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PREFERENCE AGGREGATION, COLLECTIVE CHOICE AND GENERALIZED BINARY CONSTITUTIONS[1]

Nicolas-Gabriel ANDJIGA, Joël MOULEN[2],

RÉSUMÉ – Agrégation des préférences, choix collectif et constitution généralisée binaire
Ce papier, intitulé agrégation des préférences, choix collectif et constitutions généralisées binaires, a pour objectif l'étude de la notion de constitution généralisée binaire (CGB), distribution de pouvoirs définie par Ferejohn et Fishburn [1979] qui généralise les notions de jeux simples et de jeux de vote. Une CGB permet de définir une procédure d'agrégation des préférences (PAP) et nous caractérisons les CGB pour lesquelles les PAP associées conduisent à des préférences collectives qui sont toujours soit complètes, soit asymétriques, soit transitives soit acycliques lorsque les préférences individuelles sont des préordres ou des ordres totaux. Les PAP associées à des CGB étant équivalentes aux procédures d'agrégation des préférences vérifiant l'indépendance vis-à-vis des alternatives extérieures, nous faisons un tour d'horizon de quelques résultats arrowiens. Sous les mêmes hypothèses de préférences individuelles, nous caractérisons les CGB dont le coeur est non vide et obtenons les résultats classiques dont le théorème de Nakamura sur les jeux simples.

MOTS CLÉS – Coeur, Constitution généralisée binaire, Indépendance vis-à-vis des options extérieures, Jeu simple, Procédure d'agrégation des préférences

SUMMARY – *The aim of this paper is to study the notion of Generalized Binary Constitution (GBC), a distribution of power due to Ferejohn and Fishburn [1979], which generalizes some classical notions such as simple games and voting games. The GBC helps us to define a preference aggregation rule (PAR) and we characterize GBC's whose collective preferences are either complete, asymmetric, transitive or acyclic when individual preferences are weak orders or linear orders. Since the procedure of aggregation of preferences which satisfies IIA is equivalent to the preference aggregation rule of a GBC, we give relations between our results and some Arrovian results. We also characterize core-stable GBC's and therefore deduce classical results and in particular Nakamura's theorem for simple games.*

KEYWORDS – Core, General binary constitution, Independence of irrelevant alternatives, Preference agregation rule, Simple game

1. INTRODUCTION

Many results of social choice theory show that when a preference aggregation rule satisfies certain axioms, then its underlying power structure also satisfies particular properties and in particular it can be dictatorial or oligarchic. Since Arrow, the

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relation between the distribution of decision power over individuals and the properties of a preference aggregation rule is an important stream of social choice theory. This paper is devoted to study the notion of generalized binary constitution (GBC), a notion which generalizes the notion of binary constitution due to Ferejohn and Fishburn [1979]. The best introduction to this notion is their following remark:

> *"A common notion of decisive coalition is a subset of individuals such that when they unanimously prefer one alternative to another, then the first is socially preferred to the second... The point we wish to stress on is that no conception of decisive coalition that characterizes decisiveness in terms of simple subsets of individuals, even if it is made to depend explicitly on pairs of alternatives, is adequate to characterize certain interesting aggregation procedures. An example of this is the simple majority aggregation procedure, in which x is socially preferred to y if and only if more individuals prefer x to y than y to x. In this case every coalition that contains more than half of the individuals is decisive, but what about other coalitions? For example, a non empty coalition is "decisive" when all other individuals are indifferent but is not generally decisive... Our proposal to remedy the deficiencies noted above is very simple and perhaps obvious by now. It is to characterize decisiveness structures by ordered pairs of disjoint coalitions rather than by single coalitions".*

The GBC is, then a structure of distribution of power which generalizes some classical notions: the relative majority rule, simple games and voting games (Nakamura [1979], Andjiga and Mbih [2000]), the decisive GBC's (called binary game in constitutional form in Andjiga and Moulen [1989]) and the binary constitutions (BC) (Ferejohn and Fishburn (1979]). The notion of GBC is therefore an appropriate framework to unify various structures and various results of social choice theory.

We will divide our paper into two major parts:

a) the study of a GBC as defining a preference aggregation rule (PAR) through the notion of dominance in section 1;

b) the study of a GBC as defining a collective choice rule (CCR) through the notion of core in section 2.

Let us recall that with a given profile of individual preferences, a PAR associates a collective binary relation whereas a CCR associates a subset of alternatives.

In section 1, we assume that individual preferences are either weak orders or linear orders. The first significant result is the equivalence between the PAR of a GBC and a PAR which satisfies (the Arrovian) independence of irrelevant alternatives (IIA) axiom. For each domain of individual preferences, we give necessary and sufficient conditions for the dominance of a GBC (and thus for others, particular structures) to be either complete, antisymmetric, transitive or acyclic. Each of these results can be seen as a focus on the (degree of complexity of the) power of decision which underlines each of these properties on collective preference. Let us point out that few new results in general are obtained in section 1. Most of them were already obtained in Ferejohn and Fishburn [1979] and in Aleskerov and Vladimirov [1986]. We will not be interested in obtaining the Arrow theorem again (this has been done by Monjardet, [2003]). Furthermore, the relations between acyclic PAR's, and the existence of vetoers and oligarchies are very well discussed in Moulin [1985], Le Breton and Truchon [1995] and Banks [1995]. The notions of cycles of a GBC will be used to link up the two sections.

Section 2 is devoted to the study of the core-stability of a GBC. From the main theorem, we deduce classical results obtained on simple games (Nakamura, [1979]), voting games (Andjiga and Mbih, [2000]) and decisive GBC's (Andjiga and Moulen, [1989]; Truchon, [1995]).

2. NOTATIONS AND DEFINITIONS

Let N be the finite set of players and A the finite set of alternatives.

If R is a binary relation on A, R is

- **transitive** if $\forall x, y, z \in A$, xRy and $yRz \Longrightarrow xRz$,
- **reflexive** if $\forall x \in A$, xRx,
- **irreflexive** if $\forall x \in A, not\ (xRx)$,
- **complete** if $\forall x, y \in A$, $x \neq y \Longrightarrow xRy$ or yRx,
- **strongly complete** if $\forall x, y \in A$, xRy or yRx,
- **antisymmetric** if $\forall x, y \in A$, xRy and $yRx \Longrightarrow x = y$,
- **asymmetric** if $\forall x, y \in A$, $xRy \Longrightarrow not\ (\ yRx)$,
- **negatively transitive** if $\forall x, y, z \in A$, $zRx \Longrightarrow yRx$ or zRy.

$(a_1, a_2, \cdots, a_m)$ is a **cycle of order** m of R if: $\forall k = 1, ..., m, a_k R a_{k+1}$ where $a_{m+1} = a_1$.

One point on which papers in social choice theory differ is the nature of individual preferences. For some authors (as Arrow, [1963]), individual preference relations are complete while others (as Ferejohn and Fishburn, [1979]) assume that individual preference relations are asymmetric (and so irreflexive).

Let us compare these two formalizations of the individual preferences:

If R is a binary relation, **the asymmetric component of** R is the (asymmetric) binary relation P defined by: $\forall x, y \in A, xPy \Leftrightarrow xRy$ and $not(yRx)$ and its **symmetric component** is the (symmetric) binary relation I defined by: $\forall x, y \in A, xIy \Leftrightarrow xRy$ and yRx. Since P and R are disjoint relations, we will write $R = P + I$. When R is a complete relation, one has $xPy \Leftrightarrow not(yRx)$.

Conversely if Q is an asymmetric relation, we define a (complete) relation R as follows: $\forall x, y \in A, xRy \Leftrightarrow not(yQx)$.

Observe that Q is the asymmetric component of R.

We will give the usual definitions on individual preferences first of all, in the irreflexive and asymmetric version, and then in the complete version.

DEFINITION 1. *a) R is a* **strict linear** *order if R is asymmetric, complete and transitive.*

b) R is a **strict weak order** *if R is asymmetric, negatively transitive and transitive.*

c) R is a **strict partial order** *if R is asymmetric and transitive.*

d) R is **acyclic** *if R is a relation without cycles.*

Let us note that since an irreflexive and transitive relation is asymmetric, a) b) and c) of the above definition can therefore be stated with irreflexivity instead of asymmetry. Furthermore, R has no cycle of order 2 if and only if R is asymmetric.

DEFINITION 2. *a) R is a* **linear order** *if R is antisymmetric, strongly complete and transitive (or R is strongly complete and its asymmetric component is a strict linear order).*

b) R is a **weak order** *if R is strongly complete and transitive (or R is strongly complete and its asymmetric component is a strict weak order).*

Let us remark that in general each author denominates notions in definition 1 and 2. Our choice of names is therefore personal, for example our notion of strict weak order is called asymmetric weak order by Ferejohn and Fishburn [1979].

We denote by:

- $\mathcal{B}$ (resp. $\mathcal{B}^N$) the set of binary relations on A (resp. the set of profiles of binary relations on A);
- $\mathcal{C}$ (resp. $\mathcal{C}^N$) the set of complete binary relations on A (resp. the set of profiles of complete binary relations on A);
- $\mathcal{H}$ (resp. $\mathcal{H}^N$) the set of complete and asymmetric binary relations on A (resp. the set of profiles of complete and asymmetric binary relations on A);
- $\mathcal{L}^*$ (resp. $\mathcal{L}^{*N}$) the set of strict linear orders on A (resp. the set of profiles of strict linear orders on A);
- $\mathcal{W}^*$ (resp. $\mathcal{W}^{*N}$) the set of strict weak orders on A (resp. the set of profiles of strict weak orders on A);
- $\mathcal{L}$ (resp. $\mathcal{L}^N$) the set of linear orders on A (resp. the set of profiles of linear orders on A);
- $\mathcal{W}$ (resp. $\mathcal{W}^N$) the set of weak orders on A (resp. the set of profiles of weak orders on A);
- $P(K)$ (resp. 2^K) the set of subsets (resp. non empty subsets) of K;
- $|K|$ the cardinality of a finite set K;
- $\mathcal{A} = \{(x, y) \in A \times A / x \neq y\}$;
- $\mathcal{N} = \{(S, T) \in P(N) \times P(N) / S \cap T = \varnothing\}$.

If R^N and Q^N are profiles of binary relations and $\{x, y\}$ a pair of A, we denote by:

- $R^N(x, y) \subseteq Q^N(x, y)$ if : $\forall i \in N$, $xR^iy \Rightarrow xQ^iy$,
- $R^N(x, y) = Q^N(x, y)$ if : $\forall i \in N$, $xR^iy \Longleftrightarrow xQ^iy$,
- $R^N\{x, y\} = Q^N\{x, y\}$ if : $R^N(x, y) = Q^N(x, y)$ and $R^N(y, x) = Q^N(y, x)$,
- $N(x, y, R^N) = \{i \in N/xP^iy\}$ and $\pi(x, y, R^N) = \big(N(x, y, R^N), N(y, x, R^N)\big)$.

Let us first define various distributions of power:

DEFINITION 3. *a)* **A generalized binary constitution (GBC)** *is a mapping* Δ *from* $\mathcal{A}$ *to* $\mathcal{N}$.

b) Δ *is*

i) **complete** *if:* $\forall (x, y) \in \mathcal{A}, \forall (S, T) \in \mathcal{N}, (S, T) \in \Delta(x, y)$ *or* $(T, S) \in \Delta(y, x)$,

ii) **asymmetric** *if:* $\forall (x, y) \in \mathcal{A}, \forall (S, T) \in \mathcal{N}, (S, T) \in \Delta(x, y) \Longrightarrow (T, S) \notin \Delta(y, x)$,

iii) **decisive** *if:* $\forall (x, y) \in \mathcal{A}, \forall (S, T) \in \mathcal{N}, \forall U \subseteq N, (S, T) \in \Delta(x, y)$ *and* $(S, U) \in \mathcal{N} \Longrightarrow (S, U) \in \Delta(x, y)$,

iv) **neutral** *if: there exists* $\mathcal{U} \subseteq \mathcal{N}, \forall (x, y) \in \mathcal{A}, \Delta(x, y) = \mathcal{U}$,

v) **monotonic** *if :* $\forall (x, y) \in \mathcal{A}, \forall (S, T), (U, V) \in \mathcal{N}, (S, T) \in \Delta(x, y), S \subseteq U$ *and* $V \subseteq T \Longrightarrow (U, V) \in \Delta(x, y)$,

vi) **Paretian** *if:* $\forall (x, y) \in \mathcal{A}, (N, \emptyset) \in \Delta(x, y)$.

We can notice that these previous notions on a GBC are pairwise independent. But Δ is Paretian if Δ is monotonic and $\forall (x, y) \in \mathcal{A}, \Delta(x, y) \neq \emptyset$.

Let Δ be a GBC, $(x, y) \in \mathcal{A}$ and $(S, T) \in \mathcal{N}$. Intuitively $(S, T) \in \Delta(x, y)$ means that x is socially preferred to y when the set of individuals who prefer x to y is S and the set of individuals who prefer y to x is T.

If Δ is a decisive GBC, $(S, T) \in \Delta(x, y)$ means that x is socially preferred to y when the set of individuals who prefer x to y is S (without any condition on the other individual preferences).

DEFINITION 4. *A GBC is called*

i) a **binary constitution (BC)** *if it is asymmetric,*

ii) a **voting game** *if it is decisive and neutral,*

iii) a **simple game** *if it is a monotonic voting game.*

If Δ is decisive, for every $(x, y) \in \mathcal{A}$, $\Delta(x, y)$ depends only on a family $\mathcal{F}(x, y)$ of coalitions of N (empty or not) which verifies: $\mathcal{F}(x, y) = \{S \subseteq N/\exists T \subseteq N, (S, T) \in \Delta(x, y)\}$.

A decisive GBC is therefore an application from $\mathcal{A}$ to $P(N)$.

If Δ is a voting game (i.e. decisive and neutral) this family is the same for each (x, y) in $\mathcal{A}$ and Δ is a simple game if furthermore this family is monotonic.

3. DOMINANCE OF A GBC

In this section we will first give classical properties of a preference aggregation rule. After the definition of the dominance of a GBC Δ, we will study the relationship between properties of the distribution of power Δ and properties of F_Δ, the preference aggregation rule associated to Δ.

DEFINITION 5. **A Preference aggregation rule (PAR)** *is a mapping* F *from* $\mathcal{C}^N$ *to* $\mathcal{B}$.

a) F *satisfies the* **independence of irrelevant alternatives (IIA)** *if:*
$\forall R^N, Q^N \in \mathcal{C}^N, \forall (x,y) \in \mathcal{A},\ R^N\{x,y\} = Q^N\{x,y\} \Rightarrow \left[xF(R^N)y \Leftrightarrow xF(Q^N)y\right].$

b) F *is* **decisive** *if:*
$\forall R^N, Q^N \in \mathcal{C}^N, \forall (x,y) \in \mathcal{A}, N(x,y,R^N) = N(x,y,Q^N) \Rightarrow \left[xF(R^N)y \Leftrightarrow xF(Q^N)y\right].$

c) F *is* **monotonic** *if:*
$\forall R^N, Q^N \in \mathcal{C}^N, \forall (x,y) \in \mathcal{A}, R^N(x,y) \subseteq Q^N(x,y) \Rightarrow \left[xF(R^N)y \Rightarrow xF(Q^N)y\right].$

d) F *is* **neutral** *if:*
$\forall R^N, Q^N \in \mathcal{C}^N, \forall x,y,a,b \in A,\ R^N\{x,y\} = Q^N\{a,b\} \Rightarrow \left[xF(R^N)y \Leftrightarrow aF(Q^N)b\right].$

Let us remark that if F is decisive, monotonic or neutral, then F satisfies IIA.

DEFINITION 6. *Let* Δ *be a GBC,* $R^N \in \mathcal{C}^N$ *and* $(x,y) \in \mathcal{A}$.

i) x **dominates** y *in* (Δ, R^N) *, denoted* $xd_\Delta(R^N)y$*, if* $\pi(x,y,R^N) \in \Delta(x,y)$.

ii) The **preference aggregation rule of** Δ*, called also the* **dominance of** Δ, *is the PAR, denoted* F_Δ*, defined by:*
$\forall R^N \in \mathcal{C}^N, (x,y) \in \mathcal{A}, xF_\Delta(R^N)y \Leftrightarrow xd_\Delta(R^N)y.$

When the PAR F_Δ of a GBC Δ satisfies some properties, for instance, when $F_\Delta(R^N)$ is always an acyclic relation, we will also say that the dominance of Δ is, for instance acyclic.

Let us first characterize PAR's which are dominances of GBC's.

THEOREM 1. *Let* F *be a PAR.* F *is a dominance of a GBC if and only if* F *satisfies IIA.*

Proof. i) Let F_Δ be the PAR of Δ, R^N and $Q^N \in \mathcal{C}^N$, and $(x,y) \in \mathcal{A}$, which satisfy $R^N\{x,y\} = Q^N\{x,y\}$.
$R^N\{x,y\} = Q^N\{x,y\}$ implies $\pi(x,y,R^N) = \pi(x,y,Q^N)$. Consequently $\pi(x,y,R^N) \in \Delta(x,y) \Leftrightarrow \pi(x,y,Q^N) \in \Delta(x,y)$. By the definition of F_Δ we obtain $xF_\Delta(Q^N)y$ if and only if $xF_\Delta(R^N)y$ and F_Δ is IIA.

ii) Conversely, let F be a PAR which satisfies IIA. We define Δ_0 as follows: $\forall (x,y) \in \mathcal{A}, \forall (S,T) \in \mathcal{N}, (S,T) \in \Delta_0(x,y) \Leftrightarrow \exists R^N \in \mathcal{C}^N, xF(R^N)y, (S,T) = \pi(x,y,R^N)$. It is easy to prove that: $F = F_{\Delta_0}$.

The equivalence between the preference aggregation rule of a GBC and a preference aggregation rule which satisfies IIA is one of the major achievements of the notion of GBC which can therefore be used as an unified notion for Arrovian results. Let us deduce the following more classic characterizations of particular PAR's:

COROLLARY 1. *a) F is a PAR of a decisive GBC if and only if F is decisive.*

b) F is a PAR of a neutral GBC if and only if F is neutral.

c) F is the PAR of a voting game if and only if F is decisive and neutral.

d) F is the PAR of a simple game if and only if F is decisive, neutral and monotonic.

Moreover one has the following obvious result:

PROPOSITION 1. *Let* Δ *be a GBC.*

i) $\forall R^N \in \mathcal{C}^N, F_\Delta(R^N)$ *is complete on* $A \Longleftrightarrow \Delta$ *is complete.*

ii) $\forall R^N \in \mathcal{C}^N, F_\Delta(R^N)$ *is asymmetric on* $A \Longleftrightarrow \Delta$ *is asymmetric.*

REMARK 1. Let Δ be a GBC and Δ^* , the asymmetric component of Δ be defined as follows: $\forall (x,y) \in \mathcal{A}$, $\forall (S,T) \in \mathcal{N}, (S,T) \in \Delta^*(x,y) \Longleftrightarrow (S,T) \in \Delta(x,y)$ and $T = N - S$. We can notice that Δ and Δ^* can have very different power distributions. For example, let us consider that Δ is the simple majority GBC as quoted in the introduction. Then coalitions have relative power of decision, but Δ^* is the strict majority simple game with coalitions which are decisive. When individual preferences are complete and asymmetric, it is easy to prove the following important relation between Δ and Δ^* : $\forall R^N \in \mathcal{H}^N, F_\Delta(R^N) = F_{\Delta^*(R^N)}$.

3.1. TRANSITIVITY

One of the most usual properties of a preference aggregation rule F is to associate with each profile of individual preferences, a collective relation which is transitive. Let us characterize such a rule when F is a PAR of a GBC (i.e. when F satisfies IIA) for various domains.

LEMMA 1. *Let* R^N *be a profile of weak orders on* $x, y, z, (S_0, S_1) = \pi(x, y, R^N), (T_0, T_1) = \pi(y, z, R^N)$ *and* $(U_0, U_1) = \pi(x, z, R^N)$*.*

$$\text{then} \left\{ \begin{array}{c} S_0 \cap T_0 \subseteq U_0 \subseteq S_0 \cup T_0 \\ S_1 \cap T_1 \subseteq U_1 \subseteq S_1 \cup T_1 \\ S_0 \backslash (U_0 \cup U_1) = T_1 \backslash (U_0 \cup U_1) \\ T_0 \backslash (U_0 \cup U_1) = S_1 \backslash (U_0 \cup U_1) \end{array} \right.$$

Proof. Left to the reader:

THEOREM 2. *Let* Δ *be a GBC with* $|A| \geq 3$*. The following assertions are equivalent:*

i) $\forall R^N \in \mathcal{W}^N, F_\Delta(R^N)$ *is transitive on* A*.*

ii) $\forall x, y, z \subset A, \forall (S_0, S_1), (T_0, T_1), (U_0, U_1) \in \mathcal{N}$

$$(\alpha) \left\{ \begin{array}{c} (S_0, S_1) \in \Delta(x, y) \\ (T_0, T_1) \in \Delta(y, z) \\ S_0 \cap T_0 \subseteq U_0 \subseteq S_0 \cup T_0 \\ S_1 \cap T_1 \subseteq U_1 \subseteq S_1 \cup T_1 \\ S_0 \backslash (U_0 \cup U_1) = T_1 \backslash (U_0 \cup U_1) \\ T_0 \backslash (U_0 \cup U_1) = S_1 \backslash (U_0 \cup U_1) \end{array} \right. \Longrightarrow (U_0, U_1) \in \Delta(x, z).$$

Proof.
i) $\Longrightarrow$ ii) Let $x, y, z \subset A, (S_0, S_1), (T_0, T_1)$ and $(U_0, U_1) \in \mathcal{N}$ and satisfying conditions (α). It is sufficient to show that there exists a profile R^N such that $(S_0, S_1) = \pi(x, y, R^N), (T_0, T_1) = \pi(y, z, R^N)$ and $(U_0, U_1) = \pi(x, z, R^N)$. Indeed, since $(S_0, S_1) \in \Delta(x, y)$ and $(T_0, T_1) \in \Delta(y, z)$, then $xF_\Delta(R^N)y$ and $yF_\Delta(R^N)z$. So, since $F_\Delta(R^N)$ is transitive on $A, xF_\Delta(R^N)z$ which implies (by IIA) that $(U_0, U_1) \in \Delta(x, z)$. Observe that it is also sufficient to define the restriction of such a profile on x, y, z.

Using (α), one can check that N is partitioned into at most 13 subsets given below. We assign one of the 13 possible weak orders on $\{x, y, z\}$ to every voter in one of these 13 subsets as follows:

[We will use the following notations: $R = xyx$ if $xPyPz; R = (xy)z$ if $xIyPz$ and $R = (xyz)$ if $xIyI$].

$$If \quad i \in S_0 \cap U_0 \cap T_0, R^i = xyz \qquad If \quad i \in S_0 \cap U_0 \cap T_1, R^i = xzy$$
$$If \quad i \in S_0 \cap U_0 \cap T_0^c \cap T_1^c, R^i = x(yz) \quad If \quad i \in S_0 \cap U_1 \cap T_1, R^i = zxy$$
$$If \quad i \in S_0 \backslash (U_0 \cup U_1), R^i = (xz)y$$

$$If \quad i \in S_1 \cap U_1 \cap T_1, R^i = zyx \qquad If \quad i \in S_1 \cap U_1 \cap T_0, R^i = yzx$$
$$If \quad i \in S_1 \cap U_1 \cap T_0^c \cap T_1^c, R^i = (yz)x \quad If \quad i \in S_1 \cap U_0 \cap T_0, R^i = yxz$$
$$If \quad i \in S_1 \backslash (U_0 \cup U_1), R^i = y(xz)$$

$$If \quad i \in T_0 \backslash (S_0 \cup S_1), R^i = (xy)z \qquad If \quad i \in T_1 \backslash (S_0 \cup S_1), R^i = z(xy)$$
$$If \quad i \notin S_0 cup S_1 \cup T_0 \cup T_1, R^i = (xyz)$$

We have defined the restriction of R^N on x, y, z and it is easy to check that it satisfies the required conditions.
ii) $\Longrightarrow$ i) Conversely let $R^N \in \mathcal{W}^N$ such that $xF_\Delta(R^N)y$ and $yF_\Delta(R^N)z$. Let us prove that $xF_\Delta(R^N)z$.

Let us denote by :

$$\begin{cases} (S_0, S_1) = \pi(x, y, R^N \\ (T_0, T_1) = \pi(y, z, R^N \\ (U_0, U_1) = \pi(x, z, R^N \end{cases}$$

$(S_0, S_1), (T_0, T_1)$ and (U_0, U_1) are in $\mathcal{N}$ and by Lemma 1 satisfy (α). But $xF_\Delta(R^N)y$ and $yF_\Delta(R^N)z$, then $(S_0, S_1) \in \Delta(x, y)$ and $(T_0, T_1) \in \Delta(y, z)$ and by ii), we obtain $(U_0, U_1) \in \Delta(x, z)$ and thus $xF_\Delta(R^N)z$, since $(U_0, U_1) = \pi(x, z, R^N)$.

We deduce the following result for linear orders:

COROLLARY 2. *Let Δ be a GBC with $|A| \geq 3$. The following assertions are equivalent:*
i) $\forall R^N \in \mathcal{L}^N, F_\Delta(R^N)$ is transitive on A.
ii) $\forall x, y, z \subset A, \forall (S_0, S_1), (T_0, T_1), (U_0, U_1) \in \mathcal{N}$

$$\begin{cases} (S_0, S_1 \in \Delta(x, y) \\ (T_0, T_1 \in \Delta(y, z) \\ S_0 \cap T_0 \subseteq U_0 \subseteq S_0 \cup T_0 \end{cases} \Longrightarrow (U_0, U_1) \in \Delta(x, z)$$

REMARK 2. a) The previous theorem is equivalent to the following which is due to Ferejohn and Fishburn [1979]:

Let Δ be a BC (i.e. an asymmetric GBC) with $|A| \geq 3$. The following assertions are equivalent:

i) $\forall R^N \in \mathcal{W}^{*\ N}, F_\Delta(R^N)$ is a strict partial order on A.

ii) $\forall x, y, z \subset A, \forall (S_0, S_1), (T_0, T_1), (U_0, U_1) \in \mathcal{N}$

$$(\beta) \begin{cases} (S_0, S_1) \in \Delta(x, y) \\ (T_0, T_1) \in \Delta(y, z) \\ U_0 = K_0 \cup [(S_0 \cup T_0) - (S_1 \cup T_1)] \\ U_1 = K_1 \cup [(S_1 \cup T_1) - (S_0 \cup T_0)] \\ K_0 \cap K_1 = \emptyset, K_0 \subseteq (S_0 \cap T_1) \cup (S_1 \cap T_0) \\ K_1 \subseteq (S_0 \cap T_1) \cup (S_1 \cap T_0) \end{cases} \Longrightarrow (U_0, U_1) \in \Delta(x, z).$$

Indeed it is easy to check that conditions (α) and (β) are equivalent. Moreover, by Proposition 1 ii), $F_\Delta(R^N)$ is always asymmetric if and only if Δ is a BC.

This result is also obtained by Aleskerov and Vladimirov [1986] using the framework of binary choice rules.

b) When Δ is a GBC which is decisive and $|A| \geq 3$, the following assertions are equivalent:

i) $\forall R^N \in \mathcal{W}^N, F_\Delta(R^N)$ is transitive on A.

ii) $\forall \{x, y, z\} \subset A, \forall (S_0, S_1), (T_0, T_1), (U_0, U_1) \in \mathcal{N}$

$$\begin{cases} (S_0, S_1) \in \Delta(x, y) \\ (T_0, T_1) \in \Delta(y, z) \\ S_0 \cap T_0 \subseteq U_0 \subseteq S_0 \cup T_0 \end{cases} \Longrightarrow (U_0, U_1) \in \Delta(x, z).$$

Therefore the notion of decisive GBC gives, for profiles of weak orders, a condition of transitivity which is the same as the one obtained for profiles of linear orders but for general GBC.

REMARK 3. Let Δ be a voting game (i.e. a decisive and neutral GBC), and $\mathcal{U}$ be the family of (decisive) coalitions of N which satisfies $\forall \{x, y\} \subset A, \Delta(x, y) = \mathcal{U}$. We can deduce the following properties of the preference aggregation rule of Δ from the preceding results and definitions :

1) Δ is Paretian $\Leftrightarrow N \in \mathcal{U}$.

2) $\forall R^N \in \mathcal{W}^N, F_\Delta(R^N)$ is complete $\Leftrightarrow [\forall S, T \subseteq N, S \cap T = \emptyset \Rightarrow S \in \mathcal{U}$ or $T \in \mathcal{U}]$.

3) $\forall R^N \in \mathcal{L}^N, F_\Delta(R^N)$ is complete $\Leftrightarrow [\forall S \subseteq N, S \notin \mathcal{U} \Rightarrow S^c \in \mathcal{U}]$.

4) $\forall R^N \in \mathcal{L}^N$ (or $\mathcal{W}^N$), $F_\Delta(R^N)$ is transitive
$\Leftrightarrow [\forall S, T, U \subseteq N, S \in \mathcal{U}, T \in \mathcal{U}$ and $S \cap T \subseteq U \subseteq S \cup T \Rightarrow U \in \mathcal{U}]$.

We can also obtain the following results:

5) If $\forall R^N \in \mathcal{L}^N$ (or $\mathcal{W}^N$), $F_\Delta(R^N)$ is transitive, then $[\forall S, T, \subseteq N, S \in \mathcal{U}, T \in \mathcal{U} \Rightarrow S \cap T \in \mathcal{U}]$.

6) If Δ is Paretian and $\forall R^N \in \mathcal{L}^N$ (or $\mathcal{W}^N$), $F_\Delta(R^N)$ is transitive, then Δ is monotonic.

Using the classic notions of prefilters, filters and ultrafilters one can thus obtain the Arrovian results on neutral PAR's for profiles of weak orders or profiles of linear orders (see [Monjardet, 2003] for such results).

3.2. ACYCLICITY

For the characterization of GBC's with acyclic dominance we need the following notions:

DEFINITION 7. *Let Δ be a GBC.*
a) $C = (\{a_1, a_2, \cdots, a_m\}; (S_k, T_k)_{k=1,\cdots,m})$ *is an AM-cycle of order m of Δ if:*

$$\begin{cases} 1)\ \forall k = 1, \cdots, m,\ (S_k, T_k) \in \Delta\ (a_k, a_{k+1}) \text{ with } a_{m+1} = a_1 \\ 2)\ \bigcap_{k=1}^{m} K_k \neq \emptyset \Longrightarrow \forall k = 1, \cdots, m, K_k = I_k \\ 3)\ \bigcap_{k=1}^{m} L_k \neq \emptyset \Longrightarrow \forall k = 1, \cdots, m, L_k = I_k \end{cases}$$

where $\forall k = 1, \cdots, m, I_k = N \backslash (S_k \cup T_k), K_k \in \{S_k, I_k\}$ *and* $L_k \in \{T_k, I_k\}$.

b) $C = (\{a_1, a_2, \cdots, a_m\}; (S_k, T_k)_{k=1,\cdots,m})$ *is a FF-cycle of order m of Δ if:*

$$\begin{cases} 1)\ \forall k = 1, \cdots, m, (S_k, T_k) \in \Delta(a_k, a_{k+1}) \text{ and } a_{m+1} = a_1 \\ 2)\ \forall k = 1, \cdots, m, \qquad S_k \subseteq \bigcup_{j=1, j\neq k}^{m} T_j \\ 3)\ \forall k = 1, \cdots, m, \qquad T_k \subseteq \bigcup_{j=1, j\neq k}^{m} S_j \end{cases}$$

c) $C = (\{a_1, a_2, \cdots, a_m\}; (S_k)_{k=1,\ldots,m})$ *is an asymmetric cycle (or a-cycle) of order m of Δ if:*

$$\begin{cases} 1)\ \forall k = 1, \cdots, m, (S_k, S_k^c) \in \Delta(a_k, a_{k+1}) \text{ with } a_{m+1} = a_1 \\ 2)\ \bigcap_{k=1}^{m} S_k = \emptyset \text{ and } \bigcap_{k=1}^{m} S_k^c = \emptyset \text{ where } S_k^c \text{ is the complementary of } S_k \text{ in } N \end{cases}$$

d) $C = (\{a_1, a_2, \cdots, a_m\}; (S_k)_{k=1,\ldots,m})$ *is a decisive cycle (or d-cycle) of order m of Δ if:*

$$\begin{cases} 1)\ \forall k = 1, \cdots, m, \exists T_k \subseteq N, (S_k, T_k) \in \Delta(a_k, a_{k+1}) \text{ with } a_{m+1} = a_1 \\ 2)\ \bigcap_{k=1}^{m} S_k = \emptyset \end{cases}$$

We can notice that an a-cycle is a FF-cycle and an AM-cycle. Furthermore, we have the following result (the proof is left to the reader):

LEMMA 2. *Let Δ be a GBC with $|A| \geq 2$. Δ has a FF-cycle if and only if Δ has an AM-cycle.*

The characterization of GBC's with acyclic dominance is given in the following Theorem 3. Let us recall that this theorem is due to Ferejohn and Fishburn [1979] with the minor difference that they use strict weak orders instead of weak orders as individual preferences. We will give a proof of this theorem, since the above notion of AM-cycle allows to shorten Ferejohn and Fishburn's proof. We will also use the two following lemmas:

LEMMA 3. *Let R be a strongly complete binary relation, P the asymmetric component of R and I the symmetric component of R.*

R is a weak order if and only if every cycle of R is a cycle of I.

Proof. Left to the reader:

LEMMA 4. *Let R be binary relation, P the asymmetric component of R and I the symmetric component of R.*

There exists a weak order Q on A such that $R \subseteq Q$ if and only if every cycle of R is a cycle of I.

Proof. The necessary condition is an obvious consequence of Lemma 3.
Assume that $R = P + I$ is such that every cycle of R is a cycle of his symmetric component I. Say that an element x of A is R-**maximal** if for every $y \in A, yRx$ implies xRy. We denote by $Max_R A$ the set of R-maximal elements of A. Observe that due to the property of R this set is not empty, and that this property is hereditary. Then we can consider the partition $(C_j)_{j \leq t}$ of A defined by:

$C_j = Max_R A_j$, with $A_0 = A, A_j = A - \bigcup_{h=0}^{j-1} C_h$ and $C_{t+1} = \emptyset$.

To this partition is (canonically) associated the weak order Q on A defined by: $\forall x, y \in A, xQy \Leftrightarrow x \in C_i, y \in C_j$ and $i \leq j$.

Now, by construction, xRy implies xQy, and so Q is a weak order containing R.

THEOREM 3. *Let Δ be a GBC with $|A| \geq 2$.*

$\forall R^N \in \mathcal{W}^N, F_\Delta(R^N)$ is acyclic on $A \Leftrightarrow \Delta$ has no FF-cycle.

Proof. By Lemma 2 we can prove this theorem by proving that the following assertions are equivalent:

i) $\exists R^N \in \mathcal{W}^N$ such that $\mathrm{F}_\Delta(R^N)$ has a cycle

ii) Δ has an AM-cycle.

i) $\Rightarrow$ ii) Let $R^N \in \mathcal{W}^N$, such that $F_\Delta(R^N)$ contains a cycle. Then there exists $\{a_1, a_2, \cdots, a_m\}$ such that for each $k \in \{1, 2, \cdots, m\}, a_k F_\Delta(R^N) a_{k+1}$ where $a_{m+1} = a_1$. Let $S_k = N(a_k, a_{k+1}, R^N)$ and $T_k = N(a_{k+1}, a_k, R^N)$ and $I_k = N - (S_k \smile T_k) = \{i \in N / a_k I^i a_{k+1}\}$.

Then $C = (\{a_1, a_2, \cdots, a_m\}; (S_k, T_k)_{k=1,\ldots,m})$ is an AM-cycle. Indeed, first since for each $k \in \{1, 2, ..., m\}, a_k F_\Delta(R^N) a_{k+1}, (S_k, T_k) \in \Delta(a_k, a_{k+1})$. Now, assume that, for example, there exists $i \in \bigcap_{k=1}^{m} K_k$. Then the weak order $R^i = P^i + I^i$ contains a cycle. By Lemma 3 this cycle must be a cycle of I^i, so $K_k = I_k$ for each $k \in \{1, 2, ..., m\}$.

ii) ⇒i) Let $C = (\{a_1, a_2, \cdots, a_m\}; (S_k, T_k)_{k=1,\ldots,m})$ be an AM-cycle of order m of Δ. Let us define for each $i \in N$,

$M_i^+ = \{k = 1, \cdots, m/i \in S_k\}, M_i^- = \{k = 1, \cdots, m/i \in T_k\}$ and $M_i^0 = \{k = 1, ..., m/i \in I_k\}$.

We define a relation M^i on $B = \{a_1, a_2, \cdots, a_m\}$ by:

$M^i = \{(a_k, a_{k+1}), k \in M_i^+\} \cup \{a_{k+1}, a_k), k \in M_i^-\} \cup \{a_k, a_{k+1}), (a_{k+1}, a_k), k \in M_i^0\}$

By construction M^i is a relation on B such that every cycle of M^i is a cycle of his symmetric component. So by Lemma 4 there exists a weak order Q^i on B containing M^i.

Let us define for each $i \in$N a relation R^i on A by:

$$\forall x, y \in B, xR^iy \Longleftrightarrow xQ^iy$$
$$\forall x \in B, \forall y \notin B, xR^iy$$
$$\forall x \notin B, \forall y \notin B, xR^iy \qquad and \qquad yR^ix$$

R^i is a weak order on A such that $S_k = N(a_k, a_{k+1}, R^N)$ and $T_k = N(a_{k+1}, a_k, R^N)$ for each $k \in \{1, \cdots, m\}$, then (by Definition 6 i)) for each $k \in \{1, \cdots, m\}$ $a_k F_\Delta(R^N) a_{k+1}$ and therefore $F_\Delta(R^N)$ contains a cycle.

COROLLARY 3. *a) Let Δ be a GBC with $|A| \geq 2$.*
$\forall R^N \in \mathcal{L}^N, F_\Delta(R^N)$ is acyclic on $A \Leftrightarrow \Delta$ has no a-cycle.

b) Let Δ be a decisive GBC with $|A| \geq 2$.
$\forall R^N \in \mathcal{L}^N$ (or $\mathcal{W}^N$), $F_\Delta(R^N)$ is acyclic on $A \Leftrightarrow \Delta$ has no d-cycle.

REMARK 4. If Δ has no FF-cycle of order 2, then Δ is asymmetric, the hypothesis of asymmetry is therefore redundant in Ferejohn and Fishburn [1979] for their result on acyclicity.

When Δ is a neutral GBC, pairs of coalitions are independent from pairs of alternatives. We will therefore define the notions of neutral AM-cycle, neutral FF-cycle, neutral a-cycle and neutral d-cycle by deleting any reference to alternatives.

We will, for example, obtain the following definition of neutral AM-cycle of order m of a **neutral** GBC Δ such that: $\forall \{x, y\} \subset A, \Delta(x, y) = \mathcal{U}$.

DEFINITION 8. *$C = ((S_k, T_k)_{k=1,\cdots,m})$ is a neutral AM-cycle of order m of Δ if:*

$$\begin{cases} 1)\ \forall k = 1, \cdots, m, (S_k, T_k) \in \mathcal{U} \\ 2)\ \bigcap_{k=1}^{m} K_k \neq \emptyset \Longrightarrow \forall k = 1, \cdots, m, K_k = C_k \\ 3)\ \bigcap_{k=1}^{m} L_k \neq \emptyset \Longrightarrow \forall k = 1, \cdots, m, L_k = C_k \end{cases}$$

where $\forall k = 1, \cdots, m, C_k = N \backslash (S_k \cup T_k), K_k \in \{S_k, C_k\}$ and $L_k \in \{T_k, C_k\}$

Furthermore let us define, for a neutral GBC Δ, the following numbers which are generalizations of the Nakamura's number for simple games:

DEFINITION 9. *Let Δ be a GBC.*

a) $\nu_1(\Delta) = \begin{cases} +\infty & if\ \Delta\ has\ no\ AM-cycle \\ \min\{m/\Delta\ has\ an\ AM-cycle\ of\ order\ m\} & if\ \Delta\ has\ an\ AM-cycle \end{cases}$

b) $\nu_2(\Delta) = \begin{cases} +\infty & if\ \Delta\ has\ no\ a-cycle \\ \min m/\Delta\ has\ no\ a-cycle & if\ \Delta\ has\ an\ a-cycle \end{cases}$

c) $\nu_3(\Delta) = \begin{cases} +\infty & if\ \Delta\ has\ no\ d-cycle \\ \min m/\Delta\ has\ no\ d-cycle & if\ \Delta\ has\ \ a\ \ d-cycle \end{cases}$

The results on acyclicity of neutral BGC are therefore the following:

COROLLARY 4. *1) Let Δ be a neutral GBC with* $|A| \geq 2$.
$\forall R^N \in \mathcal{W}^N$ *(or* $\mathcal{W}^{*N}$*)*, $F_\Delta(R^N)$ *is acyclic on* $A \Leftrightarrow \nu_1(\Delta) > |A|$.
2) Let Δ be a neutral GBC with $|A| \geq 2$.
$\forall R^N \in \mathcal{L}^N, F_\Delta(R^N)$ *is acyclic on* $A \Leftrightarrow \nu_2(\Delta) > |A|$.
3) Let Δ be a voting game with $|A| \geq 2$.
$\forall R^N \in \mathcal{W}^N, F_\Delta(R^N)$ *is acyclic on* $A \Leftrightarrow \nu_3(\Delta) > |A|$.

REMARK 5. The above results on acyclic GBC's have been the basis of several classic results on vetoers. A player k is called a vetoer for a PAR F if for each profile R^N, xP^ky implies that not $(yF(R^N)x)$. Le Breton and Truchon [1995] deduce, from the previous Ferejohn and Fishburn's theorem, the results of Blair and Pollak [1982] and Kelsey [1985]) on the existence of players and coalitions who have veto power over alternatives. They also give a precise answer to the problems of the minimum size of the coalitions that must have a veto power under any acyclic preference aggregation rule and of the minimum number of pairs of alternatives on which these coalitions may exercise their power. From the same theorem of Ferejohn and Fishburn, Banks [1995] extends many results such as Sen's [1970] liberal paradox, Blau and Deb's [1977] theorem on the existence of vetoers and Brown's [1975] theorem on the non-emptiness of a collegium.

4. CORE-STABILITY OF A GBC

Let us start this section on the study of core-stability of a GBC by the definition of the notion of a core of a GBC, notion which generalizes the notions of core of a decisive GBC [Andjiga and Moulen, 1989] and [Truchon, 1995], core of a voting game [Andjiga and Mbih, 2000] and core of a simple game [Nakamura, 1979].

DEFINITION 10. *Let Δ be a GBC,* $R^N \in \mathcal{B}^N$ *and* $\mathcal{G}^N \subseteq \mathcal{B}^N$.
i) An alternative x is **undominated** *in* (Δ, R^N) *if* $\forall y \in A, y \neq x, (y,x) \notin d_\Delta(R^N)$.
i) The **core of** Δ *in* R^N*, denoted* $\mathcal{C}(\Delta, R^N)$*, is the set of undominated alternatives in* (Δ, R^N)*.*
ii) Δ *is* $\mathcal{G}$-**core-stable** *if :* $\forall R^N \in \mathcal{G}^N, \mathcal{C}(\Delta, R^N) \neq \emptyset$.

The study of core-stable GBC's will need the following notions :

DEFINITION 11. *Let* Δ *be a GBC.*
a) $\Gamma = ((S_k, T_k, (x_k, y_k))_{k=1,\cdots,m}$ *is a strong AM-cycle of order* m *of* Δ *if :*
1) $\forall k = 1, \cdots, m,\ (S_k, T_k) \in \Delta(x_k, y_k)$,
2) $\bigcup_{k=1}^{m} \{y_k\} = A$,
3) $\forall \{k_1, k_2, \cdots, k_p\} \subset \{1, 2, ..., m\} / x_{kj+1} = y_{kj}\ \forall j = 1, \cdots, p$ *where* $x_{k_{p+1}} = x_{k_1}$

$$3.1) \bigcap_{j=1}^{p} K_{k_j} \neq \emptyset \Longrightarrow \forall j = 1, \cdots, p, K_{kj} = I_{kj}$$
$$3.2) \bigcap_{j=1}^{p} L_{k_j} \neq \emptyset \Longrightarrow \forall j = 1, \cdots, p, L_{kj} = I_{kj}$$

where $\forall j = 1, ..., p, I_{k_j} = N \backslash (S_{k_j} \cup T_{k_j}), K_{k_j} \in \left\{ S_{k_j, I_{k_j}} \right\}$ *and* $L_{k_j} \in \left\{ T_{k_j, I_{k_j}} \right\}$.
b) $\Gamma = ((S_k, (x_k, y_k))_{k=1,\cdots,m}$ *is a strong a-cycle of order* m *of* Δ *if:*
1) $\forall k = 1, \cdots, m,\ (S_k, S_k^c) \in \Delta(x_k, y_k)$,
2) $\bigcup_{k=1}^{m} \{y_k\} = A$,
3) $\forall \{k_1, k_2, \cdots, k_p\} \subset \{1, 2, \cdots, m\} / x_{kj+1} = y_{kj},\ \forall j = 1, \cdots, p$ *where* $x_{k_{p+1}} = x_{k_1}$

$$\bigcap_{j=1}^{p} S_{k_j} = \emptyset \text{ and } \bigcap_{j=1}^{p} S_{kj}^{c} = \emptyset$$

c) $\Gamma = ((S_k), (x_k, y_k))_{k=1,\cdots,m}$ *is a strong d-cycle of order* m *of* Δ *if:*
1) $\forall k = 1, \cdots, m, \exists T_k \subset N, (S_k, T_k) \in \Delta(x_k, y_k)$,
2) $\bigcup_{k=1}^{m} \{y_k\} = A$,
3) If there exists a subset $\{k_1, k_2, \cdots, k_p\}$ *of* $\{1, 2, \cdots, m\}$ *such that* $x_{k_{j+1}} = y_{k_j \forall j=1,\ldots,p}$ *where* $x_{k_{p+1}} = x_{k_1}$, *then* $\bigcap_{j=1}^{p} S_{k_j} = \emptyset$.

Let us give the relation between the previous notions and the notions of cycles of a GBC :

PROPOSITION 2. *Let* Δ *be a GBC.*
a) If Γ *is a strong AM-cycle of* Δ *of order* m, *then there exists an AM-cycle of order* p *of* Δ *with* $p \leq m$.
b) If Γ *is a strong a-cycle of* Δ *of order* m, *then there exists an a-cycle of order* p *of* Δ *with* $p \leq m$.
c) If Γ *is a strong d-cycle of* Δ *of order* m, *then there exists a d-cycle of order* p *of* Δ *with* $p \leq m$.

By a slight modification of the proof of the previous Ferejohn and Fishburn theorem, we obtain the following result:

THEOREM 4. *Let* Δ *be a GBC with* $|A| \geq 2$.
Δ *is* $\mathcal{W}$*-core-stable* $\Leftrightarrow$ Δ *has no strong AM-cycle.*

We deduce the following results on the core-stability of particular GBC's in particular domains of individual preferences :

COROLLARY 5. *Let* Δ *be a GBC. with* $|A| \geq 2$.
i) Δ *is* $\mathcal{L}$*-core-stable* $\Leftrightarrow$ Δ *has no strong a-cycle.*

ii) If Δ *is a decisive GBC with* $|A| \geq 2$, *then*
Δ *is* $\mathcal{L}$*-core-stable* $\Leftrightarrow$ Δ *is* $\mathcal{W}$*-core-stable* $\Leftrightarrow$ Δ *has no strong a-cycle.*

iii) If Δ *is neutral GBC with* $|A| \geq 2$, *then*
Δ *is* $\mathcal{W}$*-core-stable* $\Leftrightarrow \nu_1(\Delta) > |A|$.
Δ *is* $\mathcal{L}$*-core-stable* $\Leftrightarrow \nu_2(\Delta) > |A|$.

iv) If Δ *is a voting game with* $|A| \geq 2$, *then*
Δ *is* $\mathcal{W}$*-core-stable* $\Leftrightarrow \nu_3(\Delta) > |A|$.

v) If Δ *is a simple game with* $|A| \geq 2$, *then*
Δ *is* $\mathcal{L}$*-core-stable* $\Leftrightarrow$ Δ *is* $\mathcal{W}$*-core-stable* $\Leftrightarrow \nu_3(\Delta) > |A|$.

vi) If Δ *is Paretian GBC with* $|A| \geq 2$, *then*
Δ *is* $\mathcal{L}$*-core-stable* $\Leftrightarrow \forall R^N \in \mathcal{L}^N, F_\Delta(R^N)$ *is acyclic on* A
Δ *is* $\mathcal{W}$*-core-stable* $\Leftrightarrow \forall R^N \in \mathcal{W}^N, F_\Delta(R^N)$ *is acyclic on* A.

The above corollaries are due respectively to Andjiga and Moulen [1989]; Truchon [1995] for ii); Andjiga and Mbih [2001] for iv); Andjiga and Mbih [2000]) for v) and Nakamura [1979] for vi).

Let us finally notice that if Δ is a simple game, $\nu_3(\Delta)$ is the Nakamura's number.

5. CONCLUDING REMARKS

1) Section 1 of this paper can be seen as a continuity of the study done by Ferejohn and Fishburn [1979] when the individual preferences are strict weak orders to the case where they are weak orders or linear orders. As in Le Breton and Truchon [1995] or in Truchon [1996], the results on the acyclicity of the preference aggregation rule of a GBC can therefore also be used to obtain (classic and new) results on vetoers and oligarchies.

2) The equivalence between the dominance of GBC's and PAR's which satisfy IIA is one of the major achievements of the Ferejohn and Fishburn paper. This gives us a tool to unify the results obtained on various domains. For example, one good exercise is to deduce versions of Arrow's Theorem (with various types of dictators) as corollaries of the results of Section 1 on transitive and complete dominance of GBC's or to obtain new results on this area. For example, in Andjiga and Moulen [2003]

we obtain a characterization of lexicographic dictatorship (result also obtained in Hild [2001] and Fishburn [1974]).

3) Section 2 gives a unified study of cores defined on main distributions of power: GBC's, binary constitutions, voting games and simple games. It is obvious that non binary decisive structures as effectivity functions [Keiding, 1985], constitutions and generalized constitutions [Moulen, Andjiga and Brissaud, 1999] are not concerned with the present study. Let us notice that the Keiding's [1985] theorem on core-stability of effectivity functions, which is a corollary of Andjiga and Moulen's [1988] result on core-stability of constitutions, is a generalization of the Nakamura's [1979] theorem on core-stability of simple games.

4) In recent papers Felsenthal and Machover [1987, 2001] emphasize on the fact that some real-life decision rules, in particular UN security council and US legislature, are misrepresented as simple games and they define the notion of ternary voting games to model these situations. Let us notice that these decision rules can be (also) seen as neutral GBC's. The UN security council can be modelled as the neutral GBC Δ defined by: (S,T) is a majority for Δ if and only if S has strictly more than 8 members and T has no permanent member.

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BIBLIOGRAPHY

ALESKEROV, F.T., VLADIMIROV, A.V., “Hierarchical voting”, *Information Sciences* 39, 1986, p. 41-86.

ANDJIGA, N.G., MBIH, B., “A note on the core of voting games”, *Journal of Mathematical Economics* 33, 2000, p. 367-372.

ANDJIGA, N.G., MBIH, B., *A note on the core of voting games when individual preferences are weak orders*, Working paper, Université de Yaoundé I, 2001.

ANDJIGA, N.G., MOULEN, J., “Binary games in constitutional form and collective choice”, *Mathematical Social Sciences* 16, 1988, p. 189-201.

ANDJIGA, N.G., MOULEN, J., “Necessary and sufficient conditions for l-stability of games in constitutional form”, *International Journal of Game Theory* 18, 1989 , p. 189-201.

ANDJIGA, N.G., MOULEN, J., *Théorème d’Arrow sur les préordres totaux*, Working paper, Université de Yaoundé I, 2003.

ARROW K.J., *Social Choice and Individual Values*, New York, Wiley, 1963.

BANKS, J.S., "Acyclic social choice from finite sets", *Social Choice and Welfare* 12, 1995, p. 293-310.

BLAIR, D.H., POLLAK, R.A., "Acyclic collective choice rules", *Econometrica* 50, 1982, p. 931-943.

BLAU, J.H., DEB, R., "Social decision functions and the veto", *Econometrica* 45, 1977, p. 871-879.

BROWN, D., "Aggregation of preferences", *Quarterly Journal of Economy* 89, 1975, p. 456-469.

FEREJOHN, J.A., FISHBURN, P.C., "Representations of binary decision rules by generalized decisiveness structures", *Journal of Economic Theory* 21, 1979, p. 28-45.

FELSENTHAL, D.S., MACHOVER, M,. "Ternary voting games", *International Journal of Game theory* 26, 1997, p. 335-351.

FELSENTHAL, D.S., MACHOVER, M., "Models and reality: The Curious Case of the Absent Abstention", in : Holler, M.J. and G.Owen (eds), *Power indices and coalition formation*, Kluwer Academic Publishers, 2001.

FISHBURN, P.C., "Lexicographic orders, utilities and decision rules: a survey", *Management science* 20, 1974, p. 1442-1471.

HILD, M., *A note on impossibility theorems and seniority rules*, Working paper SSWP1123, California Institute of Technology, 2001.

KEIDING, H., "Necessary and sufficient conditions for stability of effectivity functions", *International Journal of Game Theory* 14, 1985, p. 93-101.

KELSEY, D., "Acyclic choice and group veto", *Social Choice and Welfare* 2, 1985, p. 131-137.

LE BRETON, M., TRUCHON, M., "Acyclicity and the dispersion of the veto power", *Social Choice and Welfare* 12, 1995, p. 43-58.

MONJARDET, B., "De Condorcet à Arrow via Guilbaud, Nakamura et les 'jeux simples', *Mathématiques et Sciences humaines* 163, 2003, p. 41-68.

MOULEN, J., ANDJIGA, N.G., BRISSAUD, M., *Préférences individuelles et choix collectifs*, Paris, Hermès, 1999.

MOULIN, H., "From social welfare orderings to acyclic aggregation of preferences", *Mathematical Social Sciences* 9, 1985, p. 1-17.

NAKAMURA, K., "The vetoers in a simple game with ordinal preferences", *International Journal of Game Theory* 8, 1979, p. 55-61.

SEN, A., *Collective choice and social welfare*, San Francisco, Holden-Day, 1970.

TRUCHON, M., "Voting games and acyclic collective choice rules", *Mathematical Social Sciences* 29, 1995, p. 165-179.

TRUCHON, M., "Acyclicity and decisiveness structures", *Journal of Economic Theory* 69, 1996, p. 447-469.