Building models for social space: neighourhood-based models for social networks and affiliation structures

Construire l'espace social : des modèles de voisinage pour les réseaux sociaux et les structures d'affiliation

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BUILDING MODELS FOR SOCIAL SPACE: NEIGHBOURHOOD-BASED MODELS FOR SOCIAL NETWORKS AND AFFILIATION STRUCTURES[1]

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RÉSUMÉ – Construire l'espace social☐ des modèles de voisinage pour les réseaux sociaux et les structures d'affiliation

Nous proposons un cadre pour une analyse quantitative relationnelle de l'espace social. Nous suggérons que l'espace social ne peut pas être défini simplement en termes géographiques ou socio-culturels mais que cette définition suppose de comprendre l'interdépendance entre différents types d'entités sociales telles que des personnes, des groupes, des ressources et des positions socio-culturelles. Nous suggérons également que l'espace social ne peut pas être vu comme figé☐ à la différence de l'espace euclidien de la mécanique newtonienne, l'espace social est construit au moins en partie par le processus social dont il est le support. Dans le modèle stochastique général que nous proposons, les relations entre entités sociales sont considérées comme les éléments fondamentaux de l'espace social et les échanges observés sont conçus comme les produits de processus qui agissent dans des voisinages relationnels qui se recouvrent. Chaque voisinage correspond à un ensemble d'entités relationnelles et est conçu comme un lieu d'interactions sociales. Nous montrons comment des spécifications particulières de ce cadre théorique produisent des hiérarchies de modèles pour les réseaux sociaux et pour les structures d'affiliation. Nous évoquons également de futurs développements de ce cadre.

MOTS-CLÉS – Espace social, Dynamique, Affiliation, Voisinage, Graphe aléatoire

SUMMARY – *We propose a quantitative relational framework for social space. We suggest that social space cannot be specified simply in geographical, network or sociocultural terms but, rather, requires an understanding of the interdependence of relationships among different types of social entities, such as persons, groups, sociocultural resources and places. We also suggest that social space cannot be regarded as fixed: unlike the Euclidean space of Newtonian mechanics, social space is constructed, at least in part, by the social processes that it supports. In the general stochastic relational framework that we propose, relationships among social entities are regarded as the fundamental elements of social space and observed relational entities are viewed as the outcome of processes that occur in overlapping local relational neighbourhoods. Each neighbourhood corresponds to a subset of possible relational entities and is conceived as a possible site of social interaction. We show how special cases of this framework yield hierarchies of models for social networks and for affiliation structures. We also sketch some next steps in the development of this framework.*

KEY WORDS – Social space, Dynamic, Affiliation, Neighbourhood, Random graph

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INTRODUCTION

Few social scientists would argue with the claim that individual action needs to be understood in terms of the *social context* in which it occurs. It is widely acknowledged that an individual's social context imposes constraints and provides opportunities for the individual's future possible actions. Indeed, in a vast number of empirical studies in the social sciences, context is cast in the role of an exogenous variable – either by experimental design, or by analytic framework – and the impact of context on individual behaviour (as dependent variable) is examined.

In this paper, we argue for a new analytic role for context: a role in which social contexts – or *social space* as we shall term contexts that are combined across multiple actors – are best seen as *both* independent and dependent variables. That is, we argue that social contexts are not merely exogenous factors that influence individual behaviours, but also need to be seen as outcomes generated by social processes that occur among those individuals. We claim that this approach is essential if we are to develop richer and dynamic understandings of individual behaviour, and if we are to answer the question of how the actions of individuals in context cumulate to create properties of social systems. Moreover, we claim that these two issues – the dynamics of individual action-in-context and relationship between "micro" actions and "macro" systemic properties – are inextricably linked and require a joint approach.

Conceptually, we make three claims about the nature of social space. First, we argue that it is a complex theoretical construction that invokes, across multiple analytic levels, geography and social settings, social relations and network ties, affiliations (group memberships), social institutions and cultural resources (including values and beliefs). In the broadest terms, we claim that social space can be seen as relational in form, and that it can be modelled in terms of regularities in interdependencies among its constituent relational entities. (We can think of these regularities as constituting *social structure*). Second, we propose that social space is substantially *non-deterministic*, and so that it is desirable to construct stochastic representations of social space. Third, we argue that social space does not simply constrain and enable the actions of the individuals who inhabit it, but that it is, as many social theorists have suggested, both reproduced and transformed by such actions. This last claim reflects "the capacity of social actors to transform as well as reproduce long-standing structures, frameworks and networks of interaction" [Emirbayer, Goodwin, 1994].

Methodologically, we draw on approaches for modeling interdependent systems of random variables [Besag, 1974; Frank & Strauss, 1986; Pattison & Robins, 2002; Wasserman & Pattison, 1996; Robins & Pattison, in press] We sketch the modelling framework in outline, and show how it can be used in several instances to develop a hierarchy of models for the relational constituents of social space. In particular, we build models for social networks and affiliation structures. We conclude with an outline of important next steps in the articulation of the general framework that we propose.

DOES SOCIAL SPACE MATTER?

First, though, we address the question of why social space might matter in understanding social processes. In fact, it may seem odd that this claim needs to be defended. As Anthony Giddens [1979] observed:

At first sight, nothing seems so banal and uninstructive than to assert that
social activity occurs in time and space. But neither time nor space have been
incorporated into the centre of social theory (p. 201).

We focus here on notions of space, but return briefly to the joint consideration of space and time towards the end of the paper.

When Giddens made this observation in the late 1970s, social space was being treated in "bare bones" terms by many psychologists and sociologists, often as a crudely measured predictor or covariate. Certainly, social space had been much more prominent in many earlier psychological and sociological works than it was in the second half of the 20th century. In psychology, for example, William James adopted the term "social worlds" to refer to the circles of friendships within which an individual's activities were located; and Kurt Lewin's field theory was rich in spatial metaphors. Later, though, as many have documented [e.g., Arrow *et al*, 2000; Lindenberg, 1997], the rise of cognitive accounts of social psychological phenomena led to the demise of contextual considerations in social psychology, except in so far as they were mirrored through psychological representations[3]. To some extent, these developments were reinforced by the widespread adoption of statistical models for independent observations, as well as by the difficulty of developing more precise contextual accounts.

In sociology, Georg Simmel also adopted the metaphor of "social circles" in the early part of the 20th century, according particular importance to the dual constitution of individuals and the social circles to which they belong [Breiger, 1974]. But, in parallel with the rise of the individual-focused theoretical positions in psychology, contextual orientations to sociology also gave way to what Andrew Abbott [1997] has termed "the variables revolution". By this Abbott refers to a methodological shift in sociological inquiry that encouraged the study of individuals in isolation from their social locations in neighbourhoods and networks. This shift was associated with a reliance on sample surveys and the same statistical models already mentioned; as Abbott observed, it is an approach to the description of social phenomena in which social facts are decontextualised from their locations in "social space-time". But as he asserts:

One cannot understand social life without understanding the arrangements of
particular social actors in particular social times and places … Every social
fact is situated, surrounded by other contextual facts and brought into being
by a process relating it to past contexts [Abbott, 1997, p. 1152].

Psychology has also recognized the importance of "context". Indeed, a well-known text on social cognition concludes with the claim:

The social perceiver … has been viewed as somewhat of a hermit, isolated
from the social environment. Missing from much research on social cognition
have been other people *in a status other than that of stimulus"* [Fiske, Taylor, 1990, p. 556; our emphasis][4].

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[3] These social cognitivist accounts were also reminiscent of earlier individualist traditions in social psychology that treated group-level occurrences as epiphenomenal [e.g. Allport, 1924].

[4] As an aside, it is worth commenting briefly on the common psychological strategy of representing social context in terms of psychological representations of contextual features. There are many examples of this approach: for example, the "subjective norm" of Ajzen and Fishbein's [1970] theory; or perceived group norms in social identity theory. While such psychological representations have been helpful in explaining the momentary psychological processes hypothesised to underlie a particular individual action, it is worth noting that their usefulness will often be restricted to the briefest intervals of time. Consider Harrison White's example of changing lanes on a freeway. It might suffice to describe a person's decision to

Individualist-based approaches to social cognition were also confronted by the demands of organisational research and theory, where it is often the systemic aspects of the context that are the important issues, rather than the simple accretion of individual-level responses [e.g. Weick, Roberts, 1993].

GIVING UP FAMILIAR SPATIAL IMAGES

Before discussing how, more precisely, we might conceptualise social space, it is useful to review the familiar Euclidean notion of "physical" space. As the mathematician R. H. Atkin [1972] observe:

> *When Descartes married the geometry of Euclid to the algebra of the reals,*
> *he started a revolution in mathematical and scientific thinking which has*
> *lasted for 400 years* (p. 139-140).

Descartes postulated that the space of our physical experiences can be *identified* with the three-dimensional geometry of Euclid. Atkin argued that although this assumption was accepted uncritically by many mathematicians of the 18th and 19th centuries (Leibniz was a notable exception), in fact it raised a number of measurement problems even for Newtonian mechanics (essentially because our measurement devices yield only rational measurements). There are various ways of approaching these measurement difficulties, but Atkin argued that they can be avoided if we reject the Newtonian-Galilean view that "actual-space is absolute and real objects are observed in it" and adopt instead the Leibnizian view that "actual-space is the set of relationships between objects" [Atkin, 1972, p.143]. Atkin pointed out that this shift in conceptualisation allows the topology of space – notions of "nearness" – to be embedded in the process of observation[5]. Atkin went on to develop algebraic topological foundations for physical systems that are consistent with this approach and that allow physical laws to be expressed independently of the Cartesian postulate. Atkin's fundamental point was that space may be more usefully construed in terms of *(observable) relationships between entities* – and it is exactly this approach to the conceptualisation of social space that we wish to pursue here[6].

WHAT IS SOCIAL SPACE?

Social space requires attention to many types of entities, including those referring to geography, social settings, affiliations, social relationships, and the distribution of cultural resources. We suggest that a broad relational conceptualization can capture the

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change lanes on the freeway in terms of their psychological representation of surrounding traffic, but in the moments after a lane-changing action, what matters most for the outcome is where the nearby vehicles actually were. ("Only a fool eager for hospital stay would perceive this as a process in one's own mind", writes White [1995, p.3]. Of course, substantial successes in social psychology have been achieved by the joint use of experimental methods and a focus on cognitive representations. But dynamic descriptions of both individual outcomes and entire social systems will become considerably more powerful when these insightful psychological accounts are *combined* with a dynamic understanding of interdependent social contexts.

[5] "What kind of topology, or nearness, would we ascribe to real-space if we observed it by touch, or by probing it with feather dusters?" he writes [Atkin, 1972, p.142].

[6] In fact, Atkin also extended his framework to social systems, which he construed as sets of observable relations among different types of social entities (in a similar fashion to that advocated here). Atkin's approach differs from the present approach in being deterministic and focused exclusively on patterns of connectivity among relations.

most important observable constituents of social space, including network ties among individuals, memberships of individuals in groups at local, community and national levels, links among people, places, settings and neighbourhoods, and connections between people and cultural values and orientations. Indeed, a number of social theorists have made similar claims. Perhaps the most direct proponent of this view is Emirbayer [1997], who suggests that the social world *consists primarily* in "dynamic, unfolding relations" rather than in substances and "static 'things'" (p. 781).

But before elaborating the relational view further, let us briefly consider how social space has been conceptualised by other social theorists. Arguably, there have been five major trends:

- Social space is construed simply as *geographical space* (as, for example, in spatial epidemiology); individuals are assumed to occupy geographical locations (e.g., from Geographic Information Systems), or somewhat more broadly, regional settings.

- Social space is a multidimensional space whose axes correspond to various social, psychological and even regional attributes, such as gender, class, ethnicity [e.g., Blau, 1977] and residential area. The key role of this space in Peter Blau's theory of social structure has led to the adoption of the term *Blau space* for the sociological version of this construction; distance in Blau space is a function of the dissimilarity of individuals' profiles of social attributes.

- Social space is construed as a *network*, in which individuals are *nodes*, connected by various forms of interpersonal ties, represented as nondirected or directed *edges* in the network [e.g., Barnes, 1954; Nadel, 1952; Bott, 1957; White, Boorman, Breiger, 1976]. The distance between two individuals is then conceptualized in terms of network paths that link them (e.g., the length of the shortest path).

- Social space is represented by patterns of overlapping *affiliations* [e.g., Breiger, 1974; Simmel, 1955; Pescosolido & Rubin, 2000]; individuals are close to the extent that they share many affiliations.

- Social space is described in *sociocultural* terms, for example, in terms of shared "tool-kits" of symbols, stories, rituals and world-views [e.g., Swidler, 1986], or of shared cultural "resources" [Sewell, 1992].

In addition, a number of theorists have argued for more complex constructions that depend on the interpenetration of several of these representational forms. For example, the social historian Chuck Tilly emphasises the importance of jointly considering social and cultural relations. "Culture and social relations empirically interpenetrate with and mutually condition one another so thoroughly that it is well-nigh impossible to conceive of the one without the other. This is the respect in which culture can … be said to constitute, in Charles Tilly's felicitous formulation, the very "sinews" of social reality" [Emirbayer, Goodwin, 1994, p. 1438; also White, 1992]. Feld [1981] addresses contingencies between affiliations, foci of activity and network ties. Grannis [1998] has developed an account of contingencies between racial homogeneity, network ties and urban structures. In other words, there appears to be a growing consensus that social space involves contingencies across different types of relations involving personal, interpersonal, organisational, cultural and geographical entities [e.g., Fararo, Doreian, 1984; Mische, Pattison, 2000; Mische, Robins, 2000][7].

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[7] A compelling case study that illustrates this claim is Padgett and Ansell's [1993] analysis of the interdependent personal, geographical, relational and social structural bases for the rise to power of Cosimo de' Medici. They make a convincing case that it is only when all of these factors are considered jointly that Cosimo's rise to power can be fully understood.

We therefore adopt the premise here that the elements of social space are relations – relations among personal, social, cultural and geographical entities. We also take the view that these elements should be regarded as stochastic: as White [1992] observes:

> *no larger ordering which is deterministic either in cultural assertion or social arrangement could sustain and reproduce itself across so many and such large network populations as in the current world. Some sort of stochastic environment must be assumed and requires modeling* (p. 164-165).

The topology of the model of social space that we construct is determined by hypotheses about the "nearness" of its stochastic relational elements; these hypotheses indicate which relational elements occupy the same *social neighbourhood*. We take the approach here of articulating a general framework that can accommodate many different hypotheses about this neighbourhood structure, and we give several illustrations of hypotheses and their consequences. It is important to note, though, that these hypotheses about the neighbourhood structure of social space are indeed *just* hypotheses, and need to be the subject of a significant program of empirical inquiry. It follows that the notion of "nearness" in social space may not be a universal as is distance within Euclidean space. Different empirical contexts may require different construals of "nearness". As a result there is no "one" social space. Moreover, because social contexts are both constituted by and constitutive of individual actions, a social space may require recursively-based notions of "nearness", whereby new social neighbourhoods may emerge from existing constellations of relationships.

Social structure may be construed as regularities across different regions of the space in the nature of interdependence among entities within neighbourhoods. In the most general terms, we conceive of relations as dynamic as well as stochastic, and we view the various structural regularities in which these dynamic relational entities participate as overlapping, existing at different levels and scales, and being subject to creative interpretations by the actors whom they link[8].

BUILDING MODELS FOR SOCIAL SPACE

As we have observed, individuals are linked by various forms of relational tie not only to other individuals, but also to other social entities, such as groups, organizations, institutions, cultural commitments or meeting places. These ties serve as a fundamental medium for social processes, and establish longer paths of interpersonal connections through which social activity may be channelled. A potential relational tie between two social entities can be regarded as a discrete-valued random variable. For example, if we let $N = \{1,2,...,n\}$ be a set of network nodes representing social entities, we can define the variable X_{ij} to have the value 1 if the link from entity i to entity j is present, and the value 0, otherwise; where $i, j \in N$. If X_{ij} and X_{ji} are distinguished, the relational tie variables are *directed*, otherwise, they are *nondirected*. As indicated earlier, we view these random variables as fundamental constituents of social space. A *generalized random network* (or *random graph*) is represented by a two- or higher-way array $\mathbf{X}$ of these random variables; a particular set of realised values is denoted by $\mathbf{x}$. Below, we give examples where $\mathbf{X}$ is either a single nondirected network on a set of individuals or a single affiliation network, but the models can be readily generalised to discrete-valued and multivariate relational observations, and to multiple types of entity [e.g., Skvoretz,

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[8] In other words, although we propose that social space may be organised by certain recurrent forms – that is, that structure exists – it is important to realise that this is not a strong structuralist claim.

Faust, 1999; Koehly, Pattison, in press; Mische, Robins, 2000; Pattison, Wasserman, 1999; Robins, Pattison, in press; Robins, Elliott, Pattison, 2001; Robins, Pattison, Wasserman, 1999].

A potential problem with constructing models for $Pr(\mathbf{X} = \mathbf{x})$ is that an assumption of independent relational variables is unlikely to be tenable; rather, the modeling approach needs to give explicit recognition to possible interdependencies among variables. Frank and Strauss [1986] recognized that some fundamental theorems for interdependent observations developed in spatial statistics could be applied to arbitrary dependence structures, including structures specifying assumed interdependencies among relational variables. Application of these results yields a general expression for $Pr(\mathbf{X} = \mathbf{x})$ from a specification of which pairs of relational variables are conditionally independent, given the values of all other relational variables.

Specifically, we define two relational variables to be *neighbours* in social space if they are conditionally dependent given the values of all other variables (and thereby we explicitly link interactivity with nearness in social space). This neighbourhood relation can be represented in the form of a *dependence graph* **D** [Frank & Strauss, 1986] whose nodes are the random variables X_{ij} and whose edges link pairs of neighbouring variables. Note that each relational variable X_{ij} can, in principle, act as both dependent and independent variable in this system of interacting variables, since its value both affects and is affected by the values of neighbouring variables. This may sound like an intractably complex system, but several assumptions render it manageable. The first is the dependence structure itself: the Hammersley-Clifford Theorem [Besag, 1974] establishes that the structure of a consistent probability model for the entire system of variables depends only on the neighbourhoods in the dependence structure, where by *neighbourhood* we mean a subset of relational variables, every pair of which are neighbours[9]. (Note that, as a result, the assumed dependence structure has a critical impact on the form of the model). The second simplifying assumption is generally some form of homogeneity – an assumption that absolute locations in space have no impact on the form of interaction among neighbouring variables; rather, it is the status of variables in local neighbourhoods that matter. These two assumptions lead to a specific parametric form for a probability model $Pr(\mathbf{X} = \mathbf{x})$ for the space from an hypothesis about its neighbourhood structure (that is of its *topology*). Non-zero parameters correspond to subsets P of variables that are either singletons or for which every pair of variables in the subset are neighbours; these subsets define what we term *local relational neighbourhoods*. Specifically, the model has the form:

$$Pr(\mathbf{X} = \mathbf{x}) = \exp(\Sigma_P \lambda_P z_P(\mathbf{x}))/\kappa \qquad (1)$$

where P is a local relational neighbourhood defining a *configuration* of possible ties; λ_P is a parameter associated with the neighbourhood P; the quantity $z_P(\mathbf{x}) = \Pi_{X_{ij} \in P} x_{ij}$ is the *relational statistic* for P indicating whether all ties in the relational configuration defined by P are present in the network $\mathbf{x}$; and κ is a normalizing quantity [Frank, Strauss, 1986; Wasserman, Pattison, 1996]. Since there may be many overlapping local relational neighbourhoods P, the model expresses the probability of a network as a function of self-organising interactive processes occurring in overlapping local regions of the relational system. Note that the statistic $z_P(\mathbf{x})$ is a binary-valued measure corresponding to P that is computed from $\mathbf{x}$. It takes the value 1 if all of the possible ties in the subset P are present in $\mathbf{x}$, and 0, otherwise. The subset P corresponds to a

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[9] A single variable is also a neighbourhood.

subgraph configuration in x, namely the subgraph obtained when all possible relational ties in P are present in x. Thus, if the parameter λ_P is large and positive, the probability of observing the relational system x is enhanced if the configuration corresponding to P is present in x (net of other effects).

In *homogeneous* models, parameters for isomorphic configurations are assumed equal and the statistic for a parameter is then the number of corresponding configurations observed in $\mathbf{x}$; a positive value of λ_P then indicates that relational systems with more configurations of type P are more probable, counts of all other configuration types being equal[10]. Exogenous variables may also be assumed to affect relational variables and hence interact with model parameters in ways that can be determined from an extended dependence graph incorporating directed dependencies [Robins, Elliott, Pattison, 2001; Robins, Pattison, Elliott, 2001].

MARKOVIAN AND EXTRA-MARKOVIAN NEIGHBOURHOODS

A critical step in model formulation is the specification of the relational topology, since the neighbourhood relation determines the *form* of the relational configurations parameterised in the model. Pattison and Wasserman [1999] argued for dependencies that were at least *Markovian* [Frank, Strauss, 1986], with possible relational ties X_{ij} and X_{lm} as neighbours whenever they shared an entity (i.e., $\{i,j\} \cap \{l,m\} \neq \varnothing$). More recently, Pattison and Robins [2002] have argued that, despite substantial evidence for Markov-like dependencies in network structures [e.g., Lazega, Pattison, 1999; Pattison, Wasserman, 1999; Robins, Pattison, Wasserman, 1999; Wasserman, Pattison, 1996], two alternative processes may influence network and other relational topologies.

First, neighbourhoods may emerge from the network or relational processes themselves, with new neighbourhoods created as relational ties are generated. For instance, X_{ij} and X_{kl} might *become* conditionally dependent if there is an observed tie between j and k or between l and i. The dependence of a set of conditional dependence assumptions on observed values of other variables led Baddeley and Möller [1989] to term the resulting models *realisation-dependent*, and Pattison and Robins referred to the underlying assumptions as *partial conditional dependence* assumptions.

The hypothesis just described (that X_{ij} and X_{kl} *become* conditionally dependent if there is an observed tie between j and k or between l and i) can be construed as an assumption of conditional dependence involving relational variables that form a semipath of length 3: for example, X_{ij}, X_{jk}, X_{kl}; or X_{ij}, X_{il}, X_{kl}. (A *semipath of length m* in this context can be defined as a sequence of m relational variables, for which *every* adjacent pair X_{ij} and X_{kl} in the sequence have a node in common, that is, $(i,j) \cap (k,l) \neq \varnothing$). The semipath assumption can be seen as a natural extension of the Markovian assumption, which posits conditional dependence for relational variables that form a semipath of length 2. More complex assumptions could also be entertained, leading to a hierarchy of models for relational structures in which network processes are assumed to reach across increasingly long path- and cycle-like structures.

Second, exogenous factors may limit the extent to which potential neighbourhoods can be realized; for instance, two potential ties might be regarded as neighbours only if the entities concerned occupy a common setting. Accordingly,

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[10] Two configurations P and P' are *isomorphic* if there is a 1-1 mapping φ on N such that $(i,j) \in P$ iff $(\varphi(i),\varphi(j)) \in P'$.

Pattison and Robins [2002] introduced exogenous settings as structures that may impose boundaries on neighbourhoods. Each *setting* corresponds to a subset of possible relational ties, and model parameters are assumed to be non-zero (or to have a common distinct value) if all possible relational variables in the configuration lie within a single setting. Settings can be used to represent external spatial or organisational constraints on relational processes. Together, these two developments lead to a hierarchy of increasingly complex models that can be used to explore hypotheses about the relational structure of social space.

TWO EXAMPLES

In order to apply the general and quite abstract approach that we have just described to the task of building models for social space, we must begin with some observed collection of relations among various types of social entities. Exactly which entities and which relations are to be used in any such effort are deeply theoretical choices that depend on the particular aspect of social space under investigation. We believe that we know too little about the nature of social space in order to make strong recommendations here, although it is likely to be important to take many of the relational forms described earlier into consideration.

Here we present illustrative applications of our approach using two different choices of relational structures – a social network and an affiliation network – and we build probabilistic models for the social spaces associated with these relational structures. The examples are based on two classic studies of network and affiliation relations: the Bank Wiring Room [Roethlisberger, Dickson, 1939]; and the Southern Women [Davis Gardner, Gardner, 1941; see also Homans, 1951].

MARKOV RANDOM GRAPH MODELS FOR NONDIRECTED NETWORKS

The data for the first example come from Roethlisberger and Dickson's [1939] classic study, *Management and the Worker*, and we use information supplied by the authors and discussed by Homans [1951] concerning the network of friendships observed in the Bank Wiring Room (see Figure 6 in [Homans, 1951]).

We construct a homogeneous Markov model for the friendship network. For nondirected graphs, the configurations corresponding to Markovian neighbourhoods take one of a relatively small number of forms: edges; triads; or star-like structures (see Figure 1). If the homogeneity assumption is also made (that parameters corresponding to isomorphic neighbourhoods are equal)[11], then the probability model for X has a single parameter corresponding to each of the distinct configurations in Figure 1[12]. Each configuration corresponds to an isomorphism class $[P]$ of neighbourhoods P, and the statistic $z_{[P]}(x)$ corresponding to class $[P]$ is then a count of the number of observed configurations of that form. The model for the global network structure X expresses the

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[11] In general, two configurations A and A' are *isomorphic* if there is a 1-1 mapping φ on N such that $(i,j) \in A$ iff $(\varphi(i),\varphi(j)) \in A'$). If we assume that $\lambda_A = \lambda_{A'}$ whenever A and A' are isomorphic, the model then takes the form $\Pr(X=x) = (1/c) \exp\{\Sigma_{[A]} \lambda_{[A]} n_{[A]}(x)\}$, where $[A]$ is the class of cliques in $\mathbf{D}$ isomorphic to A, and $n_{[A]}(x) = \Sigma_{A \in [A]} z_A(x)$ is the corresponding *sufficient statistic*.

[12] An implication of the topology is that the model could in fact contain up to an n-1 "star" parameters (with each parameter corresponding to a k-star configuration, that is, a central node connected to k other nodes, for some $k \leq n$-1), although such a model cannot be estimated. Here we have restricted the star parameters for k no greater than 3.

probability of a network in terms of propensities for these triadic and star-like configurations to occur.
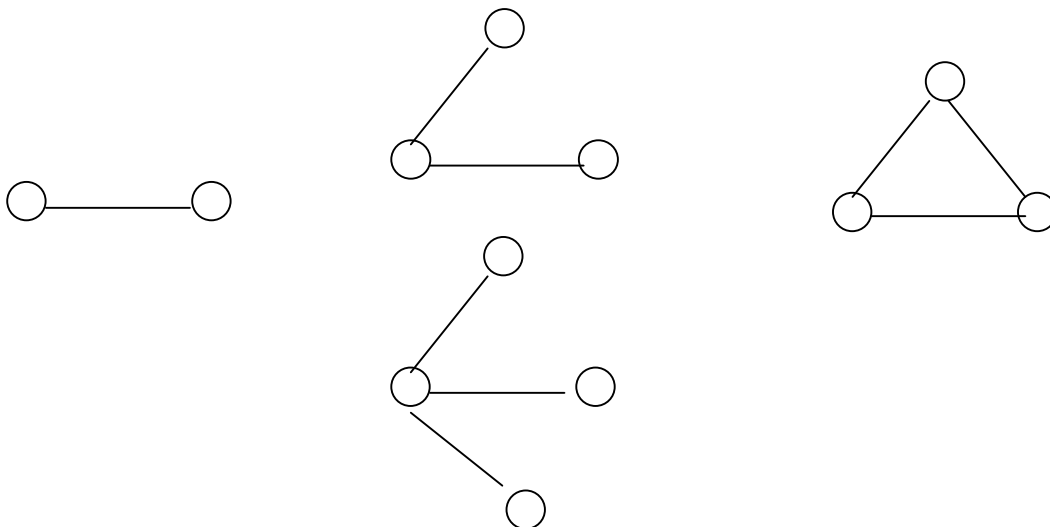

Figure 1. Configurations with three or fewer edges corresponding to Markovian neighbourhoods for a graph

Parameters for Markov random graphs have often been estimated in the past using a maximum pseudo-likelihood procedure [e.g., Strauss, Ikeda, 1990; see also Wasserman, Pattison, 1996], but new Markov Chain Monte Carlo estimation approaches are now available for some models and have the considerable additional advantage of yielding reliable standard errors for parameter estimates [e.g., Handcock, Hunter, Butts, Goodreau, Morris, 2004; Snijders, 2002].

In Table 1 we present the estimates of model parameters. The maximum likelihood estimates and their standard errors have been computed using Handcock et al's [2004] ergm program [Handcock *et al*, 2004] for estimation of exponential random graph models. Pseudo-likelihood estimates for the parameters are also shown for comparative purposes[13].

Table 1. Parameter estimates for Markov model for Bank Wiring Room (friendship ties)

parameter	PLE	MLE[a]	s.e.
edge	-6.08	-5.50	2.00
2-star	2.48	2.52	1.37
3-star	-2.02	-2.30	1.09
triangle	0.76	1.94	0.61

[a] MLEs are computed using ergm [Handcock *et al*., 2004]

The positive parameter estimate for 2-star configurations suggests a tendency for individuals to be linked by network ties to multiple network partners, and hence for ties to become centred on popular nodes (in a type of "local" preferential attachment

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[13] The PLEs and MLEs are reasonably close in this case and suggest similar qualitative interpretations. Caution must be exercised, though, since small changes in parameter values can lead to marked changes in global model properties if the parameters are near degenerate regions of parameter space (see [Handcock, 2003; 2004; Robins, Pattison, Woolcock, in press]).

process, [Barabási, Albert, 1999; Albert, Barabasi, 2002]). However, the negative three-star parameter suggests that these tendencies are quickly dampened once degrees become too high. The very negative edge parameter signifies that network ties are particularly unlikely to link otherwise isolated individuals. The positive triangle parameter reflects the tendency for ties to be clustered, that is, for network partners to be tied to the same third parties.

From this model, the social space of the Bank Wiring Room can be characterized in simple terms: a connected structure in which the variation in nodal degree is moderate but not high and in which local relational clustering prevails.

A CLUSTERING MODEL FOR BIPARTITE GRAPHS

The second application is to a model for affiliation networks represented in the form of bipartite graphs. A *bipartite graph* is a graph with two sets of nodes in which every edge in the graph links a pair of nodes from distinct sets. (In other words, there are no edges linking nodes from the same set). Bipartite graphs provide a graphical representation for two-mode binary relations, such as affiliation data structures in which relational ties are observed *only* between entities of different types [Wasserman, Faust, 1994]. They are the simplest form of relational structure linking distinct types of social entities and hence are an important case to consider here, given our general relational framework for social space in terms of relations within and among distinct types of social entity.

Typical examples of two-mode relations include the case where a relation records whether each of a set of persons is a member of each of a set of groups, or the case where a relation records whether each of a set of persons was present at each of a set of events. A well-known example of such a data structure is taken from Davis, Gardner and Gardner's [1941] *Deep South* and describes the attendance of 18 women at 14 events (see also [Homans, 1951; Breiger, 1974])[14]. The so-called *Southern Women* data is reproduced in Table 2; an entry of 1 in the matrix indicates that the woman in the designated row attended the event designated by the column; otherwise, she was absent.

Skvoretz and Faust [1999] showed that relational models can be formulated for bipartite graphs provided that (a) the set of possible ties is restricted to those linking nodes from different sets; and (b) account is taken of set type in formulating suitable homogeneity constraints[15]. Skvoretz and Faust presented the fit of homogeneous Markov models to the Southern Women data, as well as several models with theoretically motivated higher-order parameters (measures of actor and event overlap, and average distance measures between events and between actors). A model with two of these additional parameters provides a much improved fit to the data compared to the homogeneous Markov model, but the link between dependence assumptions and model parameters is not explicit for such a model. Here, we present a model developed from an explicit conditional dependence assumption, and show that the fit is at least as good.

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[14] More generally, *k*-partite graphs can be used to describe the relations among elements of k distinct sets; see Fararo and Doreian [1984] and Mische and Pattison [2000].

[15] In particular, if the two sets are denoted by N_1 and N_2, then two configurations P and P' are *isomorphic* if there is a 1-1 mapping φ on $N = N_1 \cup N_2$ such that (a) i and $\varphi(i)$ belong to the same set (N_1 or N_2) for all i, and (b) $(i,j) \in P$ iff $(\varphi(i),\varphi(j)) \in P'$). In a homogeneous model, we assume that $\lambda_P = \lambda_{P'}$ whenever P and P' are isomorphic.

Table 2. The Southern Women Bipartite Graph
[Davis, Davis, Gardner, 1941; see also Skvoretz, Faust, 1999]

Woman	Events attended													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1	1	1	1	1	1	**0**	1	**1**	0	0	0	0	0
2	1	1	1	**0**	1	1	1	1	0	0	0	0	0	0
3	**0**	1	1	1	1	1	1	1	**1**	0	0	0	0	0
4	1	**0**	1	1	1	1	1	1	0	0	0	0	0	0
5	0	0	1	**1**	1	**0**	1	**0**	0	0	0	0	0	0
6	0	0	1	0	1	1	**0**	1	0	0	0	0	0	0
7	0	0	**0**	0	1	**1**	1	1	**0**	0	0	0	0	0
8	0	0	0	0	0	**1**	0	1	1	0	0	0	0	0
9	0	0	0	0	**1**	0	1	1	1	0	0	0	0	0
10	0	0	0	0	0	0	**1**	1	1	0	0	**1**	0	0
11	0	0	0	0	0	0	0	1	1	**1**	0	**1**	0	0
12	0	0	0	0	0	0	0	1	1	1	**0**	1	1	1
13	0	0	0	0	0	0	**1**	1	1	1	**0**	1	1	1
14	0	0	0	0	0	**1**	**1**	0	1	1	1	1	1	1
15	0	0	0	0	0	0	1	1	**0**	**1**	**1**	1	0	0
16	0	0	0	0	0	0	0	1	1	0	0	0	0	0
17	0	0	0	0	0	0	0	**0**	1	0	**1**	0	0	0
18	0	0	0	0	0	0	0	**0**	1	0	**1**	0	0	0

Note: bold values associated with mean absolute residuals in clustering model ≥ .6

For the Southern Women bipartite graph, a Markov dependence structure assumes that the attendance of woman i at event j is conditionally dependent on the attendance of woman i at other events l, as well as on the attendance of other women k at event j. Parameters of a homogeneous Markov model correspond to an edge or to a k-star for $k \geq 2$ that either links a woman to k events, or an event to k women (see Figure 2, and note that 3-cycles are impossible in a bipartite graph).

Under what circumstances might one expect the Markovian assumption not to suffice, and so expect that the attendance of woman i at event j might be conditionally dependent on the attendance of another woman k at a second event l? One possibility is that these two seemingly distinct possible ties could share an extended common setting when they are linked by the joint presence of women i and k at either event j or l. Such an event creates a potential for contingency between the two possible attendance ties. Thus, it would be reasonable to propose that X_{ij} and X_{kl} are conditionally dependent when $x_{il} = 1$ or $x_{kj} = 1$, but conditionally independent for distinct i, j, k, l, otherwise. This is an example of a *partial conditional dependence* assumption [Pattison, Robins, 2002] and leads to a homogeneous model with non-Markovian parameters corresponding to configurations presented in Figure 3. We term this model a *clustering model* for bipartite graphs, since it allows a contingency to be created between two possible ties by virtue of their connection to an observed tie, and hence allows for a collection of attendance ties to become mutually contingent. (In fact, every possible complete bipartite subgraph corresponds to a non-zero model parameter, but we restrict attention

here to the simpler forms shown in Figure 3: the *3-path* structure on the left and the *complete bipartite* structure on the right). Indeed, (unobserved) higher order forms, such as groups, or clusters of mutually connected women and events, provide an alternative possible account for the assumed partial conditional dependence: X_{ij} and X_{kl} are conditionally dependent when they belong to the same (unobserved) higher-order entity. Of course, such a condition is likely to be strongly associated with the condition that x_{il} = 1 or x_{kj} = 1, so we can view the realization-dependent neighbourhood that includes both X_{ij} and X_{kl} as a possible local indicator of some emergent higher order form involving women i and k and events j and l. In this way, realization-dependent neighbourhoods of the form we are proposing provide one rudimentary but promising approach to the problem of analysing emergent forms within multiple levels of analysis.
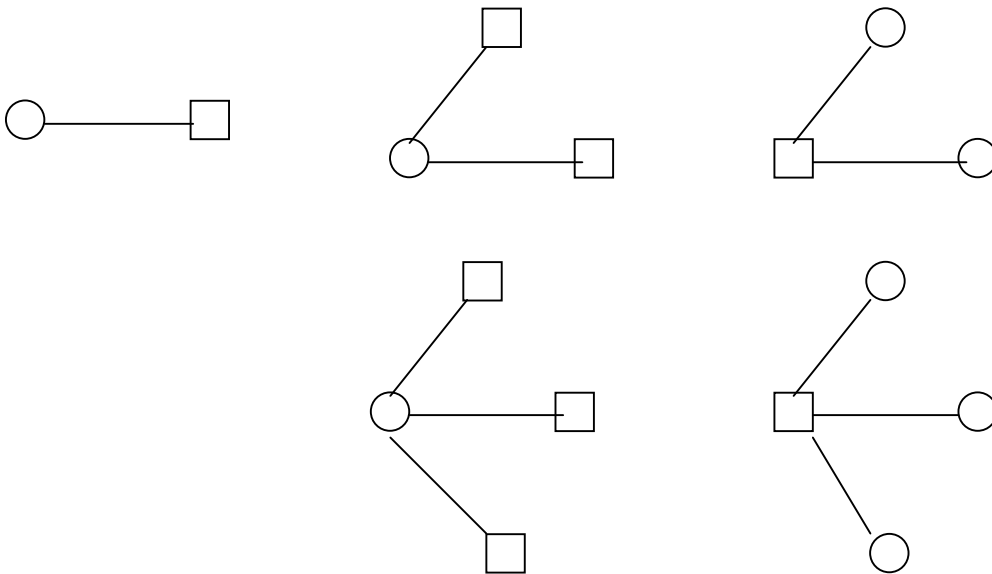

Figure 2. Some configurations corresponding to neighbourhoods in a Markovian model for a bipartite random graph

Figure 3. Some non-Markovian configurations in the clustering model for a bipartite random graph

Heuristic indices of fit[16] of the Markov model and the clustering model just described are presented in the third and fourth rows of Table 3; a reduced form of the non-Markovian model is shown in the fifth row of the table. We rely on heuristic measures of fit since the distribution of –2LPL is not known (as for other applications of

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[16] For each of the fitted models, we compute two heuristic indices of model fit: -2 times the log of the maximized pseudolikeliohood function (-2LPL); and the mean absolute residual (MAR) for each possible tie. The mean absolute residual is computed as the average value of $\left| x_{ij} - z_{ij} \right|$, where z_{ij} is the estimated value of $Pr(X_{ij} = 1)$ conditional on the remaining values of **X** (for details, see [Strauss, Ikeda, 1990]).

maximum pseudolikelihood estimation (see [Besag, 1977, and Strauss & Ikeda, 1990]). For comparison, we also present two Bernoulli models[17]: in the first row, a homogeneous Bernoulli model; and in the second, an analogue of the p_1 model for a bipartite graph [Wasserman, Iaccobucci, 1986] in which each woman has an individual propensity to attend events, and each event has an individual propensity to attract attendances.

Table 3. Fit of models to the Southern Women data

Model	no. of parameters	2LPL	MAR
Bernoulli	1	327.3	.457
p_1^* for two-mode relational data	31	256.2	.338
Markov (edges, 2-stars)	3	304.8	.416
Markov (edges, 2-stars, 3-stars)	5	302.6	.411
clustering (edges, 2-stars, 3-paths, clustering)	5	228.6	.293

A comparison of the two Bernoulli models shows that the fit of the homogeneous model is substantially worse than the fit of the model possessing node-specific degree tendencies (although at the cost of a large number of parameters). The homogeneous Markov model is also a modest improvement over the homogeneous Bernoulli model. But the model with the "clustering" parameter corresponding to the complete bipartite subgraph shown on the right in Figure 3 appears to provide the most elegant model of those fitted: with just 5 parameters, it provides a fit at least as good as that of other models presented in Table 5 (for comparison, the best fitting model identified by [Skvoretz and Faust, 1999], had 5 parameters and -2LPL = 264.0). Pseudolikelihood estimates for the clustering model are presented in Table 4.

Table 4. Parameter estimates for clustering model for the Southern Women data

Configuration	PLE	approx s.e.
edge	-2.647	.603
2-star (1 woman, 2 events)	1.416	.248
2-star (2 women, 1 event)	1.063	.173
3-path	-0.168	.025
4-cycle (cluster)	0.331	.049

What does the clustering model fitted to the attendance array reveal about the social space of the Southern Women? The parameters of the model suggest that while there is variability in the tendency for women to attend multiple events and in the tendency for many women to be in attendance at a given event (and hence for local preferential attending patterns), there is also an enhanced tendency for two women to attend both events if they are already connected by co-presence at one of the events. Thus, local clustering in the relational space is also evident.

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[17] i.e. models where all edges are independent of each other.

It should be noted that the overall goodness of fit of the clustering model is only moderate, with a mean absolute residual of .293. In Table 2 we indicate where the fit of the clustering model is especially poor. It can be seen that the attendance patterns for events 7 and 11 are not well fitted, and nor are those for events 6, 8 and 9. Events 6, 7, 8 and 9 stand out as "bridging" events, that is, as events that attract attendances from women in both of the two main clusters of women that are evident in the data. The model tends to under-estimate cross-cluster attendances and over-estimate within-cluster attendances, a pattern that suggests that the bridging pattern associated with these events is not well captured by the model. Event 11 is also associated with a bridging pattern, in this case, the overlap in attendances between the more involved women 12, 13, 14 and 15 and the more peripheral 17 and 18. Thus, although the model captures the clustering effect quite well, it appears to need an even more complex characterization to reproduce appropriate forms of overlap between clusters. Potentially suitable characterisations are described by Snijders, Pattison, Robins and Handcock [2004].

NEXT STEPS

We have addressed here the question of how to conceptualise and model the social spaces (i.e., the structure of social locations) within which interactive social behaviour occurs. Plausible characterisations of social spaces are a precondition for adequate models of social behaviour (the literature on HIV transmission provides a compelling case study), yet the social process modelling literature has, in general, been slow to respond to this point. Here we have presented some initial steps in the elaboration of a multi-layered conceptualisation of social space in terms of generalized network structures that possess self-organising network topologies that can be modified by overlapping social settings.

MODEL SPECIFICATION

There is still much to be done in explicating new classes of models based on this conceptualisation and in systematically evaluating the evidence for, and the properties of, models within the class. The clustering model applied to the Southern Women data illustrates the potential value of the careful articulation and analysis of neighbourhood assumptions. For network structures, Pattison and Robins [2002] have proposed a hierarchy of models based on progressively more complex structural forms that might be used to systematically explore how different network topologies can account for properties of global network structures. More recently, Snijders *et al* [2004] have extended this approach further in order to develop potentially more robust model specifications. It will be important to determine empirically the extent to which models based on these higher-order configurations are required in order to reproduce important global properties of observed relational structures.

A related question is the extent to which homogeneous relational characteristics for a single set of entities are sufficient for the development of plausible models. There are strong theoretical grounds to suppose that relational ties are not only dependent on other relational ties or attributes but also on other forms of social organization [Feld, 1981]. By introducing *settings* as an abstract mathematical representation of these forms, Pattison and Robins have developed simple models in which spatial arrangements, group memberships, and organisational structures can modify relational processes. However, these models require elaboration in two major respects. The first

is to recognize that individuals are located in geographical space and to use this information in model construction. Indeed, the increasing availability of detailed geographical information, an upsurge of interest in the spatial bases of social processes [e.g., Raftery, 2001], and the surprisingly small amount of work on spatially based network processes (though see [Hoff, Raftery, Handcock, 2002]) should provide a compelling impetus for these developments. Such models should provide important guidance on the extent to which both network and geographical information is important to the characterisation of social space. A second focus for the elaboration of setting-dependent models is the relationship between networks and other social entities, such as groups. If we regard groups as (potentially overlapping) settings, we can formulate models in which network and group processes make potentially distinctive contributions to the nature of social space. It is also important to recognize that relationships are of many types, and characterizations of the interdependence of different types of relational tie can yield important insights into the nature of social spaces and social structures, as Nadel [1952] foreshadowed. In particular, application of the general approach outlined here to multiple relational structures [e.g., Koehly, Pattison, in press; Lazega, Pattison, 1999; Pattison, Wasserman, 1999] yields stochastic models that incorporate the type of regularities in social structural forms that were revealed by earlier algebraic constructions based on aggregated blockmodel data [e.g., Boorman, White, 1976; Pattison, 1982; 1993].

THE DYNAMICS OF SOCIAL PROCESSES

Finally, we observe that a broad framework for modelling social space in terms of regularities in observable relationships among social entities is only a small step towards the greater methodological challenge put forward by Abbott [1997]: a challenge to develop "a general empirical approach founded on action in context" (p. 1158). In his analysis of the Chicago School, Abbott [1997] describes the notion of an interactional field:

> *In the concept of an interactional field, we must … move away from the level of individual cases and begin to describe the rules and regularities of interaction throughout the field* (p. 1157).

According to Abbott:

> *we require … ways of investigating complex spatial interdependence, and of making this spatial interdependence more and more temporally structured, till again we arrive at the description and measurement of interactional fields* [Abbott, 1997, p. 1166].

We agree, and accordingly note that a vital next step is to introduce an explicit dynamic framework so that we can model the evolution of relational structures, and ultimately the joint evolution of interdependent social processes *at multiple levels of analysis*. As many social theorists have argued, there are likely to be important interdependencies involving actions or attributes at the level of *individual* social entities; *relational* connections at the level of pairs of individual entities; and *groups* or *settings* at the level of subsets of entities or relational ties. Social processes are shaped by but also shape the social contexts in which they are embedded. We therefore need to elaborate and assess dynamic models for the joint evolution of social processes across levels, and thereby develop models for processes that both depend on social space while at the same time they transform it (for some significant steps in these directions, see [Snijders, 2001; Snijders, 2004]).

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