



Master's Degree Thesis

On the automatic planning of healthy and balanced menus

Planificación automática de menús saludables y
equilibrados

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Abstract

With the raise of diseases related with unhealthy lifestyles such as heart-attacks, overweight, diabetes, etc., encouraging healthy and balanced patterns in the population is one of the most important action points for governments around the world. Furthermore, it is actually even a more critical situation when a high percentage of patients are children and teenagers whose habits consist merely in eating fast or ultra-processed food and a sedentary life.

The development of healthy and balanced menu plans becomes a typical task for physicians and nutritionists, and it is at this point that Computer Science has taken an important role. Discovering new approaches for generating healthy and balanced, as well as inexpensive menu plans will play an important part in banish of diseases from actual and new generations.

In this Master Thesis, a recently proposed Evolutionary Algorithm has been compared to other state-of-art evolutionary algorithms for solving the Menu Planning Problem. In order to evaluate the performance of the developed algorithm, an exhaustive experimental assessment was made. Firstly, we focused on evaluating the parameter setting of the algorithm so afterwards the best configuration found could be compared with other well-known algorithms.

Keywords: menu planning, computer science, evolutionary computing, multi-objective optimisation

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Chapter 1

Motivation

1.1 Description of the Master Thesis

In this Master Thesis, the main intention is to develop an evolutionary algorithm for solving the well-known *Menu Planning Problem (MPP)*. The MPP is an optimisation problem which is based on designing menu plans under some restrictions. Although there is a lot of different kinds of algorithms for solving such a problem, a high percentage of published papers use evolutionary computation due its large benefits like robustness, reliability, global-search ability and its simplicity [12–14, 18, 21].

Concretely, this Master Thesis will be focused on solving a recently proposed MPP formulation with an evolutionary algorithm called *Multi-objective Evolutionary Algorithm based on Decomposition* [28] and compare its performance with other state-of-art algorithms such as *Nondominated Sorting Genetic Algorithm II (NSGA-II)* [5] and *Strength Pareto Evolutionary Algorithm 2 (SPEA 2)* [15].

1.2 State of the art

The Menu Planning Problem (*MPP*) is a well-known NP-Hard problem, which was firstly proposed in 1960 [19]. In essence, the MPP consists of finding a set of dishes combination which satisfies some restrictions of budge, variety and nutritional requirements for a period of n days. In addition, it can include other constraints such as user preferences, cooking time or the number of meals each day. Even though there is not consensus about the number of objectives that a MPP's formulation may have, in almost every formulation the cost of the menu plan is considered as one of the main objectives to be optimised [18, 19]. But, it also supports other objective functions, like maximising the variability or minimising the cooking time.

Furthermore, the MPP can be studied as a multi-objective problem [20] if

the amounts of nutrient requirements and cost of the meals are considered as independent objectives. This approach leads to reduce the MPP to a Multi-dimensional Knapsack Problem (*MDKP*) where the maximum amount of each nutrient define the limits of the multiple dimensions. However, the MPP has also been studied as a single-objective problem where the total cost of the meals is considered as the typical objective function. For instance, a single-objective approach for the MPP is in [18]. In this particular research, the authors proposed an evolutionary approach to solving the 5-day Single-Objective Menu Planning Problem composed by three meals daily. In addition, the set of constraints that the researchers defined to this problem are moderately different from the usual constraints set for the typical MPP. In this occasion, the authors set the student age group, the school category, school duration time, school location, variety of preparations, the maximum amount to be paid for each meal and finally, and the lower and upper limits of macro-nutrients as the constraints set to be satisfied for each solution to be considered feasible. Within this research, the authors used the standard Genetic Algorithm (GA) for the computational experiments. The results obtained compared with a Greedy-based approach demonstrated that the GA was able to outperform the Greedy-based approach when the limit values of the meals are fixed at R\$ 2.00 for breakfast, R\$ 4.00 for lunch and R\$ 2.00 for the snack. (BRL - R\$ 1.0 USD - \$ 0.31).

At the same time, in [10], the authors referred to the Two-phase Cooking N-day Menu Planning Problem where the objective is to maximise the preferences among the selected foods in the menu plan. The conditions which shape the set of constraints that must be satisfied are three. The total cooking time of any day must not exceed the limit specified, only foods that which allow two-phase cooking can be selected for two-phase cooking and finally, the food cannot be repeated more times than a certain repetition constraint. In order to face this problem, the researchers used a simple greedy method prioritising the user-specified preferences with the cooking time of each food.

Eventually, another study where the MPP is faced as a single-objective problem was considered in [24]. Here, the authors set up a mathematical model to solve the MPP considering only one objective function. The goal of the model is to minimise the budget provided by the government subject to the restriction of trying to maximise the variety of dishes. Furthermore, the model tries to create menus in such a way they maximise the nutritional requirements. For the computational experiments, the researchers implemented an Integer Programming algorithm in Matlab using LPSolve. Furthermore, the given results, taking into account that the optimal solution was found within one second, are compared to other heuristics, like GA.

As it can be seen, there is a certain variety within the optimisation methods for solving the single-objective MPP approach. Despite that, Evolutionary Computation (*EC*) techniques, such as GA, are mostly cited in the related bibliography as a suitable choice [18–20].

This Master Thesis is focused on developing, analysing and comparing different well-known multi-objective evolutionary algorithms for solving real-world instances of a multi-objective variant of the MPP.

Chapter 2

Background

2.1 Optimisation Problems

An Optimisation Problem (OP) is a problem which has a score function and bounds where the main task is to find a input that optimises the score function. Optimisation problems can be categorised as *discrete optimisation problem (DOP)* or *Continuous Optimisation Problem (COP)* whether the variables of the problem are discrete or continuous. Formally speaking, an OP can be described as follows:

$$\min f(x), x \in \chi, \quad s.t. \Omega$$

where $\chi \subset \mathbb{Z}^n$ is the search space defined over a set of n decision variables $x = (x_1, x_2, \dots, x_n)$, $f : \chi \rightarrow \mathbb{R}$ is the score function and Ω is the restrictions set in x . This is the definition for a minimisation DOP, although it would be equivalent for a maximisation DOP changing $\min f(x)$ by $\max f(x)$. The same happens for a COP, if in addition the decision variables are set over \mathbb{R}^n instead of \mathbb{Z}^n .

Furthermore, optimisation problems may have more than one score function and they are called *Multi-Objective Optimisation Problems (MOOP)*. This is the primary field of study in this work so, hereinafter all the references to optimisation problems in this work will be to MOOP.

2.1.1 Multi-Objective Optimisation Problems

Multi-Objective Optimisation Problems (MOOPs) are optimisation problems which have two or more objective functions to optimise and those objective functions can take opposite directions (thinking about directions as *minimise or maximise*). Besides, a MOOP can be discrete or continuous considering whether the variables of the problem are discrete or continuous.

Formally speaking, a MOOP can be described as finding a vector x inside the problem's search space χ in such way that optimises the vector of objective

functions $f(x)$ [6]:

$$\begin{aligned} \min f(x) &= (f_1(x), f_2(x), \dots, f_k(x)), x \in \chi \\ g_i(x) &\leq 0, i = 1, 2, \dots, q. \\ h_i(x) &\leq 0, i = 1, 2, \dots, p. \end{aligned}$$

where $x = (x_1, x_2, \dots, x_n) \in \mathbb{Z}^n$, are the objective functions to optimise $f_i : \mathbb{Z}^n \rightarrow \mathbb{R}$, $i = 1, \dots, k$ being n the number of decision variables and $g_i : \mathbb{Z}^n \rightarrow \mathbb{R}$, $i = 1, \dots, q$ and $h_i : \mathbb{Z}^n \rightarrow \mathbb{R}$, $i = 1, \dots, p$ are the problem's restriction functions.

Moreover, the standard method for evaluating and distinguish the quality between solutions for a MPP is the well-known *Pareto method optimality* [6]. The Pareto optimality is based on the non-dominance principle [6, 25]. On the one hand, dominance means that given two solutions for a MOOP, one solutions dominates the other one when it has as least the same quality for every objective and, it has strictly more quality for one of them than the other solution. Formally, this can be expressed as follows [6]:

$$\begin{aligned} A \succeq B &\Leftrightarrow \forall i \in \{1, 2, \dots, n\} a_i \leq b_i, \\ &\text{and } \exists i \in \{1, 2, \dots, n\}, a_i < b_i \end{aligned}$$

On the other hand, it is the direction conflict between objectives which leads to solutions with trade-offs between those objectives. So, at this point it is where the *non-dominance* appears. The non-dominance refers the situation where a solution it is not dominated by any other solution of the problem. That means that it can not be found any other solution to the problem which increases the quality of any objective without irredeemably decreases the quality of another one. Non-dominated solutions may be found at the limits of the search space (χ) and are those which shape the Pareto set.

2.2 Evolutionary Algorithms

Nowadays, there are many methods for solving MOOPs but they can be classified merely in two types of methods: *approximated methods and exacts methods*. The different categories of exact and approximated methods can be seen in the Figure 2.1.

On the one hand, *exacts methods* are those which ensure that, if there is an optimal solution to the facing problem they will be able to find it. However, even though these methods guarantee reaching the optimal solution they have a important drawback on its performance. Assuring the optimal solution implies increasing the computational work and hence more time to obtain the solution.

On the other hand, *approximated methods* are very popular nowadays even though they do not guarantee reaching the optimal solution for a problem.

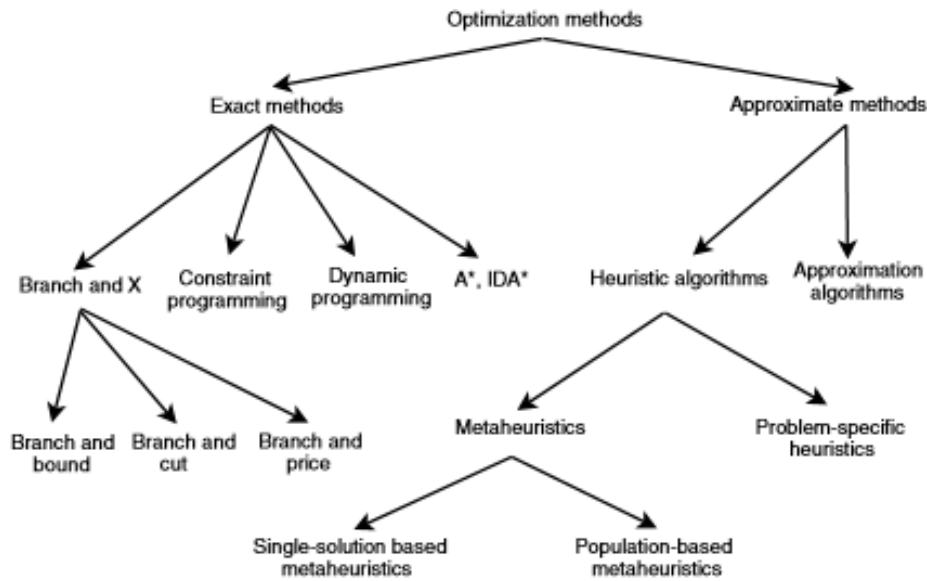


Figure 2.1: Optimisation methods

Nevertheless, approximated methods can obtain high quality solutions in an assumable time due they set a balance between computational performance and solution quality.

Approximated methods can be divided in two categories: *heuristic algorithms* and *meta-heuristics algorithms*. However, this work it is focus primarily in *meta-heuristics algorithm* and even more specifically in the field of *evolutionary algorithms*.

Evolutionary algorithms (EA) develop the metaphor of natural evolution, the survival of the fittest individual [8]. This is, given a population of individuals in some environment with limited resources, the competition for surviving causes natural selection and the fittest individuals are more likely to survive and reproduce. Nevertheless, there are several variants of EA, some of them are:

1. Genetic Algorithms [2, 22, 27].
2. Evolutionary Strategies [4, 11].
3. Differential Evolution [1, 9, 26, 29].

Moreover, there is a specific group of evolutionary algorithms for solving multi-objective problems known as *Multi-Objective Evolutionary Algorithms (MOEAs)*. MOEAs can be classified in many different subgroups considering the main approach underlying the algorithm [30]:

- MOEAs based on decomposition, i.e., MOEA/D [17, 28].
- Pareto-based MOEAs, i.e., NSGA-II [5] and SPEA-2 [15].

- Indicator-based MOEAs, i.e., IBEA [31].

Considering the field of optimisation problems, the natural evolution metaphor can be specified as the following set of steps:

1. The problem to solve and its bounds is the environment with limited resources.
2. A set of random initial solutions for the given problem are the first individuals at generation zero.
3. The population of individuals reproduce between each other applying genetic operators to generate offspring. Commonly combination and mutation.
4. At each generation, the individuals within a population compete and the fittest individuals (the better quality solutions) survive.
5. Steps three and four are repeated until reaching the stop condition.

Generally, the aforementioned metaphor can be shown as a pseudocode [8]:

Algorithm 1 Pseudocode of an EA.

- 1: INITIALISE population with random candidate solutions
 - 2: EVALUATE each candidate
 - 3: **while** not StopCriteria satisfied **do**
 - 4: SELECT parents
 - 5: RECOMBINE pairs of parents to obtain the offspring
 - 6: MUTATE the offspring
 - 7: EVALUATE new offspring
 - 8: SELECT individuals from among parents and offspring for the next generation
 - 9: **end**
-

2.3 Considered Formulation for the Menu Planning Problem

In this particular case, a novel formulation of the Menu Planning Problem proposed for school cafeterias is considered. The authors defined two objectives: meal cost and variety of dishes. On the one hand, as usual in MPP, one goal is to minimise the total cost of the meal plan generated. Since the meal plan is designed for school cafeterias, the authors considered three meals in each menu: first course, second course and dessert. Formally, the meal plan cost can be defined as follows:

$$\min C = \sum_{i=1}^n c_{fc_i} + c_{sc_i} + c_{d_i}$$

where C is the total cost of the menu plan and $c_{fc_i}, c_{sc_i}, c_{d_i}$ represent the cost of the first course, second course and dessert in n days.

On the other hand, an assorted menu plan is a must for children in order to avoid them to annoy about the food. For that reason, the second objective is to minimise the level of repetition of dishes and food groups in a certain menu plan.

$$\min L_{Rep} = \sum_{i=1}^n v_{table_i} + \frac{p_{fc}}{d_{fc_i}} + \frac{p_{sc}}{d_{sc_i}} + \frac{p_{ds}}{d_{ds_i}} + v_{FG_i}$$

Where L_{Rep} is the level of repetition to be minimise, v_{table_i} represents the compatibility between the courses $c_{fc_i}, c_{sc_i}, c_{d_i}$ for day n , p is a penalty constant for every kind of course and d stands for the number of days since the course was repeat for the last time. Finally, v_{FG_i} is the penalty value for repetition of food groups in the last five days.

Additionally, in order to consider that a menu plan as feasible, it must satisfy some restrictions about a set of nutritional requirements (N). The nutritional requirements considered in this formulation are the following:

- Folic acid.
- Calcium.
- Energy (Kcal).
- Phosphorus.
- Fat.
- Carbohydrate.
- Iron.
- Magnesium.
- Potassium.
- Protein.
- Selenium.
- Sodium.
- Vitamin A.
- Vitamin B1.

- Vitamin B2.
- Vitamin B6.
- Vitamin B12.
- Vitamin C.
- Vitamin D.
- Vitamin E.
- Iodo.
- Zinc.

There is also a vector R in which the minimum and maximum amount of each nutritional requirement is stored. So formally speaking, a menu plan is feasible only if

$$\forall n \in N : R_{min_n} \leq I_n \leq R_{max_n}$$

where I_n is the amount of the n-nutritional requirement in the menu plan.

Lastly, the set G of food groups considered for the available meals is:

- Meat.
- Cereal.
- Fruit.
- Dairy.
- Fish.
- Vegetable.
- Shellfish.
- Legume.
- Pasta.
- Others.

Chapter 3

Algorithms

At this point, it will be introduced the Evolutionary Algorithms compared in this Master Thesis. With a view to have variety of MOEAs, every algorithm it is based on a different MOEA approach [30]:

- Based on decomposition: Multi-objective Evolutionary Algorithm Based on Decomposition.
- Based on Pareto optimality: Non-dominated Sorting Genetic Algorithm II and Strength Pareto Evolutionary Algorithm 2.

3.1 Multi-objective Evolutionary Algorithm Based on Decomposition

Multi-objective Evolutionary Algorithm Based on Decomposition (MOEA/D) is an evolutionary algorithm for multi-objective optimisation proposed by Qingfu Zhang and Hui Li in 2007 [28]. The underlying idea behind this algorithm is to decompose a multi-objective optimisation problem into a number of scalar optimisation sub-problems and optimise them simultaneously. It also harnesses in the well-known feature of Pareto optimal solutions to a MOOP, which sustains that an optimal solution for a scalar optimisation problem with an objective function as the aggregation of all the f_i could be the same as the Pareto optimal solution for the MOOP [28].

The decomposition approach of MOEA/D takes place where the algorithm decomposes a MOOP into N sub-problems and simultaneously optimises every single sub-problem at each generation. Furthermore it establish some relations between sub-problems and organised them in neighbourhoods. These neighbourhoods are shaped by sub-problems which coefficient vectors are very similar to each other and every single sub-problem is optimised based on its neighbouring sub-problems information. Therefore, the optimal solution for two neighbouring sub-problems should be very similar [28].

On the other hand, the process of decompose a MOOP into N sub-problems can be done from some different approaches. However, as the authors referred in [28], in this Master Thesis the MOEA/D uses the Tchebycheff Approach [17] to decompose a MOOP. Formally, the Tchebycheff Approach it is defined as follows:

$$\min g^{te}(x|\lambda, z^*) = \max_{i=1}^m \{\lambda_i |f_i(x) - z_i^*|\}$$

where $z^* = (z_1^*, \dots, z_m^*)$ is the reference point with the best solution founds so far for each sub-problem and $\lambda_i = (\lambda_{i,1}, \dots, \lambda_{i,m})$ is a even spread weight vector for each sub-problem i .

In addition, MOEA/D version implemented in this paper works as follows. It takes a MOOP, the population size, a stopping criterion and the number of neighbours for each neighbourhood. The number of sub-problems in this implementation are the MOOP's objectives. Then, it starts by randomly generate N even spread weight vectors and compute the Euclidean distance between each others to shape the neighbourhoods, generates an initial random population and computes the reference point Z^* . After the initialisation phase, it goes into the main loop where, until the stopping criteria is not satisfied, the algorithm preforms theses steps for each individual of the population:

- **Reproduction:** generates a new child individual from two randomly selected neighbours l, k .
- **Improve:** maintains the new child under the limits of the problem's search space.
- **UpdateZ:** updates the reference point by comparing it with each new child individual.
- **Update Neighbours:** if the new child individual performs better than any neighbours, replaces the neighbour with the brand new individual.

Finally, MOEA/D returns the Pareto Front points found.

Concretely, the algorithm can be outlined as follows:

Algorithm 2 MOEA/D.

```

1: SetRandomWeightVectors
2: EuclideanDistance
3: GenerateRandomPopulation
4: InitializeZ
5: while not StopCriteria satisfied do
6:   for all sub-problem do
7:     l,k = getRandomNeighbours
8:     child = reproduce(l, k)
9:     child = improve(child)
10:    updateZ(child)
11:    updateNeighbouringSolutions(child)
12: end

```

3.2 Non-dominated Sorting Genetic Algorithm II

Nondominated Sorting Genetic Algorithm II, as well known as *NSGA-II* was proposed in 2002 by K. Deb and A. Pratap and S. Agarwal and T. Meyarivan to mitigate the major difficulties of nondominated sorting MOEAs [5]. In fact, this algorithm is an improvement of the previously algorithm is an improvement of the previously suggested algorithm NSGA [16] algorithm NSGA [23] in 1994 by N. Srinivas and Kalyanmoy Deb.

The main improvements of NSGA-II over NSGA is a fast nondominated sorting approach with complexity $\mathcal{O}(MN^2)$ that replaces the previous one which has complexity $\mathcal{O}(MN^3)$ [5](considering M the number of objectives and N the population size) and the selection operator of NSGA-II which comes to solve the lack of elitism of the previous NSGA version.

The fast nondominated sorting procedure starts by computing the domination count n_p which is the number of solutions that dominates p and next, the set of solutions dominated by p called S_p . Then the procedure continues identifying all Pareto Fronts and ranking the solutions in different fronts by its n_p [5]. An example of the fast nondominated sort procedure can be seen at the following pseudocode.

Algorithm 3 Fast Nondominated Sort.

```

1: for all individual  $p$  in population do
2:    $S_p = \emptyset$ 
3:    $n_p = 0$ 
4:   for all individual  $q$  in population do
5:     if  $p \prec q$  then
6:        $S_p = S_p \cup \{q\}$ 
7:     else
8:       if  $q \prec p$  then
9:          $n_p = n_p + 1$ 
10:  if  $n_p = 0$  then
11:     $p_{rank} = 1$ 
12:     $F_1 = F_1 \cup \{p\}$ 
13:   $i = 1$ 
14:  while  $f_i \neq \emptyset$  do
15:     $Q = \emptyset$ 
16:    for all  $p \in F_i$  do
17:      for all  $q \in S_p$  do
18:         $n_q = n_q - 1$ 
19:        if  $n_q = 0$  then
20:           $q_{rank} = i + 1$ 
21:           $Q = Q \cup \{q\}$ 
22:     $i = i + 1$ 
23:     $F_i = Q$ 
24: end

```

NSGA-II algorithm is quite simple and it can be seen in the pseudocode down below.

Algorithm 4 NSGA-II.

```

1:  $P = \text{CreateInitialPopulation}(N)$ 
2:  $\text{FastNondominatedSorting}(P)$ 
3: while not StopCriteria satisfied do
4:    $\text{BinaryTournamentSelection}(P)$ 
5:    $Q = \text{CreateOffspring}(P)$ 
6:    $R = \text{Combine}(P, Q)$ 
7:    $\text{FastNondominatedSorting}(R)$ 
8:    $P = \text{SelectNIndividuals}(R, N)$ 
9: end

```

3.3 Strength Pareto Evolutionary Algorithm 2

Likewise *NSGA-II* is an improvement of its predecessor NSGA, the *Strength Pareto Evolutionary Algorithm 2 (SPEA-2)* was published in 2001 by Eckart Zitzler, Marco Laumanns and Lothar Thiele as a new version of SPEA algorithm proposed in 1999 by Zitzler and Thiele [7]. Essentially, the SPEA 2 differs with SPEA in a fine-grained fitness assignment strategy, a density estimation technique, and an enhanced archive truncation method [7]. The basis of both SPEA and SPEA 2 algorithm are that they use a standard population (P) and also an *archive* (\bar{P}) or external population and follows these steps:

- Create an initial random population of size N and an empty archive.
- Then all nondominated individuals are sent to the archive.
- If the size of the archive increase over the limit \bar{N} , new archive individuals are deleted preserving the nondominated front.
- Both population and archive individuals are evaluated and a fitness value is assigned to each of them.
- After the evaluation, the mating selection phase comes.
- When the parents are selected, genetic operators are applied to generate offspring and replace the old population.

Although those are the foundations of SPEA and SPEA 2, SPEA 2 has two improvements in *fitness assignment* and *environmental selection*. On the one hand, in an effort to avoid that individuals dominated by the same archive individuals have the same fitness, each individual i in \bar{P} and P have a strength value $S(i)$ indicating the number of individuals it dominates [7].

$$S(i) = |\{j | j \in P_t + \bar{P}_t \wedge i \succ j\}|$$

After computing the strength of each individual, a *Raw fitness value* (R) is calculated for each individual.

$$R(i) = \sum_{j \in P_t + \bar{P}_t, j \succ i} S(j)$$

In the case where most individuals do not dominate each other, R it is not enough so an adaptation of the k -the nearest neighbour algorithm is included for additional density information. In this particular case, authors use k as the

result of the square root of the sample size, so $k = \sqrt{N + \bar{N}}$. Then, the density of each individual is calculated as follows.

$$D(i) = \frac{1}{\sigma_i^k + 2}$$

Finally, the fitness value (F) of each individual is defined as the sum of its raw fitness plus its density.

$$F(i) = R(i) + D(i)$$

On the other hand, the archive updating procedure of SPEA 2 is slightly different from SPEA. Primarily, it ensures two aspects [7]:

- The number of individuals in the archive maintain is regular.
- The truncation method prevents boundary solutions being removed.

The environmental selection begins by copying all nondominated individuals from archive and population which have a fitness value F lower than one to the archive for the next generation.

$$\overline{P}_{t+1} = \{i | i \in \overline{P}_t + \overline{P}_t \wedge F(i) < 1\}$$

After finishing this step, if the archive is fully filling, the environmental selection is completed. Under other conditions, new individuals are added to the archive if it is too small or deleted in other case to fit the size \bar{N} .

All the previous description of SPEA 2 algorithm can be outlined in the following pseudocode.

Algorithm 5 SPEA 2.

- 1: $P = \text{CreateInitialPopulation}(N)$
 - 2: $\overline{P} = \text{CreateEmptyArchive};$
 - 3: **while** not StopCriteria satisfied **do**
 - 4: $\text{ComputeFitness}(P, \overline{P})$
 - 5: EnvironmentalSelection;
 - 6: BinaryTournamentSelection;
 - 7: Recombination;
 - 8: Mutation;
 - 9: **end**
-

Chapter 4

Experimental Evaluation

In this chapter, the experimental evaluation of MOEA/D will be introduced. The algorithm and the experimental evaluation were developed through the same framework called Metaheuristic-based Extensible Tool for Cooperative Optimisation (METCO) ¹ proposed in [16]. In addition, the experiment were executed on a Debian GNU/Linux computer with four AMD Opteron processors at 2.8GHz and 64 GB RAM. Each run was repeated 25 times considering $1e8$ evaluations as the stop criteria.

Furthermore, with the aim of statistically supporting the conclusions extracted, the following the evaluation procedure was applied. The *hypervolume (HV)* [3] was the metric selected to compare the different configurations of MOEA/D and the *Shapiro-Wilk*, *Levene*, *ANOVA* or *Welch* test were considered for results which follow a normal distribution or *Kruskal-Wallis* test otherwise.

4.1 Instances

For this evaluation, a total number of 67 different courses were available group together in three different files:

- l_{st} : 19 starters.
- l_{mc} : 34 main courses.
- l_{ds} : 14 desserts.

Besides, the structure of every file is an CSV file with the following fields:

- Name of the course.
- Price of the course.

¹Available at: <https://github.com/PAL-ULL/software-metco>

- Binary list of different allergens in case whether the course contains or not the allergen.
- Incompatibilities.
- Amount of the different nutrients.
- Food groups which belongs the course.

4.2 Parameter Setting

In this preliminary experiment, the main goal was to find which values of the MOEA/D [28] parameters provide the best configuration facing the MPP formulation considered in this Master Thesis. The list of MOEA/D parameters is:

- Population size.
- Neighbourhood size.
- Mutation probability that was set at 0.05.
- Crossover probability which was set at 1.

At this point, it is worth mentioning that MPP takes one parameter which defines the number of different days, for which the menu plan will be designed, that in this preliminary experiment was set to **20 days**. The comments of the authors of MOEA/D algorithm in [28] about the performance of the algorithm with very small or large neighbourhood sizes and the effect of the population size on its performance were taken into account when designing this experiment. Bearing the above in mind, a wide range of values were considered. Particularly, five different values for the population size and neighbourhood size were set in order to obtain 25 different configurations of MOEA/D. The values are:

- Population size: 25, 80, 140, 190, 250.
- Neighbourhood size: 0.4, 0.3, 0.25, 0.2, 0.16 percentage of the total population size.

Table 4.1 shows the ranking of all MOEA/D configuration for a 20-days MPP related to the hypervolume values obtained at the end of the executions. The ranking (R) was calculated considering the number of times that one configuration statistically outperforms other configurations (W) and the number of times that it was outperformed by other configurations (L):

$$R = W - L$$

Configuration A statistically outperforms configuration B if the p-value, obtained after performing a pairwise comparison of both approaches by following the statistical testing procedure described at the beginning of this chapter, is lower than the significance level $\alpha = 0.05$, and if at the same time, A provides a higher mean and median of the hypervolume at the end of the runs.

Even though there is not a significant control of the best ranked configuration in Table 4.1 with only 9 wins over 24 other configurations, the MOEA/D configuration with a population size of 140 individuals and 42 individuals per neighbourhood seems to be the best one. Thus, this is the configuration chosen for the next experiment.

Configuration	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	W	L	Ranking
MOEA_D_PopSize_140_Neihb_42	0.7161	0.7733	0.7839	0.7834	0.8087	0.8435	9	0	9
MOEA_D_PopSize_250_Neihb_50	0.7270	0.7635	0.7737	0.7775	0.7930	0.8213	2	0	2
MOEA_D_PopSize_80_Neihb_16	0.7306	0.7655	0.7787	0.7774	0.7945	0.8342	2	0	2
MOEA_D_PopSize_140_Neihb_35	0.7388	0.7583	0.7671	0.7728	0.7914	0.8188	1	0	1
MOEA_D_PopSize_190_Neihb_48	0.7216	0.7521	0.7670	0.7688	0.7842	0.8034	0	0	0
MOEA_D_PopSize_25_Neihb_10	0.7228	0.7563	0.7669	0.7686	0.7805	0.8250	0	0	0
MOEA_D_PopSize_25_Neihb_4	0.7363	0.7490	0.7628	0.7682	0.7817	0.8174	0	0	0
MOEA_D_PopSize_80_Neihb_13	0.7231	0.7563	0.7690	0.7691	0.7874	0.8143	0	0	0
MOEA_D_PopSize_25_Neihb_8	0.7384	0.7490	0.7725	0.7716	0.7884	0.7989	0	0	0
MOEA_D_PopSize_140_Neihb_28	0.7299	0.7583	0.7715	0.7715	0.7892	0.8005	0	0	0
MOEA_D_PopSize_80_Neihb_32	0.7256	0.7428	0.7708	0.7675	0.7840	0.8190	0	0	0
MOEA_D_PopSize_250_Neihb_62	0.7244	0.7577	0.7733	0.7702	0.7847	0.8189	0	0	0
MOEA_D_PopSize_80_Neihb_24	0.7234	0.7540	0.7710	0.7712	0.7843	0.8158	0	0	0
MOEA_D_PopSize_140_Neihb_22	0.7292	0.7504	0.7728	0.7684	0.7813	0.8231	0	0	0
MOEA_D_PopSize_250_Neihb_100	0.7328	0.7501	0.7775	0.7721	0.7925	0.8253	0	0	0
MOEA_D_PopSize_25_Neihb_6	0.7211	0.7506	0.7673	0.7706	0.7897	0.8300	0	0	0
MOEA_D_PopSize_250_Neihb_40	0.7158	0.7507	0.7737	0.7674	0.7811	0.8155	0	1	-1
MOEA_D_PopSize_190_Neihb_57	0.6755	0.7441	0.7698	0.7655	0.7809	0.8135	0	1	-1
MOEA_D_PopSize_140_Neihb_56	0.7149	0.7472	0.7704	0.7664	0.7799	0.8311	0	1	-1
MOEA_D_PopSize_250_Neihb_75	0.7078	0.7443	0.7664	0.7637	0.7765	0.8143	0	1	-1
MOEA_D_PopSize_190_Neihb_76	0.7374	0.7546	0.7694	0.7685	0.7818	0.8018	0	1	-1
MOEA_D_PopSize_25_Neihb_5	0.7299	0.7494	0.7683	0.7672	0.7802	0.8180	0	1	-1
MOEA_D_PopSize_80_Neihb_20	0.7269	0.7484	0.7676	0.7663	0.7732	0.8217	0	1	-1
MOEA_D_PopSize_190_Neihb_38	0.7224	0.7515	0.7643	0.7634	0.7780	0.8113	0	3	-3
MOEA_D_PopSize_190_Neihb_30	0.7280	0.7494	0.7581	0.7607	0.7698	0.7978	0	4	-4

Table 4.1: Ranking of all MOEA/D configurations

4.3 Problem size variation

In this experiment, the main goal was to analyse how the problem dimension affects in the performance of the best MOEA/D configuration found so far for MPP. For that reason, other 25 independent executions of MOEA/D with 140 individuals in the population and neighbourhoods of 42 individuals were run for instances of the MPP of 5, 10 and 40 days MPP. Table 4.2 below shows the minimum, median, mean and maximum hypervolume values obtained. The best MOEA/D configuration is compared to the best results for NSGA-II and SPEA-2 for every different MPP instance. As it can be observed, both NSGA-II and SPEA-2 outperformed MOEA/D in every different MPP instance with statistically significant differences.

Menu plannings for 5 days				
Configuration	Min.	Std	Mean	Max.
NSGA2_PopSize_250_pm_0.2_pc_0.8	0.956507	0.006186	0.969247	0.977835
SPEA2_ps_100_ArchSize_100_pm_0.2_pc_0.8	0.937224	0.007334	0.951471	0.964441
MOEA_D_PopSize_140_Neihb_42	0.747678	0.030129	0.827381	0.873078
Menu plannings for 10 days				
Configuration	Min	Std	Mean	Max.
NSGA2_PopSize_250_pm_0.2_pc_0.8	0.934024	0.008141	0.948577	0.961192
SPEA2_PopSize_100_ArchSize_100_pm_0.2_pc_0.8	0.9237	0.008851	0.941078	0.955088
MOEA_D_PopSize_140_Neihb_42	0.743725	0.030455	0.783656	0.83404
Menu plannings for 20 days				
Configuration	Min.	Std	Mean	Max.
SPEA2_PopSize_100_ArchSize_100_pm_0.1_pc_0.8	0.906556	0.011438	0.925087	0.945195
NSGA2_ps_250_pm_0.05_pc_0.8	0.940483	0.014064	0.921332	0.940483
MOEA_D_PopSize_140_Neihb_42	0.7161	0.03205	0.7834	0.8435
Menu plannings for 40 days				
Configuration	Min.	Std	Mean	Max.
SPEA2_ps_100_ArchSize_100_pm_0.025_pc_0.8	0.891074	0.012357	0.910226	0.929
NSGA2_ps_250_pm_0.05_pc_0.8	0.886774	0.008034	0.9019	0.918159
MOEA_D_PopSize_140_Neihb_42	0.65815	0.033453	0.716339	0.783212

Table 4.2: MOEA/D performance comparison against best results of NSGA-II and SPEA-2 with different MPP instance sizes.

Additionally, the Figure 4.1 shows how the average hypervolume value evolves and both NSGA-II and SPEA-2 reach considerably higher values than MOEA/D. With less than $0.2e8$ evaluations, the average hypervolume value is noticeably higher than the average value reached by MOEA/D at $1e8$ evaluations. The same comparison it is done in Figures 4.2, 4.3 and 4.4 for 10, 20 and 40 days MPP instances, respectively.

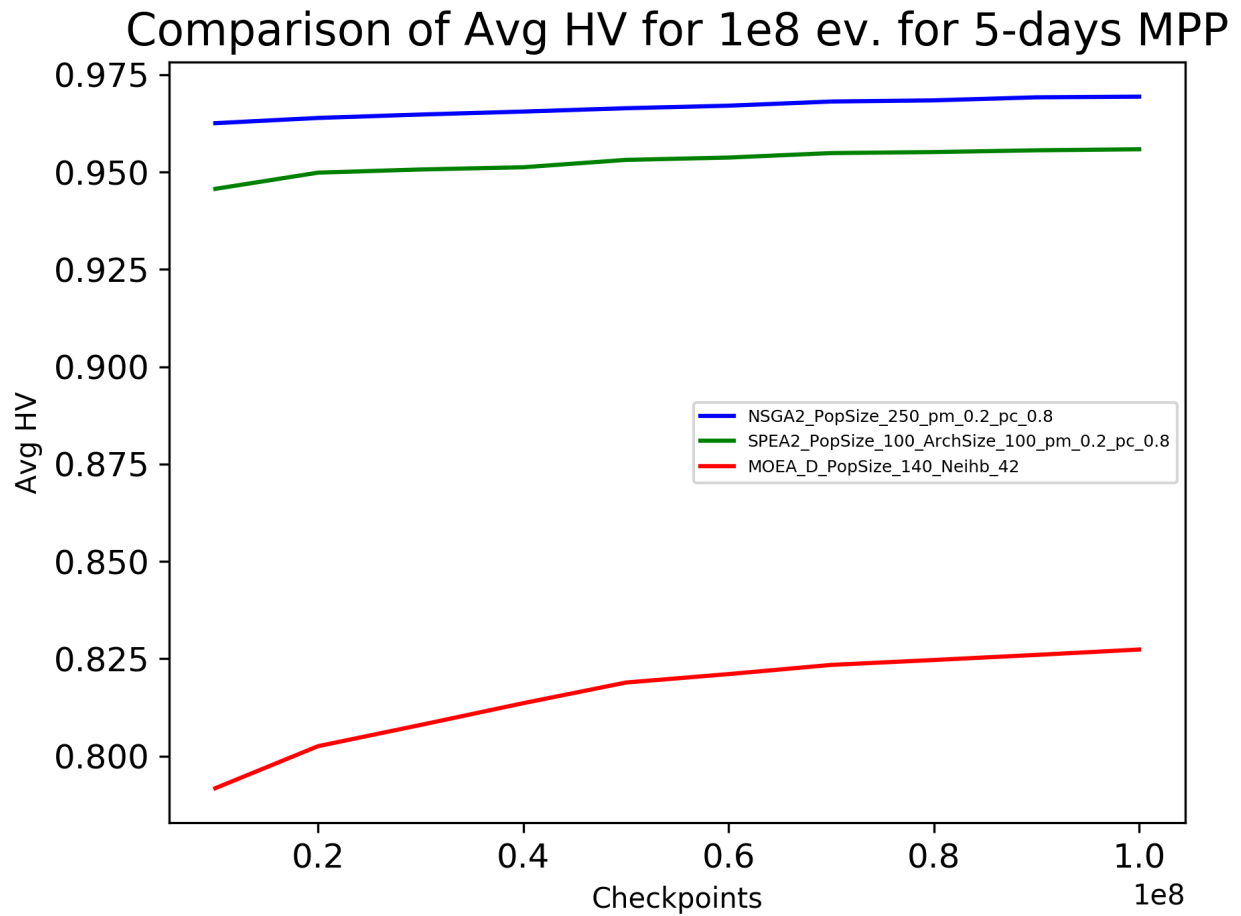


Figure 4.1: Evolution of the average HV value for 5-days MPP at 1e8 evaluations.

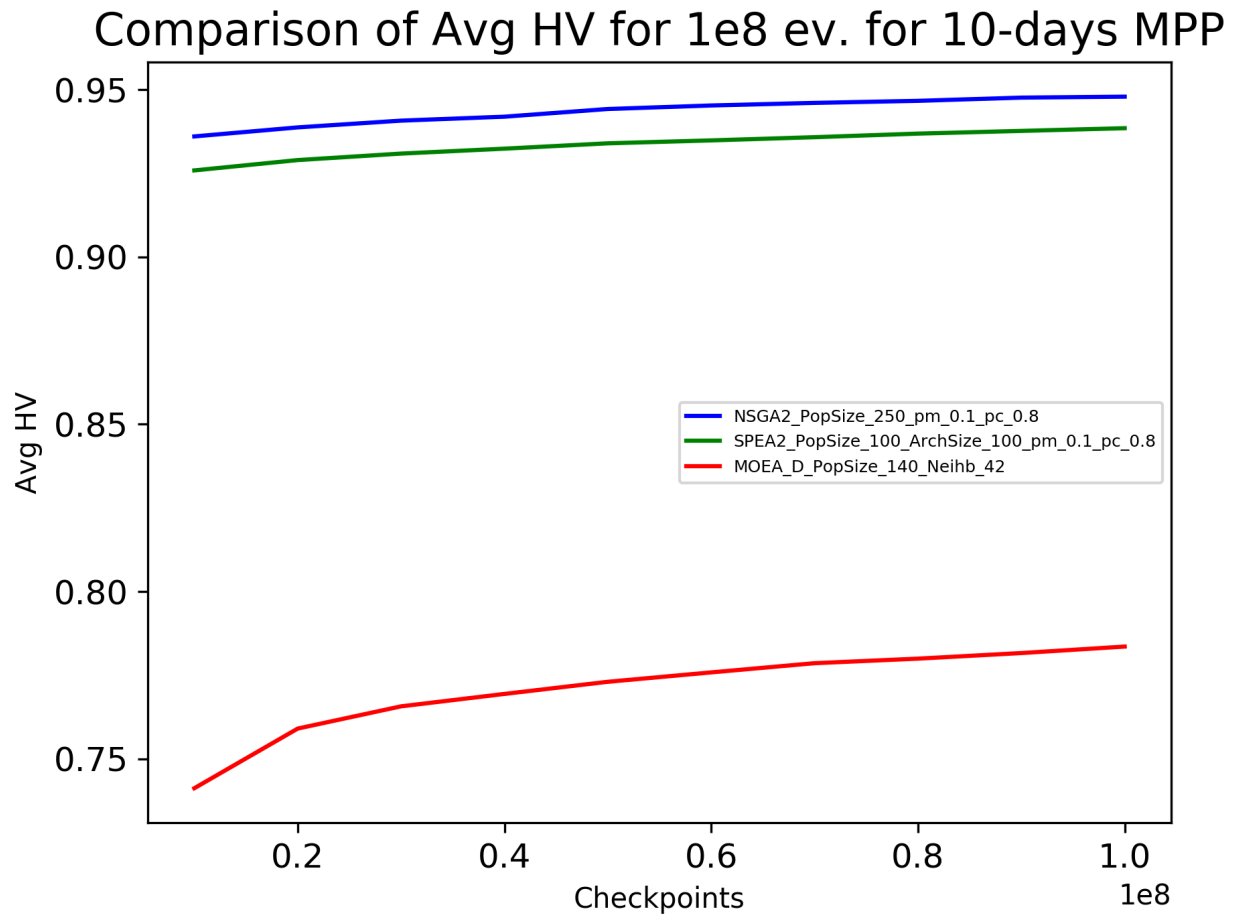


Figure 4.2: Evolution of the average HV value for 10-days MPP at 1e8 evaluations.

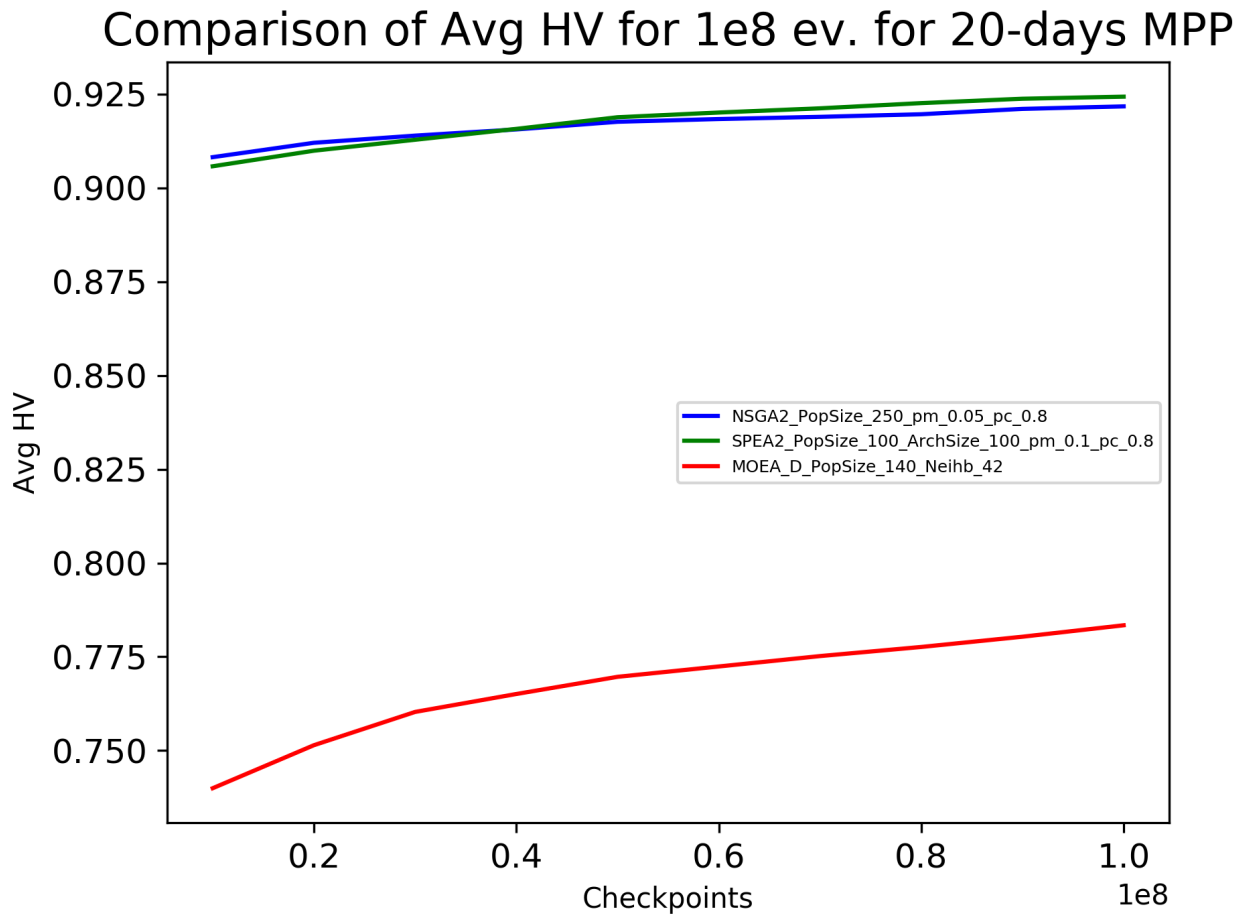


Figure 4.3: Evolution of the average HV value for 20-days MPP at 1e8 evaluations.

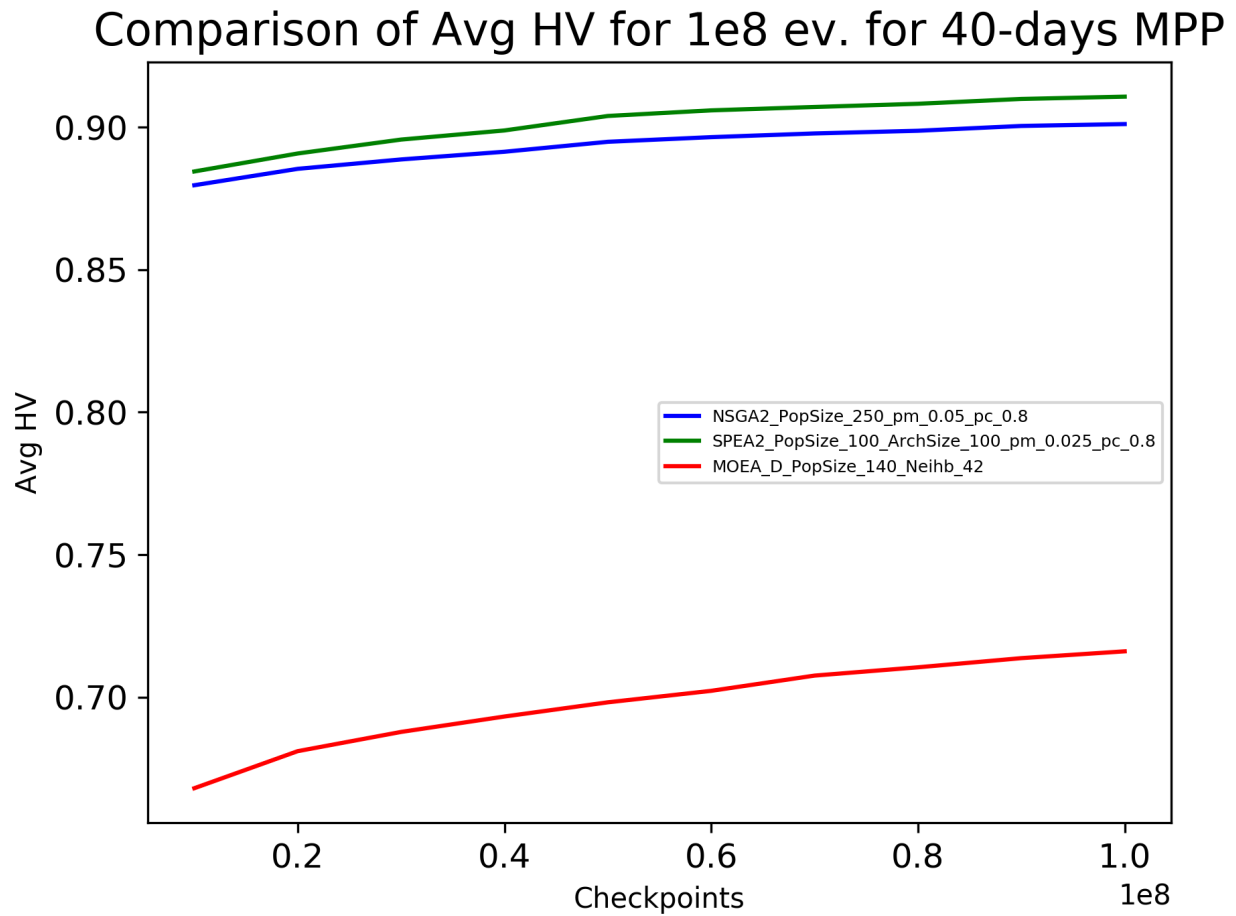


Figure 4.4: Evolution of the average HV value for 40-days MPP at 1e8 evaluations.

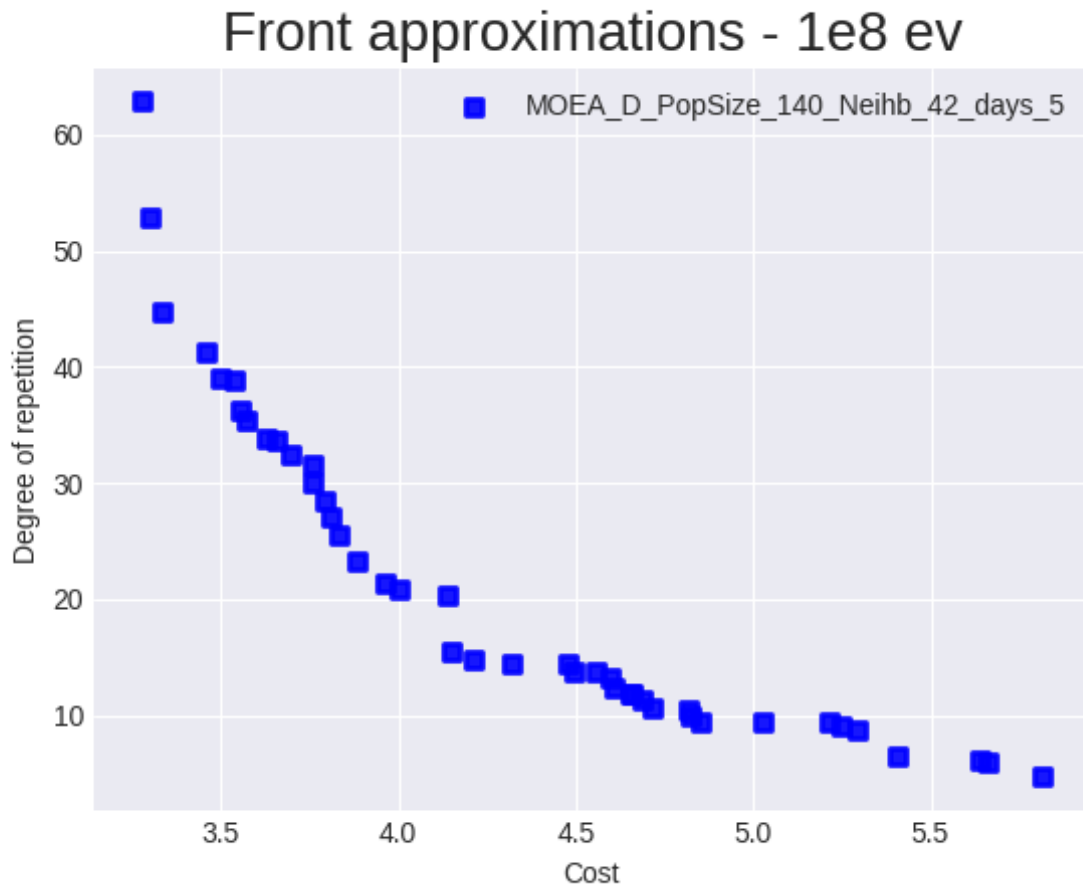


Figure 4.5: Front approximation at 1e8 evaluations from best MOEA/D configuration found facing 5-days MPP.

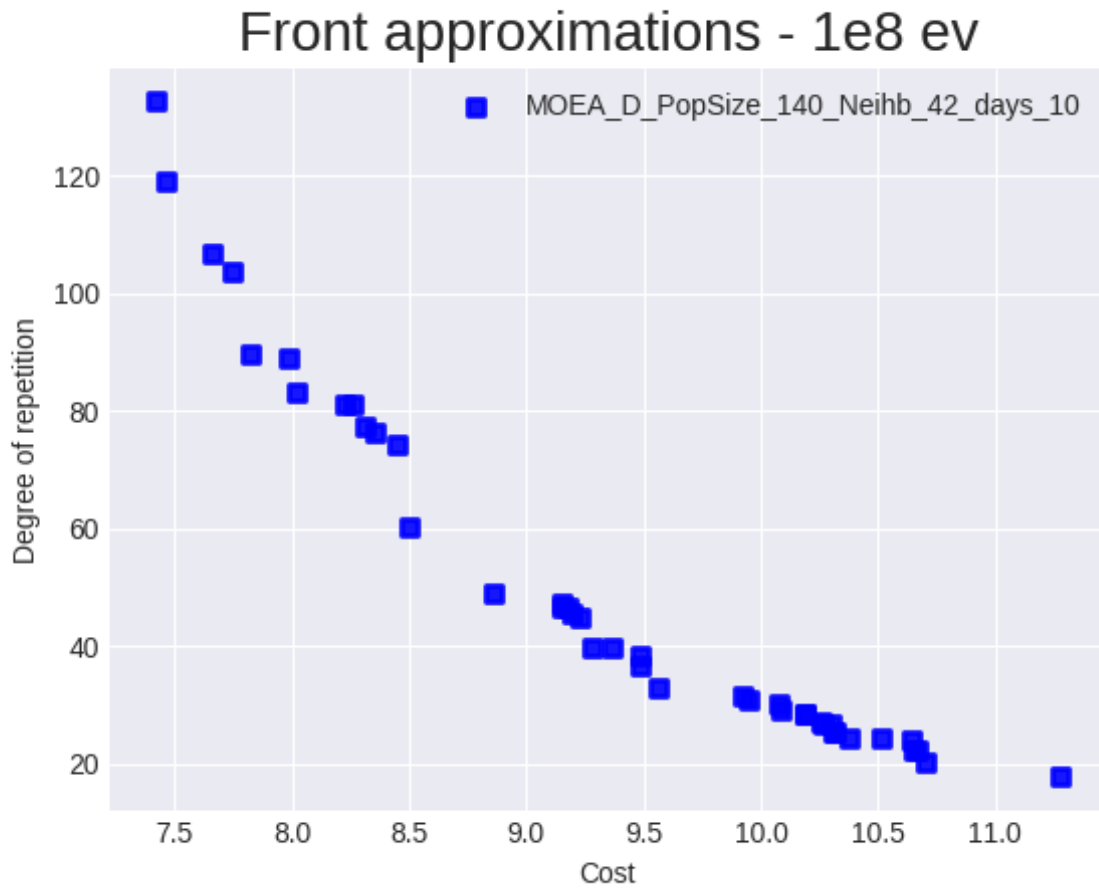


Figure 4.6: Front approximation at 1e8 evaluations from best MOEA/D configuration found facing 10-days MPP.

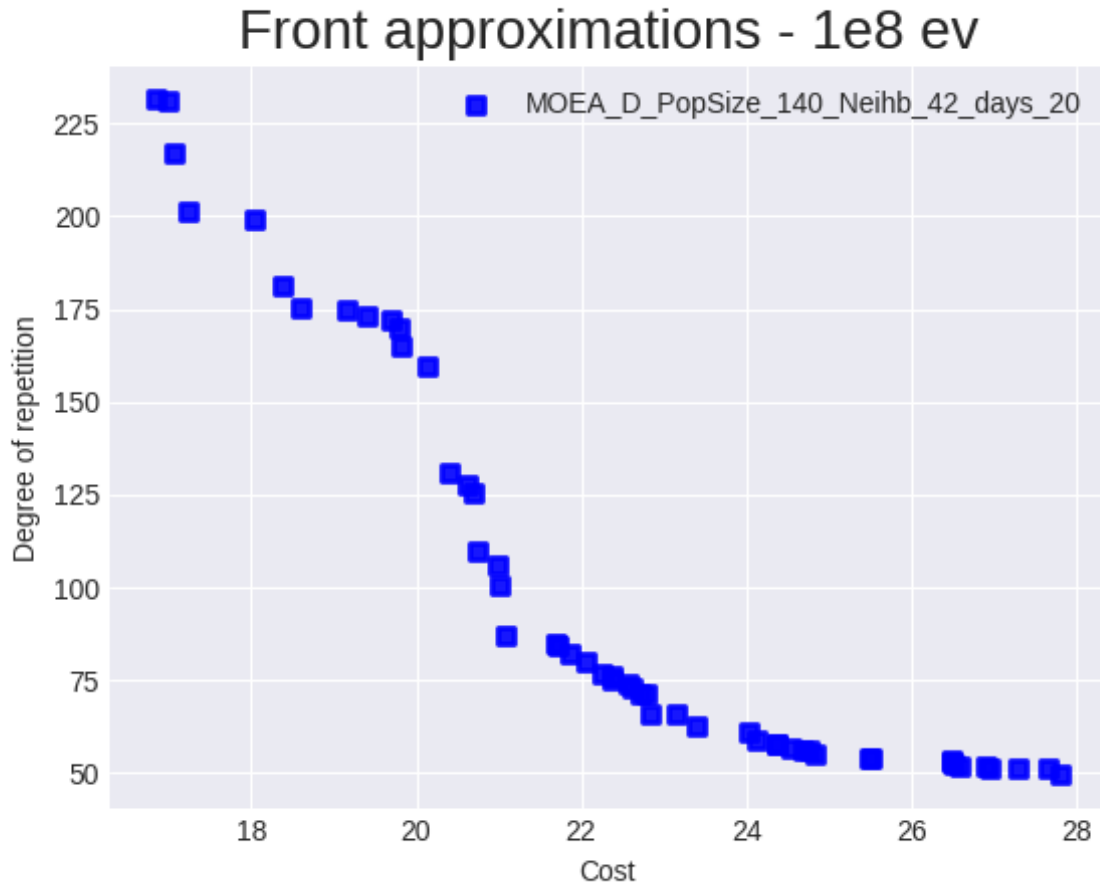


Figure 4.7: Front approximation at 1e8 evaluations from best MOEA/D configuration found facing 20-days MPP.

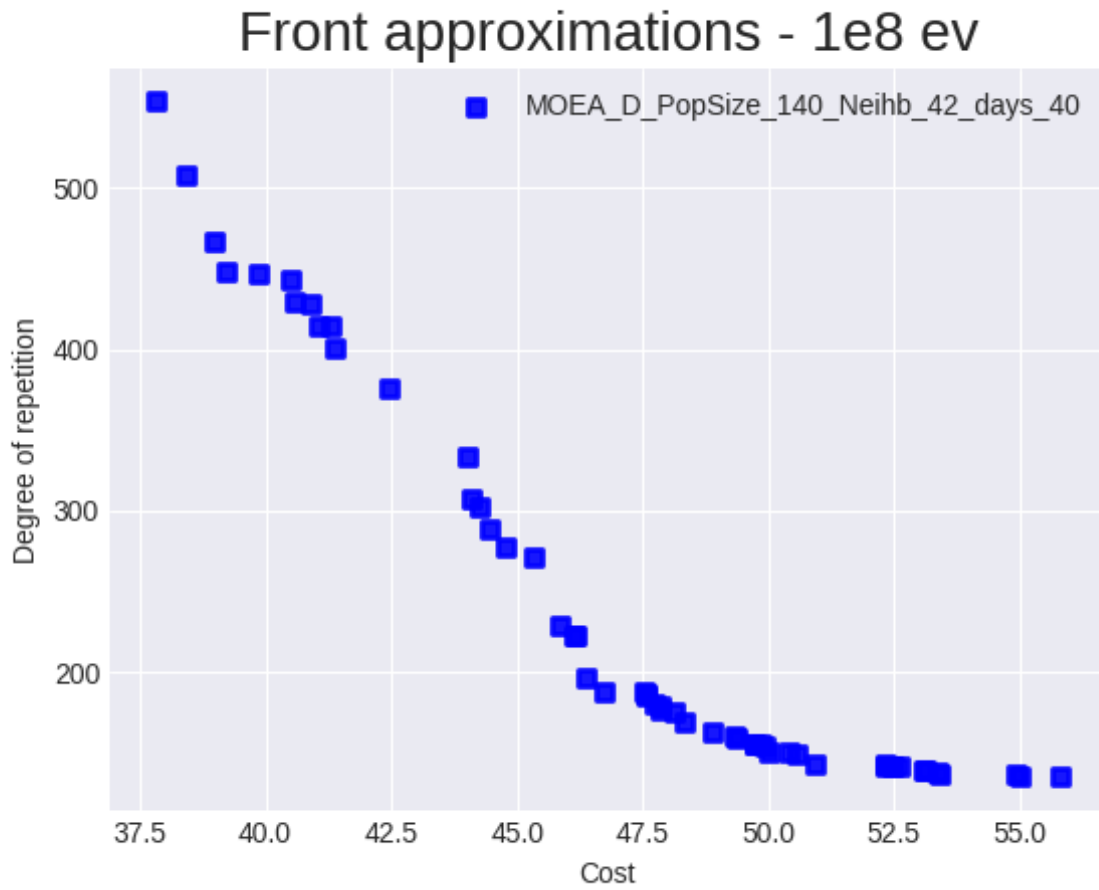


Figure 4.8: Front approximation at 1e8 evaluations from best MOEA/D configuration found facing 40-days MPP.

Chapter 5

Summary and conclusions

5.1 Conclusions and future work

As seen in Chapter 4, NSGA-II still being the state-of-art in Multi-Objective Evolutionary Algorithms since it outperforms both SPEA-2 and MOEA/D with statistically significant differences for this recently proposed MPP formulation.

Regarding MOEA/D algorithm, the quite simple version developed for this Master Thesis does not obtain as high quality solutions as NSGA-II or SPEA-2. In the experimental evaluation explained in Chapter 4, the population size and neighbourhood size seems not to have a high impact into the performance of the MOEA/D algorithm as it can be appreciated in the ranking from the preliminary experimental evaluation in Table 4.1.

For further work, considering a new approach for initial weight generation may be a interesting choice as well as a more depth experimental evaluation with MOEA/D considering the mutation and crossover probability rate and increasing the evaluation limit to 4e8 evaluations.

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Appendices

Appendix A

Resources

All the work done in this Master Thesis can be found in the following [Github repository](#).