higgs dynamics in the early universe. We show that stabilization of the Higgs field can be obtained in models of inflation if we allow the existence of Higgs-inflaton couplings or non-minimal coupling with gravity. The same models are then studied at the reheating stage, when all the particles that compose the present day universe are produced. On the other hand,

we also explore the possibility that the Higgs mixes with the inflaton. The mixing can stabilize the Higgs potential at all energies and generates two scalar eigenstates. The lighter one is identified with the boson discovered in 2012 and the other could be observed at the LHC or at future colliders.

This thesis is organized as follows. Chapter 1 serves as a brief introduction to the topics in cosmology and particle physics relevant for this thesis. In Chapter 2 we show how the problem of the Higgs metastability arises in the SM, what the implications for the present universe are and demonstrate how the Higgs coupling with a hidden scalar can cure the instability. Implications of metastability for the Higgs dynamics during inflation are discussed in Chapter 3, where we also consider some mechanisms induced by BSM operators which can stabilize the Higgs. In the same chapter we show that a mixing with the inflaton can cure the metastability problem of the SM. In Chapter 4 we study what happens to a metastable universe after inflation, namely during reheating and preheating. In fact, during those stages, couplings of the Higgs to gravity can lead to destabilization of the Higgs field.

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List of Included Papers

The three articles [1–4] included in this thesis are:

1. C. Gross, O. Lebedev and M. Zatta,
   Higgs–inflaton coupling from reheating and the metastable Universe,
   Phys. Lett. **B753** 178-181, [arXiv: 1506.05106]

2. K. Enqvist, M. Karciauskas, O. Lebedev, S. Rusak and M. Zatta,
   Postinflationary vacuum instability and Higgs-inflaton couplings,
   JCAP **1611** (2016) 025, [arXiv: 1608.08848]

3. Y. Ema, M. Karciauskas, O. Lebedev and M. Zatta,
   Early Universe Higgs dynamics in the presence of the Higgs-inflaton and non-minimal Higgs-gravity couplings,
   JCAP **1706** (2017) 054, [arXiv: 1703.04681]

4. Y. Ema, M .Karciauskas, O. Lebedev, S. Rusak and M. Zatta,
   Higgs-Inflaton Mixing and Vacuum Stability,
   Under peer review, [arXiv: 1703.04681]

The articles will be referred to as Paper 1-4 throughout this thesis.

Author's Contribution

Paper 1: The author of the present manuscript performed the analytic calculations included in the paper and prepared notes that were later included in the paper.

Paper 2: The author took care of the numerical simulations included in the paper by modifying the software `Latticeasy`. The author also wrote a draft of Section 6 of the paper.

Paper 3: Numerical simulations with a modified version of `Latticeeasy` were performed by the present author. In addition, the author contributed in writing Section 4 and Appendix A of the paper.

Paper 4: The author participated in the numerical and analytical computations of the paper. For numerical computations a self-written `Python` code were used. The draft of Section IV was written by the author.

Contents

Chapter 1

Cosmology and particle physics

This chapter serves as an introduction to the topics of cosmology and particle physics which are relevant for the studies performed in this thesis and in the papers attached. We begin with a description of the homogeneous and isotropic universe. We then introduce the motivations for having a period of inflation in the early universe and describe the properties of particle physics models of inflation. We end the chapter with a brief introduction to the standard model of particle physics and introduce the Higgs boson and the Higgs mechanism.

In this thesis we use natural units, by setting $\hbar = c = k_B = 1$, and take $M_{\rm Pl} = (8\pi G_N)^{-1/2} = 1$.

1.1 The Friedman-Robertson-Walker universe

The most important fact about our universe is that it looks homogeneous and isotropic on large scales. Homogeneous since the physical conditions are the same at each point, and isotropic since the physical conditions look the same in all directions when viewed from a given point. This implies that the evolution of the universe can be represented geometrically as a sequence of three-dimensional space-like hypersurfaces, where each hypersurface is homogeneous and isotropic.

The metric of such homogeneous and isotropic universe is called Friedman-Robertson-Walker (FRW) metric and is determined by the following equation

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right)\right], \tag{1.1}$$

where k is related to the spatial curvature of the space-like hypersurface, with $k = -1, 0, 1$ corresponding to an open, flat or closed universe. The scale factor $a(t)$ characterizes the relative size of hypersurfaces at different times.

Observations tell us that the universe is expanding according to the Hubble law, that is the distance between two objects A and B evolves according to

$$\dot{r}_{AB} = H(t)\, r_{AB}\,. \tag{1.2}$$

The quantity H is called the Hubble parameter and represents the velocity of expansion of the universe. The relation between the scale factor and the Hubble parameter is

$$\frac{\dot{a}}{a} = H\,, \tag{1.3}$$

where a dot means derivative with respect to the time t.

According to General Relativity, the dynamics of the universe can be determined starting from the Einstein-Hilbert action

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{2}R + \mathcal{L}_{\text{matter}}\right)\,, \tag{1.4}$$

where g is the determinant of the metric $g_{\mu\nu}$ that determines the spacetime interval $\mathrm{d}s^2$ in (1.1), R is the Ricci scalar of curvature and $\mathcal{L}_{\text{matter}}$ is the Lagrangian density of the matter content of the universe. The variation of the action with respect to the metric gives the Einstein equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = T_{\mu\nu}\,, \tag{1.5}$$

where R is the Ricci tensor and $T_{\mu\nu}$ is the stress-energy tensor of matter.

The Einstein equations determine the dynamics of the universe as soon as the explicit form of the stress-energy tensor is given. Assuming that matter behaves like a perfect fluid, equations (1.5) take the form of the *Friedman equations*

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\rho - \frac{k}{a^2}\,, \tag{1.6}$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3p)\,, \tag{1.7}$$

where ρ and p are the energy density and pressure of the fluid in its rest frame.

If we combine the Friedman equations (1.6) and (1.7) and assume that there is no intrinsic curvature of space $k = 0$, we can write the continuity equation

$$\frac{\mathrm{d}\ln\rho}{\mathrm{d}\ln a} = -3(1+w)\,, \tag{1.8}$$

where the equation of state parameter is defined as

$$w \equiv \frac{p}{\rho}\,. \tag{1.9}$$

Integrating equation (1.8) yields

$$\rho \propto a^{-3(1+w)}\,. \tag{1.10}$$

The most important classes of perfect fluid that we will encounter in this thesis are three: non-relativistic matter (or dust), radiation and vacuum energy. We briefly describe their properties.

Non-relativistic matter Non relativistic matter, or dust, is characterized by an equation of state parameter $w = 0$, which gives $\rho \propto a^{-3}$. Introducing this energy scaling into the first Friedman equation (1.6) we get that the universe expands as $a \propto t^{2/3}$ if it is dominated by dust.

Radiation Relativistic matter, or radiation, is characterized by $w = 0$, which implies $\rho \propto a^{-4}$. We immediately notice that the energy density of radiation decreases faster with the expansion of the universe than the energy density of non-relativistic matter. For a radiation dominated universe, the first Friedman equation (1.6) yields $a \propto t^{1/2}$.

Vacuum energy The equation of state parameter of vacuum energy is $w = -1$. Consequently its energy density does not scale with the expansion of the universe and remains constant. From (1.6) we observe that the expansion of the universe is exponential $a \propto e^{Ht}$ if vacuum energy dominates.

1.1.1 The Concordance Model

The Concordance Model, also known as the ΛCDM model, is the theoretical model that best fits cosmological observations. In this section we describe what its properties are.

Observations of the cosmic microwave background (CMB) and large-scale structures find that the universe is flat. An equivalent statement is that the energy density of the universe is very close to the critical density ρ_c, which is defined from the first Friedman equation (1.6) by taking $k = 0$ as

$$\rho \sim \rho_c = 3H_0^2 \,, \tag{1.11}$$

where the subscript 0 denotes the present value of the Hubble rate[i]. In particular, observations show that the energy density of the universe is composed of 30% matter, 70% vacuum energy and a negligible amount of radiation [5]. The contribution of matter is further divided in ordinary baryonic matter, which amounts only to 5% of the total energy density of the universe, and the remaining 25% in the form of invisible matter, the so-called cold dark matter. In particular, cold dark matter (CDM) and vacuum energy Λ are the reason for the name ΛCDM.

An important consequence of the domination of vacuum energy is that the universe expansion is accelerating. In fact, the equation of state parameter for vacuum energy is $w = -1$, which implies $p = -\rho$. If we introduce this value into the second Friedman equation (1.7) we get

$$\frac{\ddot{a}}{a} = \frac{2\rho}{3} > 0 \,. \tag{1.12}$$

We note that this is true only for vacuum energy. A matter dominated or radiation dominated universe experiences decelerated expansion.

[i] We note here that the value of the critical density is not a constant in time but depends on the value of the Hubble parameter, which is an evolving quantity.

In the previous section we noted that ρ for dust and radiation decreases as the universe expands while that of vacuum energy remains constant. In particular, the energy density of radiation decreases faster than that of dust. This implies that, if we go backward in time, the universe was initially dominated by radiation and subsequently by non-relativistic matter. Only at a later time vacuum energy began to dominate.

1.2 Inflation

1.2.1 ΛCDM puzzles

Although the concordance model is successful in describing many aspects of our universe, there are some things which are left unexplained. Among these *puzzles* we discuss here the horizon and the flatness problems.

Horizon problem

The horizon problem is the problem to explain why the universe was extremely homogeneous at the time of last-scattering, as observed from the CMB. This fact seems to imply that the whole portion of the universe which is probed by the CMB was in causal contact at the time of CMB formation. Unfortunately, according to the ΛCDM model the CMB sky was composed of many causally disconnected regions and it is not possible to justify its homogeneity.

The maximum distance that a light ray can travel between time 0 and time t is called the *particle horizon* τ, it determines the causal structure of space-time and is defined as [6, 7]

$$\tau \equiv \int_0^t \frac{\mathrm{d}t'}{a(t')} = \int_0^a \frac{\mathrm{d}a}{a^2 H} \,. \tag{1.13}$$

For a universe dominated by a perfect fluid with equation of state parameter w, the particle horizon scales as

$$\tau \propto a^{\frac{1}{2}(1+3w)} \,. \tag{1.14}$$

We immediately notice that for matter and radiation the particle horizon increases monotonically with time. In the concordance model, the universe was dominated first by radiation and then by matter, and only relatively recently by vacuum energy. The particle horizon thus has always increased its size, meaning that regions of the sky that are in causal contact today were not in the past. In particular at the time of CMB formation, large portions of the sky were not in causal contact and therefore there is no reason why the temperature of two arbitrary points in the CMB sky should look so homogeneous.

Flatness problem

The flatness problem is related to the question: why the energy density today is so close to the critical density ρ_c? In fact, the first Friedman equation (1.6) can be divided by H^2 to get

$$1 - \Omega = -\frac{k}{(aH)^2}\,, \tag{1.15}$$

where we defined $\Omega \equiv \rho/\rho_c$. The value of Ω observed today is very close to one which implies that the universe is almost flat. From eq. (1.15) we see that if the universe is not exactly flat the right-hand side grows in time for a matter or radiation dominated universe. This means that if Ω is close to one today, in the past it was even closer. In particular, to explain the value of Ω observed today we would have to fine-tune its value in the early universe in such a way that $|\Omega(a_{\rm Pl}) - 1| \leq \mathcal{O}(10^{-61})$, where Ω is taken at the Planck scale. This is a huge fine-tuning in the initial conditions which the concordance model fails to explain.

1.2.2 Inflation to the rescue

The idea of a period of accelerated expansion in the early universe, dubbed *inflation*, was developed to solve the problems that we have described in the previous section [8–10]. The idea comes from a very simple observation. The horizon and flatness problems stem essentially from the fact that the comoving Hubble radius, $(aH)^{-1}$, increases with time in a matter dominated universe, where the expansion is decelerating. On the other hand, when the universe expands with increasing velocity, the comoving Hubble radius becomes smaller.

When the comoving Hubble radius shrinks, regions that are in causal contact lose their causal relation. The horizon problem can then be solved if we assume that the whole CMB sky was in causal contact in the early universe and that a period of inflation made it causally disconnected later on. With this assumption the fact that the CMB has approximately the same temperature everywhere stops to be surprising.

For what concerns the horizon problem, the crucial observation is that when the universe expands exponentially the right-hand side of (1.15) becomes smaller and smaller, pushing to zero the value of $|1 - \Omega|$. This can explain why we find today that $\Omega \approx 1$.

In order to get inflation the comoving Hubble radius has to decrease:

$$\frac{\mathrm{d}}{\mathrm{d}t}(aH)^{-1} < 0\,. \tag{1.16}$$

Some algebra can show this to be equivalent to require accelerated expansion of the universe

$$\ddot{a} > 0\,. \tag{1.17}$$

From the second Friedman equation (1.7) we immediately notice that accelerated expansion requires $p < -\rho/3$, or equivalently an equation of state parameter $w < -1/3$. This condition is fulfilled by vacuum energy. Thus, inflation can be obtained if we find a mechanism that can generate a vacuum energy-like equation of state.

1.2.3 Slow-roll inflation

The simplest model of inflation involves a scalar field ϕ. Since this field is responsible for inflation it is usually called *inflaton*. The Einstein-Hilbert action (1.4) with the matter sector completely dominated by a scalar field has the form

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] , \tag{1.18}$$

where $V(\phi)$ is the inflaton potential and we assume that the scalar field is minimally coupled with gravity, i.e. that there are no direct couplings between ϕ and R. The stress-energy tensor for the scalar field is

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}\partial^\sigma\phi\partial_\sigma\phi + V(\phi)\right) , \tag{1.19}$$

and the equation of motion

$$\frac{\delta S}{\delta\phi} = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\partial^\mu\phi) + V'(\phi) = 0\,, \tag{1.20}$$

where a prime denotes derivative with respect to the field ϕ. Assuming FRW metric (1.1) and homogeneity in the spatial component of the inflaton field $\phi(t, \mathbf{x}) = \phi(t)$, the energy density and pressure take the form

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)\,, \tag{1.21}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)\,. \tag{1.22}$$

The equation of state parameter is then given by

$$w_\phi = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)}\,. \tag{1.23}$$

It is easy to see that accelerated expansion, or equivalently $w_\phi < -1/3$, can be obtained if the potential energy is much larger than the kinetic energy of the inflaton. We note that, if the potential energy dominates, we have $w_\phi \approx -1$, which is the same equation of state parameter as that of vacuum energy.

The inflaton equation of motion and the Hubble rate evolution can be obtained from (1.6) and (1.20)

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \quad \text{and} \quad 3H^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi)\,. \tag{1.24}$$

The inflaton slowly rolls if the conditions $|\ddot{\phi}| \ll |3H\phi|$ and $\dot{\phi}^2 \ll V$ are fulfilled. When this happens the *slow-roll parameters*

$$\varepsilon_V(\phi) = \frac{1}{2}\left(\frac{V'}{V}\right)^2 , \tag{1.25}$$

$$\eta_V(\phi) = \frac{V''}{V} , \tag{1.26}$$

are much smaller than unity[ii]. Inflation ends when $\varepsilon \approx 1$ or $|\eta| \approx 1$.

We now turn to consider a simple model of inflation by considering a scalar potential $V(\phi) = m_\phi^2\phi^2/2$. This is known as squared chaotic inflation. In this model inflation ends when $\varepsilon_V(\phi_{\rm end}) \approx 1$, which happens when the inflaton takes the value

$$\phi_{\rm end} = \sqrt{2} . \tag{1.27}$$

Therefore, in order to get inflation, we must consider super-Planckian values for the inflaton field.

The number of e-folds to the end of inflation is defined as

$$N(\phi) \equiv \ln \frac{a_{\rm end}}{a} . \tag{1.28}$$

In order to solve the horizon and flatness problem the required number of e-folds should be larger than $N_{\rm tot} \geq 40 - 60$, depending on the uncertainty in the reheating dynamics and the scale of inflation. The requirement puts a lower bound on the initial value of the inflaton ϕ_i, that in the model at hand translates into $\phi_i > 15$.

1.2.4 Quantum fluctuations during inflation

So far, in our discussion of inflation we considered only the homogeneous inflaton. However, the world we live in is not classical and thus we expect the inflaton to be a quantum field. Probably the biggest success of inflation is determined by its ability to explain the generation of the small inhomogeneities that later evolved into the structures that we observe in the universe today. This is made possible by the fact that, during inflation, quantum fluctuations are stretched to macroscopic scales [13, 14]. These fluctuations become classical and eventually lead to the formation of galaxy clusters and other large scale structures.

The best probe for the quantum fluctuations are the temperature perturbations observed in the CMB. A discussion of how these fluctuations are generated is beyond the scope of this thesis. Nevertheless, quantum fluctuations have a very important implication on the issue of the Higgs

[ii]Strictly speaking these are necessary but not sufficient conditions to ensure slow-roll of the inflaton. An alternative version of the slow-roll approximation based on the Hamilton-Jacobi formulation [11] exists which is both necessary and sufficient [12].

instability, that is the primary focus of this thesis, because we can determine the value of the Hubble rate from them.

Quantum fluctuations of wavelength k evolve only when $k > aH$, that is when the scale k is *inside the horizon.* During inflation aH increases and an increasing number of ultraviolet scales exits the horizon. When fluctuations are outside the horizon, i.e. $k < aH$, they get frozen to a constant value and stop evolving. In particular, scales that exit the horizon at the time of CMB formation determine completely the value of the temperature perturbations. It turns out to be possible to relate the power spectrum of the CMB perturbations $\mathcal{P}_{\mathcal{R}}$ with the Hubble rate at the time of CMB formation

$$\mathcal{P}_{\mathcal{R}} = \left(\frac{H^2}{2\pi\,\dot{\phi}} \right)^2 , \tag{1.29}$$

where both H and $\dot{\phi}$ are evaluated at the time of CMB formation. The measured value of the power spectrum $\mathcal{P}_{\mathcal{R}} = 2.2 \times 10^{-9}$ [15] is thus an important constraint not only on the parameters of the inflaton potential but also on the value of the Hubble rate during inflation. In Section 3.1 we will see that the size of the Hubble rate is relevant for the dynamics of the Higgs during inflation.

1.3 The Standard Model of particle physics

After the discussion of the cosmological topics which are relevant for this thesis, it is now time to turn our focus to particle physics, in particular to the Standard Model.

The Standard Model (SM) of particle physics is the theory that describes interactions between particles, the fundamental indivisible components of matter. It has been proven successful in describing the physics of particles up to energies of today's most powerful collider, the LHC. The mathematical framework on which the SM is built is provided by quantum field theory. The construction of any quantum field theory proceeds by first postulating the symmetries of the system and then writing down the most general renormalizable Lagrangian that respects these symmetries. The symmetries of the SM are Lorentz and gauge invariance. In particular, the gauge group of the SM is $SU_c(3) \otimes SU_L(2) \otimes U_Y(1)$. The different subgroups are related to three of the four fundamental forces of nature, respectively strong, weak and electromagnetic. Thus, the SM does not unify all three fundamental forces under the same description. In fact, it leaves out gravity, which is described by General Relativity.

Particles in the SM are completely characterized by their spins, masses and charges. Quarks and leptons are SM particles with half-integer spins and interact via the strong, weak and electromagnetic interactions, which are mediated by the gauge bosons that have integer spin equal to 1. The Higgs is another boson, the only one that does not mediate any gauge interactions and the only spin-0 particle of the SM. All fermions of the SM except neutrinos are massive[iii]. On the other hand, only two gauge bosons have non-zero mass: the $W^{\pm}$ and Z bosons, which are the mediators of the weak forces.

The way in which gauge bosons and fermions get their masses in the SM is very peculiar. An explicit mass term for these particle would in fact break the gauge invariance of the theory. Therefore, we have to resort to a trick in order to introduce fermions, leptons and weak gauge boson masses. This trick is the Higgs mechanism.

1.4 The Higgs Mechanism

The Higgs mechanism was formulated independently by Higgs [18, 19], Brout and Englert [20], Guralnik, Hagen and Kibble [21] in order to generate masses for the gauge bosons and fermions via the so-called *spontaneous symmetry breaking* of the gauge symmetry. This mechanism is useful in order to break the gauge subgroup $SU_L(2) \otimes U_Y(1)$ of the SM into the electromagnetic group $U_{\text{em}}(1)$ and is at the foundation of the electroweak (EW) theory.

Spontaneous symmetry breaking is obtained in the SM with the introduction of a complex scalar doublet H, the Higgs doublet, charged under $SU_L(2) \otimes U_Y(1)$, that is made of four scalars

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \eta_1 + i\eta_2 \\ \eta_3 + i\eta_4 \end{pmatrix} . \tag{1.30}$$

The potential for this field is gauge invariant and can be written as

$$V(H) = -\mu_h^2 H^\dagger H + \lambda_h \left(H^\dagger H\right)^2 . \tag{1.31}$$

However, the parameters of the potential are chosen in such a way that the doublet develops a non-zero vacuum expectation value (VEV)

$$\langle H \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} , \tag{1.32}$$

that does not respect the gauge symmetry. The choice of which component obtains non-zero VEV is dictated from the requirement that the vacuum must have zero electric charge, in such a way to preserve electromagnetic gauge invariance.

In this thesis we will usually use gauge symmetry by rewriting the Higgs doublet in the so-called unitary gauge as

$$H(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} , \tag{1.33}$$

[iii] This turns out to be a shortcoming of the SM. In fact, neutrino oscillations indicate that at least some of the neutrinos have non-zero mass [16, 17]

where $h(x)$ is a real scalar degree of freedom, the Higgs boson, and the other three degrees of freedom of H are rotated in the gauge boson sector, where they become the longitudinal polarization of the $W^{\pm}$ and Z gauge bosons.

The Higgs mechanism introduces two parameters in the SM: the Higgs VEV v, which is determined by the weak gauge boson masses, and the Higgs self-coupling λ_h, which can be related to the Higgs boson mass as $m_h = \sqrt{2\lambda_h}v$. The measurement of the Higgs boson mass at the LHC on July 2012 [22, 23] allowed us to determine all the parameters of the SM model. We will show in the next chapter that the value of the mass is deeply connected with the problem of the EW vacuum metastability.

Cosmological drawbacks of the Standard Model

Since this thesis focuses on the connections between particle physics and cosmology, we briefly discuss what the cosmological drawbacks of the SM are.

In the previous sections we have encountered the notion of dark matter. This is a particular kind of non-relativistic matter that does not interact with light. Unfortunately, the SM does not accommodate any candidate that can explain the observed abundance of dark matter. The only particles in the SM that do not interact with photons are neutrinos but unfortunately their abundance is much less than that of dark matter. We are then forced to consider beyond the SM (BSM) theories in order to explain dark matter.

Another shortcoming of the SM is that it cannot explain how the matter-antimatter asymmetry generated. In fact, we observe a large amount of baryonic matter but no anti-baryonic matter in the universe. The SM lacks a mechanism that provides the origin of the observed matter-antimatter asymmetry [24].

Finally, the SM does not have a mechanism that can provide inflation. The only scalar particle of the SM, the Higgs boson, has an unstable potential at high energies, as we will see in the next chapter, and therefore some BSM physics is required in order to get a flat potential that can sustain inflation. Therefore, the SM must be extended in order to get inflation.

We thus see that the SM is far from a complete theory and extensions to the SM must be postulated if we want to be able to describe the universe we live in. In this thesis we follow a very minimalistic approach. We do not try to find a solution to all problems of the SM, but we decide to focus only on inflation and on the Higgs vacuum metastability, leaving out from our discussion the nature of dark matter and the origin of the matter-antimatter asymmetry.

Chapter 2

The Higgs instability

In this chapter we introduce the issue of the EW vacuum metastability in the SM. Metastability means that the EW vacuum is not the global minimum of the SM Higgs potential and that the decay time is much longer than the age of the universe. The problem of the EW vacuum metastability is deeply connected with the history of our universe. As we will see in Chapters 3 and 4, it has important implications on the Higgs dynamics in the early universe. In this chapter we review the literature on this topic and in the final section provide a mechanism that can stabilize the Higgs potential by adding just an additional singlet to the SM. This mechanism turns out to be useful in connecting the Higgs boson with cosmology when we identify the singlet with the inflaton.

2.1 Stability of the Higgs effective potential

The discussion of the EW vacuum metastability is deeply connected with the quantum nature of the SM. The SM is a quantum field theory which cannot be solved exactly and thus has to be computed using perturbation theory. During the computation of higher orders of the perturbative expansion, we encounter divergences in some processes due to the so-called loop integrals. The systematic approach for eliminating divergences from measurable quantities is called *renormalization*. Renormalization requires to regularize a theory by introducing sensitivity to a cutoff: a parameter that allows to make the divergences finite. In the limit that the cutoff disappears, physical quantities diverge. Thus, it seems that the procedure of regularization introduces an additional parameter to the theory, the cutoff. However, when we compute measurable quantities, we see that a change in the cutoff can be compensated by a change in the couplings so that physical quantities are left invariant. Therefore, couplings develop a dependence on the energy scale at which the renormalization procedure is performed. This dependence is given by the *renormalization group equations*.

The EW vacuum metastability originates precisely from the scale dependence of the Higgs self-coupling introduced by the renormalization group. Let us show how this scale dependence is computed in the SM.

Quantum corrections can be found by computing the effects of virtual particle emission and reabsorption on the interaction energy. The result of this computation is the effective potential [25]. In the Landau gauge at one loop, the SM effective potential takes the form [26]

$$V_{\rm eff} = V_0 + V_{\rm gauge} + V_{\rm scalar} + V_{\rm fermion} \tag{2.1}$$

where

$$\begin{aligned} V_0 &= -\frac{1}{2}\mu_h^2 h^2 + \frac{1}{4}\lambda_h h^4\,, \quad V_{\rm gauge} = \frac{3\left(2g^4 + (g^2 + g'^{\,2})^2\right)}{1024\pi^2} h^4 \ln\frac{h^2}{M^2} \\ V_{\rm scalar} &= \frac{1}{64\pi^2}\left(-\mu_h^2 + 3\lambda_h h^2\right)^2 \ln\frac{-\mu_h^2 + 3\lambda_h h^2}{M^2} + \frac{3}{64\pi^2}\left(-\mu_h^2 + \lambda_h h^2\right)^2 \ln\frac{-\mu_h^2 + \lambda_h h^2}{M^2}\,, \\ V_{\rm fermion} &= -\frac{3y_t^4}{64\pi^2} h^4 \ln\frac{h^2}{M^2}\,, \end{aligned} \tag{2.2}$$

where g and g' are the gauge couplings, y_t is the top Yukawa coupling and M is the renormalization scale. The expression of the effective potential has been divided into four different parts, where V_0 is the tree level potential. The other three parts are the loop contributions which correspond to the different virtual particles that we are considering when computing loop corrections. In particular, we see that all particles that couple with the Higgs contribute. The scalar part is further divided into the contribution coming from the Higgs boson itself and the Goldstones, the other scalar degrees of freedom of the Higgs doublet. Contributions from other fermions have been neglected since their Yukawa couplings are much smaller than y_t.

We immediately notice from equations (2.2) that there are two competing effects. On the one hand, Higgs and gauge bosons loops give a positive contribution to the effective potential. On the other hand, fermions give a negative contribution. For very large h these can push the potential towards positive or negative values depending on the relative size of the couplings. It has long been noted that fermions could destabilize the potential, and in fact many attempts to find upper bounds on the fermion masses, or equivalently lower bounds on the Higgs mass, were made in the literature by demanding stability of the SM vacuum up to some arbitrary scale [27–35].

Unfortunately, the effective potential as is written in (2.2) cannot give reliable predictions on the large field behavior of the model. The reason is that the loop expansion depends on the combination

$$\alpha^{(n+1)}\left[\ln\!\left(h^2/M^2\right)\right]^n\,, \tag{2.3}$$

where α is the largest coupling in the model and n is the number of loops. If we suppose the renormalization scale M is much smaller than h the logarithm can become very large and consequently the combination (2.3) bigger than unity. This would make the perturbative expansion unreliable because higher order terms would matter as much as lower order terms.

In order to give reliable predictions we can resort to the renormalization group to find a *renormalization group improved* expression for the effective potential. As we mentioned at the beginning of the section, the renormalization group provides the dependence on the renormalization scale of the different couplings. The dependence can be obtained as follows.

The renormalization scale M in (2.2) is completely arbitrary and therefore the effective potential cannot be affected by a change in M. This statement allows us to write the following renormalization group equation for the effective potential

$$\left[M\frac{\partial}{\partial M}+\beta_{\lambda_h}\frac{\partial}{\partial\lambda_h}+\beta_{g_i}\frac{\partial}{\partial g_i}+\beta_{y_t}\frac{\partial}{\partial y_t}+\beta_{\mu_h^2}\mu_h^2\frac{\partial}{\partial\mu_h^2}-\gamma h\frac{\partial}{\partial h}\right]V_{\rm eff}=0\,, \tag{2.4}$$

where the beta functions define the dependence of the couplings with the respect to the renormalization scale M. For instance, the Higgs self coupling beta function is given by

$$\beta_{\lambda_h}=M\frac{\partial\lambda_h}{\partial M}\,, \tag{2.5}$$

and there are similar expressions for the mass parameter μ_h^2, for the gauge couplings g_i and for the Yukawa couplings. The γ function is the anomalous dimension and depends on the wave function renormalization Z [36]. Equation (2.4) is a special case of the Callan-Symanzik equation [37,38].

It is possible to solve (2.4) and find a general expression for $V_{\rm eff}$ that is solution to the Callan-Symanzik equation[i]. Since the instability scale of the SM appears only for very large values of the field h, for which $h\gg\mu_h$, we can consider just an approximated form for the renormalization group improved effective potential. At one loop it takes the form [40]:

$$V_{\rm eff}(h)\simeq\frac{1}{4}\lambda_h(t)\,[G(t)h]^4\;, \tag{2.6}$$

where $t=\ln(h/M)$ and

$$\begin{aligned}&\frac{{\rm d}\lambda_h}{{\rm d}t}=\beta_{\lambda_h}(g_i(t),\lambda_h(t)),\quad\frac{{\rm d}g_i}{{\rm d}t}=\beta_{g_i}(g_i(t),\lambda_h(t)),\\&G(t)=\exp\left(-\int_0^t{\rm d}t'\gamma(g_i(t),\lambda_h(t))\right)\,.\end{aligned} \tag{2.7}$$

We see from (2.6) that the study of the SM instability reduces to the study of the sign of $\lambda_h(t)$ for large t. In particular, we can solve the differential equation determined by the beta function β_{λ_h} giving the evolution of λ_h and check at what scale $\Lambda_{\rm inst}$ the Higgs self coupling turns negative

$$\lambda_h(\Lambda_{\rm inst})\simeq 0\,. \tag{2.8}$$

[i]For details on the computation we refer for instance to [39].

The beta function for λ_h cannot be solved exactly because it depends on all other couplings of the model for which there are similar beta functions governing their evolution. Numerical methods are thus needed for solving a system of coupled differential equations in order to find Λ_{inst}.

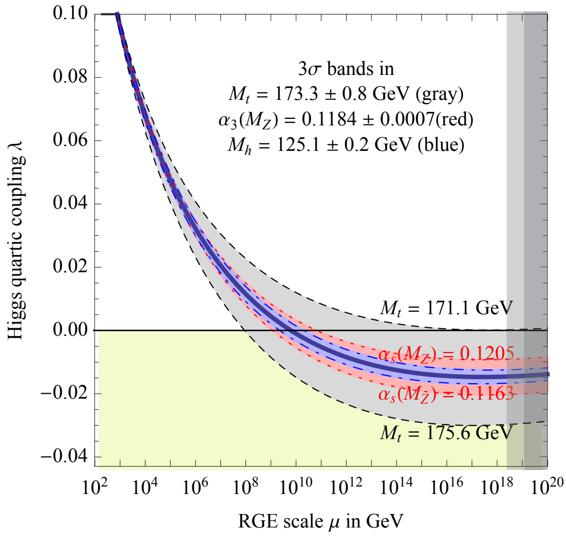

Figure 2.1: Evolution of λ_h as a function of the energy scale. Shaded regions represent the different uncertainties. Image taken from [41].

In Fig. 2.1 we report the result that we would obtain by solving the system of coupled differential equations determined by the beta functions of the couplings and assuming that the SM is valid up to the Planck scale. The result is taken from [41][ii], where the extrapolation of the SM model parameters is performed with 3-loop precision. We observe that, for the current measured value of the QCD coupling constant, Higgs boson and top mass the Higgs self coupling turns negative at a scale which is roughly $\Lambda_{\text{inst}} \simeq \mathcal{O}(10^{10})$.

Much of the uncertainty in the quoted results comes from the determination of the SM parameters at the EW scale. Results in Fig. 2.1 are obtained from a 2-loop extraction from physical observables. We see that the largest uncertainty on the instability scale is given by the value of the top mass. This has basically to do with the difficulty to define a pole mass for the top quark because, being a colored particle, it does not actually exist as an asymptotic state. Supposing that the top is about 1 GeV lighter than the current measured value, the SM can still be completely stable. Thus, a better measurement of the top quark mass would be very important in determining the stability of the SM. Better precision on the top quark mass will be reached only at future electron-positron colliders where the uncertainty will be pushed from one GeV to

[ii]Similar studies can be found for instance in [42–48].

hundreds of MeV [49–52].

2.2 The metastable universe

The existence of a deeper minimum in the SM opens up the possibility for the Higgs to decay via quantum tunneling. Perhaps surprisingly, the parameters of the SM are such that the decay time is much longer than the age of the universe. This peculiar state goes with the name of *metastability.* The long lifetime implies that no new physics is needed to explain why we live in such an energetically disfavored state and the existence of our universe is perfectly consistent with metastability.

The theory of vacuum decay for a scalar theory with two classically stable minimum configurations was first developed in [53–55] and applied to the case of the SM for example in [45,49,56]. The qualitative picture of the decay is similar to that of a superheated fluid. Bubbles of true vacuum can form due to quantum fluctuations and, if a large enough bubble forms, it starts to grow and expands over the whole universe converting the false vacuum to true. The bubble has to be large enough such that the internal energy is larger than the surface energy, because the formation of the bubble wall has an energy cost that can kill it.

The computation of the decay rate per unit volume γ of the false vacuum decay yields the following expression

$$\gamma = \mathcal{A} e^{-\mathcal{B}}\,, \tag{2.9}$$

where $\mathcal{B}$ is the Euclidean action of the *bounce*, the solution of the classical field equations that interpolates between the false vacuum and the opposite side of the barrier, i.e. the bubble, and $\mathcal{A}$ is a pre-factor which takes account of the fluctuations around the bounce.

The bounce action is easily computed in the case of the SM Higgs. Let us denote with h the radial component of the Higgs doublet. The bounce is a solution $H = h(r)$ that depends only on the radius r of the 4D Euclidean space which obeys

$$-\partial_\mu \partial_\mu h + V'(h) = -\frac{\mathrm{d}^2 h}{\mathrm{d}r^2} - \frac{3}{r}\frac{\mathrm{d}h}{\mathrm{d}r} + V'(h) = 0\,, \tag{2.10}$$

satisfying boundary conditions

$$h'(0) = 0, \quad h(\infty) = 0\,, \tag{2.11}$$

where in the second condition we assume the false minimum of the potential is at 0 rather than at v. This is due to the large h assumption for which we approximate $V(h) \simeq \lambda_h h^4/4$. For $\lambda_h < 0$ the tree level computations gives [57]

$$h(r) = \sqrt{\frac{2}{|\lambda_h|}}\frac{2R}{r^2 + R^2}, \quad \mathcal{B} = \frac{8\pi^2}{3|\lambda_h|}\,, \tag{2.12}$$

where R is an arbitrary scale. The arbitrariness is a consequence of the scale invariance of the potential in the quartic approximation. There is also an ambiguity on which scale μ we have to take to evaluate the running coupling $\lambda_h(\mu)$. These can be resolved when a complete loop computation is performed. In particular, the scale R is integrated to get an expression for the decay rate and, to make the integral finite in the infrared, μ has to be chosen as the scale at which the beta function $\beta_{\lambda_h}(\mu)$ vanishes [58].

We note that taking $\lambda_h < 0$ makes $h = 0$ a maximum of the potential. This might seem quite odd but in quantum field theory the tunneling requires non-zero kinetic energy, and therefore it is suppressed even in absence of a barrier[iii].

The computation of the pre-factor $\mathcal{A}$ is instead much more involved, and the complete treatment would be out of the scope of this thesis. For details on the most recent computations of the SM vacuum decay we refer to [58, 60]. Here we limit to quote some of the results obtained in those papers.

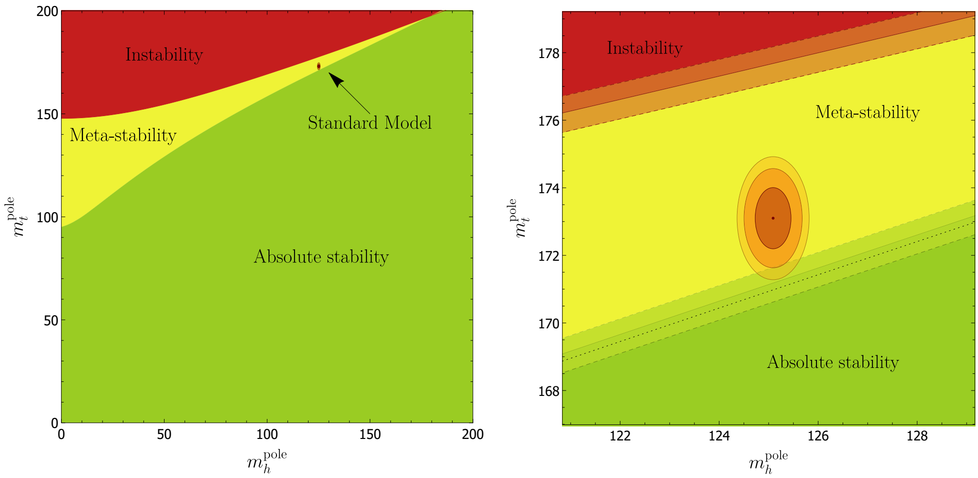

Figure 2.2: Phase diagram in the top and Higgs mass plane. Ellipses show the 68%, 95% and 99% contours based on the experimental uncertainties on the Higgs and top pole masses. The shaded bands on the phase boundaries, framed by the dashed lines and centered on the solid lines, are combinations of the α_s experimental uncertainty and the theory uncertainty. Images taken from [58].

Figure 2.2 shows the phase diagram in the top and Higgs mass plane. The SM lies in a small band of metastability between absolute stability and complete instability.

[iii] The same argument can be used to explain why cosmic rays or a single Higgs particle with momentum larger than Λ_{inst} do not destabilize our vacuum [59].

The lifetime of the universe is given by $\gamma^{-1/4}$, for which the SM parameters yield [58]

$$\tau_{\rm SM} = 10^{139}\,\text{years}\,. \tag{2.13}$$

This has to be compared with the age of our universe which is of the order of 10^9 years. We thus see that the difference is really huge and the metastability does not endanger to the existence of our universe today. The largest uncertainty in the determination of $\tau_{\rm SM}$, which is not reported in (2.13), comes from the determination of the top mass, strong coupling constant and theory uncertainty due to the matching of the pole masses and $\overline{\rm MS}$ parameters at the EW scale.

2.3 Stabilization of the EW vacuum by a singlet

While the existence of a much deeper minimum in the SM potential is not a problem in itself, it is hard to justify how the Higgs ended up in the false vacuum in the early universe. Understanding the implications of metastability on the cosmological history of our universe will be in fact the main subject of the following chapters. However, here we would like to introduce a very simple mechanism that can provide a way to stabilize the Higgs EW vacuum completely. This mechanism requires just a minimal assumption of physics beyond the SM, namely the addition of a new scalar. Scalars give a positive contribution to the Higgs effective potential and thus can modify the RGE running of λ_h by giving a positive correction to its beta function. The mechanism that we introduce, however, is more powerful and relies on the fact that the singlet develops a large VEV. In that case the singlet scalar gives a tree level correction that can make the Higgs self-coupling positive all the way up to the Planck scale [61, 62].

Let us suppose that the SM plus singlet scalar potential in the unitary gauge is

$$V(\phi, h) = \frac{\lambda_h}{4}h^2 - \frac{\mu_h^2}{2}h^2 + \frac{\lambda_{h\phi}}{2}h^2\phi^2 + \frac{\lambda_\phi}{4}\phi^4 - \frac{\mu_\phi^2}{2}\phi^2\,, \tag{2.14}$$

where we have assumed a $\mathbb{Z}_2$ symmetry for the singlet. We are interested in the situation where both fields develop a VEV: $\langle h\rangle = v$ and $\langle\phi\rangle = u$. Provided that the minimum of the potential is situated in (v, u), the VEVs generate a mixing between the Higgs and the singlet into two scalar eigenstates with masses given by

$$m_{1,2}^2 = \lambda_h v^2 + \lambda_\phi u^2 \mp \sqrt{(\lambda_\phi u^2 - \lambda_h v^2)^2 + 4\lambda_{h\phi}^2 u^2 v^2}\,. \tag{2.15}$$

Assuming that the lighter state is "Higgs-like" and expanding for $u \ll v$, we have

$$m_1^2 \simeq 2\left(\lambda_h - \frac{\lambda_{h\phi}^2}{\lambda_\phi}\right)v^2\,. \tag{2.16}$$

The presence of the Higgs-singlet coupling generates a negative correction to the mass squared of the Higgs implying, for a given Higgs mass, a larger value of λ_h than in the SM case. The Higgs self coupling therefore can remain positive at all scales, ensuring stability of the potential.

A similar effect can be obtained for a singlet scalar with no discrete symmetries. Let us consider for example the following potential

$$V(\phi, h) = \frac{\lambda_h}{4} h^2 - \frac{\mu_h^2}{2} h^2 + \frac{\lambda_{h\phi}}{2} h^2 \phi^2 + \sigma h^2 \phi + \frac{\lambda_\phi}{4} \phi^4 + \frac{b_3}{3} \phi^3 - \frac{\mu_\phi^2}{2} \phi^2 + b_1 \phi \,. \tag{2.17}$$

We have included all terms up to dimension four and allowed by the symmetries of the SM. This potential has a symmetry $\phi \to \phi + c$, with c constant, accompanied by the corresponding shift in the parameters which leaves the potential formally invariant [63, 64]. The symmetry can be used to get $u = \langle \phi \rangle = 0$. Even in the absence of singlet VEV we obtain a mixing between the Higgs and the singlet due to the presence of the trilinear coupling σ that can stabilize the Higgs potential. This mechanism was actually employed in [4] by assuming that the stabilizing singlet is the inflaton field. This is an exciting example of how the Higgs can act as a link between cosmology and particle physics, enabling direct detection of the inflaton at colliders. The inflationary dynamics of such model will be analyzed towards the end of the next chapter.

Chapter 3

Higgs dynamics during inflation

The inflationary period can be dangerous for a metastable universe. The main reason is that, if no new physics enters before the inflationary scale, the Higgs is a light field and its fluctuations are proportional to the Hubble rate. If this is too large, the Higgs might be pushed beyond the instability barrier of the SM. In this chapter we describe the evolution of a metastable Higgs during inflation: under which conditions the field decays into the true vacuum and what are the minimal solutions to this problem. At the end of the chapter we briefly discuss the evolution of a stable Higgs and show the implications of mixing with a light inflaton.

3.1 Higgs fluctuations and vacuum decay

Quantum fluctuations are dangerous for the EW vacuum because they can push the Higgs over the instability barrier. During inflation, the wavelength of quantum fluctuations grows as the universe expands. When the wavelength of a particular fluctuation crosses the horizon, its amplitude freezes and remains unchanged. This corresponds to the creation of a classical field with an average amplitude proportional to $H/2\pi$. Every fluctuation that exits the horizon contributes to the classical field, but the phases of different fluctuations are not the same. This makes the evolution of the classical field resemble that of a particle in Brownian motion. The evolution of the Higgs can therefore be computed with a stochastic approach, where the long wavelength modes behave as a classical field, and the quantized short wavelength modes provide a stochastic term for the equation of motion of the classical field. The classical part is assumed constant within each causally connected patch that forms during inflation and evolving in time according to the stochastic nature of the process.

Understanding the behavior of h is crucial for understanding whether the Higgs decays into the true vacuum of its potential during inflation. The nature of the decay depends on the size of H with respect to the instability scale of the SM Higgs potential Λ_{inst}. If H is smaller than $\Lambda_{\rm inst}$, the EW vacuum can decay according to the Coleman-de Luccia bubble nucleation process [65], while the transition is well described by the Hawking-Moss instanton [66] if H is comparable to $\Lambda_{\rm inst}$. In the literature these methods have been applied to the case of the EW vacuum instability for instance in [67–69]. In this thesis, we describe the Higgs dynamics using the so-called stochastic approach. This is done by modeling the probability distribution to find the Higgs at a given value h at time t, which is denoted with $P(h,t)$. This computation had been performed in the literature either by studying the evolution of $P(h,t)$ using the Fokker-Planck equation [70–73], or by computing the evolution of h via the Langevin equation [74,75]. Here we discuss the approach followed in [75] and model the evolution of h using the Langevin equation. The noise source of the equation is given by the quantum fluctuations of the Higgs field.

The Langevin equation for the long wavelength modes for the effectively massless Higgs can be written as

$$\frac{\mathrm{d}h}{\mathrm{d}N} + \frac{1}{3H^2}V'(h) = \eta(N)\,, \tag{3.1}$$

where for simplicity we have replaced the time with the number of e-folds $N = Ht$, and where $\eta(N)$ is Gaussian random noise satisfying

$$\langle \eta(N)\,\eta(N')\rangle = \frac{H^2}{4\pi^2}\delta(N-N')\,. \tag{3.2}$$

The Langevin equation allows to generate iteratively many random realizations of the Higgs evolution at a given number of e-folds N starting from an initial value $h(0)$. This is done using

$$h(N+\mathrm{d}N) = h(N) - \frac{V'}{3H^2}\mathrm{d}N + r\,, \tag{3.3}$$

where r are random numbers which are extracted from a Gaussian distribution with zero mean and variance $\sigma^2 = H^2\,\mathrm{d}N/4\pi^2$. The quantity r is generated by the Gaussian noise and depends completely on the properties of the quantum fluctuations.

Suppose that at the beginning of inflation the Higgs is found at the origin, that is $h(0) = 0$. Using (3.3) we find the position of the Higgs at a given e-fold N. By repeating this process many times, we are able to extract the probability distribution $P(h,N)$. Provided that we take $H \simeq \Lambda_{\rm inst}$, what we find is a Gaussian with mean 0 and variance that grows with $\sqrt{N}$. The motivation for this behavior is the following: the quantum noise term in eq. (3.3) dominates the Higgs evolution since, near the instability scale, the Higgs quartic coupling vanishes making the gradient of the potential negligible. This allows the Higgs to fluctuate beyond the instability barrier during inflation without feeling the slope of the potential and preventing the decay into the true vacuum. The decay happens when the Higgs value becomes so large that the potential term dominates.

When the Higgs starts to fall into the true vacuum of the SM potential, the energy density of the causally connected patch becomes negative creating an AdS bubble. This stops inflation locally and initiates a crunching process inside the bubble. However, this happens only inside the bubble, in fact the shell of the negative energy region expands into the surrounding spacetime making it possible for the bubble to persist after inflation is over. Therefore, at the end of inflation, this AdS bubble starts to expand at the speed of light and swallows up all the other regions that enter in causal contact with it. The presence of one of such bubbles in our past light-cone would be incompatible with the existence of our universe. Requiring that no bubble formed in our past lightcone results in the following bound on the Hubble rate and the value of the Higgs at the local maximum of its potential: $H/h_{\max} \lesssim 0.07$ [76].

The situation differs for patches of the universe where the Higgs is found close to the maximum $h_{\max}$ at the end of inflation but where the fall into the true vacuum has not started yet. The fate of these regions strongly depends on post-inflationary dynamics. Thermal effects, which would kick in during reheating, might lift the true vacuum and push the Higgs towards the origin. In that case we can expect that, as the temperature decreases, the Higgs will find itself trapped in the EW vacuum.

Before closing this section we would like to make one final remark. Changing the initial Higgs value from $h(0) = 0$ to a different value increases the probability of the decay because the resulting Gaussian has a shifted mean towards the instability barrier. This would require a lower Hubble rate during inflation in order to avoid vacuum decay. What is more important is that the actual initial Higgs value is not fixed by any dynamics and, thus, it is completely arbitrary. As a matter of fact, assuming that the Higgs starts in the EW vacuum corresponds to a fine-tuning of the Higgs initial conditions. This introduces an additional problem connected to the vacuum stability related to the initial conditions of the Higgs field. The assumption that the Higgs starts at the origin is in principle just an assumption. Requiring that the Hubble rate is small during inflation is not enough to ensure stability if we cannot explain why the Higgs started its evolution in the EW vacuum and not in the true vacuum in the first place.

3.2 Stabilizing the Higgs during inflaton

The problem described in the previous section stems essentially from the fact that the Higgs mass is typically much smaller than the Hubble rate during inflation. Therefore, the simplest solution would be to assume the existence of a mechanism that generates a positive effective mass for the Higgs which is larger than the Hubble rate during inflation. In this thesis we focus on two BSM operators that can produce such a mass: a direct coupling of the Higgs to the inflaton in the form $\lambda_{h\phi}\phi^2 H^\dagger H$, and a non-minimal coupling of the Higgs with gravity $\xi_h H^\dagger H R$. It is remarkable that both couplings are dimension 4 and also perfectly consistent with the symmetries of the SM.

We begin by discussing the implications of a Higgs mass m_h larger than the Hubble rate on the Higgs dynamics in a toy model. In this model we have only the Higgs field and the inflaton. In the unitary gauge the scalar sector potential takes the form

$$V(\phi, h) = V(\phi) + \frac{1}{2} m_h^2 h^2 + \frac{1}{4} \lambda_h h^4 \,. \tag{3.4}$$

Note that we do not specify any particular form for the inflaton potential. The Hubble rate for the Higgs-inflaton system is determined from the first Friedman equation (1.6) and can be written as

$$3H^2 = \frac{1}{2}\dot{h}^2 + \frac{1}{2}\dot{\phi}^2 + V(\phi, h) \,. \tag{3.5}$$

Since we are interested in the inflationary stage, the time derivative of the inflaton is negligible because it is slowly rolling. In addition, we require that $m_h^2 h^2/2$ dominates the Higgs potential and that the Higgs self-interaction is negligible.

When h is large, the Hubble rate is dominated by the Higgs effective mass term and the Higgs equation of motion becomes

$$\ddot{h} + \sqrt{\frac{3}{2}} \sqrt{\dot{h}^2 + m_h^2 h^2}\, \dot{h} + m_h^2 h = 0 \,. \tag{3.6}$$

Let us follow the solution of this equation presented in [6]. In this regime the Hubble rate evolution is given by

$$6H^2 \simeq \dot{h}^2 + m_h^2 h^2 \,. \tag{3.7}$$

This allows us to use the Hubble rate H and θ as the radius and the angle of a new polar coordinate system satisfying the following relations

$$\dot{h} = \sqrt{6} H \sin\theta \,, \tag{3.8}$$

$$m_h h = \sqrt{6} H \cos\theta \,. \tag{3.9}$$

In terms of these new variables and from (3.6) we get the following differential equations

$$\dot{H} = -3H^2 \sin^2\theta \,, \tag{3.10}$$

$$\dot{\theta} = -m_h - \frac{3}{2} H \sin 2\theta \,. \tag{3.11}$$

The first of these equations tells us that H decays in time. Thus, for large t, we can neglect the last term on the right-hand side of (3.11) and integrate it

$$\theta \simeq -m_h t \,. \tag{3.12}$$

Substituting this into (3.10) we obtain

$$H(t) \simeq \frac{2}{3t} \left(1 - \frac{\sin(2 m_h t)}{2 m_h t} \right) \,. \tag{3.13}$$

This solution is valid only in the limit $m_h t \gg 1$, therefore we can expand the term in round brackets in powers of $(m_h t)^{-1}$. Substituting (3.12) and (3.13) into (3.9), we obtain

$$h(t) \simeq \sqrt{\frac{8}{3}} \frac{\cos(m_h t)}{m_h t} + O\left((m_h t)^{-2}\right) . \tag{3.14}$$

We see from (3.14) that the amplitude of h decreases with time, thus pushing the Higgs towards the origin. When h decreases enough and $m_h^2 h^2/2$ becomes smaller than the inflaton potential $V(\phi)$, we enter in another regime, where the Hubble rate evolution is well approximated by

$$3H^2 \simeq V(\phi) . \tag{3.15}$$

During inflation the inflaton slowly rolls and maintains approximately a constant value, implying that also the Hubble rate remains constant in time. The Higgs equation of motion

$$\ddot{h} + 3H\dot{h} + m_h^2 h = 0 \tag{3.16}$$

during that regime has the two solutions

$$h_\pm = C_\pm \exp\left[-\frac{t}{2}\left(3H \pm \sqrt{9H^2 - 4m_h^2}\right)\right] . \tag{3.17}$$

Since we are assuming $m_h^2 \gg H$ we get $|h(t)| \simeq |h(0)|\, e^{-3Ht/2}$, showing that the Higgs amplitude goes to zero exponentially.

We have seen that the easiest way to overcome problems related to the EW vacuum instability during inflation is to assume a mechanism that generates a Higgs effective mass larger than the Hubble rate. This mechanism allows us to solve the issues that were raised in the previous section: the problem of the Higgs initial conditions and the decay due to large quantum fluctuations. At the beginning of this section we quoted two BSM operators that can generate a large mass term for the Higgs. Let us see how these work in details.

3.2.1 Stabilization via Higgs-inflaton couplings

Let us first consider the Higgs-inflaton coupling and take quadratic chaotic inflation as the inflationary model [77]. This way of stabilizing the Higgs was first discussed in [78]. In this model the scalar potential has the form

$$V(\phi, h) = \frac{m^2}{2}\phi^2 + \frac{\lambda_{h\phi}}{2}\phi^2 h^2 + \frac{\lambda_h}{4} h^4 . \tag{3.18}$$

As the first constraint we should require that the portal coupling $\lambda_{h\phi}$ does not lead to large radiative corrections to the inflaton potential, as that would spoil the predictions of the inflationary model. The radiative corrections can be estimated, for instance, via the computation of the Coleman-Weinberg effective potential [25]. The largest correction comes from the term

$$\Delta V \simeq \frac{\lambda_{h\phi}^2}{16\pi^2}\phi^4 \log \frac{\lambda_{h\phi}\phi^2}{m_\phi^2} . \tag{3.19}$$

When we take the typical values $m \simeq 10^{-5}$ and $\phi \sim 10$, we get the following upper bound for the portal coupling

$$\lambda_{h\phi} \lesssim 10^{-6} \,. \tag{3.20}$$

Secondly, we require that the Higgs potential is dominated by the coupling with the inflaton. This puts a lower bound on the inflaton field value:

$$\phi > \sqrt{\frac{|\lambda_h|}{2\lambda_{h\phi}}}\, h \,. \tag{3.21}$$

A well motivated assumption is to take the Higgs initial value smaller than the Planck scale, that is $h(0) \simeq 0.1$, in order to keep higher dimensional operators unimportant. In addition, taking also $|\lambda_h| \simeq 0.1$ we get from (3.21) that $\phi \gtrsim 20$. We just note here that this is just a representative value, in fact the initial value of the Higgs can be lower and thus also the lower bound of the inflaton can decrease accordingly. However, the values we find are perfectly consistent with those we quoted in Section 1.2.3.

The portal coupling $\lambda_{h\phi}$ has a lower bound which is given by requiring the Higgs effective mass to be larger than the Hubble rate. Clearly, if the Hubble rate is dominated by $m_h^2 h^2$ the condition is automatically satisfied for $h < 1$. On the other hand, when the Hubble rate is dominated by the inflaton mass, the Higgs effective mass is larger than the Hubble rate only if

$$m_h^2 > H^2 \longrightarrow \lambda_{h\phi} > \frac{m_\phi^2}{6} \simeq 10^{-10} \,. \tag{3.22}$$

The coupling $\lambda_{h\phi}$ is not the only Higgs-inflaton coupling affecting the Higgs dynamics. In principle, also a trilinear coupling of the form $\sigma H^\dagger H \phi$ is present if the inflaton does not possess any symmetry preventing it. We would like, however, to point out that stabilization with that coupling is a bit trickier in the sense that the mechanism is operative depending on the sign of the inflaton field and on the sign of σ. Therefore, although it is possible to stabilize the Higgs with the trilinear coupling, we prefer to focus only on the quartic coupling.

To summarize, we have seen that the Higgs-inflaton coupling can provide a natural mechanism for explaining why the universe ends up in the EW vacuum at the end of inflation. The coupling must be found in the range

$$10^{-10} \lesssim \lambda_{h\phi} \lesssim 10^{-6} \,, \tag{3.23}$$

where the upper bound is to avoid too large radiative corrections to the inflaton potential, and the lower bound by requiring that the Higgs effective mass is larger than the Hubble rate during inflation.

3.2.2 Stabilization via non-minimal coupling with gravity

In this section we explain how a Higgs non-minimally coupled with gravity can develop a mass which is Higher than the Hubble rate. This mechanism was first suggested in [71] and later discussed also in [79]. The non-minimal coupling of the Higgs with gravity has the interesting property of being generated via loop corrections and thus it runs with the renormalization scale according to equations that can be found in [79–82]. The fact that it runs implies that even if we suppose that at some scale is set to zero, a non-zero value will be generated at a different scale. Therefore, if we want to consider a realistic model for the Higgs evolution, we must take its effect into consideration. Besides, we would like to point out that also the Higgs-inflaton coupling runs with the energy [1], and thus must be taken into account for the same reasons. In a realistic setting we thus expect both couplings to be present at the same time. Nevertheless, in this section we restrict our discussion to the case where the Higgs-inflaton coupling $\lambda_{h\phi}$ is neglected.

The starting point is the action of the system in the so-called Jordan frame

$$S = \int \mathrm{d}^4x \sqrt{-\hat{g}} \left[\frac{1}{2}(1-\xi_h h^2)R - \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2}\partial_\mu h \, \partial^\mu h - \frac{m_\phi^2}{2}\phi^2 - \frac{\lambda_h}{4}h^4 \right], \tag{3.24}$$

where ξ_h is a dimensionless coupling constant and a hat denotes the metric in the Jordan frame. Again we consider quadratic chaotic inflation even though the results is applicable to other models as well.

The usual procedure to follow when we have a scalar field non-minimally coupled with gravity is to perform a conformal transformation of the metric [83],

$$g^{\mu\nu} = \Omega^2 \hat{g}^{\mu\nu}, \tag{3.25}$$

where the rescaling factor is defined as

$$\Omega^2(h) = 1 - \xi_h h^2 . \tag{3.26}$$

The resulting frame is called the Einstein frame and the action becomes

$$S = \int \mathrm{d}^4x \sqrt{-g} \left[\frac{1}{2}R - \frac{1}{2\Omega^2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2}\frac{6(\xi_h h)^2 + \Omega^2}{\Omega^4}\partial_\mu h \, \partial^\mu h - \frac{m_\phi^2}{2\Omega^4}\phi^2 - \frac{\lambda_h}{4\Omega^4}h^4 \right]. \tag{3.27}$$

In this frame the gravity part of the action takes the usual Einstein-Hilbert form that we introduced in (1.4), at the expense of introducing non-canonical kinetic terms for the inflaton and the Higgs. Unfortunately, the field space is curved and therefore it is not possible to get two canonically normalized fields at the same time. In our discussion we choose to canonically normalize only the Higgs field via the transformation

$$\mathrm{d}h_c = \sqrt{\frac{6\xi_h^2 h^2 + \Omega^2}{\Omega^4}} \, \mathrm{d}h , \tag{3.28}$$

where the canonically normalized field is denoted with the subscript c. The differential equation 3.28 can be solved analytically. However, we are interested only in the regime where h_c is much smaller than the Planck scale and we make the approximation

$$\xi_h h_c^2 \,,\, \xi_h^2 h_c^2 \ll 1 \,. \tag{3.29}$$

In this regime the Higgs in the Jordan frame and the canonically normalized Higgs in the Einstein frame are related as follows

$$h_c \simeq h \left[1 + \left(\xi_h + \frac{1}{6}\right)\xi_h h^2\right] \,. \tag{3.30}$$

The potential in the Einstein frame becomes

$$U(\phi, h_c) = \frac{V(\phi, h_c)}{\Omega^4} = \frac{m_\phi^2}{2}\phi^2 + \xi_h m_\phi^2 \,\phi^2 h_c^2 + \frac{\lambda_h}{4} h_c^4 + \ldots \,, \tag{3.31}$$

where dots denote higher than 4 dimensional operators which are small and therefore neglected.

It is then apparent from (3.31) that the Higgs-inflaton and the non-minimal coupling with gravity are equivalent in this model. In fact, the equivalence between the model described in the previous section and the one we are discussing becomes manifest when we perform the substitution

$$\xi_h m_\phi^2 \to \frac{\lambda_{h\phi}}{2} \tag{3.32}$$

in the potential. The constraints we found in (3.23) can then be applied also to ξ_h and the resulting allowed range becomes

$$0.1 \lesssim \xi_h \lesssim 10^4 \,, \tag{3.33}$$

where again the upper bound is given by requiring small radiative corrections to the inflaton potential and the lower bound by requiring that the Higgs effective mass is larger than the Hubble rate during inflation. We note that in this approximation additional corrections come from other operators such as $h_c^2\, \partial_\mu \phi \,\partial_\nu \phi$. These can be neglected because they lead only to higher derivative interactions and small corrections to the kinetic terms.

We return to the relation between the EW vacuum stability and the Higgs-inflaton and non-minimal coupling with gravity in the next chapter, when we discuss the Higgs evolution during reheating.

3.3 Higgs evolution in a stable potential

So far we have discussed the implications of the EW vacuum instability on the Higgs evolution during inflation. We have discussed temporary solutions to the instability problem, where the Higgs potential is modified during inflation but left untouched at lower energies. However, we showed in Section 2.3 that just adding a SM gauge singlet is enough to make the Higgs completely stable. In such a case, the quantum fluctuations and the initial conditions would not be a problem for the universe evolution because the EW vacuum would be the global minimum of the Higgs potential.

At the beginning of inflation the Higgs value can be slightly below the Planck scale and thus much higher than the Hubble rate. In that case the Higgs is effectively massive and oscillates with decreasing amplitude similar to the one we described in Section 3.2. Eventually the Higgs value becomes small such that its effective mass drops below the Hubble rate. From that point on the Higgs becomes effectively massless and its dynamics can be described following the stochastic approach introduced in Section 3.1. The resulting probability distribution for the Higgs value has zero mean, due to the symmetricity of the potential, but non-zero variance [74, 84] which corresponds to the creation of a Higgs condensate. This condensate will eventually decay into SM particles after inflation. This is what happens if the stable Higgs is just a spectator during inflation. However, its role can be much more active in the early universe.

The exciting possibility that the Higgs itself can be the inflaton opens up if the SM Higgs potential is completely stable [82, 85–88]. In this model the Higgs is coupled non-minimally with gravity in the same fashion as in (3.24) with the field ϕ absent. In order to get the correct size of temperature perturbations in the CMB spectra, the non-minimal coupling with gravity has to take a very large value, that is of the order of 10^4 [89]. A very nice feature of the model, besides explaining inflation with just the particle content of the SM, is that the predicted tensor-to-scalar ratio r lies right in the sweet spot of the data observed by PLANCK [15]. Unfortunately, models with a non-minimally coupled Higgs-like field[i] have cutoff scale lower than the Planck scale [90–93]. The problem lies essentially in the fact that the energy scale of inflation in these models $1/\sqrt{\xi_h}$ is higher than the lowered cutoff scale $1/\xi_h$. However, some authors disagree on the actual existence of the unitarity problem. The proposed argument is that the cutoff of the model depends on the background field value of the Higgs. During Higgs-inflation that value is large and this increases the cutoff to values larger than $1/\xi_h$ [94, 95]. Apart from these considerations, it is easy to find solutions to the unitarity problem by adding new degrees of freedom. For instance, we can assume the existence of an additional real scalar that completes the theory of Higgs-inflation in the ultraviolet [96, 97]. Completing the theory increases the cutoff, thus preserving unitarity during inflation. Unfortunately, the addition of new degrees of freedom makes the Higgs-inflation scenario less appealing because the biggest advantage, that is the minimality of the model, is lost.

3.4 Higgs-inflaton mixing

In the last section of this chapter we discuss a very simple model that shows how the Higgs can relate collider physics to early universe cosmology. In this model the Higgs potential is made completely stable via the mechanism described in Section 2.3, where the role of the singlet field

[i] We refer to fields with a quartic potential as Higgs-like.

is played by the inflaton. We suppose that both fields are non-minimally coupled with gravity and that all operators up to dimension 4, Lorentz invariant and allowed by the SM gauge groups are present in the model. The action in the Jordan frame takes the form

$$S = \int \mathrm{d}^4x\sqrt{-\hat{g}}\left[\frac{1}{2}\Omega^2\hat{R} - \frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu h\partial_\nu h - V(\phi,h)\right], \tag{3.34}$$

where

$$\Omega^2 = 1 + \xi_\phi\phi^2 + \xi_h h^2\,, \tag{3.35}$$

$$V(\phi,h) = \frac{\lambda_h}{4}h^4 - \frac{\mu_h^2}{2}h^2 + \frac{\lambda_{h\phi}}{2}h^2\phi^2 + \sigma h^2\phi + \frac{\lambda_\phi}{4}\phi^4 + \frac{b_3}{3}\phi^3 - \frac{\mu_\phi^2}{2}\phi^2 + b_1\phi\,. \tag{3.36}$$

Actually, also the term $\phi\hat{R}$ should be included in the action (3.34) since it respects all the criteria of dimensionality and symmetry that we are considering. However, it is possible to eliminate it by a field redefinition of ϕ[ii].

The study of the model proceeds in the usual way. We first make a conformal transformation $g^{\mu\nu} = \Omega^2\,\hat{g}^{\mu\nu}$ in order to go to the Einstein frame and then introduce canonically normalized fields. Unfortunately, as was the case in Section 3.2.2, it is not possible to canonically normalize both fields simultaneously. Moreover, the non-minimal couplings introduce kinetic mixing in the form $\xi_\phi\,\xi_h\,g^{\mu\nu}\partial_\mu h\,\partial_\nu\phi$ [83], which makes the disentanglement of the two fields even more complicated. To keep the model tractable we are thus forced to make some simplifying assumptions.

We first assume that inflation is driven only by the inflaton in a way similar to that of Higgs-inflation. This assumption has two main benefits. The first one is that the model, as is the case for Higgs-inflation, fits PLANCK data very well [15], since the predicted tensor-to-scalar ratio is very small. The second is that λ_ϕ is a free parameter and it is possible to find a sizable region of parameter space for which the energy scale of inflation is lower than the cutoff scale, thus preserving unitarity. A second assumption is that the Higgs is heavy during inflation, which makes its value smaller than that of the inflaton, $h \ll \phi$, via the mechanism described in Section 3.2. Also, we assume that the Higgs non-minimal coupling is smaller than that of the inflaton, $|\xi_h| \ll \xi_\phi$. Finally, assuming the dimensionful parameters are far below the Planck scale, all terms except $\lambda_\phi\phi^4$ and $\xi_\phi\phi^2 R$ can be neglected during inflation [4].

Our assumptions allow us to introduce a canonically normalized inflaton field via the transformation

$$\frac{\mathrm{d}\chi}{\mathrm{d}\phi} = \frac{\sqrt{1+\xi_\phi(1+6\xi_\phi)\phi^2}}{1+\xi_\phi\phi^2}\,, \tag{3.37}$$

which, for large $\xi_\phi\phi \gg 1$, gives $\chi \simeq \sqrt{\frac{2}{3}}\ln\xi_\phi\phi^2$ and scalar potential

$$U(\chi) \simeq \frac{\lambda_\phi}{4\xi_\phi^2}\left(1 - e^{-\sqrt{\frac{2}{3}}\chi}\right)^2, \tag{3.38}$$

[ii] See [98] for a recent study with also ϕR present.

where $U \equiv V/\Omega^4$. The constraints on ξ_ϕ and λ_ϕ coming from the CMB normalization [89] and unitarity are

$$\lambda_\phi \lesssim 2 \times 10^{-5}\,, \tag{3.39}$$

$$\xi_\phi \lesssim 2 \times 10^{2}\,. \tag{3.40}$$

Further constraints are imposed on the parameter $\lambda_{h\phi}$. One is imposed by requiring that radiative corrections to the inflaton potential generated by Higgs loops are small. Secondly, $\lambda_{h\phi}$ contributes to the RGE running of λ_ϕ with a positive sign. This makes the value of λ_ϕ grow with the energy scale and violate the unitarity bound during inflation. In [4] it was found that the second requirement gives a more severe constraint.

This model introduces a mixing between the Higgs and the inflaton at low energies which opens up the exciting possibility to observe the inflaton at colliders such as the LHC. In fact, the mixing creates two scalar particles with different masses and the lighter one can be identified with the scalar particle that has been observed at the LHC. On the other hand, the heavier scalar state could be observed in the future via its decay into the lighter state [64, 99–101]. Furthermore, the mixing can be observed as a universal reduction of the Higgs couplings to gauge bosons and fermions [102].

Chapter 4

Higgs dynamics after inflation

The reheating stage is very important for the universe evolution. This is the moment in the cosmic history when particles we observe today are produced. Unfortunately, reheating dynamics strongly depends on the underlying particle theory we are assuming, making it very hard to draw generic conclusions. Nevertheless, it is possible to identify salient features of preheating and their implications on the issue of vacuum metastability.

4.1 Inflaton evolution after inflation

Our study of reheating starts from understanding the inflaton evolution at the end of inflation. After inflation the universe is in a cold and empty state where the energy density is dominated by the inflaton ϕ. The inflaton can still be regarded as homogeneous since spatial inhomogeneities have been washed away by the accelerated expansion. This means that the inflaton equation of motion can be written as

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\mathrm{d}V}{\mathrm{d}\phi} = 0\,, \tag{4.1}$$

where $V(\phi)$ is the inflaton potential. The Hubble rate is given by the first Friedman equation (1.6) and, since the universe is dominated by the inflaton field, is equal to

$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)\,. \tag{4.2}$$

For definiteness we consider a simple model of chaotic inflation where the inflaton potential has the form $m_\phi^2\phi^2/2$. We choose to work with a very simple model to highlight all the relevant features of reheating dynamics. The results that we obtain here and in the following sections can then be generalized to other large field inflationary models.

We immediately notice from the inflaton equation of motion (4.1) and the form of the Hubble rate (4.2) that the system evolution resembles that we described in Section 3.2 for the case of a massive Higgs. In the limit $m_\phi t \gg 1$, we get from (3.14) that the inflaton evolves as

$$\phi(t) \simeq \frac{\Phi_0}{m_\phi t} \cos(m_\phi t)\,, \tag{4.3}$$

where in this chapter the subscript 0 denotes quantities at the initial stage of reheating. Introducing (4.3) in the Friedman equation (4.2) and averaging over several oscillations, we see that the evolution of the scale factor is $a(t) \approx a_0(t/t_0)^{2/3}$, which is that of non-relativistic matter. The amplitude of oscillations of the inflaton decreases with the expansion as $\Phi = \Phi_0/a^{3/2}$. Reheating occurs when the energy density of the inflaton is transferred to the energy density of other particles and its amplitude decreases much faster than $a^{-3/2}$.

4.2 Perturbative reheating

The theory of perturbative reheating was first developed in [103–105]. Nowadays it is known this is usually preceded by a stage of non-perturbative production of particles which can be very efficient. These considerations are always model dependent but usually the perturbative decay of the inflaton remains the last stage of reheating, in which the transfer of the energy density into other particles completes. In this section we describe the features of perturbative reheating and in the next sections we study the non-perturbative stage.

Soon after inflation and before particle production becomes effective, the inflaton is a homogeneous scalar field oscillating with frequency m_ϕ. This can be considered as a coherent wave of ϕ-particles with zero momenta and with particle density $n_\phi = \rho_\phi/m_\phi$, where $\rho_\phi = (\dot{\phi}^2 + m_\phi^2\phi^2)/2$ is the inflaton energy density. We note that in absence of particle production the inflaton energy density decreases as $\rho_\phi \sim a^{-3}$ as we would expect from non-relativistic matter.

Let us assume that the inflaton couples with two other fields, a fermion ψ and a scalar h, with bare mass much smaller than that of the inflaton, in such a way that the inflaton perturbative decay is allowed. Specifically, the inflaton coupling with the two additional fields is given by the following operators:

$$\mathscr{L}_{\rm int} = -\sigma\phi h^2 - g\phi\bar{\psi}\psi\,, \tag{4.4}$$

where σ and g are the couplings.

For the homogeneous inflaton, the equation of motion including non-gravitational quantum corrections is [106]

$$\ddot{\phi} + 3H\dot{\phi} + \left[m_\phi^2 + \Sigma(\omega)\right]\phi = 0\,, \tag{4.5}$$

where $\Sigma(\omega)$ is the self-energy of the inflaton field with four-momentum $k_\mu = (\omega, 0, 0, 0)$, where $\omega = m_\phi$. The self-energy operator has a real part that contributes to the renormalization of the inflaton mass and, for $m_\chi, m_h < m_\phi/2$, an imaginary part, which we denote with $\mathrm{Im}\,\Sigma(\omega)$.

Since after inflation $m_\phi^2 \gg H^2$, assuming that $m_\phi^2 \gg \mathrm{Im}\,\Sigma$ and neglecting the time dependence of H and $\Sigma(\omega)$, we find from (4.5) that near the point $\phi = 0$ [106]

$$\phi \approx \Phi_0 \exp(i m_\phi t) \exp\left[-\frac{1}{2}\left(3H + \frac{\mathrm{Im}\,\Sigma(m_\phi)}{m_\phi}\right) t\right] . \tag{4.6}$$

From the unitarity relations [36, 107], it follows that

$$\mathrm{Im}\,\Sigma(m_\phi) = m_\phi \Gamma \,, \tag{4.7}$$

where Γ is the total decay rate of ϕ particles. Thus we see that two regimes are possible. When $\Gamma \ll 3H$, the inflaton decreases as $a^{-3/2}$ and no particle production occurs. On the other hand, when $\Gamma \gg 3H$, the decrease of the inflaton amplitude is mainly driven by its decay. Note that in this regime the inflaton energy density decreases exponentially, i.e. $\rho_\phi \sim e^{-\Gamma t}$, which is what we would expect interpreting ϕ as a coherent wave.

The total decay rate can be computed as the sum of the two decay channels available for the inflaton, which are given by the two operators in eq. (4.4). Assuming the inflaton mass is much larger than that of the decay products, we obtain the decay rates:

$$\Gamma(\phi \to \psi\psi) = \frac{g^2 m_\phi}{8\pi} \,, \qquad \Gamma(\phi \to hh) = \frac{\sigma^2}{8\pi m_\phi} \,. \tag{4.8}$$

The couplings that enter into these quantities cannot be arbitrarily large. In fact, they are bounded from above by requiring that radiative corrections do not spoil the inflaton potential during inflation. This implies that initially $\Gamma < H$ and the decrease in the energy density of the inflaton ϕ is determined only by the expansion of the universe. During this period particles can still be produced but their contribution to the energy density remains negligible. This is due to the fact that they are relativistic and their energy density decreases faster than that of the inflaton. Hence in this model, the energy density will be dominated by the produced particles only when the inflaton decay is complete. This happens when $\Gamma \sim 3H$ and at that point the energy density takes the value

$$\rho_r = \frac{\Gamma^2}{3} \,, \tag{4.9}$$

where the subscript r tells us that this is the energy density when reheating is complete. If the relativistic particles that are produced interact strongly enough with each other, they reach thermodynamic equilibrium and acquire a temperature T_r, the so-called reheating temperature. If thermalization is instantaneous, ρ_r does not have time to decrease because of the expansion and the reheating temperature is determined by the relation [106]

$$\rho_r \sim \frac{\pi^2}{30} N(T_r) T_r^4 \sim \frac{\Gamma^2}{3} \,, \tag{4.10}$$

where $N(T_r)$ represents the number of relativistic degrees of freedom at temperature T_r. For realistic models we have $N(T_r) \sim 10^2 - 10^3$, so that the relation between the reheating temperature and the inflaton decay rate is given by

$$T_r \sim 0.2\sqrt{\Gamma}\,. \tag{4.11}$$

4.2.1 Reheating and the Higgs inflaton couplings

Here we would like to briefly discuss what are the implications of reheating for the Higgs boson. Having successful reheating means that all the energy density stored in the inflaton sector transfers completely into the SM sector. This requires the inflaton to couple in some way to SM particles via some BSM operator. An important consequence is that these operators induce the Higgs-inflaton couplings radiatively. In fact, if we take any realistic reheating model and compute quantum corrections we observe that operators of the form $\lambda_{h\phi}\phi^2 H^\dagger H$ and $\sigma\phi H^\dagger H$ are generated at loop level [1].

In these models we introduce couplings of the inflaton with BSM or SM fields in order to provide a decay channel for the inflaton into SM particles. The Higgs-inflaton couplings are then generated via loops because the Higgs couples with SM particles. The loops are divergent and the introduction of counterterms is needed in order to renormalize the model. The renormalization group thus implies that the couplings run with the energy, meaning they cannot be set to zero at all energies.

These considerations tell us that a consistent study of the Higgs dynamics in the early universe must be done taking into account the Higgs-inflaton couplings. In fact, they can have dramatic consequences during inflation and, as we will see in the next sections, during the subsequent stage of non-perturbative particle production.

4.3 General theory of preheating

In Section 4.2 we discussed the formalism of perturbative reheating. That formalism however does not take into account the coherent nature of the inflaton field. For this reason a new formulation of reheating was developed in [108, 109] that took into account quantum mechanical particle production in a classical background inflaton field. These ideas were then developed further in [110, 111] and then analyzed in detail in [112]. In those papers the authors discovered that at the beginning of the inflaton oscillations the inflaton decays explosively into other particles. In particular, they noted that this stage of particle production, that they called *preheating*, cannot be described by the usual approach based on perturbation theory. In the following subsections we are going to describe preheating in detail and show how this non-perturbative phenomenon proceeds[i]. Later we will also discuss what are the implications on the Higgs instability of explosive particle production.

4.3.1 Preheating in Minkowski space

In this section we start by considering the simplified situation of a non-expanding universe. This approximation contains all the interesting features of preheating and the translation to the expanding universe will require only slight modifications. Preheating was first introduced for the case of a scalar field coupled to the inflaton. It is possible to have preheating for fermions as well [116–119], however we neglect them here because we are ultimately interested in the Higgs dynamics only. Let us consider the operator

$$\mathcal{L}_{\text{int}} = -\frac{1}{2}\lambda_{h\phi}\phi^2 h^2 \,. \tag{4.12}$$

The dynamics of the quantum field h can be studied in Fourier space by expanding it in its eigenmodes h_k as

$$h(t,\mathbf{x}) = \int \mathrm{d}^3 k \left(\hat{a}_k h_k(t) e^{-i\mathbf{kx}} + \hat{a}_k^\dagger h_k^*(t) e^{i\mathbf{kx}}\right) \,. \tag{4.13}$$

where $\hat{a}_k^\dagger$ and $\hat{a}_k$ are the usual creation and annihilation operators. The functions h_k are solutions to the classical equation of motion,

$$\ddot{h}_k + \left(k^2 + \lambda_{h\phi}\Phi^2\cos^2 m_\phi t\right) h_k = 0 \,, \tag{4.14}$$

where we substituted in an explicit form the inflaton field. Note that the amplitude of oscillations of Φ is constant because we are not taking into account the universe expansion.

It is possible to rewrite eq. (4.14) in the following form

$$h_k'' + \left(A_k - 2q\cos(2z)\right) h_k = 0 \,, \tag{4.15}$$

where primes denote derivative with respect to the rescaled time $z = m_\phi t - \pi/2$ and the parameters A_k and q are defined as

$$\begin{aligned} A_k &\equiv \left(\frac{k}{m_\phi}\right)^2 + 2q \,, \\ q &\equiv \frac{\lambda_{h\phi}\Phi^2}{4m_\phi^2} \,. \end{aligned} \tag{4.16}$$

Equation (4.15) is known in the literature as the Mathieu equation [120]. Solutions to the Mathieu equation are periodic functions multiplying the exponential of $\mu_k z$, where μ_k is a complex number, which depends on the parameters A_k and q, usually known as the Floquet exponent. When μ_k is purely imaginary the solution h_k is just a product of two periodic

[i]For reviews on preheating, see for instance [113–115]

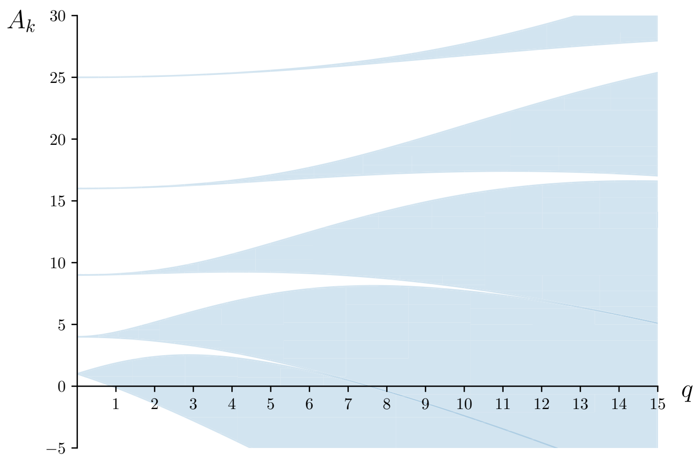

Figure 4.1: Stability and instability regions of the Mathieu equation. The horizontal axis is the parameter q, while the vertical axis A_k. The shaded regions corresponds to exponentially growing solutions.

functions, whereas if μ_k has a non-zero real part, the amplitude of oscillation of the solution h_k grows exponentially as

$$h_k \propto e^{\mathrm{Re}(\mu_k)z} . \tag{4.17}$$

This growth is known in the literature as *parametric resonance* [121].

Since the exponential growth depends exclusively on the value of the parameters A_k and q, one can plot charts like that in Fig. 4.1, where the shaded regions represent values of the parameters that lead to parametric resonance. The strength of the resonance is determined by the parameter q. In fact, it can be seen from the plot that when $q \ll 1$ the resonance happens only for narrow bands of modes k, while instability bands become much broader when $q \gg 1$. The case $q \gg 1$ is the more interesting since it is relevant for preheating in an expanding universe.

The Higgs effective mass is given by $m_h(t) = \sqrt{\lambda_{h\phi}}\phi(t)$. This is much larger than the inflaton mass m_ϕ and during one inflaton oscillation the field h oscillates many times. This means that the effective mass $m_h(t)$ is changing adiabatically. However, adiabaticity is lost when ϕ becomes small, and when this happens particle production occurs. In particular, adiabaticity is violated when

$$|\dot{\omega}_k| \gtrsim \omega_k^2 , \tag{4.18}$$

where $\omega_k = \sqrt{k^2 + m_h^2(t)}$. By taking the explicit form of the inflaton $\phi(t) = \Phi \cos m_\phi t$, we see that $\phi \approx 0$ when t is close to $t_j = m_\phi^{-1}(j+1)\pi/2$. Condition (4.18) can be rewritten in the

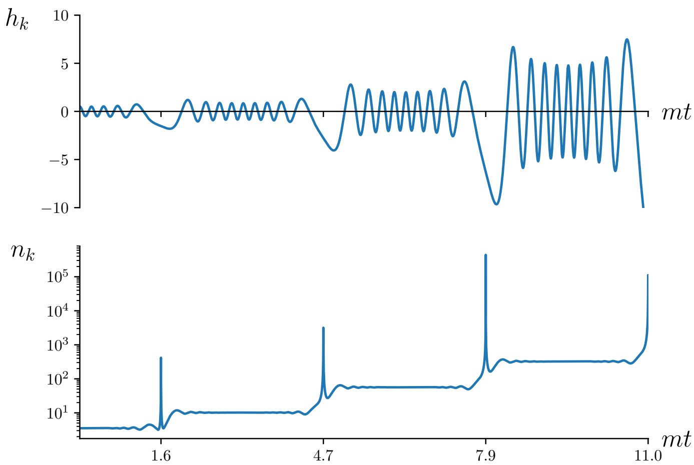

Figure 4.2: Evolution of the eigenmode h_k and its occupation number n_k as a function of $m_\phi t$ in the broad resonance regime. The resonance appears in bursts when the inflaton crosses the minimum of its potential as described in the text. Note that the occupation number n_k is an adiabatic quantity and it is well defined only when ϕ is far from zero. Ticks on the horizontal axes correspond to $\phi = 0$.

vicinity of t_j for $\Delta t \ll 1/m$ as

$$\frac{\Delta t/\Delta t_*}{(k^2\Delta t_*^2 + (\Delta t/\Delta t_*))^{3/2}} \gtrsim 1\,, \tag{4.19}$$

where

$$\Delta t_* \simeq (\sqrt{\lambda_{h\phi}}\Phi m_\phi)^{-1/2}\,. \tag{4.20}$$

The adiabaticity condition is thus violated only for modes

$$k < k_* \simeq \Delta t_*^{-1} \simeq m_\phi(\sqrt{\lambda_{h\phi}}\Phi/m_\phi)^{1/2}\,, \tag{4.21}$$

during short time intervals $\Delta t \sim \Delta t_*$.

The growth of the modes h_k leads to a growth of the occupation number of the h particles. This can be seen from the explicit expression of the number density n_k, which is defined as usual as the energy density of the particles with momentum $\mathbf{k}$ divided by the energy of each particle:

$$n_k = \frac{1}{2\omega_k}\left(\dot{h}_k^2 + \omega_k^2|h_k|^2\right) - \frac{1}{2}\,, \tag{4.22}$$

where the last term accounts for the subtraction of the zero-point energy of the mode. In the broad resonance regime the occupation number n_k remains constant during most of the inflaton oscillation. When the change in the frequency of oscillation ω_k ceases to be adiabatic, particles with the corresponding momentum are produced. This happens at each zero crossing of the inflaton field. In Fig. 4.2 we plot the evolution in time of a solution of the Mathieu equation h_k and its occupation number n_k for k in the range (4.21).

4.3.2 Preheating in an expanding universe

The case of an expanding background can be studied with the methods applied in the previous section, by expanding the field h in its Fourier modes h_k and writing their equations of motion. It turns out that their evolution equations take a much simpler form if we introduce the rescaled modes X_k defined as $X_k \equiv a^{3/2} h_k$, where a is the scale factor. In a matter dominated universe, we obtain the following equation

$$\ddot{X}_k + \omega_k^2 X_k = 0\,, \tag{4.23}$$

where

$$\omega_k^2 = \left(\frac{k}{a}\right)^2 + \lambda_{h\phi}\Phi^2\cos^2(m_\phi t)\,, \tag{4.24}$$

and $\Phi(t) \approx \Phi_0/m_\phi t$ as found in section 4.1. The equation of motion can be rewritten in the form of a Mathieu equation provided that we perform the substitutions $A_k = (k/am_\phi)^2 + 2q$, $q = \lambda_{h\phi}\Phi^2/4m_\phi^2$ and $z = m_\phi t - \pi/2$.

The parameters A_k and q decrease in time because of their dependence on k/a and Φ. Thus, in contrast to the situation of a static universe, a generic solution X_k of eq. (4.23) moves along a line on the Mathieu plane of Fig. 4.1. The movement points toward the origin of the plane because both A_k and q decrease. When they get inside the stability region near the origin, preheating for that particular mode stops and its occupation number remains constant.

To get a better understanding of the physics involved let us start by looking at numerical solutions to the system. In Fig. 4.3 we plot the evolution in time of a particular X_k and its corresponding occupation number n_k. The whole evolution of the mode up to the end of preheating is depicted on the left-hand side of the figure, whereas on the right-hand side we find the initial stages of preheating.

We begin the discussion by first focusing on the initial instants of the time evolution. In the simplest models of preheating, A_k and q start with values much higher than $\mathcal{O}(100)$. In this situation the behavior of the system resembles that of broad resonance but with an important modification. As can be seen from the right plot of Fig. 4.3 the occupation number n_k does not increase every time the inflaton crosses the minimum of the potential. In fact, sometimes it decreases. This behavior is mainly due to the fact that ω_k changes dramatically at every half oscillation of the inflaton due to its dependence on k/a and Φ. We can compare the behavior of X_k in Fig. 4.3 with that in Fig. 4.2. We see that the number of X_k oscillations changes every half inflaton oscillation because of the changing in ω_k. This makes it hard to find X_k with the right phase in order to be excited when adiabaticity is broken. Sometimes destructive interference

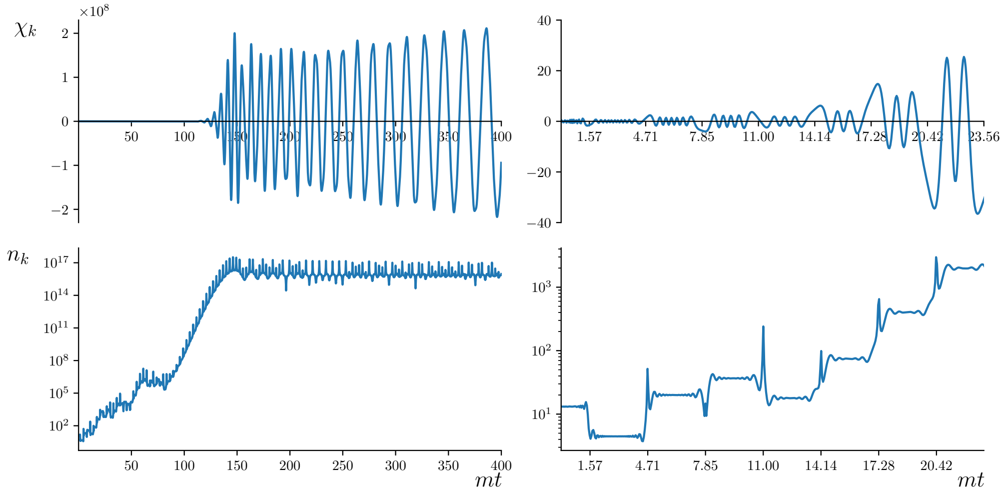

Figure 4.3: Evolution of a particular X_k and its occupation number n_k in an expanding background as a function of $m_\phi t$. One the left-hand side the evolution is followed up to $m_\phi t = 400$, while on the right-hand side only the first 8 zero crossing of the inflaton are plotted. Parameters are the same in both sides and correspond to an intial $q_0 \sim 3 \times 10^3$. The different features of the plots are described in the text. Ticks on the horizontal axes of the plots in the right-hand side correspond to t_j for which $\phi(t_j) = 0$.

occurs and the occupation number decreases. Unfortunately, when adiabaticity is broken it is not possible to predict whether the mode will be excited or not. This gives some randomness to the process and for this reason this is called *stochastic resonance*. The random nature of the process is relevant only at the first moments of preheating, when $q \gg 100$.

As q becomes smaller the behavior of the system gets closer to that of broad resonance in Minkowski space. Unstable regions become less dense and sometimes it is possible to find the mode X_k inside stable bands at the inflaton crossing. This can be seen in the left panel of Fig. 4.3. In the evolution of n_k we have a plateau for $m_\phi t \sim 60 - 80$ where the occupation number remains constant. At that moment the momentum X_k is crossing the stable band before the last unstable region closer to the origin of the plane in Fig. 4.1. After the plateau, the mode X_k crosses the last instability band where the stochastic behavior of the resonance disappears and the growth remains exponential during the whole crossing. This continues until the mode reaches the stable region close to the origin, where the resonance stops and the magnitude of the occupation number remains constant.

4.3.3 Analytic theory of preheating

In this section, we present an analytic study of preheating in an expanding background as done in ref. [112]. The results will be relevant later when we apply them to the Higgs dynamics during preheating.

We have seen from the previous section that the modes X_k evolve adiabatically when the inflaton is far from the minimum of its potential. Using the adiabatic approximation the modes can be rewritten as

$$X_k(t) \equiv \frac{\alpha_k(t)}{\sqrt{2\omega}} e^{-i\int^t \omega\, \mathrm{d}t} + \frac{\beta_k(t)}{\sqrt{2\omega}} e^{i\int^t \omega\, \mathrm{d}t}, \tag{4.25}$$

where the coefficients $\alpha_k(t)$ and $\beta_k(t)$ are analogous to the coefficients of the Bogolyubov transformation that diagonalize the Hamiltonian of the field h at each instant t. We assume a vacuum initial condition for the h modes with $\alpha_k = 1$ and $\beta_k = 0$.

With our choice of the initial conditions, the occupation number of the mode X_k defined by

$$n_k = \frac{\omega_k}{2}\left(\frac{|\dot{X}_k|^2}{\omega_k^2} + |X_k|^2\right) - \frac{1}{2}. \tag{4.26}$$

is given by $n_k = |\beta_k|^2$. The coefficients $\alpha_k(t)$ and $\beta_k(t)$ are constant in the adiabatic regime and change their value only when the inflaton crosses the minimum of its potential. We label the evolution during two consecutive zero crossing t_{j-1} and t_j with the label j, for $j = 1, 2, 3, \dots$.

The interaction term $\lambda_{h\phi}\phi^2(t)$ in eq. (4.23) can be approximated in the vicinity of t_j as

$$\lambda_{h\phi}\phi^2(t) \approx k_*^4 (t - t_j)^2, \tag{4.27}$$

where $k_* = (\sqrt{\lambda_{h\phi}}\Phi m)^{1/2}$ as in eq. (4.21), and Φ is the current amplitude of the inflaton oscillations.

We introduce a rescaled time variable $\tau = k_*(t - t_j)$ and momentum $\kappa = k/ak_*$ for simplicity. With this notation eq. (4.21) reduces for each j to

$$\frac{\mathrm{d}^2 X_k}{\mathrm{d}\tau^2} + \left(\kappa^2 + \tau^2\right) X_k = 0. \tag{4.28}$$

We see that near the zeros of the function $\lambda_{h\phi}\phi(t)$ the problem reduces to the scattering of the plane wave $X_k(\tau)$ on a parabolic potential.

A general solution to eq. (4.28) is the linear combination of the parabolic cylinder function [122]: $W(-\kappa^2/2; \pm\sqrt{2}\tau)$. The reflection and transmission coefficients for the scattering on the parabolic potential can be found from these analytic solutions and are

$$R_k = -\frac{i e^{i\varphi_k}}{\sqrt{1 + e^{\pi\kappa^2}}}, \tag{4.29}$$

$$D_k = \frac{e^{-i\varphi_k}}{\sqrt{1 + e^{\pi\kappa^2}}}, \tag{4.30}$$

with the angle φ_k given by

$$\varphi_k = \arg\Gamma\left(\frac{1+i\kappa^2}{2}\right) + \frac{\kappa^2}{2}\left(1 + \ln\frac{2}{\kappa^2}\right) , \tag{4.31}$$

where Γ is the Gamma function.

The reflection and transmission coefficients are used to determine the relation between the incoming wave X_k^j and the outgoing wave X_k^{j+1}. In particular, they give the following mapping between the Bogolyubov coefficients:

$$\begin{pmatrix} \alpha_k^{j+1} \\ \beta_k^{j+1} \end{pmatrix} = \begin{pmatrix} \frac{1}{D_k} & \frac{R_k^*}{D_k^*} e^{-2i\theta_k^j} \\ \frac{R_k}{D_k} e^{2i\theta_k^j} & \frac{1}{D_k^*} \end{pmatrix} \begin{pmatrix} \alpha_k^j \\ \beta_k^j \end{pmatrix} , \tag{4.32}$$

where $\theta_k^j = \int_0^{t_j} \mathrm{d}t\, \omega(t)$ is the phase accumulated at t_j.

The occupation number $n_k^{j+1} = |\beta_k^{j+1}|^2$ after t_j can be obtained from n_k^j:

$$n_k^{j+1} = e^{-\pi\kappa^2} + \left(1 + 2e^{-\pi\kappa^2}\right) n_k^j - 2e^{-\pi\kappa^2/2}\sqrt{1+e^{-\pi\kappa^2}}\sqrt{n_k^j(1+n_k^j)}\,\sin\theta_{tot}^j , \tag{4.33}$$

where the phase $\theta_{\mathrm{tot}}^j = 2\theta_k^j - \varphi_k + \arg\beta_k^j - \arg\alpha_k^j$. The Floquet exponent μ_k^j is defined by the formula

$$n_k^{j+1} = n_k^j \exp\left(2\pi\mu_k^j\right) , \tag{4.34}$$

and, in the limit $n_k \gg 1$, it takes the form

$$\mu_k^j = \frac{1}{2\pi}\ln\left(1 + 2e^{-\pi\kappa^2} - 2\sin\theta_{\mathrm{tot}}^j\, e^{-\pi\kappa^2/2}\sqrt{1+e^{-\pi\kappa^2}}\right) . \tag{4.35}$$

The first two terms in the logarithm are always positive and contribute to the growth of the modes. The second term can have either signs depending on the argument of the sine. In particular, its sign and size determine whether the occupation number increases or decreases. The variable θ_{tot}^j has a complicated time dependence and can be treated as a random variable. It it is thus clear from where the randomness of the process arises.

4.3.4 Estimating fluctuations of h and the end of the resonance

The growth in the occupation number of the modes contributes to the size of the field fluctuations $\langle h^2\rangle$, where the average is taken on the vacuum. Using the methods developed in the previous section we have the possibility to find an estimate for the fluctuations. This turns out to be useful when we want to understand the implications of preheating for the Higgs instability. Strictly speaking $\langle h^2\rangle$ is a divergent quantity in quantum field theory, due to the presence of the vacuum fluctuations, that has to be renormalized. However, we restrict our study only to momenta that are excited during preheating, for which $n_k \gg 1$. This implies that we have a natural cutoff on the momentum which gives us a finite $\langle h\rangle^2$ without the need to renormalize it.

In the adiabatic regime, $\dot{\omega}_k \ll \omega_k^2$, the occupation number n_k and the eigenmodes h_k are related by eq. (4.26) as follows:

$$n_k \simeq a^3 \omega_k \left|h_k\right|^2 . \tag{4.36}$$

Using eq. (4.13) we get the following expression for the field fluctuations

$$\langle h^2 \rangle = \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \left|h_k\right|^2 \simeq \frac{1}{(2\pi a)^3} \int \mathrm{d}^3 k \, \frac{n_k}{\omega_k} . \tag{4.37}$$

In order to estimate the size of the fluctuations it is necessary to find the expression of n_k in order to perform the integral. The occupation number n_k can be estimated from the results obtained in the previous section, in particular eqs. (4.34) and (4.35). For simplicity we take the term involving the sine in eq. (4.35) to be zero and consider the fluctuations at late times ($a_j \gg 1$).

Within our approximations the Floquet exponent at the j-th zero crossing can be expressed from eq. (4.35) as

$$2\pi \mu_k^j \simeq \ln 3 - \frac{2\pi}{3} \kappa^2 , \tag{4.38}$$

where $\kappa = k/ak_*$ and k_* was defined in eq. (4.21). The occupation number is then

$$n_k^{j+1} = n_k^j e^{2\pi \mu_k^j} \simeq 3^j e^{-2\pi \sum_i \mu_k^i} \equiv 3^j \, e^{-\bar{\mu}_j k^2/m_\phi^2} , \tag{4.39}$$

where we defined

$$\bar{\mu}_j = \frac{\sqrt{6}\, a}{3\sqrt{q_0}\Phi_0} , \tag{4.40}$$

with q_0 being the value of the Mathieu parameter of eq. (4.16) at the beginning of preheating. Higgs fluctuations can then be estimated as

$$\langle h^2 \rangle \simeq \frac{3^j m_\phi^2}{4\pi^2 a^3} \int_0^\infty \mathrm{d}k \frac{e^{-\bar{\mu}_j \left(k/m_\phi\right)^2 + 2\ln\left(k/m_\phi\right)}}{\cos\sqrt{\left(k/m_\phi \, a\right)^2 + 4q\cos^2 m_\phi t}} . \tag{4.41}$$

The integrand function has a maximum at some $k_{\rm max}$ that gives the biggest contribution to the integral. The value of $k_{\rm max}$ is well approximated by the minimum of the function in the exponential and is found to be $k_{\rm max}^2 \approx m_\phi^2/\bar{\mu}_j$. The integrand function can then be expanded around this value and the problem is reduced to the computation of a Gaussian integral. The fluctuations are easily obtained

$$\langle h^2 \rangle \simeq \frac{3^j k_{\rm max}^3}{2^{3/2} m_\phi \, e \, \pi^{3/2} a^3 \sqrt{\left(k_{\rm max}/m_\phi a\right)^2 + 4q\cos^2 m_\phi t}} . \tag{4.42}$$

4.4 Higgs evolution during preheating

The copious production of particles during preheating can be dangerous for the unstable Higgs since it can lead to vacuum decay. The decay during preheating happens in a different way compared to that during inflation and a qualitative description can be given as follows. The Higgs effective mass squared during preheating is

$$m_h^2 = \lambda_{h\phi}\phi^2 + 3\lambda_h h^2 \,, \tag{4.43}$$

where the first term on the right-hand side comes from the Higgs-inflaton coupling and the second from the Higgs potential. At the beginning of preheating, the Hartree approximation can be used and the last term can be written as $3\lambda_h\langle h^2\rangle$. In this case the equations of motion for different momentum modes decouple and can be studied according to the theory that we developed in the previous sections. The EW mass term for the Higgs is neglected here because it is much smaller than the other two terms we are considering.

At the beginning of preheating the Higgs-inflaton coupling dominates the effective mass because the Higgs variance is small. However, preheating increases the value of the fluctuations and the Higgs self interaction term becomes more and more relevant each time the inflaton crosses the zero of its potential. The Higgs self coupling runs according to the renormalization group and the appropriate value that we have to consider is $\lambda_h(\sqrt{\langle h^2\rangle})$, where the relevant energy scale is set by the Higgs variance. When the energy scale becomes larger than the SM instability scale $\Lambda_{\rm inst}$, a negative contribution to the Higgs effective mass (4.43) appears. If the negative term comes to dominate, the Higgs field becomes tachyonic and gets exponentially amplified leading to vacuum decay [2, 123].

The subtlety here is that the inflaton oscillates and therefore the first term of the Higgs effective mass (4.43) gets very close to zero during a short interval of time. During these instants the negative term dominates the effective mass. Clearly, the time interval in which the Higgs self interaction dominates grows in time because the inflaton amplitude of oscillations decreases and preheating increases the Higgs variance. The EW vacuum can survive only if the resonance stops before the exponential amplification becomes too effective.

We can quantify these statements as follows. The Higgs effective mass is dominated by the tachyonic term during the time interval Δt given by

$$|\Delta t| < \sqrt{\frac{3|\lambda_h|\langle h^2\rangle}{\lambda_{h\phi}\Phi^2 m_\phi^2}} \,. \tag{4.44}$$

When this relation is fulfilled, the Higgs field is exponentially amplified by a factor $e^{|m_h\Delta t|}$, where the modulus is taken because m_h is imaginary in this regime. The amplification is insignificant as long as

$$|m_h\Delta t| \simeq \sqrt{3|\lambda_h|\langle h^2\rangle}|\Delta t| < 1 \,. \tag{4.45}$$

If we plug (4.44) and (4.42) into (4.45), and use the fact that at the end of the resonance $q = \lambda_{h\phi}\Phi^2/4m^2 \approx 1$, we obtain the following upper bound [2]

$$\lambda_{h\phi} \lesssim 1.5\times 10^{-8} \,. \tag{4.46}$$

This is lower than the upper bound we obtained in (3.23) by requiring small radiative corrections to the inflaton potential. The Higgs-inflaton coupling stabilizes the Higgs potential during inflation but during preheating it has the opposite effect. For this reason the upper bound on $\lambda_{h\phi}$ must be lowered.

Understanding whether destabilization is triggered at later times is much more involved. The Higgs particles that are produced during the resonance can decay into other SM particles which can then create a thermal bath. This would generate a thermal mass for the Higgs that can stabilize the EW vacuum [123]. In addition, the Higgs variance might be reduced via non-perturbative production of particles via the Higgs couplings to gauge bosons and fermions [124]. These and possibly other effects may enter into play after preheating making it difficult to draw any conclusions without specific models and a careful investigation. However, we have seen that the situation remains quite well understood during the preheating stage.

In the next section we are going to show how to generalize the discussion to the other important couplings such as the non-minimal coupling with gravity ξ_h and the trilinear Higgs-inflaton coupling σ.

4.4.1 Gravitational and tachyonic preheating

The non-minimal coupling of the Higgs with gravity ξ_h plays an important role during preheating because it can lead to destabilization as in the case of the Higgs-inflaton coupling. Studies of preheating with only ξ_h applied to the Higgs instability can be found in [123, 125–129]. Let us consider here the system where the Higgs-inflaton coupling $\lambda_{h\phi}$ and the non-minimal coupling ξ_h are considered simultaneously as was done in [3] and let us show how the dynamics change from that we described so far.

The action in the so-called Jordan frame has the form

$$S = \int \mathrm{d}^4x \, \sqrt{g} \left[\frac{1}{2}(1 - \xi_h h^2)R - \frac{1}{2}\partial_\mu\phi \, \partial^\mu\phi - \frac{1}{2}\partial_\mu h \, \partial^\mu h - V(\phi, h) \right] , \tag{4.47}$$

where again h is the Higgs in the unitary gauge. The inflaton evolves as in the previous section, whereas the Higgs evolution gets modified due to the presence of ξ_h. The additional term we introduced can be viewed as a mass term proportional to the Ricci scalar R.

Since the universe is dominated by the inflaton, the Ricci scalar can be written as

$$R = \left(2m_\phi^2\phi^2 - \dot{\phi}^2 \right) . \tag{4.48}$$

Substituting $\phi \approx \Phi \cos(mt)$ into R we immediately notice that the sign of the Ricci scalar changes in time because of the inflaton oscillation. This generates a negative mass term for h. When this happens, particle production is very efficient due to the tachyonic nature of the field h.

Expanding the field h in its comoving eigenmodes yields an equation for X_k's similar to (4.23), where now the eigenfrequency takes the form

$$\omega_k^2 = \left(\frac{k}{a}\right)^2 + \left(\lambda_{h\phi} + 2\xi_h m^2\right)\Phi^2 \cos^2 mt - \xi_h m^2 \Phi^2 \sin^2 mt\,. \tag{4.49}$$

Here we have neglected terms proportional to H since they are small at the beginning of preheating and decrease with time. Also, the term due to the Higgs self-interaction is neglected because of the initial smallness of the Higgs variance. With the help of some trigonometric identities we can rewrite the equation of motion of X_k as a Mathieu equation with parameters

$$A_k \equiv \left(\frac{k}{a\,m}\right)^2 + \frac{\Phi^2}{2m^2}\left(\lambda_{h\phi} + \xi_h m^2\right)\,, \tag{4.50}$$

$$q \equiv \frac{\Phi^2}{4m^2}\left(\lambda_{h\phi} + 3\xi_h m^2\right)\,. \tag{4.51}$$

Two main differences arise from the situation described in the previous sections. The first one is that the resonance can be killed completely. This happens when $\lambda_{h\phi} \approx -3\xi_h m^2$ and thus the resonance parameter q becomes very close to zero. In this case the system evolves very close to the vertical line in the (A_k, q) plane of Fig. 4.1 and never crosses any instability bands. The second difference is that, as we noted earlier, ξ_h introduces the possibility to get tachyonic particle production. For some values of the parameter the system evolves in the (A_k, q) plane much deeper in the instability bands making particle production much stronger.

The joint effect of ξ_h and $\lambda_{h\phi}$ can therefore be dual. On the one hand, parametric resonance can be enhanced by tachyonic particle production and, on the other hand, the value of the couplings can be such that particle production is completely suppressed. Thanks to this second feature the parameter space for which the Higgs remains in the EW vacuum throughout preheating is much larger [3] than in the case where only $\lambda_{h\phi}$ or ξ_h are present [2,123]. In particular, the upper bounds on the couplings is essentially set by requiring small radiative corrections to the inflaton potential rather than avoiding excessive particle production.

4.4.2 The effect of the trilinear coupling

The trilinear coupling is the last missing piece in our picture of the Higgs evolution during preheating. Preheating of a scalar field with trilinear coupling to the inflaton was considered in [130], while the simultaneous presence of quartic and trilinear interactions was considered in [131]. For simplicity, let us put aside for a moment the Higgs non-minimal coupling with gravity and focus only on the trilinear and quartic coupling. The Higgs effective mass squared in this case takes the form

$$m_h^2 = \lambda_{h\phi}\phi^2 + 2\sigma\phi + 3\lambda_h h^2\,. \tag{4.52}$$

The first thing we observe is that the term dependent on the trilinear interaction is negative during half oscillation of the inflaton field because ϕ enters linearly in the effective mass. This, analogously to the non-minimal coupling case, can generate a tachyonic effective mass leading to to very efficient particle production and exponential amplification of the Higgs field. We thus see that the presence of the trilinear coupling creates additional danger for the EW vacuum stability.

The decomposition into Higgs comoving eigenmodes allows us to write an equation of motion for X_k in the form (4.23). The eigenfrequency of the modes is

$$\omega_k^2 = \left(\frac{k}{a}\right)^2 + \lambda_{h\phi}\Phi^2 \cos^2 mt + 2\sigma\Phi \cos mt \,, \tag{4.53}$$

where we neglect the Higgs-self interaction terms because at the beginning of preheating is very small. We note immediately that it is not possible to reduce (4.53) to the Mathieu equation because there are two different frequencies of oscillation: mt and $2mt$, since $\cos^2 mt = (1 + \cos 2mt)/2$. Nevertheless, we can rescale the time as $mt = z$ and do some trigonometric manipulation in order to rewrite the equation of motion as

$$X_k'' + (A_k + 2p\cos z + 2q\cos 2z)\, X_k = 0 \,. \tag{4.54}$$

Here A_k and q are defined as in (4.16), while

$$p \equiv \frac{\sigma\Phi}{m^2} \,. \tag{4.55}$$

Equation (4.54) is known as the *Whittaker-Hill equation*. Detailed analyses of the Whittaker-Hill equation can be found in [131–134]. The Whittaker-Hill equation does not have a simple two dimensional representation for the instability bands, since it depends on three parameters. Nevertheless, the qualitative picture remains the same. The parameters move in a three dimensional space and during their evolution pass through many instability bands where they get exponentially amplified.

Studies of preheating in this setup with without the non-minimal coupling to gravity can be found in the attached papers [2,3]. The behavior of the Higgs field does not change qualitatively from the Mathieu case. Particle production increases the value of the Higgs variance, which in turn increases the importance of the self-interaction term generating a large tachyonic mass for the Higgs. The difference is that the strength of the resonance is determined by two parameters, q and p. In order to avoid EW vacuum decay the resonance must be suppressed, and this can be achieved only for small initial q and p. This sets an upper bound on the trilinear coupling [2]:

$$\sigma \lesssim 10^8\,\mathrm{GeV} \,. \tag{4.56}$$

Chapter 5

Conclusions

In this thesis we have considered the implications of metastability for the Higgs dynamics in the early universe. We have seen in Chapter 2 that metastability is not dangerous because the decay time exceeds by far the age of our universe. For this reason physics beyond the SM is not needed in order to stabilize the Higgs potential and our existence is completely consistent with metastability. However, the situation is different if we want to justify how the universe evolved in the EW vacuum during its cosmological history.

In Chapter 3 we showed that large Higgs fluctuations that form during inflation are harmful for the EW vacuum, and that avoiding the decay into the true vacuum puts strong constraints on the Hubble rate during inflation. In our discussion we also highlighted the fact that the problem of the fluctuations is not the only one connected to the EW vacuum metastability. In fact, there is no reason to assume that the Higgs started its evolution in the false minimum, implying that also small field inflationary models are not exempt from complications deriving from the metastability. In fact, they cannot justify the fine-tuning required to put the Higgs in such an energetically disfavored state.

Solutions to these issues may lie in the scalar nature of the Higgs and in the possibility to write interactions with BSM particles or with gravity at the renormalizable level. In this thesis we discussed the case of direct Higgs-inflaton couplings and non-minimal coupling with gravity. These interactions can modify the Higgs potential at all scales or during inflation only. We have shown that mixing of the Higgs with a SM gauge singlet can stabilize the Higgs potential completely and that identifying the singlet with the inflaton allows for the exciting possibility to observe such a particle at the LHC or at future colliders. On the other hand, we have also demonstrated that Higgs couplings to the inflaton or gravity can modify the shape of the Higgs potential during inflation by generating a large effective mass that pushes the field to very small values. Unfortunately, while being able to stabilize the Higgs during inflation, they can create an additional source of instability at a later time, namely during preheating.

In Chapter 4 we have shown that the couplings allow for copious production of Higgs particles, leading to large field fluctuations. These can trigger the decay into the true vacuum if their size gets comparable to that of the instability scale of the SM. In our work we have shown that it is possible to find reasonable parameter space in order to stabilize the Higgs during inflation and avoid destabilization during preheating.

The Higgs instability remains a very interesting topic which has important implications for early universe physics. If the instability is there, we are led to the conclusion that some operators which are not part of the SM must be added if we want to justify the existence of the universe we observe today. In this work we explored the simplest realistic scenario which allows to treat consistently the Higgs evolution in the framework of an inflationary universe. We have found that consistency is made possible by the special nature of the Higgs which allows us to write renormalizable BSM couplings with the inflaton or gravity. Some uncertainty still remains as to whether metastability of the SM is there or not. An improvement in the accuracy of the top quark mass measurement and a better understanding of its relation with the top Yukawa coupling will certainly help us to determine the existence of this problem. Besides, a discovery of BSM physics at colliders might change the picture we have of particle physics from the TeV to the Planck scale and show us that the Higgs potential gets modified at lower energies than the SM instability scale. In addition, cosmology might give us further insights of the particle physics laws that govern the dynamics in the early universe. Therefore, we must watch closely what comes from experiments at many different scales, from the infinitesimally small to the infinitely large, because they can give us new hints on the nature of the Higgs boson.

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