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Introduction: From Practice to Results in Mathematics and Logic

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1 Mathematical practice: a short overview

This volume is a collection of essays that discuss the relationships between the practices deployed by logicians and mathematicians, either as individuals or as members of research communities, and the results from their research. We are interested in exploring the concept of 'practices' in the formal sciences. Though common in the history, philosophy and sociology of science, this concept has surprisingly thus far been little reflected upon in logic and mathematics. Yet, such practice-based approach would be most crucial for a critical study of these fields. Indeed, in their daily work, mathematicians and logicians do deploy sets of practices, some intimately tied to mathematics and logic as such and others relating rather to material, theoretical, intellectual and social issues in their

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environments. For these reasons, this volume pertains to the interdisciplinary trend that has gained more and more interest in recent years, and which is known as the *philosophy of mathematical practice.*

To give an all too short account of this recent approach, one might look for its roots in Imre Lakatos's 1976 seminal *Proofs and Refutations* [Lakatos 1976], a work that was in part inspired by Georg Polya's *How to Solve It* [Polya 1945] which discusses heuristics and problem-solving techniques in an educational setting. Later on, Philip Kitcher proposed in his book *The Nature of Mathematical Knowledge* [Kitcher 1985] a more or less formal model of how mathematics as an activity can be described. Several authors embraced that trend, as is shown in subsequent volumes that made connections between philosophy and history of mathematics, as did Thomas Tymoczko's *New Directions in the Philosophy of Mathematics* [Tymoczko 1986] and, later and more explicitly, Bill Aspray and Philip Kitcher's *History and Philosophy of Modern Mathematics* [Aspray & Kitcher 1988]. It must be said here that historians of mathematics have long paid attention to practices and produced significant studies with often implicit, but also sometimes explicit, claims about their philosophical relevance.

The above description may suggest a rather uniform approach to the study of mathematical practice, but such is not the case. In the introduction to *The Philosophy of Mathematical Practice* [Mancosu 2008], Paolo Mancosu identifies two main traditions. The first is the 'maverick' tradition which remains close to the Lakatosian approach, while the second settles itself within the modern analytical tradition, namely the naturalizing programme that started with Williard Van Orman Quine, and where Penelope Maddy has played and still plays an important role.

This is not yet the end of the story however. Independently of these developments in the philosophy of mathematics, but also partially inspired by Lakatos as well as Thomas Kuhn's *The Structure of Scientific Revolutions* [Kuhn 1962], some researchers developed a sociology of mathematics, where one of the major focuses was mathematical practice as a group or community phenomenon. Two works should be mentioned in this line: David Bloor's *Knowledge and Social Imagery* [Bloor 1976] and Sal Restivo's *The Social Relations of Physics, Mysticism, and Mathematics* [Restivo 1985]. In contrast with history and philosophy of mathematics, the sociological approach did not merge easily with the mentioned traditions, although some 'brave' attempts in this direction should be noticed [Restivo, Van Bendegem, & Fischer 1993], [Rosental 2008], [Löwe & Müller 2010]. Plenty of reasons can be listed but surely one of the corner elements is the internal-external debate: does mathematics develop according to 'laws' or patterns that are internal to mathematics, or do external elements contribute?

One outcome of these discussions has been to draw the attention to other, so far neglected, areas where mathematics is involved, prominently mathematics education and ethnomathematics. In 2002, a conference was organized in Brussels where the organizers, Jean Paul Van Bendegem and Bart Van Kerkhove, tried to realize their ambition in bringing representatives of all these disciplines together. This conference has led to the 2007 *Perspectives on Mathematical Practices*, with the overambitious subtitle *Bringing Together Philosophy of Mathematics, Sociology of Mathematics, and Mathematics Education* [Van Kerkhove & Van Bendegem 2007]. Another important outcome from all these developments, overall, has been the confirmation of the rather heterogeneous character of this field, as can be seen in the recently founded *Association for the Philosophy of Mathematical Practice (APMP)* (see the website of the association at `http://institucional.us.es/apmp/`), wherein both of Mancosu's traditions are clearly present and at times happily interacting.

More generally, so as to draw a complete picture of the philosophy of mathematical practice, it has to be noted that all of the above would have to be looked at from a wider perspective, in order to see how these developments relate to the whole domain of the philosophy of mathematics, including its mainstream sub-domain, the foundational studies of mathematics. In a sense this is a tension within the internal approach: even if one accepts that mathematics is subjected to something like an internal development, it will remain to find out how this development is best characterized. Such a complete picture will not be drawn here but will need to be clarified in the future.

2 Themes and issues in the philosophy of mathematical practice

In the light of what has been said, how does the problem agenda of the philosophy of mathematical practice look like? The main focus will be on exploring the internal issues, while indicating along the way connections with external issues and themes. An elegant way to obtain some coherence in such an agenda is to look at the various levels where the practices are situated: macro-, meso- and micro-level, as a first rough classification.

At the macro-level, the discussion is about the global development of mathematics. One specific question that arises is whether or not 'revolutions' take place and, if so, of what kind. On this issue, the standard volume remains Donald Gillies's *Revolutions in Mathematics* [Gillies 1992]. Are there Kuhnian parallels to be drawn or does a 'philosophy of mathematical practice' approach also confirm the special status of mathematics vis-à-vis the sciences? As has happened in post-Kuhn philosophy of science, the sociological elements enter into the picture, as one asks why particular mathematical developments took place in their place and time. Rephrased, the issue is that of how well mathematics is or is not closed off from the rest of society. The relevance of the educational and ethnomathematical dimensions is obvious at this level.

At the meso-level, the concern is to deal with research programmes (traditions, styles, etc.). Are there such 'research programmes'? How do they guide research, motivate people, and so forth? The educational dimension is present here as well but on a more internal level: how is the next generation of mathematicians initiated in a particular research programme? A lot of work has been done on this level and addressed features include the issues of being deemed, interesting, fruitful, provider of explanation, promising solutions, or indicative of heuristics. These are all features that are usually neglected in foundational studies.

At the micro-level, the core theme is the nature of mathematical proof. Besides the notion of proof itself and its inner difficulties, there is a wealth of related themes to explore. A first group includes issues related to presentation—concerning the (non-)formalized character of a proof, its (in-)formality, the use of diagrams and their status —, and accessibility—who is able to grasp the proof?—and, perhaps most importantly, why is a proof convincing? A second group of themes concerns what 'accompanies' proofs. Mathematicians perform 'experiments', but are such quotation marks necessary? Why (not)?; they crunch numbers in the hope of finding interesting patterns, performing inductive reasoning; they support their proofs by arguments, leading to the questions whether proofs themselves can be seen as a particular type of argument, and whether, fascinatingly, a rhetorics of mathematical texts is possible. Quite interestingly, at this latter micro-level, other fields and disciplines than history, philosophy and sociology, also contribute to the picture. Such is the case with cognitive psychology, crucial to understand reasoning processes as they take place in the human (social?) brain. Such is the case too with evolutionary biology and psychology, necessary to establish the roots and origins of mankind's 'number sense', if any. It is to be noted too that, at this micro-level, the impact of the particular tradition, whether 'maverick' or 'naturalized', is not overwhelmingly present, making a close collaboration perfectly possible.

More generally, there is one issue that is for sure: there is no shortage of themes to explore and problems to be solved when looking at the philosophy of mathematics from the standpoint of practices. The list of subjects that we presented above is not exhaustive and additional questions—the use of computers in mathematics certainly being one of them—are expected to be addressed by the (hopefully) growing and striving community of philosophers interested in this practice-based approach.

3 The essays in this volume

The essays in this volume are contributions to the philosophy of mathematical practice. Two kinds of papers are to be found. The first set of essays is more conceptual and theoretical, with authors discussing the notion of 'practice' and its connections with concepts, understanding and representations. The second set of essays contains rather detailed analyses and discussions of case studies.

In the first group of essays, first, as a general discussion, Mark Smith defends a conception of mathematical practice and mathematical subject matter that puts together inferential pluralism and a form of concept-realism. The second essay by Danielle Macbeth is a close examination of the relation between the practice of proving and understanding. Fully formalized mathematical proofs usually do not advance our mathematical understanding: does this mean that form does not correspond to content? In her view, to avoid such an opposition, it is necessary to develop a different notion of (formal) proof, and eighteenth-century algebraic proofs can provide an example of fully rigorous and fully content-filled mathematical proofs. The third essay, by Jessica Carter, is an analysis of the role of representations in mathematical proofs, illustrated by an example from contemporary mathematical practice where the value of an expression is found by gradually breaking it down into simpler parts. More generally, and referring to the Peircean terminology, she also discusses the role of icons and indices in such a practice. Finally, the fourth essay by Catarina Dutilh Novaes delineates a practice-based philosophy of logic and illustrates it by focusing on the role played by formal languages in logic. In her view, formal languages have an operative role; paper-and-pencil and hands-on technology trigger specific cognitive processes, which psychology discloses.

In the second group of essays, the first three are analyses of case studies exploring specific mathematical practices. First, Daniele Molinini focuses on the explanatory character of Euler's proof of his Theorem, which, according to the author, is not recognized by Steiner's approach to mathematical explanation. Secondly, Irina Starikova shows how the representation of groups through graphs leads to a new conceptual perspective on the geometry of groups. Thirdly, Baptiste Mélès defends the thesis that the concepts of paradigm and thematization used by Jean Cavaillès find an illustration and a formalization in category theory and a precedent in the Hegelian dialectic. In the fourth and last essay, Vincent Ardourel explores possible interactions between mathematics and physics, by focusing on instances of practices of mathematical research that consist in reformulating constructively physical theories.

All in all, the essays in this volume give an idea of how many different routes can be taken by the investigation on the mathematical and logical practices. Although one might wish the opposite, the general impression is that it is not likely that a far-reaching integration of the internal and external viewpoints may occur in the very near future. Nevertheless, and following the recent creation of the APMP, one may hope that, at the very least, historians and philosophers of mathematics and logic may have found one another, not that it would mean that the heterogeneity of this field of research may not be an issue to be dealt with.

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