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Hugh MacColl after One Hundred Years

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Was Hugh MacColl a logical pluralist or a logical monist ? A case study in the slow emergence of metatheorising

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Résumé : Dans la seconde moitié des années 1900, Bertrand Russell et Hugh MacColl échangèrent sans s'entendre sur les questions de l'implication et de l'existence, dans le cadre d'un débat plus général sur la nature de la logique. Il est tentant de voir dans cet échange une opposition entre le moniste logique Russell et le pluraliste MacColl. Dans cet article, j'affirme que cette interprétation est inexacte, et que les deux hommes étaient tous deux monistes, bien qu'ayant des allégeances différentes. La transition du monisme au pluralisme ne s'effectue en réalité qu'à partir du début des années 1910, peu après la mort de MacColl en 1909. Les premiers signes de cette transition sont à trouver particulièrement chez le philosophe américain C. I. Lewis, le mathématicien néerlandais L. E. J. Brouwer, et le logicien polonais Jan Łukasiewicz. Ces auteurs sont des exemples de l'avènement graduel de la métalogique.

Abstract: In the mid- and late 1900s Bertrand Russell and Hugh MacColl had a non-discussion about implication and existence, as parts of a dispute over the nature of logic. We are tempted to see this debate in terms of logical monist Russell against logical pluralist MacColl, but I argue that this interpretation is inaccurate; each man was a logical monist, but with different allegiances. The transition from monism to pluralism began to occur from the early 1910s onwards, soon after MacColl's death in 1909; early traces will be found especially in the American philosopher C. I. Lewis, the Dutch mathematician L. E. J. Brouwer, and the Polish logician Jan Łukasiewicz. They form examples of the gradual rise of metalogic.

Explicanda

The word 'logic' refers here to the theory (or theories) of correct and incorrect reasoning, and valid and invalid deductions. 'Logical monism' is the claim that only one theory, L, deserves to be called logic: all other candidate theories either are misconceived in some way, or are legitimate bodies of knowledge

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but not logic, or are special cases of L. 'Logical pluralism' is the stance that allows for many logics, although the pluralist may prefer some over others. Both positions are to be understood to be *very general,* not just, for example, concerning the difference between a word and its referent. The presence or absence of symbolism in a logic under consideration is a separate issue.

The history of logic up to the early 20th century shows that logical monism has usually reigned supreme, with L assigned to some version or other of classical true-or-false time-independent logic (hereafter, 'C'), usually involving predicates and quantification over variables as well as propositions. Examples include (post-)Boolean algebras, and a mathematical logic including a logic of relations as practised by Russell. MacColl was a pioneer among logicians in wishing to break clear from the dominance of C. Some commentators have seen him as a logical pluralist; they include [Rescher 1974], [Grattan-Guinness 1998], and [Rahman & Redmond 2008] (who elaborate MacColl's logic in an impressive way, and also achieve the no mean feat of supplying a full bibliography for him). However, I shall argue here that he was a logical monist, adhering to an L that was wider than C. For recent historical commentary, see [Cavaliere 1996], further articles in [Astroh & Read 1998], and [Anellis 2009]. [1]

I restrict the account to the last five years of MacColl's life, including the publication of his logic book [MacColl 1906a] and his discussions with Russell in the philosophy journal *Mind.* These included Russell's review [Russell 1907] of the book there with the reply [MacColl 1907], and notes on existence ([MacColl 1905b; 1905c; 1905d], [Russell 1905]) and on implication ([MacColl 1905a; 1906b; 1908a; 1908b], [Russell 1908]). [2] He and Russell also corresponded quite extensively during the 1900s, and 26 of MacColl's letters are preserved in the Russell Archives at McMaster University, Canada [MacColl 1901-1909]. The main points are made several times over in their written and published exchanges; usually one typical reference is given below. While Russell was rather dismissive of MacColl to other correspondents, such as Louis Couturat [Schmid 2001, 499–500] and Philip Jourdain [Grattan-Guinness 1977, 101; 119], at least he kept these letters, which was not always his normal practice with correspondence at that time (for example, Jourdain's).

Russell's classical position

During the period under consideration here Russell was busily working with A. N. Whitehead on the exposition in *Principia Mathematica* of their logicist programme for deriving (some) mathematics from their $L = C$ while

1. Among histories of logic covering that period, the appropriate parts of [Mangione & Bozzi 1993] and [Grattan-Guinness 2000] are useful for context, but [Haaparanta 2009] is far too brief.

2. [Russell 1905] and four of MacColl's pieces are reprinted in [Russell 1973]; one hopes that in some year or decade Russell's papers for the period 1906-1908 will reappear in volume 5 of his *Collected Papers.*

avoiding the paradoxes. Part of the preparation was the formalisation of C itself; Russell reported on progress, especially concerning the propositional calculus, in a paper on 'The theory of implication' published as [Russell 1906]. While he emphasised the role in his theory of the logic of Gottlob Frege, which he had studied in detail only from 1902, his approach was still dominated by his chief mentor Giuseppe Peano. In this context the influence was not a happy one; this paper is the most unclear one by Russell that I have ever read, on any subject. There are two main reasons: that for Russell logic is an all-embracing theory, so that there is no "room" anywhere to talk *about* it, nor was room sought; and that implication, inference, entailment, consequence, reasoning and deduction tended to be merged, although at least in [Russell 1908] he used implications between names of propositions.

Russell followed Frege in taking implication and negation as the primitive connectives, and stressed that the former connective held between propositions of whichever truth-value: '*Therefore* is distinguished from *implies* by being only applicable to implications between *true* propositions' [Russell 1906, 165]. One major consequence, which will come under attack from MacColl, stated that for propositions p and q,

> 'p implies q' is logically equivalent to 'not-p or q',
> where the disjunction is inclusive. (1)

In fact Russell took (1) in the form of defining 'p or q' as 'not-p implies q' [Russell 1906, 176]. Here are some further examples of his exegesis.

'Propositional function' referred to logical combinations of propositions, such as (p and p) [Russell 1906, 163]; this use was additional to that of referring to predicates fx of individual variables x.

The first 'primitive proposition', itself a conflation of axiom and rule of inference, stated that 'Anything implied by a true proposition is true' [Russell 1906, 164]. His use of 'anything', not 'any proposition' was deliberate; for him 'Aristotle implies Aristotle', for example, was not badly formed or meaningless, but false.

Truth-values were "included" in the conception of a proposition; for example not-p was expressed by 'p is not true' [Russell 1906, 164]. However, 'p implies p' meant 'for *any* value of p, p implies p' [Russell 1906, 163].

The law of excluded middle was 'p or not-p'; in words, 'Everything [sic] is true or not true' [Russell 1906, 167; 177]. The law of contradiction was handled similarly [Russell 1906, 178].

No explicit statement of inference was made; for example, the phrase 'modus ponens' never appeared. A closely related proposition was:

> 'If p is true, then, if p implies q, q is true'. [Russell 1906, 169] (2)

Another candidate is this consequent drawn several pages later, 'an important principle of inference, which I shall call the "principle of assertion"' [Russell 1906, 180]:

'p is true, and if p is true then q is true, then q is true'. (3)

There was plenty here to puzzle the readers (and the audience of *Principia Mathematica* from 1910 onwards), although at that time some of these conflations were also committed by other logicians, including by MacColl. However, his reaction against the status that Russell gave to C was sweeping.

MacColl's claims

MacColl's two main points of disagreement concerned existence, and affirming or rejecting (1) about implication [MacColl 1908b]; other features that Russell affirmed and MacColl at least doubted included the need for propositional functions, relations and quantification in logic, treating a proposition as a truth-bearer rather than just as a form of words, and the legitimacy of non-Euclidean geometries ([MacColl 1907, 1901-1909, letters of 26 January 1905, 3 February 1905]). Let us note some main features.

MacColl insisted that propositions should not be classifiable only as true and false ones but also be divisible into necessary and possible ones. This brought him into territory that we now recognise as modal logic, and constitutes his principal claim to fame. He wished to restrict implication between propositions to those that exhibited some semantic or circumstantial connection; this position also drew him towards relevance logic. In [MacColl 1908a] he used as an example the propositions:

p := 'He is a doctor' and q := 'He is red-haired' (4)

and denied that 'p implies q' could obtain between them. Russell did not deny implication, since under the conception of logic that he inherited from Peano and found backed in this respect by Frege implication satisfied (1) and so could be asserted between *any* two well-formed propositions whether or not they exhibited any connection. In [Russell 1908], he offered to MacColl the option of asserting that formal implication held between the corresponding propositional functions by treating 'he' as a variable; but this move involved another point of dissent, since for MacColl 'he' was a parameter, and propositions such as those given in (4) were sometimes true and sometimes false. He showed no appreciation of quantification over individuals, propositional functions, relations or propositions, all of which were key to Russell's logic.

The two propositions in (4) are also examples of MacColl's fifth category, 'variable' propositions, which are possible but uncertain; an example was:

'Mrs. Brown is not at home'. (5)

He associated this kind of proposition with probability theory [MacColl 1906a, 7], in that one might assess the likelihood of Mrs Brown's residence at any given time. It also brought him towards what we now regard as epistemic logic, for he stressed that such propositions would have a truth value but we did not know which ([MacColl 1906a, 19] for (5)). For [Russell 1907], (5)

should be converted to the propositional function 'Mrs. Brown is not at home at time x'. Later he defined a propositional function fx to be necessary or possible according as it is satisfied respectively by all pertinent values of x or by at least one of them [Russell 1919, 163]; he used modality in a few other contexts, though always in the confines of C [Dejnozka 1999].

MacColl missed an opportunity to strengthen his case against $L = C$ by also interpreting (5) as belonging to temporal logic, even though he mentioned that (5) could be true in the morning and false in the afternoon [MacColl 1907, 470]! Thus he helped to sustain an enormous oversight in the history of the fight for logics beyond C. To explain, take one frequently encountered context that C cannot handle: we often make statements about taking sequences of actions and decisions *in some order in time,* with 'and' meaning 'and then'. For example, if I fulfil my promise to the court to tell the truth, the whole truth and nothing but the truth, and I assert of the plaintiff that 'he walked down the stairs and opened the door', I definitely do not assert that 'he opened the door and walked down the stairs'. Now this difference belongs to temporal logic, which differs from C in not being commutative. The long history of over-looking temporal logic still has to be recorded; we note here the irony that MacColl belongs to it. [3]

MacColl and Russell also differed on the theory of collections, in that Russell used Cantorian set theory while MacColl seemed to draw upon traditional part-whole theory [MacColl 1905d]; the main reason for Russell's advocacy of set theory was its central role in logicism, whereas MacColl had no special plan or place for mathematics in his exposition of logic. Their disagreements did not hinge upon this difference of aim. More problematic for Russell, and indeed also for Shearman and every reader, was MacColl's handling of classes: for example, misattributing the predicate 'existent' as a class in [MacColl 1905b]. Also unfortunate was his use of '0' to identify the class of 'unrealities', that is, of individuals that do not exist, such as mermaids ([MacColl 1905a] & [MacColl 1906a, 42–43]); for '0' had long been identified with the empty class (or set). Russell's reaction is quite reasonable [Russell 1905], though compromised by his own overuse of 'existence' as a predicate, with a variety of senses that sometimes contradict each other: for him an individual could exist 1) as in existential quantification, or 2) as the referent of a denoting phrase (an important special case of 1)); and a Cantorian set could exist as in 3) the existential quantification of sets, or 4) if associable with a propositional function, or 5) in being non-empty! In particular,

3. On the history of temporal logics see [Øhrstrom & Hasle 2006a] & [Øhrstrom & Hasle 2006b]. We distinguish temporal logics from the expression *within* C of temporal order of events in terms of ordered sets—an exercise of which, curiously, is pursued in [Russell 1936]. W. V. Quine, a famed logical monist for C, seems to fail to make the distinction in his treatment of tense in [Quine 1986, 30–31], the only attention paid to time in this book on the 'philosophy of logic'. His criticisms elsewhere of use-mention muddles infecting the understanding of implication are often justified independently of the issue of monism or pluralism.

the empty set existed in the first two senses but not in the third [Grattan-Guinness 1977, 70–74].

Assessments

The exchanges attracted a little attention from others at the time. Shearman was critical of MacColl, though not very profitably ([Shearman 1906a] & [Shearman 1906b, chap. 5]). Jourdain was judicious in his appraisal of the bearing upon mathematics of MacColl's contributions to logic, which MacColl read and accepted shortly before his death in 1909; however, Jourdain had quoted from Russell much more than from MacColl [Jourdain 1912]. Couturat took Russell's side in a letter of March 1906 to Peano [Luciano & Roero 2005, 102–103], who even omitted MacColl from his bibliography of logicians [Peano 1908, xxiv–xxxvi].

At this distance of time it looks as if MacColl was foraging into various non-classical logics; but there is a fundamental difference between his and modern conceptions of such activity. Today logical pluralists see themselves as shopping around in the various stores of the logics market [Beall & Restall 2005]; but I am sure that MacColl was *extending* his preferred logic L into a department store, with one floor occupied especially by modal logic and other floors by some forms of C, relevance and probability logics. Presumably he saw the danger that his jumbo logic would be inconsistent, but felt safe if its constituent sub-logics were kept apart. While in his letters to Russell he hoped for reconciliation of their views, in his criticisms he did not see Russell prosecuting *a* logic different from his own but doing some things *wrong* in logic: 'the whole subject of formal logic needs recasting' [MacColl 1908b, 454], not supplementing with alternatives to C. He was seeking for replacements within L and additions to it, not attempting to choose between C and competitor logics.

As they both recognised, the chief difference between the two men lay in the area of deduction, and rested on the status of modality. The centre of his department store was the boutique for the necessary and the possible. In the cases of both 'implication' 'If A then B' between propositions A and B and 'inference' (logical consequence) between them, he saw the conjunction of A and 'the denial of B' as an impossibility, not just as a falsehood as Russell read it (for example, [MacColl 1906a, 7–8; 80–83]). Such a stance is not pluralism as we now understand it but a monism, albeit of a scope wider than Russell's where there is a *pares* among which modality is *primus.* Further, the clarity of discussion was handicapped on both sides by the absence of a hierarchy of theories and languages in which the various issues and stances can be sited.

When, then, did logical pluralism emerge, and with whom?

The emergence of logical pluralism

While MacColl and Russell were disagreeing in the late 1900s, C. I. Lewis was preparing a doctoral thesis in philosophy at Harvard University under the direction of Josiah Royce; he completed it in 1910 and soon secured a post at the University of California at Berkeley. He had taken Royce's lecture course in logic, and from 1912 he started quickly to produce papers in the subject, publishing in *Mind* or in American journals in philosophy. His chief target was the newly published first volume of *Principia Mathematica,* of which Royce had given him a copy; like MacColl he disliked the treatment there of implication.[4] The opening sentence of his first paper, published in *Mind,* referred to 'two somewhat startling theorems' in 'the algebra of logic', as he called the traditional approaches based upon C: '(1) a false proposition implies any proposition, and (2) a true proposition is implied by any proposition' [Lewis 1912, 522]. He named as 'extensional disjunction' the logically equivalent formulation (1) of implication in terms of negation and disjunction, and contrasted it with its 'intensional' counterpart, for which (1) did *not* hold; in order to highlight the difference he named as 'strict' the intensional kind [Lewis 1912, 524; 526]. These moves sound like pluralism, but in fact he was monist, for he made this analogy with geometry: 'The present calculus of propositions is untrue in the sense in which non-Euclidian geometry is untrue' [Lewis 1912, 530], which was *not* the pluralist understanding of geometries then normally held by mathematicians. In [Lewis 1913a], he was still more severe about C: 'Not only does the calculus of implication contain false theorems, but all its theorems are not *proved*' [Lewis 1913a, 242].

Clearly Lewis was in MacColl territory; presumably he got there independently, for only in [Lewis 1913b, 430] did he note MacColl's work in a short footnote. Later he mentioned MacColl's 'highly complex system', where for him 'the fundamental symbols represent propositional functions rather than propositions' [Lewis 1918, 108]; MacColl would not have been pleased.

A new paper for *Mind* is crucial for our concerns [Lewis 1914b]. Lewis began by recalling from its predecessor the analogy with Euclidean and non-Euclidean geometries, in the same monistic spirit. But in the next paragraph he suddenly went pluralist: 'The relation of these two [calculi of propositions] sufficiently resembles that of a Euclidean and a non-Euclidean geometry to make the analogy worth bearing in mind. Like two geometries, material and strict implication are equally self-consistent mathematical systems; but they apply to different worlds' [Lewis 1914b, 240–241], with *the tradition* relegated to non-Euclidean status and preference given to strict implication, which 'has a wider range of applications' [Lewis 1914b, 241]. Thereafter Lewis remained pluralist; in particular, in his textbook on logic he presented the chapter on 'the calculus of strict implication' as concerned with 'two entirely different

4. Interestingly, Lewis did not question the role of C in logicism in his admiring review [1914a] of the second volume of *Principia Mathematica* (1912).

meanings of "implies"' [Lewis 1918, 292]. In his own reminiscence of those early years as a logician he recalled a reluctant pluralist, although he did make an analogy with Henri Poincaré choosing a geometry; interestingly, he used the names ' "metalogics" or "pseudo-logics" ' as synonyms, a known though infrequent use of the former word [Lewis 1930, 41–43]. [5]

Another attack on the dominance of C had been made at this time in the doctoral thesis of L. E. J. Brouwer [Brouwer 1907]. On rather mysterious grounds embedded in a metaphysics about the flow of time and the importance of languagelessness he banned the law of excluded middle from mathematics and worked his own version of constructive mathematics (and logic), which he called 'intuitionism'. In terms of his later definitive position on logic and mathematics this was an indeterminate position in which, for example, the infinite numbers of Cantor's second number-class were admitted but the class as such not. As his context was quite separate from the concerns of MacColl and Russell ([van Dalen 1999, chap. 3] & [Hesseling 2003, chap. 2]), the details need not delay us here; but it is worth noting the coincidence in timing of another case of logical pluralism—or, perhaps for Brouwer himself in his most polemical passages, logical monism affirming intuitionistic logic as L.

Very gradually a few non-classical logics began to develop during the 1910s and the 1920s. Nicolai Vasiliev introduced his 'imaginary logic' from 1910, Brouwer continued with intuitionism in his own eccentric way, Lewis formulated several different modal logics and compared them with each other from 1912, Jan Łukasiewicz helped to launch many-valued logics from 1917 [Mangione & Bozzi 1993, 465–487]; elsewhere, for example, quantum mechanics was to breed a logic of its own. Opposition was quite vigorous, and for a very long time; [6] but how could a discussion about logic(s) take place at all? The ground floor of the home of logics was extending with the construction of new rooms; but did not this bungalow need an upper storey?

The slow recognition of metalogic

Another feature of young Brouwer's thesis is the subject of this final section: in order to discuss logical monism and pluralism properly we need metalogic as a "place" in which we can talk. [7] As we have seen, such mouthroom

5. On Lewis's logic see [Parry 1968] & [Murphey 2005, chap. 3]; however, neither author explicitly raises the issue of monism and pluralism. On Lewis's next stage concerning possible worlds, see [Sedlar 2009]. In 1920 he came back to Harvard University, where he had Whitehead as colleague for two decades from 1924; some aspects of Whitehead's philosophy influenced him, but they do not seem to have interacted over logics [Murphey 2005, chaps. 4–7].

6. For a personal witness of the expression of dissent by logical monists over [Rescher 1974] on MacColl, see [Grattan-Guinness 1998, 11–12].

7. It is not practical to cite all the original sources in this brief survey. The appropriate parts of [Grattan-Guinness 2000], [Haaparanta 2009] & [Gabbay & Woods 2009] contain much pertinent information and many further references.

was painfully lacking in both MacColl and Russell, and Lewis did not (yet) perceive the need for it either. However, Brouwer explicitly formulated 'mathematics of the second order, *which consists of the mathematical consideration of mathematics or of the language of mathematics*' [Brouwer 1907, 61]; the corresponding extension of logic is not far away, for his metamathematics itself was classical.

Brouwer was partly influenced by the first phase of David Hilbert's programme of metamathematics, which lasted from 1898 to around 1905. The place of metatheorising was actually one of its weaker sides, for he had got his arithmetic and his logic somewhat intertwined. But the second phase, which started in 1917, was far more thrusting, and highlighted metathinking about mathematical theories. Throughout he relied upon C for his logic.

Another important source of metalogic lay in Polish logic. Already in 1913 Łukasiewicz published, in German, a short book on the philosophical foundations of probability theory. Seeking an objective interpretation of probability, he plumped on the truth-values of propositions determined from a propositional function (or 'indefinite proposition') fx by assigning specific values to x. The proportion of true judgements that result gave the probability value; in particular, if *only* true judgements are obtained, then the indefinite proposition is true; if never, it is false. Thus, as an offshoot of this approach to probability, a propositional calculus was developed in which truth values of propositions played a central role, as a 'calculus of truth-values' [Łukasiewicz 1913, 16–18; 20–23].

This was a somewhat marginal though nevertheless direct example of the importance of metalogic for the propositional calculus. In the following years Łukasiewicz and some other Polish logicians (for example, Stanislav Lesniewski and Leon Chwistek) became sensitive to metalogic, seemingly via logics themselves, through model theory, and/or sorting out syntax from semantics in natural or formalised languages. The most famous manifestation was to be Alfred Tarski's semantic definition of the truth of a proposition of a formal language in terms of satisfaction in a metalanguage (his term), which was published in the early 1930s. Tarski lectured in Vienna in February 1930, and seems to have alerted his friends in the Vienna Circle to the fundamental importance of metatheory.[8] Thus in 1931 Rudolf Carnap took over the word 'metalogic' from the normal meaning that we saw Lewis use at exactly this time, and referred it instead to the logic of logic, the sense that soon became customary. Carnap had been motivated by Kurt Gödel's newly proved first

8. Despite substantial historical literature such as [Wolenski 1989] on the Poles, [Luschei 1962] on Lesniewski, [Feferman & Feferman 2004] on Tarski, [Stadler 2001] on the Vienna Circle, and [Szaniawski 1989] & [Wolenski & Köhler 1999] on the links between the two communities, the histories during the 1920s of their recognitions of metalogic/theory/language as *central* concerns are still somewhat obscure; in giving priority to the Poles I am somewhat influenced by [Menger 1994, chap. 12] and the letters in [Tarski 1992].

theorem on the incompletability of first-order arithmetic, where the distinction between logic and its metalogic plays an essential role.

The absence of Russell from these developments is ironic. We saw above that the distinction was lacking from his logic; however, when in 1921 he wrote his introduction to Ludwig Wittgenstein's *Tractatus* he rejected the distinction made there between showing and saying and proposed instead 'that every language has, as Mr. Wittgenstein says, a structure concerning which, *in the language,* nothing can be said, but that there may be another language dealing with the structure of the first language, and having itself a new structure, and that to this hierarchy of languages there may be no limit' [Russell 1922, xxii]. This was one of the highlights of his philosophical career; sadly, he did not notice, and in particular did not apply his insight to perceive the need for a corresponding hierarchy of logics and their metalogics. So it played no part in his preparation in 1923 and 1924 of the new material for the second edition of *Principia Mathematica*; hence, when the new material appeared in 1925, the Harvard logician Henry Sheffer justifiably pointed in his review to the 'logocentric predicament' that in talking about logic one had to use logic, a conundrum that he could not resolve [Sheffer 1926]. [9] Similarly, Russell himself never understood Gödel's first theorem, always misstating it as applicable to all mathematical theories and especially thinking that his hierarchy of languages solved the 'puzzle' that he thought it posed rather than being required to allow the theorem to be stated in the first place. [10] He always remained the philosopher of one all-embracing logic that he had been in the 1900s when disagreeing with another logical monist, Hugh MacColl.

References

Anellis, Irving N.

2009 Russell and his sources for non-classical logics, *Logica Universalis,* 3, 153–218.

Astroh, Michael & Read, Stephen (eds.)

1998 Hugh MacColl and the tradition of logic, special issue of the *Nordic Journal of Philosophical Logic,* 3 (1), Dec.

Beall, J. C. & Restall, Greg

2005 *Logical Pluralism,* Oxford: Oxford University Press.

9. Sheffer's colleague Lewis did not raise any issues about metalogic, or even about implication, in his extensive review of the second edition of *Principia Mathematica* [Lewis 1928].

10. For evidence from his nineties see [Russell 1971], and his letter of 1963 published in [Grattan-Guinness 2000, 592–593] (see also [Grattan-Guinness 2000, 542; 565]). Russell seems to have ignored Gödel's second theorem.

Brouwer, Luitzen E. J.

1907 *Over de Grondslagen der Wiskunde,* Amsterdam: Mass & van Suchtelen. Quoted from L. E. J. Brouwer, *Collected Works,* 1, Amsterdam: North-Holland, 1975, 11–101.

Cavaliere, Fania

1996 L'opera di Hugh MacColl: Alle origini delle logiche non-classiche, *Modern Logic,* 6, 373–402.

Dejnozka, Jan

1999 *Bertrand Russell on Modality and Logical Relevance,* Aldershot: Ashgate.

Feferman, Anita B. & Feferman, Solomon

2004 *Alfred Tarski: Life and Logic,* Cambridge: Cambridge University Press.

Gabbay, Dov M. & Woods, John (eds.)

2009 *Logic from Russell to Church,* Amsterdam: North-Holland.

Grattan-Guinness, Ivor

1977 *Dear Russell - Dear Jourdain. A Commentary on Russell's Logic Based on his Correspondence with Philip Jourdain,* London: Duckworth, New York: Columbia University Press.

1998 Are other logics possible? MacColl's logic and some English reactions, 1905-1912, *Nordic Journal of Philosophical Logic,* 3 (1), 1–16.

2000 *The Search for Mathematical Roots, 1870-1940: Logics, Set Theories and the Foundations of Mathematics from Cantor through Russell to Gödel,* Princeton: Princeton University Press.

Haaparanta, Leila (ed.)

2009 *The Development of Modern Logic,* Oxford: Oxford University Press.

Hesseling, Dennis E.

2003 *Gnomes in the Fog: The Reception of Brouwer's Intuitionism in the 1920s,* Basel: Birkhauser.

Jourdain, Philip E. B.

1912 The development of the theories of mathematical logic and the principles of mathematics. pt. 2, *Quarterly Journal of Pure and Applied Mathematics,* 43, 219–314 (reprinted in: P. E. B. Jourdain, *Selected Essays on the History of Set Theory and Logics (1906-1918),* edited by I. Grattan-Guinness, 1991, Bologna: CLUEB, 133–228).

Lewis, Clarence Irving

1912 Implication and the algebra of logic, *Mind,* 21 (84), 522–531.

1913a Interesting theorems in symbolic logic, *Journal of Philosophy,* 10, 239–242.

1913b A new algebra of implications and some consequences, *Journal of Philosophy,* 10, 428–438.

1914a Review of A. N. Whitehead & B. Russell's *Principia Mathematica* (2, Cambridge: Cambridge University Press, 1912), *Journal of Philosophy, Psychology and Scientific Methods,* 11 (18), 497–502.

1914b The calculus of strict implication, *Mind,* 23 (90), 240–247.

1918 *A Survey of Symbolic Logic,* Berkeley: University of California Press.

1928 Review of A. N. Whitehead & B. Russell's *Principia Mathematica* (1, 2nd ed., Cambridge: Cambridge University Press, 1925), *American Mathematical Monthly,* 35 (4), 200–205.

1930 Logic and pragmatism, *in* G.P. Adams & M. P. Montague (eds.), *Contemporary American Philosophy: Personal Statements,* 2, New York: Macmillan, 31–51.

Luciano, Erika & Roero, Clara S. (eds.)

2005 *Giuseppe Peano - Louis Couturat : Carteggio (1896-1914),* Florence: Olschki.

Łukasiewicz, Jan

1913 *Die Logische Grundlagen der Wahrscheinlichkeitsrechnung,* Cracow: Wspolka Wydawnicza Polska. Quoted from: J. Łukasiewicz, *Selected Works,* edited by L. Borkowski, Amsterdam: North-Holland, Warsaw: Polish Scientific Publishers, 1970, 16–69.

Luschei, Eugene C.

1962 *The Logical Systems of Lesniewski,* Amsterdam: North-Holland.

MacColl, Hugh

1901-1909 *Letters to Bertrand Russell,* The Bertrand Russell Archives, McMaster University, Canada (unpublished).

1905a Symbolic reasoning (VI), *Mind,* 14, 74–81.

1905b Existential import, *Mind,* 14, 295–296.

1905c The existential import of propositions, *Mind,* 14, 401–402.

1905d The existential import of propositions, *Mind,* 14, 578–580.

1906a *Symbolic Logic and its Applications,* London: Longmans, Green and Co.

1906b Symbolic reasoning (VIII), *Mind,* 15, 504–518

1907 Symbolic logic (a reply), *Mind,* 16, 470–473.

1908a 'If' and 'imply', *Mind,* 17, 151–152.

1908b 'If' and 'imply', *Mind,* 17, 453–455.

Mangione, Corrado & Bozzi, Silvio

1993 *Storia della Logica da Boole ai nostri Giorni,* Milan: Garzanti.

Menger, Karl

1994 *Reminiscences of the Vienna Circle and the Mathematical Colloquium,* edited by L. Golland, B. F. McGuinness & A. Sklar, Dordrecht: Kluwer.

Murphey, Murray G.

2005 *C. I. Lewis: The Last Great Pragmatist,* Albany: State University of New York Press.

Øhrstrom, Peter & Hasle Per

2006a A. N. Prior's logic, *in* D. M. Gabbay & J. Woods (eds.), *Logic and the Modalities in the Twentieth Century,* Amsterdam: North-Holland, 399–446.

2006b Modern temporal logic: the philosophical background, *in* D. M. Gabbay & J. Woods (eds.), *Logic and the Modalities in the Twentieth Century,* Amsterdam: North-Holland, 447–498.

Parry, W. T.

1968 The logic of C. I. Lewis, *in* P. A. Schilpp (ed.), *The Philosophy of C. I. Lewis,* La Salle, Illinois: Open Court, 15–54.

Peano, Giuseppe (ed.)

1908 *Formulario Mathematico,* 5th ed., Turin: Bocca.

Quine, Willard V.

1986 *Philosophy of Logic,* 2nd ed., Englefield Cliffs, NJ: Prentice Hall.

Rahman, Shahid & Redmond, Juan

2008 Hugh MacColl and the birth of logical pluralism, *in* D. M. Gabbay & J. Woods (eds.), *British Logic in the Nineteenth Century,* Amsterdam: North-Holland, 533–604.

Rescher, Nicholas

1974 Russell and modal logic, *in* N. Rescher et al. (eds.), *Studies in Modality,* Oxford: Blackwell, 85–96 (reprinted *in* G. Andrews (ed.), *The Bertrand Russell Memorial Volume,* London: Allen & Unwin, 1979, 139–149).

Russell, Bertrand A. W.

1905 The existential import of propositions, *Mind,* 14, 398–401.

1906 The theory of implication, *American Journal of Mathematics,* 28(2), 159–202.

1907 Review of Hugh MacColl's *Symbolic Logic and its Applications* (London: Longmans, Green and Co., 1906), *Mind,* 15, 255–260.

1908 'If' and 'imply': a reply to Mr. MacColl, *Mind,* 17, 300–301.

1919 *Introduction to Mathematical Philosophy,* London: Allen & Unwin.

1922 Introduction, *in* L. Wittgenstein, *Tractatus Logico-Philosophicus,* London: Allen & Unwin, 7–23. Quoted from the 2nd ed., London: Routledge & Kegan Paul, 1961, ix–xxii.

1936 On order in time, *Proceedings of the Cambridge Philosophical Society,* 32, 216–228.

1971 Addendum to my "Reply to criticisms", *in* P. A. Schilpp (ed.), *The Philosophy of Bertrand Russell,* 4th ed., La Salle, Illinois: Open Court, xvii–xx.

1973 *Essays in Analysis,* edited by D. P. Lackey, London: Allen & Unwin; New York: Braziller.

Schmid, Anne-Françoise (ed.)

2001 *Bertrand Russell: Correspondance sur la philosophie, la logique et la politique avec Louis Couturat (1879-1913),* 2 vol., Paris: Kimé.

Sedlar, Igor

2009 C. I. Lewis on possible worlds, *History and Philosophy of Logic,* 30, 283–291.

Shearman, Arthur T.

1906a Mr. MacColl's views on logical existence, *Mind,* 15, 143–144.

1906b *The Development of Symbolic Logic: a Critico-Historical Study of the Logical Calculus,* London: Williams & Norgate.

Sheffer, Henry M.

1926 Review of A. N. Whitehead & B. Russell's *Principia Mathematica* (1, 2nd ed., Cambridge: Cambridge University Press, 1925), *Isis,* 8(1), 226–231.

Stadler, Friedrich

2001 *The Vienna Circle,* Vienna, New York: Springer.

Szaniawski, Klemens (ed.)

1989 *The Vienna Circle and the Lvov-Warsaw school,* Dordrecht: Kluwer.

Tarski, Alfred

1992 Drei Briefe an Otto Neurath, *in* R. Haller (ed.), *Grazer philosophische Studien,* 43, 1–32.

van Dalen, Dirk

1999 *Mystic, Geometer, and Intuitionist: The Life of L. E. J. Brouwer,* 1, Oxford: Oxford University Press.

Wolenski, Jan

1989 *Logic and Philosophy in the Lvov-Warsaw School,* Dordrecht: Kluwer.

Wolenski, Jan & Köhler, Eckehart (eds.)

1999 *Alfred Tarski and the Vienna Circle,* Dordrecht: Kluwer.