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What is Absolute Necessity?

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Résumé : On pourrait définir la nécessité absolue comme la vérité dans absolument tous les mondes possibles sans restriction. Mais nous devrions être capables de l'expliquer sans invoquer les mondes possibles. J'envisage trois définitions alternatives de : « Il est absolument nécessaire que p » et défends une *définition contrefactuelle généralisée* : $\forall q(q \Box\!\!\rightarrow p)$. Je montre que la nécessité absolue satisfait le principe S5 et soutiens que la nécessité *logique* est absolue. Je discute ensuite des relations entre la nécessité logique et la nécessité métaphysique, en expliquant comment il peut y avoir des nécessités absolues *non logiques*. J'esquisse également une théorie essentialiste selon laquelle la nécessité a son fondement dans la nature des choses. Je discute certaines de ses conséquences, y compris concernant l'existence nécessaire de certaines propriétés et celle de certains objets.

Abstract: Absolute necessity might be defined as truth at absolutely all possible worlds without restriction. But we should be able to explain it without invoking possible worlds. I consider three alternative definitions of 'it is absolutely necessary that p' and argue for a *generalized counterfactual definition*: $\forall q(q \Box\!\!\rightarrow p)$. I show that absolute necessity satisfies the S5 principle, and argue that *logical* necessity is absolute. I discuss the relations between logical and metaphysical necessity, explaining how there can be *non-logical* absolute necessities, and sketch an essentialist theory according to which necessity is grounded in the natures of things, and discuss some of its consequences, including the necessary existence of certain properties and objects.

Preamble

If one believes that the notions of necessity and possibility receive their most fundamental explanation in terms of worlds—so that what is necessary is what is true in all possible worlds, and what is possible is what is true in at least

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one—then one might seek to explain the notion of absolute necessity by saying that what is absolutely necessary is what is true in absolutely all worlds without restriction. The idea would be, perhaps, to contrast absolute necessity with what holds true only throughout some restricted range of worlds—say, the worlds at which our laws of physics hold, it being supposed that there are worlds which are in some sense possible, but at which our physical laws fail. Although I think there is something right about this explanation, and although I do, of course, recognise the enormous utility of a semantic account of modalities in terms of worlds, or something like them, I do not think that modal notions or modal facts can be adequately explained in terms of, much less reduced to, facts about worlds. [1] This is not because I think they can be more adequately explained—in non-modal terms—in some *other* way. Of course, there have been other attempts to explain modality. In pre-Kripkean times, in the heyday of logical empiricism and the linguistic theory of the *a priori,* necessity was seen as having its source in facts about meaning—convention was the mother of necessity. Even the most celebrated critic of meanings and analyticity could be found asserting that 'necessity resides in the way we talk about things, not in the things we talk about'. [2] I reject this kind of explanation altogether, even for the so-called [3] conceptual or analytic necessities which it seems best suited to explain.

1. A full statement of my reasons for this view is beyond the scope of this paper. In brief, I take the most plausible versions of the opposing view—that modal facts can be reduced to facts about possible worlds—to be those advocated by David Lewis and David Armstrong. Both depend, in different ways, on a combinatorialist assumption to the effect that there is a possible world corresponding to every mathematically possible combination, or arrangement, of some specified collection of fundamental bits and pieces (mereological individuals or atoms in Lewis's theory, 'thin' particulars and 'sparse' universals in Armstrong's). In brief, I think this conception can avoid circularity—and thus constitute a genuine reduction—only if the combinatorialist assumption is understood in such a way that it begs fundamental questions against broadly essentialist views; questions which, in my view, ought to be settled by philosophical argument, not stipulation. Of course, both Lewis and Armstrong are perfectly well-aware of the anti-essentialist implications of their theories of worlds, and are untroubled by them, because they are in any case opposed to essentialism. If they had independently compelling arguments against essentialism, that might be some comfort—but they don't, so it isn't.

2. [Quine 1956, 174]. In 'Reference and modality', he writes:

> Being necessarily or possibly thus and so is in general not a trait of the object concerned, but depends on the manner of referring to the object ... *Necessary* greaterness than 7 makes no sense as applied to *a number x*; necessity attaches only to the connection between '$x > 7$' and the particular method ... of specifying x. [Quine 1953, 148–149].

3. The terms can be misleading, because they suggest that the necessities so-called have their source in concepts or facts about meaning. I reject such views. I think talk of conceptual or analytic necessity is best understood in an epistemological sense—for it is plausible that such necessities can indeed be known *a priori,* on the basis of a grasp of the relevant concepts.

Although I think that modal notions and facts involving them are *irreducible*—i.e., they cannot be explained away, in other, non-modal, terms—I don't think that means we can't say anything useful about what necessity and possibility are, or about their source or basis. On the contrary, I think there is a lot to be said—much more than I can cover in a short paper. Broadly speaking, I think a non-reductive explanation of the nature of necessity can illuminate by bringing out the way in which the class of necessities as a whole is structured, with necessities of a certain kind playing a basic or fundamental role, in terms of which other necessities may be seen as derivative because consequences of more basic ones. I hope this broad idea will become clearer when I get to details later in the paper. The notion of *absolute* necessity provides a useful starting point.

1 Three conceptions of absolute necessity[4]

Relative necessity is necessity relative to some body[5] of propositions. p is necessary relative to Φ if p must be true, given that all the members of Φ are true. There are as many kinds of relative necessity as there are different bodies of propositions, nearly all of them completely uninteresting. Every proposition p is necessary relative to $\{p\}$, as well as to many other bodies of propositions. More interesting kinds of relative necessity can be got by taking the members of Φ to share some interesting property, such as all being laws of physics.[6] Whether physical and, perhaps more generally, natural necessity can be adequately explained as forms of relative necessity is a further, and rather controversial, question. I can set it aside here.

That p is necessary relative to Φ is usually expressed as a strict conditional $\Box(\Phi \supset p)$,[7] with $\Box$ interpreted as logical necessity. This formulation is not unproblematic (see [Humberstone 1981]), but it is close enough for my purposes.

That relative necessity presupposes some non-relative, or absolute, form of necessity—expressed by $\Box$ in the usual formulation—seems obvious and indisputable. On the face of it, if $\Box$ in $\Box(\Phi \supset p)$ were itself relative, there would have

4. As it happens, I think there are compelling reasons to believe that some necessities are absolute (see fn. 8 below). But the interest and importance of the question what absolute necessity is does not depend on this belief—for one cannot even sensibly deny the existence of absolute necessity unless one is clear what it would be, for something to be absolutely necessary.

5. I use this fulsome term as conveniently ambiguous between set and (proper) class.

6. What, in general, makes a form of relative necessity interesting is a good question, but not one which I can pursue here.

7. *Locus classicus*—[Smiley 1963]. In this formulation, Φ is harmlessly identified with the conjunction of its members.

to be some Ψ and some $\Box'$ such that $\Box(\Phi \supset p)$ abbreviates $\Box'(\Psi \supset (\Phi \supset p))$. Claiming that $\Box'$ is relative threatens an apparently vicious infinite regress. [8]

What is it for a kind of necessity to be *absolute*? One might say that it is simply for it to be non-relative. I doubt that this is a sufficient explanation, since our explanation of relativity presupposes a non-relative notion. I can think of three moderately plausible characterizations:

Limit of relative necessity

There is an obvious and simple way to generalise on the idea of relative necessity. A proposition p may be necessary to one body of propositions, Φ, but not necessary relative to some other body, Ψ. But there is a kind of limiting case—where no matter what set of propositions Φ we choose, p is necessary relative to Φ. In ordinary, non-limit, cases of $\Box(\Phi \supset p)$, it will often be the case that p's truth-value depends on the truth-value of the conjunctive proposition Φ—i.e., that if Φ were not true, p might not be true either. But in the limit case, since p is necessary relative to *every* Φ, its truth-value cannot depend on that of any particular Φ. In view of this, we can regard the limit case as defining a kind of absolute necessity—limit-absolute necessity:

$$\Box_{lim} p =_{df} \forall \Phi \, \Box \, (\Phi \supset p).$$

Absence of competing possibility

The basic idea here is, roughly, that p is absolutely necessary if there is no sense in which the opposite, $\neg p$, is possible:

$$\Box_{max} p =_{df} \neg \exists \diamond \, \diamond \neg p.$$ [9]

This won't quite do as it stands, since the opposite of just about any plausibly absolute necessity may be *epistemically* possible—i.e., true for all we know. But it seems clear that the fact that—as things stand—it is possible for all we know that there is a very large even number which is not the sum of two primes ought not to count as showing that Goldbach's Conjecture, if true, is not absolutely necessary. To make this idea precise, we would need to impose a suitable restriction on the range of $\diamond$ to relevantly competing kinds of possibility (see [Hale 1996]). Perhaps it would be enough to restrict attention to alethic modalities. [10] Saying

8. This is one reason for thinking that there must be some absolute necessities. I believe there are other, in some ways philosophically more important reasons. Some of these reasons are developed in [Hale 1999].

9. Here I quantify into operator position, with $\diamond$ varying over possibility-operators.

10. I am not sure that it *is* good enough. I think a modality is usually said to be alethic if it has somehow to do with the way or 'mode' in which a proposition is true—as opposed

that a kind of necessity is absolute in this sense amounts to claiming that it implies, and so is at least as strong as, any other comparable kind of necessity—hence my label 'maximal'.

Absolutely general counterfactuality: $\Box_{cfac}p =_{df} \forall q(q \Box\!\!\rightarrow p)$

In simple terms, the idea is that what is absolutely necessary is what would be the case, no matter what else was the case. The idea has two parts. First, that we can explain the unary necessity operator, $\Box$, by means of a binary counterfactual conditional operator, $\Box\!\!\rightarrow$, by generalisation on the antecedent: p is necessary iff p would be true no matter what (else) were true. [11] And second, that we can explain *absolute* necessity in terms of the *absolute* (i.e., completely unrestricted) *generality* of the propositional quantifier [12]—it is absolutely necessary that p iff for absolutely every q, $q \Box\!\!\rightarrow p$.

2 Logical necessity and absoluteness

By *logical* necessity, I mean the kind of necessity with which the conclusion of a valid inference follows from its premise. If one takes the necessity of logical consequence as basic, one can easily explain what it is for *propositions* to be logically necessary—a conditional $p \supset q$ is logically necessary (i.e., $\Box(p \supset q)$) iff q is a logically necessary consequence of p. And quite generally, $\Box p$ iff p is a logically necessary consequence of the empty set of premises.

I write $\Box_\Phi p$ to abbreviate $\Box(\Phi \supset p)$. Some obvious facts about relative and limit-necessity are:

(Inclusion) If $\Phi \subseteq \Psi$ then $\Box_\Phi p$ entails $\Box_\Psi p$

(Null) $\Box_\varnothing p \leftrightarrow \Box_{lim} p$.

to how it is known, believed, etc., or whether it ought to be true, etc. This is rather vague. A more precise statement is that a modality is alethic iff the strong modal operator is factive—but while that excludes doxastic and deontic modalities, it makes epistemic modalities alethic, so that one would need a separate stipulation to exclude them.

11. This idea is implicit in [McFetridge 1990]. It is discussed in [Hale 1996]. The idea of explaining $\Box$ in terms of the strong or counterfactual conditional goes back at least to Robert Stalnaker (see [Stalnaker 1968]). It is discussed in several recent publications by Timothy Williamson. One reader of this paper reminds me that one might distinguish different sorts of counterfactual—such as logical, metaphysical, and natural. In view of this, I should emphasise that I intend my counterfactual to be so understood as to make as strong a claim as possible.

12. This explanation assumes the standard Stalnaker-Lewis semantics for conditionals, under which strong conditionals with impossible antecedents are always automatically true. I am grateful to Sonia Roca Royes for discussion of this and several other points.

Since what is logically necessary is just what follows from the null set of premises (i.e., $\Box p \leftrightarrow \Box_{\varnothing} p$), it immediately follows that, if we interpret $\Box$ in the *definiens* for $\Box_{lim}$ as logical necessity, then limit-absoluteness collapses to logical necessity, i.e., $\Box_{lim} p \leftrightarrow \Box p$.

It is also obvious that at most one kind of necessity can be maximally-absolute. For if $\Box$ and $\blacksquare$ are both maximal, $\Box p$ iff $\neg\exists \diamondsuit\ \diamondsuit\neg p$ iff $\blacksquare p$, so that $\Box$ and $\blacksquare$ must coincide.

Under some plausible, but not uncontroversial, assumptions, one can prove that *logical* necessity is maximal, so that every absolute necessity must be logical. The most important assumptions are that any kind of alethic possibility, $\diamondsuit$, is closed under logical consequence and conforms to a form of the law of non-contradiction. That is:

($\diamondsuit$-closure) If $\diamondsuit A$ and $\Box(A \supset B)$ then $\diamondsuit B$
(LNC $\diamondsuit$) $\neg \diamondsuit (A \wedge \neg A)$.

Given these assumptions, one can prove:

(McFetridge's Lemma) If $\Box(p \supset q)$ then $\neg\exists \diamondsuit\ \diamondsuit(p \wedge \neg q)$.

Proof. Suppose $\Box(p \supset q)$. Then $\Box((p \wedge \neg q) \supset q)$. Since $\Box(\neg q \supset \neg q)$, $\Box((p \wedge \neg q) \supset \neg q)$. Hence $\Box((p \wedge \neg q) \supset (q \wedge \neg q))$. Suppose that for some $\diamondsuit$, $\diamondsuit(p \wedge \neg q)$.
By $\diamondsuit$-closure, it follows that $\diamondsuit(q \wedge \neg q)$, contrary to LNC$\diamondsuit$. So by *reductio ad absurdum,* $\neg\exists \diamondsuit\ \diamondsuit(p \wedge \neg q)$. □

Using this, we can then prove:

(Logical necessity is maximally-absolute) If $\Box p$ then $\Box_{max} p$

Proof. Suppose $\Box p$. Then $\Box((p \supset p) \supset p)$. By McFetridge's Lemma, $\neg \diamondsuit ((p \supset p) \wedge \neg p)$ for any $\diamondsuit$. But if $\exists \diamondsuit\ \diamondsuit\neg p$ then since $\Box(\neg p \supset ((p \supset p) \wedge \neg p))$, $\diamondsuit((p \supset p) \wedge \neg p)$ by $\diamondsuit$-closure. By *reductio ad absurdum,* $\neg\exists \diamondsuit\ \diamondsuit\neg p$. □

The modal principles required for these proofs are the usual Rules of Necessitation and $\Box$-distribution across $\supset$, and standard $\Box$-introduction and $\Box$-elimination. These all hold in any reasonable modal logic. The non-modal principles needed all hold in *minimal* logic.

As far as I can see, the proofs require the principle that if $\vdash A$ then $\vdash B \supset (A \wedge B)$. Relevant logicians reject this, because it allows one to prove 'irrelevant' theorems such as $\vdash q \supset (p \supset p)$. I am not much disturbed by the fact that my proofs can't be carried out in a relevant logic, since I'm inclined to regard the rejected principle as independently compelling (and so a reason not to insist upon relevance).

As I said, both main assumptions are controversial.[13] To that extent, at least, the claim that logical necessity is maximally-absolute must be viewed as problematic. But for the claim that logical necessity is absolute in our remaining sense—the generalised counterfactual sense—there is a quite straightforward semantic argument.[14]

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13. (LNC$\Diamond$) asserts, in effect, the maximally-absolute necessity of the law of non-contradiction—there is no relevant sense of 'possible' in which it is possible that a contradiction is true. It may be supposed that this simply begs the question against those—dialetheists and paraconsistent logicians—who hold that some contradictions are or could be true. But matters are not quite so simple. $\neg(A \wedge \neg A)$ is a thesis of Priest's *Logic of Paradox* and likewise of the relevant logic R. One would expect $\neg \Diamond(A \wedge \neg A)$ to hold. The real problem lies elsewhere. In contrast with the rest of us, a dialetheist may hold that *both* $\neg\Diamond(A \wedge \neg A)$ *and* $\Diamond(A \wedge \neg A)$ are true—and the truth of the latter is enough to spoil the claim of logical necessity to be maximally-absolute. But the vast majority of propositions are not *dialetheia,* so unless the dialetheist finds fault with the argument elsewhere, he should accept a qualified version of the claim, to the effect that if a *non-dialetheic* proposition is logically necessary, there is no competing sense in which its opposite is possible.

($\Diamond$-closure) is denied by Dorothy Edgington, who claims:

> [Hale's] crucial assumption is this: if A logically entails B, and it is in any sense possible that A, it is in that sense possible that B. But this assumption is question-begging. The opponent thinks that $A\&\neg B$ is logically impossible, and so entails everything, yet $A\&\neg B$ is possible in some other sense. But she does not accept that everything is possible in this other sense. [Edgington 2004, 14, fn. 9].

Two points in reply: First, it is hard to see how Edgington can reject ($\Diamond$-closure)—if what follows from $\Diamond$-possibilities need not be $\Diamond$-possible, how are we to reason about such things? Second, I think it is Edgington who begs the question—her argument requires *ex falso quodlibet,* but this does not hold in the minimal logic employed in my argument. There are good reasons to rely only on a weak logic in this context—in mounting an argument for the absoluteness of logical necessity, one should make one's demands on logic as weak as possible, and in particular, avoid reliance on putative logical principles whose status as such is open to dispute.

14. In view of my rejection of any explanation of necessity and possibility in terms of worlds (see Preamble), it might reasonably be asked what, if anything, entitles me to deploy semantic arguments such as this one, in which I deal freely in terms of possible worlds or possibilities. At the very least, it might be supposed, such arguments ought, for me, to be mere shortcuts, replaceable by others which make no use of the apparatus of world semantics. I disagree. It is beyond the scope of this paper to give details, but I think it can be shown that we can reconstruct a version of world semantics (possibility semantics) within a general theory of properties which I believe can be shown to support a reasonable form of second-order logic. This theory of properties is irreducibly modal, so there is no reduction of modality—but it does, I think, entitle me to free use of semantic arguments such as the ones given in this paper. I am indebted to Professor Kazuyuki Nomoto, of Tokyo Metropolitan University, for helpful discussion of this point.

(General counterfactual absoluteness) **If $\Box p$ then $\forall q(q \Box\!\!\rightarrow p)$**

Proof. Suppose $\Box p$. Then $\Box_{lim} p$, so that $\forall \Phi \Box (\Phi \supset p)$. Consider any particular q. Then $\Box(q \supset p)$, i.e., $q \supset p$ is true at every possibility.[15] Hence p is true at all q-possibilities, and *a fortiori* true at all closest such possibilities. So $q \Box\!\!\rightarrow p$. But q was arbitrary, so $\forall q(q \Box\!\!\rightarrow p)$. □

One might think one could prove the converse implication, i.e., that any counterfactually-absolute necessity is logical. For can't we argue as follows?

> Suppose $\forall q(q \Box\!\!\rightarrow p)$ and consider any r. Then $r \Box\!\!\rightarrow p$. Hence p is true at all closest r-possibilities. But r was arbitrary, and every possibility w is one of the closest r-possibilities, for some choice of r—so p is true at *all* possibilities. Hence $q \supset p$ is true at all possibilities, for any q. So $\Box(q \supset p)$. But q was arbitrary, so $\forall q \Box (q \supset p)$ (i.e., $\Box_{lim} p$, so that $\Box p$).

The problem lies with the step from '$q \supset p$ is true at all possibilities, for any q' to $\Box(q \supset p)$, where $\Box$ expresses *logical* necessity. This step assumes that logical necessity can be *identified* with truth at all possibilities—but this effectively assumes what is to be proved. Truth at all possibilities is *necessary* for logical necessity (if logical necessity is to be a species of absolute necessity), but the assumption that it is *sufficient* is tantamount to the assumption that there are no other, non-logical, absolute necessities in the generalised counterfactual sense—i.e., that all such absolute necessities are logical.

The failure of the converse implication is important. It leaves open the possibility that there are absolute necessities—in the generalised counterfactual sense—other than logical necessities. If the arguments I gave earlier are correct, limit-absolute and maximal-absolute necessities are always logical—i.e., there can be no *non-logical* absolute necessities in either of these senses of 'absolute'. Since I think it is very plausible that there *are* non-logical absolute necessities—for example, arithmetical ones[16]—I think this is a quite strong reason to prefer the generalised counterfactual explanation of absoluteness.

To sum up, of the three conceptions of absolute necessity considered here, the first two—limit- and maximal-absoluteness—collapse, at least on plausible assumptions, into logical necessity; but the third—general counterfactual absoluteness does not. If one believes, as I do, that some non-logical necessities are

15. By a possibility, I mean a way things could be, or might have been. Possibilities are not assumed to be fully determinate or complete, so they are not possible worlds in the usual sense. I think we can and should dispense with worlds in favour of possibilities, but we need not consider how that might be done here. Nothing in the argument under discussion turns on the distinction between worlds and possibilities.

16. In my view, arithmetical necessities are a species of metaphysical necessities, so that if I am right, at least some metaphysical necessities are absolute. Whether all the examples of metaphysical necessity often discussed—such as those prominent in Kripke's third lecture on *Naming and Necessity*—are absolutely necessary is a further question.

absolute, this gives one a strong reason to prefer my third conception, and I shall assume it in the sequel.

3 The logic of absolute necessity

It is a further advantage of the generalised counterfactual explanation that it supports a straightforward argument to show that absolute necessity conforms to the S4 and S5 principles, i.e., that $\Box p \supset \Box\Box p$ and that $\Diamond\Box p \supset \Box p$.

The argument is semantic. We assume the standard Stalnaker-Lewis semantics [17] for $\Box\!\!\rightarrow$. It is easily verified (Appendix 1) that if we define $\Box p$ as $\forall q(q \Box\!\!\rightarrow p)$, this semantics induces the standard truth-condition for $\Box p$ to be true at any given possibility: $\Box p$ is true at w iff for every w' accessible from w, p is true at w'.

This requires no special assumption about the accessibility relation. But given that our intention is to capture an *absolute* notion of necessity, it is reasonable to require that the accessibility relation be *universal*—i.e., each possibility is accessible from itself and every other. When we define $\Box p$ as a universally quantified counterfactual, $\forall q(q \Box\!\!\rightarrow p)$, the quantifier $\forall q$ is to be understood as *absolutely unrestricted*—as ranging over all propositions whatever. This gives it a kind of *modal* strength additional to that carried by a singular counterfactual. Since no proposition, whether actually true or not, is excluded from the range of its quantifier, the claim is effectively equivalent to the claim that no matter how things *might* have been, it would still have been true that p—expressed in terms of possibilities, that it is true that p at every possibility *without restriction*. As we have seen, the induced condition for $\Box p$ to be true at a possibility w is the usual one—that p be true at every possibility w' accessible from that possibility. If there were possibilities *in*accessible from w at which p is false, then, since those possibilities are *possibilities* (although *not* possibilities *relative* to w), p would not be true at all possibilities without restriction—it would be true only at all possibilities possible relative to w. In that sense, its necessity would be merely *relative* (truth throughout a *restricted* class of possibilities).

If that is right, we can assume that in any model, all possibilities are accessible from any given possibility—so that we can just suppress reference to accessibility. It is then no surprise that we can prove the characteristic S4 and S5 principles (Appendix 1). [18]

17. This assumes a space of possibilities, or possible worlds, which are taken to be complete or fully determinate ways things could be. I prefer a more modest version which avoids the assumption of completeness. This makes for some complications, but I need not trouble you with them here.

18. It is worth emphasising that I am not identifying what philosophers often call *metaphysical* necessity with absolute necessity, or assuming that metaphysical necessities are invariably absolute—although I hold that some are (see fn 16). So I am not claiming that

4 Logical and metaphysical modalities

Our discussion thus far has been quite general and abstract. With the exception of logical necessity, I have avoided reference to specific kinds of alethic modality. My aim has been to explain a notion of absolute necessity which leaves it open which specific kinds of alethic necessity are absolute. If it is agreed that logical necessities must be true at all possibilities, then the semantic argument discussed in the preceding section establishes that logical necessities are absolute.[19] But beyond that, nothing is settled—our definition of absoluteness leaves it open whether there are non-logical absolute necessities.

It is widely accepted that there are *metaphysical* necessities which are *not* logical necessities. If we also accept[20] that there are *no* logical necessities whose negations are metaphysically possible—i.e., that logical necessities are *always* metaphysically necessary—then logical necessities are a proper subclass of metaphysical necessities.

It follows that in a standard sense of 'stronger', logical necessity is *stronger* than metaphysical—if we write $\Box$ for logical and $\blacksquare$ for metaphysical necessity, whenever $\Box A$, $\blacksquare A$, but not conversely. If the corresponding kinds of possibility are related to $\Box$ and $\blacksquare$ in the usual way—so that $\Diamond A$ iff $\neg \Box \neg A$ and $\blacklozenge A$ iff $\neg \blacksquare \neg A$—then whenever $\blacklozenge A$, $\Diamond A$, but not conversely. That is, $\Diamond$ is *weaker* than $\blacklozenge$.

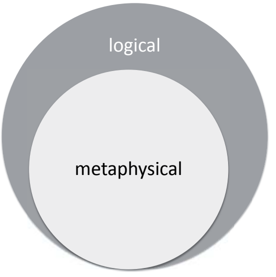
How should we picture this relationship in semantic terms? If we assume that logical possibility is not just weaker than metaphysical, but weaker than any other kind of possibility, one picture of the space of possibilities we might adopt has the *whole* space filled by logical possibilities, and a *proper* subspace filled by metaphysical possibilities (see Figure next page).

But this picture is not mandatory. It involves seeing metaphysical necessities as holding, not at *all* possibilities whatever, but only throughout a *restricted* class of them. The picture is indeed inevitable, if we think of the points in the space of possibilities as *logical* possibilities—for then our assumption that it may be metaphysically but not logically necessary that p, or logically but not metaphysically

the logic of *metaphysical* necessity is S5. On the contrary, it may be that while some metaphysical necessities are absolute, others are not, so that there is, properly speaking, no such thing as *the* logic of metaphysical necessity.

19. Here and hereafter, in the absence of explicit indication to the contrary, 'absolute' means absolute in the generalised counterfactual sense.

20. Perhaps most philosophers would accept this. But not all. It is denied by Dorothy Edgington (*op. cit.*), who identifies logical necessity with *a priori* knowability and thinks that while Leverrier knew *a priori* that if Neptune existed, it caused certain irregularities in the orbit of Uranus, that conditional is not metaphysically necessary—for Neptune might have existed but been knocked off course by an asteroid a million years ago, so that it would have been too far away to disturb the orbit of Uranus. I think we can and should reject her identification of logical necessity with *a priori* knowability. We may agree that logical necessities are knowable *a priori,* but deny that everything knowable *a priori* is logically necessary.



possible that p, demands that there be possibilities which are *not* metaphysical possibilities. However, we could instead think of the difference between logical and metaphysical modalities in a quite different way. For we are not forced to understand the difference in terms of different (more or less inclusive) kinds of possible situations in which logical and metaphysical necessities and possibilities are true. Instead, we could think of necessities of *both* kinds as being true throughout the *whole* space of possibilities, and view the difference between them as like the difference between broader and narrower kinds of logical necessity, rather than like that between, say, logical necessity and some kind of merely relative necessity such as biological or technical necessity.

Let me explain this more fully. What is biologically necessary is, roughly, what must be so, given the actual laws of biology, or the nature of living organisms. What is biologically impossible may well be logically possible. So the right picture, in terms of possibilities, identifies the biologically necessary as what holds true throughout only a restricted range of possibilities. But with logical necessity a different picture seems appropriate. We can distinguish between narrower and broader kinds of logical necessity. There are, for example, the logical necessities of propositional logic, those of first-order logic, and so on. One might think of logical necessities as those necessary propositions which can be expressed making essential use of just logical vocabulary. Alternatively, one might adopt a broader, more generous conception which encompasses what might otherwise be classed as analytic or conceptual necessities, and so recognises as logically necessary truths whose expression essentially involves non-logical vocabulary. There is no need to resolve that issue here. Even if one restricts logical necessities to truths whose expression essentially involves only logical vocabulary, there are broader and narrower classes of logical necessities. But it does not seem correct to think of the necessities of propositional logic as holding true throughout a more extensive class of possibilities than those of first-order logic which depend on their quantificational structure. Rather, we should surely think of necessities of both kinds as holding throughout the *whole* space of possibilities. There is, for example, no possibility at which it is true that my socks are red but not true that something is red, for all that the inference 'My socks are red. So something is red' is not valid in propositional logic. The difference between the necessities of propositional logic and those of first-order logic is not a difference in the ranges of possibilities throughout which they hold. At the linguistic level, it is simply a difference in the kind of vocabulary required for their adequate expression. At what one might think of as a more fundamental—ontological or metaphysical—level, it is a difference in the range of entities essentially involved in explaining why these different kinds of necessities hold.

Once we reject the more and less inclusive classes of possibilities picture for different kinds of logical necessity, we can just as easily reject it as the right picture for the relation between metaphysical and logical necessities. Logical necessities are properly included among metaphysical necessities, just as the necessities of propositional logic are properly included within those of first-order logic. But logical and metaphysical necessities alike may be true throughout the whole space of possibilities. In particular, there may be no possibilities at which some metaphysical but non-logical necessities fail to be true.[21] The difference between logical and metaphysical necessities lies, not in the ranges of possibilities throughout which they hold, but—at the linguistic level—in the kind of vocabulary essential to their expression, and more fundamentally, in the kinds of entities essentially involved in explaining them.

5 The essentialist theory of modality

The alternative picture of the relations between logical and metaphysical necessities I've just sketched fits very badly with the view that necessity and possibility are fundamentally matters of what holds true at all or some possible worlds.[22]

21. This means that a description's being free from explicit or implicit inconsistency does not guarantee that it represents a possibility—freedom from contradiction is a necessary, but not a sufficient condition. It may not be possible to give *a priori* a complete characterisation of the space of possibilities. If there are any absolute necessities which can be known only *a posteriori,* then the space of (absolute) possibilities cannot be characterised *a priori.* I am inclined to the view that the usual Kripkean examples of *a posteriori* necessities are *not* absolute, and that, more generally, that no *a posteriori* necessities are absolute. But even if I am wrong about that, it does not necessarily mean that we cannot, for purely logical purposes, restrict our attention in doing model-theoretic semantics to what we might call *logical* possibilities.

22. According to the worlds approach, distinctions between different kinds of necessity and possibility are to be explained in terms of different classes of worlds throughout which the different kinds of modal proposition hold true—but, as we have seen, the relation between logical and metaphysical necessity cannot be satisfactorily explained in these terms.

But it fits very well with, and can be explained by, what I shall call the *essentialist theory* of modality. A full exposition and defence of this theory lies well beyond the scope of this paper. Here I shall be able to give only a brief outline of the main ideas.[23]

The theory can be seen as answering the question: *What is the source, or basis, of necessity and possibility*? Some philosophers—most famously David Lewis—have answered: There are many possible worlds besides the actual world. Necessity is simply truth in all of them, and possibility truth in at least one. Other philosophers, mainly before Lewis, have said: Necessity is just truth in virtue of meaning. It is not in the world apart from us, but is simply the product of our conventions for using words. The essentialist theory rejects both these answers, and says: Necessity has its source in the natures of things.

To understand this theory, the first things one needs to know are what *things* are, and what the *nature* of a thing is.

By *things,* I mean things of *all* kinds or types—so not just *objects* (including abstract objects such as numbers and sets, as well as concrete objects such as particles and people), but also *properties, relations* and *functions* of all types and levels—properties of concrete objects, such as *being white,* or *having negative charge*; properties of abstract objects, such as *being a natural number,* or *being prime*; properties of properties of objects, such as *being rare* (i.e., *being a property possessed by few objects)*; and similarly for relations and functions. Anything we can talk about is a thing.[24]

By the *nature,* or *essence,* of a thing, I mean *what it is to be that thing.* This is what is given by a *definition* of the thing. For example, the definition of *circle* is: set of points in a plane equidistant from some given point. The definition of *mammal* is: air-breathing animal with a backbone and, if female, mammary glands.[25] This is definition in an Aristotelian sense—what is defined is *not,* or not primarily, a *word* for the thing, but *the thing itself.* A correct definition may serve to state what the word means—as with the previous examples—but it need not: for example, gold is the element with 79 protons per atom, but this is not what the world 'gold' means.

23. Broadly similar or related ideas have been put forward by various others, including especially [Fine 1994, 1995], with whose view mine has some significant affinities; see also [Zalta 2006] and [Lowe 2008]. A comparative assessment is beyond the scope of this paper, and would, however illuminating, distract from my main purposes.

24. Thus my use of 'thing' may be compared with Russell's use of 'term' in [Russell 1903], where he writes: 'Whatever may be an object of thought, or may occur in any true or false proposition, or can be counted as *one,* I call a *term.* This, then, is the widest word in the philosophical vocabulary' [Russell 1903, 43].

25. My definitions of *circle* and *mammal* leave it open what kinds of entity I take these things to be. I don't believe anything of much importance turns on this, at least for my purposes—it would be easy enough to re-state the definitions as definitions of certain first-level properties.

The simplest way to explain the theory is by examples. So consider first the propositions:

$1 < 2$
$1 + 1 = 2.$

I claim that these are necessarily true, and indeed absolutely necessary. Why is that? The essentialist answer is that the first is necessary because true in virtue of the nature of the natural numbers 1 and 2, and of the relation of *being less than*—to be the number 2 just is to be the natural number which immediately follows 1, and for a natural number x to be less than a natural number y is just for y to follow after x in the sequence of natural numbers. Similarly, the second is necessary because true in virtue of the nature of the numbers 1 and 2 and of the operation (function) of *addition*: addition (of natural numbers) is that function of two natural numbers x and y whose value is the natural number z iff z is the y^{th} successor of x.

Here is another example. Consider the proposition:

If a conjunction of propositions A and B is true, A is true.

I claim this too is absolutely necessary. Why? The essentialist answer is very simple: it is necessary because it is true just in virtue of the nature of *conjunction*. Conjunction just is that function of two propositions which has a true proposition as its value for those propositions as arguments if and only if both arguments are true.

Some necessities, like this last example, hold just in virtue of the natures of various *logical* entities—the functions of conjunction, negation, disjunction, quantification, identity, etc. These are the purely logical necessities. Other necessities depend upon the natures of non-logical entities—just as in my earlier examples, which depend upon the natures of certain *mathematical* entities. The essentialist theory is a kind of obvious generalisation of these claims. Things of all kinds have natures. Metaphysical necessities in general are just those necessities which hold in virtue of the nature of some things or other. That is, it is metaphysically necessary that p iff there are some things $X_1, \ldots, X_n$ such that p is true in virtue of the natures of $X_1, \ldots, X_n$.

This is perhaps enough to give you an idea of the essentialist theory. A fuller explanation would need to say much more about the structure of the theory, and about the kind of explanation it provides. This would require a defence of the notion of essence or nature against the charge that these notions are unintelligible, or at least too unclear to be used for philosophical purposes. There need to be limits, or restrictions, upon what counts as belonging to the nature of a thing—if just any necessary truth about a thing counts as part of its nature, the theory becomes explanatorily vacuous. If the notion of essence is to have any explanatory use, there will need to be a distinction between those necessities which hold directly in virtue of a thing's nature, and those which are more or less remote logical consequences of facts about its nature, along with the natures of other things. Among the questions that we shall need to answer are:

Can the theory account for *all* metaphysical necessities? How does it account for metaphysical *possibilities*? Are all metaphysical necessities *absolute*? Can the theory fully explain those necessities which *are* absolute, such as logical and mathematical necessities? I can't discuss all these questions here. In the next and concluding section, I'll outline my answer to the last of them.

6 Absolutely necessary beings

The first point to be clear about is that this question is distinct from the question whether the essentialist can explain all *metaphysical* necessities. It is plausible that all absolute necessities are metaphysical, so that an affirmative answer to the latter question requires an affirmative answer to our present question. But the converse does not hold. Metaphysical necessities—or at least what are commonly taken to be metaphysical necessities—may not all be absolute. Consider, for example, 'Hesperus = Phosphorus' or 'Aristotle was a man'. According to post-Kripkean orthodoxy, these are metaphysical necessities. But they are surely not absolute. For there might have been no such planet as Venus, and in that case, it would not have been true that Hesperus = Phosphorus, i.e., $\neg\forall q(q \mathrel{\Box\!\!\rightarrow}$ Hesperus = Phosphorus$)$, and similarly in the case of Aristotle.[26] An interesting question is whether other Kripkean *a posteriori* necessities, such as that water is H_2O, that light is a stream of photons, etc., are absolute. Kripke himself says they are necessary 'in the highest degree—whatever that means' [Kripke 1972, 99]. I am inclined to think they cannot be absolutely necessary—but I won't go into this here.

If a necessity is to be *absolute,* it must not depend upon any matter of contingent fact. If an essentialist explanation is correct, the *explanandum*—the necessity to be explained—depends upon the *explanans*—the relevant facts about the nature of the entities involved. It follows that, if the necessity explained is to be absolute, the *explanans* must not itself be contingent—so that in particular, it must involve no commitment to any entity whose existence is a matter of contingency. Our question is, in part, whether every absolute necessity can be given an essentialist explanation that meets this condition. Every essentialist explanation appeals to the nature of at least one entity, and so requires at the very

26. This raises the question: in what sense, if any, are these propositions necessary? One might suggest (as Kripke does, cf. [Kripke 1993, 164]) that they are *weakly* necessary, in the sense that they hold true at all the possibilities at which the relevant objects exist. One might, perhaps, agree that $\neg\Diamond$Hesperus $\neq$ Phosphorus, and that $\neg\Diamond$ Aristotle was not a man, but deny that $\neg\Diamond\neg p$ entails $\Box p$. That is, as in Prior's system Q, $\neg\Diamond\neg p$ is a weaker proposition than $\Box p$. One could then define a weak necessity operator by $\boxdot p =_{df} \neg\Diamond\neg p$. An alternative, to which I am myself inclined, is to insist that only the conditionals 'If Aristotle exists, he is a man' and 'If Venus exists, Hesperus = Phosphorus' are absolutely necessary. One could then define the weak necessity operator $\boxdot$ by $\boxdot p =_{df} \Box(e \supset p)$, where e asserts the existence of the things which must exist, if p is to be true.

least that certain natures exist. Thus if an essentialist necessity is to be absolute, the relevant nature(s) must exist of necessity. I hold that the nature of a thing is always a property,[27] so we should begin by seeing how the existence of certain properties can be necessary.

According to the modest, deflationary theory of properties I think we should accept, it suffices for the existence of a property that there should be a suitably meaningful predicate—a predicate with a well-defined satisfaction-condition. To avoid complications afflicting predicates which involve reference to contingently existing objects, I shall focus on *purely general* predicates, by which I mean predicates entirely free of any devices of singular reference. Correspondingly, by a *pure property or relation,* I mean a property or relation which is or could be the semantic value of a purely general predicate. On the modest, deflationary theory of properties, the actual existence of a suitably meaningful purely general predicate suffices for that of a corresponding pure property or relation.[28] But it is clear that this sufficient condition should not be taken to be necessary. For we should surely allow for the existence of properties and relations for which, as a matter of contingent fact, no actual language provides suitable predicates. The most that can properly be required, for the existence of a property or relation, is that there *could* be a suitable predicate—i.e., one which would express it, or have it as its semantic value. In particular, it is sufficient for the existence of a *pure* property or relation that there could be a suitably meaningful purely general predicate. But the possibility in question here—whether or not there could be a predicate with a certain meaning—is surely *absolute.* That is, if it is indeed possible that there should be a suitable predicate, that is itself necessarily so—i.e., it is *necessarily possible.* But if that is right, then the existence of any pure property or relation is always a matter of necessity. For let P be any pure property or relation. Necessarily, P exists iff there could be a predicate having P as its semantic value. But since the logic of absolute modality is S5, it is necessary that (there could be a predicate having P as its semantic value iff *necessarily* there could be such a predicate). It follows that if P exists, then necessarily P exists.[29]

27. What kind of property depends on what kind of thing is in question. The nature of an individual object—what it is to be that object—is a first-level property, but the nature of a general property or relation is a higher-level property, and so on.

28. Readers familiar with Strawson's *Individuals* may notice a close affinity between my claim and Strawson's view that no more is required for the 'introduction' of a universal (in contrast with particulars) than the existence of a suitably meaningful general term. See [Strawson 1959, 180ff].

29. If we abbreviate 'P exists' by p and 'there is a predicate having P as semantic value' by q, our inference has the form: $\Box(p \leftrightarrow \Diamond q) \therefore p \supset \Box p$. This is easily seen to be valid in S5. Suppose the premise true but the conclusion false at some world (or possibility) w_0. Then p must be true but $\Box p$ false at w_0, so that p must be false at some w_1 accessible from w_0. But $p \leftrightarrow \Diamond q$ must be true at w_1, so that $\Diamond q$ must be false at w_1. However, since $p \leftrightarrow \Diamond q$ must be true also at w_0, where p is true, we must have $\Diamond q$ true at w_0. This requires q true at some w_2 accessible from w_0. But since the accessibility relation is both symmetric and

An essentialist explanation of a certain necessity may appeal to the nature of some entity whose definition identifies it as a pure property or relation, which—as we have just observed—exists as a matter of necessity. It is clear that if an essentialist explanation appeals *only* to *pure* properties and relations, there is no reason why the necessity explained should not be absolute. For there is, in the definition of the relevant entities, no presupposition of existence of any objects at all, and so no presupposition of any objects whose existence might be a contingent matter. However, it is plausible that there are absolute necessities which cannot be explained by appeal only to (the natures of things which are) pure properties or relations. For example, it is plausible that it is not only necessary, but absolutely necessary, that $1 < 2$, and that $1 + 1 = 2$. It seems quite clear that if these necessities can be explained on the essentialist theory at all, it must explain them, much as I have suggested already, by appeal to the nature of the numbers 1 and 2, the relation of being less than, and the operation of addition. But in that case, the essentialist explanation would not appeal only to *pure* properties and relations, for the definitions of these entities essentially involve reference to particular objects. They presuppose the existence of the natural numbers, and in particular, of the number 0.

This does not mean that there are absolute necessities which cannot be explained by the essentialist theory. But it does bring to the fore a crucial question. For since these necessities require the existence of certain objects, they can be absolute only if the existence of those objects is itself absolutely necessary. The crucial question, therefore, is: *How, if at all, can the essentialist theory explain the necessary existence of those objects?*

It may seem that if the essentialist is to answer this question at all, he must be reduced to claiming that it is simply in the nature of certain objects to exist—that whereas the nature of aardvarks, say, leaves it open whether there are in fact any aardvarks, it belongs to the nature of natural numbers to exist—existing is part of what it is to be a natural number, in a way that it is not part of what it is to be an aardvark.[30] Even if there is no outright incoherence in the idea that existence can be part of a thing's essence, this is a desperate move—it amounts to an invitation to accept necessary existence as a brute, inexplicable fact. I shall argue that the essentialist can do better—that he can provide an explanation why the natural numbers, for example, exist as a matter of necessity.

We start with the fact that pure properties and relations necessarily exist. I contend that this fact can be explained in essentialist terms. The essentialist does not claim that existence is simply and irreducibly part of what it is for something to be a pure property or relation. What he claims is that a pure property or relation just is one for the existence of which it is sufficient that there could be

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transitive, w_2 is accessible from w_1, whence q must be false at w_2 (since $\Diamond q$ is false at w_1). Contradiction! It follows that there can be no counter-model to the inference.

30. Of course, there couldn't be aardvarks that don't exist—the point is that there is such a thing as being an aardvark, whether or not the general property is actually instantiated.

a suitable predicate—that is, in the case of a first-level property or relation, a predicate expressing the condition objects must meet, if they are to have that property or bear that relation to one another, and similarly for higher-levels. That is what is true, in virtue of the nature of a pure property or relation. The possibility involved here, of the existence of a suitable predicate, consists simply in its not being ruled out by any absolute necessities. Hence it is an absolute possibility. But what is absolutely possible is absolutely necessarily possible.[31] And from this, as we've already seen, it follows that the existence of any pure property or relation is necessary.

This conclusion is quite general: whatever pure properties and relations there are, of whatever level, could not fail to exist—their existence is necessary. Since it is clear that there are indefinitely many meaningful purely general first-level predicates, this implies the existence, and indeed necessary existence, of a large range of pure first-level properties and relations. This includes many pure sortal properties—i.e., properties with which are associated not only satisfaction conditions, but also identity-conditions for their instances.

I want now to extend the argument just given to purely general functions—those functions which could be represented or expressed by functional expressions entirely devoid of singular terms. It might at first appear that no *extension* of the argument is required. For functions, as usually treated in set theory, are just a special subclass of relations.[32] Since the argument already given applies to pure properties and relations quite generally, it may seem that it covers functions as a special case. But matters are not quite so straightforward.

A given relation's being a function depends, of course, upon its meeting certain additional conditions—there must be, for every element of its domain, *something* to which that element bears the relation, and there must *not be more than one* such.[33] Whether or not these conditions are met in a given case may be a matter of contingency. For example, in a society in which there are no single sex marriages, every adult is married, but no one has more than one spouse, the relation expressed by 'x is married to y' is functional, and is, indeed, not just many-one but one-one, so that 'the spouse of …' stands for a function. This may be a matter of law, or it may be just a matter of chance. Either way, it is clearly a contingent matter—nothing in the notion of marriage as such precludes

31. This (i.e., $\Diamond p \supset \Box\Diamond p$) is equivalent to the S5 law: $\Diamond \Box p \supset \Box p$. The equivalence holds in intuitionistic as well as classical S5.

32. I.e., many-one relations, where a binary relation R is many-one if anything that bears R to anything bears it to at most one thing (i.e., $\forall x \forall y \forall z((xRy \wedge xRz) \supset y = z)$), and likewise, with minor adjustments, for relations of more terms.

33. These are what are commonly called the *existence* and *uniqueness* requirements—for every thing that is a possible argument to the function, there must exist a value, and this must be unique. I am assuming the function is to be total—that is, defined for every element in its domain. A partial function, say from domain X to range Y, need not be defined for every element of X, i.e., have a value in Y for every argument in X; but its value for any argument must still be unique, wherever it is defined.

polygamy or enforces heterosexuality. My earlier argument does not, accordingly, establish the necessary existence of such a function. At most, it establishes the necessary existence of a certain binary relation—*being married to*—which may, but need not, be a function. And the point clearly generalises. That is, the general argument cannot, by itself, establish the necessary existence of any function; it may establish the necessary existence of a relation, but it cannot show that any relation is functional. How, then—if at all—can the necessary existence of any functions be established?

The obvious answer is that it may, in contrast with *being married to,* be part of the nature of a relation that it is many-one. As a simple illustration, consider the successor *relation* of addition among numbers, xSy. By the general argument given already, this relation necessarily exists. What we want to see is that it is, also as a matter of necessity, a functional relation. It will then follow that the existence of the successor *function* is necessary. I shall assume, with Frege, that a number [34] is essentially the number of things of some kind. Where F is some kind of things, I'll write $NxFx$ to mean 'the number of Fs'. We can define the successor *relation* among numbers—n follows directly after m—again with Frege, [35] as follows:

$$nSm \quad =_{def} \quad \exists F \exists x(Fx \wedge NuFu = n \wedge Nu(Fu \wedge u \neq x) = m)$$

—that is, n follows directly after m if n is the number of Fs and m is the number of all but one of the Fs. What we need to show is that if nSm and nSp, then $m = p$. To show this, we exploit the fact that the number of things of some kind F is the same as the number of things of some kind G if and only if there is a one-one correspondence between them. So suppose nSm and nSp. Then for some kind F and for some object a, $Fa \wedge NuFu = n \wedge Nu(Fu \wedge u \neq a) = m$ and for some kind G and for some object b, $Gb \wedge NuGu = n \wedge Nu(Gu \wedge u \neq b) = p$. It follows from the two middle conjuncts that $NuFu = NuGu$, so that there must be a one-one correspondence between the Fs and the Gs. And it follows from this, together with the facts that Fa and Gb, that there must be a one-one correspondence between the Fs other than a and the Gs other than b. [36] Hence $Nu(Fu \wedge u \neq a) = Nu(Gu \wedge u \neq b)$. But then $m = p$, as required. The relation xSy is many-one, so we may write it using the standard functional notation: $S(y) = x$.

This illustrates one way in which we can establish the necessary existence of a function—there may be a relation which we can prove to be many-one. In

34. That is, a cardinal number—a number we can use to say how many things of some kind there are.

35. Cf. [Frege 1884, § 76].

36. Let ϕ be the given one-one correspondence between the Fs and the Gs. If $a = b$, ϕ itself is a one-one correspondence between the Fs other than a and the Gs other than b. If $a \neq b$, $\phi(a) = c$ for some c such that Gc, and $\phi(d) = b$ for some d such that Fd. Let ϕ^* be just like ϕ except that $\phi^*(d) = c$. Then ϕ^* is a one-one correspondence between the Fs other than a and the Gs other than b.

such cases, we may think of the function as definable in terms of the relation. But not all functions can be defined in this way, on the basis of underlying relations which can be proved to be many-one. Inspection of our proof that the successor relation is many-one reveals why not. To carry out the proof, we have had to rely on the fact that numbers can be represented as themselves values of a certain function—the function represented by the number operator '$Nu \ldots u \ldots$'. Since this applies to predicates (which may be simple like '... is a horse', or complex like '... is a horse other than Bucephalus'), it stands for a function from properties to objects—it maps each property to the number of its instances. This feature of our proof is no accident. Any proof of the functionality of some relation must rely upon some more basic function. Although many functions can be defined in terms of underlying relations which can be shown to be many-one, not all functions can be so defined. Those functions which cannot be defined in terms of underlying relations, but which can themselves be used to define relations and prove their functionality, might reasonably be viewed as basic or fundamental functions. I think that the number operator is best viewed as such a function. [37]

This gives rise to an obvious question: can the (necessary) existence of basic functions be proved, and if so, how? I want to defend an affirmative answer to the first part of this question, by way of a certain kind of answer to the second part.

How may expressions for basic functions be explained? To take a certain function f as basic is to take it that it cannot be *explicitly* defined by setting $f(x) = y$ iff xRy, using some prior many-one relation R. But that does not mean that f cannot be defined at all. A now quite widely discussed proposal [38] is that we can *implicitly* define functions by means of *abstraction* principles—that is, principles of the general shape:

$$\forall a \forall b (f(a) = f(b) \leftrightarrow aEb)$$

37. It might be suggested that we could define the number operator in terms of an underlying relation, expressed by 'x numbers F', by stipulating that $NuFu$ = the x such that x numbers F. This seems to be what [Boolos 1997, 306] and [MacFarlane 2009, 448] have in mind. But, of course, this just assumes that the numbering relation is many-one. Without a proof that it is so, we are no further forward. Stipulating that 'x numbers F' is to stand for a many-one relation (i.e., that if x numbers F and y numbers F, $x = y$) just begs the question: how do we know that such a stipulation is effective (i.e., that there is such a many-one relation)?

It might be suggested that we could define the number operator using class-abstraction: $NuFu =_{df} \{x | x \approx \{u | Fu\}\}$, where $x \approx y$ iff there is a one-one correspondence between the classes x and y. Perhaps, but only at the price of taking the class-abstraction operator as basic. I think we do best to define the number operator more directly in terms of one-one correspondence, without any detour through set theory.

38. This is a central plank in the neo-Fregean version of logicism originally put forward by Crispin Wright in [Wright 1983], supported in [Hale 1987], and defended by Wright and me in numerous papers, many of them collected together in [Hale & Wright 2001].

where E is an equivalence relation on entities of the type over which a and b vary. The idea is that we may fix a function whose arguments are elements in the field of the equivalence relation (and whose values are objects) by giving, by means of that equivalence relation, a necessary and sufficient condition for the function's value to be the same for any pair arguments. The proposal is, I need hardly say, controversial. If it is to be upheld, various objections must be answered and some hard problems solved. A full defence lies beyond the scope of this discussion, in which I shall largely take it for granted that that defence can be provided, and concentrate upon those aspects of the proposal which bear especially upon my present concerns.[39]

We may, without serious loss of generality, focus on a definite example—the function invoked in our proof of the functionality of the successor relation. The abstractionist proposal here is that we may implicitly define a certain functional expression 'the number of ...' $(Nu \ldots u \ldots)$, where the gap is to be filled by a first-level sortal predicate, by means of:

Hume's Principle: $\forall F \forall G(NuFu = NuGu \leftrightarrow F \approx G)$

where $F \approx G$ abbreviates a second-order formula[40] saying that there is a one-one correspondence between F and G. If all goes well, there is, corresponding to this second-level equivalence relation on properties, a second-level function from first-level properties to objects. The value of the function, for each such property as argument, is the number of objects having that property. The bare possibility that there could be a meaningful relational expression, such as '$\approx$', is sufficient for the existence of a corresponding relation, and the definition of this relation ensures that it is an equivalence.[41] Thus the relation $\approx$ necessarily exists and it is necessarily an equivalence relation. Hume's principle ensures the existence of the corresponding function, given the existence of the equivalence relation. Hence the function $Nu \ldots u \ldots$ necessarily exists.

The example is, in one way, quite special. For if, as Wright and I have argued, Hume's principle can be laid down, with no significant epistemological cost, as

39. Perhaps the main problems confronting the claim that functions may be implicitly defined by abstraction are the so-called Julius Caesar and Bad Company Problems. These are both discussed in [Hale & Wright 2001]—see the Introduction (especially pp.14–23) and Essays 5, 8, 9–14, and the Postscript. The Bad Company problem is the subject of a special number of *Synthese* edited by Øystein Linnebo [Linnebo 2009]. Some more recent defensive efforts are included in [Hale & Wright 2008, 2009a,b].

40. Such as $\exists R(\forall x(Fx \supset \exists! y(Gy \wedge xRy)) \wedge \forall x(Gx \supset \exists! y(Fy \wedge yRx))$.

41. *Proof*: Let $F \approx G$ abbreviate the formula in the previous note. To see that $\approx$ is *reflexive,* observe that for any F, we have $\forall x(Fx \supset \exists! y(Fy \wedge x = y))$, so we can let R be identity. For symmetry, suppose that R is a relation which one-one correlates F with G. Then R's converse—the relation which G bears to F exactly when F bears R to G—one-one correlates G with F. For transitivity, suppose that R one-one correlates F with G and S one-one correlates G with H. Then the composition of R and S—the relation which F bears to H just when there is some G such that F bears R to G and G bears S to H—one-one correlates F with H.

an implicit definition of the number operator, it can usefully serve, given a suitable underlying logic, as an epistemological foundation for elementary arithmetic.[42] Other functions definable from underlying equivalence relations are unlikely to be of much philosophical, or for that matter mathematical, interest or importance. But in another way, the example is not at all special; wherever an acceptable abstraction principle can be formulated, there will be a function corresponding to its equivalence relation as $Nu \ldots u \ldots$ does to $\approx$, and wherever the relevant relation is provably and so necessarily an equivalence relation, the existence of the corresponding function will be necessary. There are many equivalence relations, and many of them are necessarily so. It follows that many functions exist as a matter of necessity.

We are now ready to give a simple and straightforward explanation of the necessary existence of certain objects, *viz.* cardinal numbers. The cardinal numbers exist necessarily because their existence is a consequence of the existence of a certain function and certain properties which themselves exist necessarily. They are the values of the purely general function $Nu \ldots u \ldots$ for a certain range of arguments—purely general first-level sortal properties—and both that function and those arguments to it exist necessarily.

This is not quite, yet, an explanation of the necessary existence of the *natural* numbers. But the extra steps needed to get such an explanation are simple. The natural numbers are just the finite cardinal numbers. The smallest finite cardinal number is 0. This certainly exists as a matter of necessity, since it is the value of $Nu \ldots u \ldots$ for any empty first-level property as argument, and there necessarily exists at least one empty first-level property, *viz.* the property an object x possesses if and only if it is self-distinct (i.e., $x \neq x$). The remaining natural numbers can then be defined, following Frege, as those cardinal numbers which succeed 0—more precisely, those cardinals which bear the ancestral of the successor relation to 0.[43] This definition ensures the validity of mathematical induction over the natural numbers, which can then be used to establish that

42. The underlying logic must be at least second-order, and if Hume's principle is to subserve a proof of the infinity of the natural numbers, it must be understood so that properties of numbers themselves are among the possible values of the variables F and G. This requires that the first-level quantifiers involved in the expanded version of $F \approx G$ must be construed impredicatively, by allowing numbers—which are defined via Hume's principle—to be possible values of the variables they bind. Some critics of the neo-Fregean programme view this as objectionable. For defence, see [Wright 1998] and [Hale 1994]. One must also, of course, defend the claim that higher-order logic really is logic, and not just—as Quine and others have complained—'set theory in sheep's clothing'.

43. Following [Frege 1879], we say that x bears the ancestral of a binary relation R to y (briefly xR^*y) iff y has every property P which belongs to x and belongs to anything to which anything that has P bears the relation R (i.e., xR^*y iff $\forall P((Px \wedge \forall u \forall v((Pu \wedge uRv) \supset Pv)) \supset Py)$. Then x is a natural number iff $x = 0 \vee xS^*0$.

each natural number has a new natural number as its successor, and hence that there are infinitely many natural numbers.[44, 45]

Appendix 1 S4 and S5 principles

$\Box p$ is true at w iff $\forall q(q \Box\!\!\rightarrow p)$ is true at w
iff for every q, $q \Box\!\!\rightarrow p$ is true at w
iff for every q and for every nearest q-possibility, w', accessible from w, p is true at w'
iff for every w' accessible from w, p is true at w'.

The final equivalence is proved as follows:
Suppose that for every q and for every nearest q-possibility, w', accessible from w, p is true at w'. Let w' be any possibility accessible from w. Then for some q, w' is a nearest q-possibility accessible from w. So p is true at w'. But w' was arbitrary, so p is true at every possibility accessible from w.

Conversely, suppose p is true at every possibility accessible from w. Let q be any proposition. If there are any q-possibilities accessible from w, p is true at all of them, and so true at all nearest q-possibilities accessible from w. But q was arbitrary. So for every q and for every nearest q-possibility, w', accessible from w, p is true at w'.

Expanded by our definition, the S4 principle in the form $\Box p \rightarrow \Box \Box p$ is:

$$\forall q(q \Box\!\!\rightarrow p) \rightarrow \forall q(q \Box\!\!\rightarrow \forall r(r \Box\!\!\rightarrow p))$$

Proof. Suppose $\forall q(q \Box\!\!\rightarrow p)$ is true at w. Let w' be any possibility. Then for some q, w' is one of the nearest q-possibilities to w, so p is true at w'. Hence p is true at all possibilities.

Now let q be any proposition. If q is impossible, there are no nearest q-possibilities to w, so (vacuously) A is true at all nearest q-possibilities to w, for any proposition A. So in particular, $\forall r(r \Box\!\!\rightarrow p)$ is true at all nearest q-possibilities to w. So $q \Box\!\!\rightarrow \forall r(r \Box\!\!\rightarrow p)$ is true at w. But since q was arbitrary, $\forall q(q \Box\!\!\rightarrow \forall r(r \Box\!\!\rightarrow p))$ is true at w.

44. As Frege explains in [Frege 1884, §§ 82–83]. See also [Wright 1983, 154ff], and [Boolos & Heck 1997].

45. This paper was written during my tenure of a Senior Leverhulme Research Fellowship. I should like to express my gratitude to the Trust for its generous support. The paper is a somewhat revised and expanded version of a talk given at a workshop on the semantics and epistemology of modality organised by the universities of Paris-Sorbonne, Rennes 1, Nantes and Nancy, and held in Nancy in December 2010. I am grateful to the organisers and the other participants, and especially Christian Nimtz, Sonia Roca Royes and Filipe Drapeau Vieira Contim, for very helpful discussion, and to two anonymous referees for useful comments which have led, I hope, to some improvements in my paper, even though I have not always felt able to follow their advice.

So suppose instead that q is possible. Let w' be one of the nearest q-possibilities to w. We must show that $\forall r(r \,\Box\!\!\rightarrow p)$ is true at w'. So we must show that p is true at the nearest r-possibilities to w', for every r. But p is true at all possibilities whatever.[46] *A fortiori,* p is true at the nearest r-possibilities to w', no matter how r is chosen. Hence $\forall r(r \,\Box\!\!\rightarrow p)$ is true at w'. So $q \,\Box\!\!\rightarrow \forall r(r \,\Box\!\!\rightarrow p)$ is true at w. Clearly this reasoning does not depend on how q is chosen. Hence $\forall q(q \,\Box\!\!\rightarrow \forall r(r \,\Box\!\!\rightarrow p))$ is true at w. □

Expanded by our definition, and using the equivalence of $\Diamond$ with $\neg \Box \neg$, the S5 principle in the form $\Diamond \Box p \rightarrow \Box p$ is:

$$\neg\forall r(r \,\Box\!\!\rightarrow \neg\forall q(q \,\Box\!\!\rightarrow p)) \rightarrow \forall q(q \,\Box\!\!\rightarrow p).$$

Proof. Suppose that for some w, $\neg\forall r(r \,\Box\!\!\rightarrow \neg\forall q(q \,\Box\!\!\rightarrow p))$ is true at w, but $\forall q(q \,\Box\!\!\rightarrow p)$ false at w. Then $\forall r(r \,\Box\!\!\rightarrow \neg\forall q(q \,\Box\!\!\rightarrow p))$ is false at w, so that for some r, $r \,\Box\!\!\rightarrow \neg\forall q(q \,\Box\!\!\rightarrow p)$ is false at w. Hence there is some w' nearest to w at which r is true but $\neg\forall q(q \,\Box\!\!\rightarrow p)$ is false, and so at which $\forall q(q \,\Box\!\!\rightarrow p)$ is true. Further, there is some w'' nearest to w at which q is true but p is false. But[47] for some s, w'' is a nearest s-possibility to w', and since $\forall q(q \,\Box\!\!\rightarrow p)$ is true at w', $s \,\Box\!\!\rightarrow p$ is true at w'. But then p must be true at w'', contradicting our earlier conclusion that p is false at w''. So if $\neg\forall r(r \,\Box\!\!\rightarrow \neg\forall q(q \,\Box\!\!\rightarrow p))$ is true at w, $\forall q(q \,\Box\!\!\rightarrow p)$ must be true there as well. □

Appendix 2 Logical necessity, logical truth and a priority

A problem for the essentialist theory[48]

According to the essentialist theory, logical necessities are a (proper) subset of metaphysical necessities. The latter comprise those propositions which are true in virtue of the essences, or natures, of some things or other, and the logical necessities are then those propositions which are true specifically in virtue of the natures of logical entities—the various logical functions, including the truth-functions, the quantificational functions, the modal functions, and so on. Similarly, the mathematical necessities are those propositions which are true in virtue of the natures of just mathematical (and logical) entities.

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46. This is the crucial step, of course. Without the assumption that the accessibility relation is universal, we would have only that p is true at all possibilities w' accessible from w, and could not get from here to the conclusion that p is true at the nearest r-possibilities to any such w'. For that step, the minimum assumption we would need is that the accessibility relation is transitive.

47. This is the crucial step, i.e., the point at which we rely upon the accessibility relation being symmetric, as well as transitive.

48. This appendix did not form part of my original paper, but resulted from a very useful exchange with Filipe Drapeau Contim, to whom I am deeply indebted.

The essentialist theory of necessity is, in a certain sense, *epistemologically neutral*—it does not imply any tight link between being necessary and being knowable *a priori,* but leaves room for necessities which may be knowable only *a posteriori.* Since it is very plausible that at least some metaphysical necessities *are* only knowable *a posteriori,* this is just as well.

It may, however, seem that this feature of the essentialist theory has some unacceptable, or at least implausible, consequences. For logical and mathematical necessities, or so it may be thought, are always and by their very nature knowable *a priori*—we do not think that of the truths of logic and mathematics, there are some which can only be known *a posteriori.* And yet it seems that, according to the essentialist theory, some propositions which can be known only *a posteriori* must qualify as logically or mathematically necessary.

Examples:

(a) Let 'Numerus' rigidly designate the actual number of solar planets, i.e., 8, and consider:

(1) $8 > 7 \supset$ Numerus > 7

(1) is *metaphysically* necessary. And since (1) makes reference to no entity other than those to which reference is made in

(2) $8 > 7 \supset 8 > 7$

the source of its truth must be the same. But (2) is *logically* necessary, since it is true just in virtue of the nature of truth-functional implication, which guarantees that any proposition whatever truth-functionally implies itself. So since the source of (1)'s truth is the same as that of (2)—indeed, according to one widely supported view, they are the same proposition in different guises[49]—(1) must likewise be *logically* necessary. But as against this, it may be claimed that (1) cannot be logically necessary, for it is surely logically possible that Numerus should not have been greater than 7.

(b)[50] While it may not be absolutely necessary that Hesperus = Phophorus, because Hesperus/Phosphorus might never have existed, it surely is absolutely necessary that:

(3) Hesperus exists $\wedge$ Phosphorus exists $\supset$ Hesperus = Phosphorus

Similarly, it is surely absolutely necessary that:

49. I am thinking, of course, of a 'direct reference' theory of names, according to which a name directly contributes an object—the bearer of the name—to any proposition expressed by a sentence in which the name is used, without the mediation of any Fregean sense, and without otherwise contributing any information about the object.

50. (a) and (b) are slightly modified versions of examples suggested by Filipe Drapeau Contim, who suggested that the problem might best be solved by adopting a Two Dimensionalist treatment which allows one to hold that logical necessities are always knowable *a priori* by maintain that while (1) is superficially necessary, it is—in contrast with (2)—deeply contingent.

(4) Hesperus exists ⊃ Hesperus = Hesperus.

But now it may be argued that, since identity is a logical relation, and (4) is true just in virtue of the nature of identity, together with that of truth-functional implication, (4) must be reckoned *logically* necessary. And since (3) makes reference to no entities beyond those to which reference is made in (4), the source of its truth must be the same, so that it too must be reckoned *logically* necessary. But while it is perhaps metaphysically necessary that if Hesperus/Phosphorus exists, Hesperus = Phosphorus, the necessity of this identity is surely not a matter of *logic*.

A Two Dimensionalist response

One possible response to examples such as these is to concede that they bring out a defect in the essentialist account of logical necessity, but to argue that the defect can be remedied by combining the essentialist theory with a form of Two Dimensionalism about modality.[51] More fully, it may be claimed that the difficulty stems from the fact that logical necessity is distinguished from other forms of metaphysical necessity not just by its source—in distinctively logical entities—but also by its being knowable *a priori*. The essentialist theory captures the first distinguishing feature of logical necessity, but fails altogether to provide for the second. But, it may further be claimed, we can respect the second feature of logical necessity, and block examples like those given above, by deploying the Two Dimensionalist distinction between two kinds of necessity. Putting the matter in terms of worlds, a statement, A, is *superficially* necessary iff the proposition A expresses (i.e., in the actual world) is true not only there, in the actual world, but also in every counterfactual alternative to the actual world; whereas A is *deeply* necessary iff, no matter which world is actual, the proposition A expresses in that world is true there. In terms of this distinction, it can be seen that while (1) is *superficially* necessary, it is *deeply contingent,* whereas (2) is *deeply* necessary. Similarly, whereas (4) is deeply necessary, (3) is, while superficially necessary, deeply contingent. Thus the unwanted conclusion that (1) and (3) are logically necessary can be blocked by insisting that metaphysical necessity in general, and specific kinds of metaphysical necessity such as logical necessity, must be understood as forms of *deep* necessity. That is, for it to be *metaphysically* necessary that p, it is required that it be *deeply* necessary that p, and that there be some entities $x_1, x_2, \ldots$ such that p is true in virtue of the nature(s) of $x_1, x_2, \ldots$; and for it to be, additionally, *logically* necessary that p, it is additionally required that $x_1, x_2, \ldots$ be *logical* entities.

51. This is the response originally favoured by Filipe Drapeau Contim (see previous note)—he claims, and he may well be right, that it is the only way to preserve an essentialist account of logical necessity whilst accepting that such necessities are always knowable *a priori*.

Whilst it may be true that it is only in this way that one can combine an essentialist account of the basis of logical necessity with the insistence that logical necessities are always knowable *a priori,* it must, I think, be allowed that this solution carries with it a heavy—and I would hold—unacceptable cost. For in Two Dimensionalist terms, many standard examples of metaphysical necessities will fail to qualify as such. What ensures that logical necessities are always knowable *a priori,* on this account, is the insistence that metaphysical necessities, including logical necessities, must always be deeply necessary. But since what is deeply necessary is *always* knowable *a priori,* this entails that there can be *no a posteriori* metaphysical necessities. Thus virtually all of the main examples of essentialist necessities discussed by Kripke in *Naming and Necessity* will fail to qualify as metaphysical necessities, on the present proposal—for necessities of origin, of substance or substantial composition, of kind membership and kind inclusion, and what he terms 'theoretical identifications', such as 'Water is H_2O', 'Heat is mean kinetic energy', etc., are all, typically, knowable only *a posteriori.*

I think a quite different kind of response is called for—one which, crucially, leaves room for essentialist necessities which are knowable only *a posteriori.* But before I go into details, it will be useful to notice that the issue is not confined to specifically *logical* necessity. Essentially the same situation arises in regard, for example, to *mathematical* necessities.

Examples:

(c) Let 'Numerus', as in example (a), rigidly designate the actual number of solar planets, and consider:

(5) Numerus > 7

Since (5) is true in virtue of the nature of the objects Numerus (i.e., 8) and 7, and the relation *greater than* among natural numbers, it is metaphysically necessary. And since (5) makes reference to no entity besides those mentioned in:

(6) $8 > 7$

the source of its truth must be the same. But (6) is true solely in virtue of the natures of purely arithmetical entities and so is *arithmetically* necessary, so that (5) must be so as well. Yet surely (5) is not a truth of *arithmetic,* so how can it be arithmetically necessary?

It should be clear that there is a general recipe for generating further examples.[52] For wherever we have a species of metaphysical necessity concerning entities of some specific kind—schematically, Φ-necessity, such that Φ-necessary truths can be known *a priori*—it should be possible to introduce new terms as rigidly designating entities of that kind, fixing their reference to those entities by relying upon contingent facts of some sort, just as we fixed the reference of 'Numerus' as the number 8, by means of the description 'the number of solar planets', which contingently applies to that number. Then, by substituting such

52. I am not claiming that further examples can *only* be generated in this way.

terms in statements expressing Φ-necessities, we may obtain statements which we are unwilling to regard as Φ-necessary.

An alternative response

We must first dispose of a red herring. In stating the objection to the essentialist theory, it was suggested that, given a certain conception of how names work, (1) and (2) would express one and the same proposition. It would then follow immediately, given that (2) states a logical necessity, that (1) must likewise do so. If the objection depended upon the claim that (1) and (2) (and likewise (3) and (4)) express the same proposition, it would admit of a simple and obvious reply. For given that (2) and (4) express logically necessary propositions, the conclusion that (1) and (3) do so is unavoidable, since they express the very same propositions! And since it is undeniable that the propositions expressed by (2) and (4) are logically necessary, the objection is simply confused. The response can be put as a dilemma: Either the pair (1)/(2) express the same proposition (likewise (3)/(4)), or they don't. If they do, the objection is incoherent, but if they don't, it collapses (since it depends upon the claim that they do).

I don't think we should be satisfied with this quick response. For the objection does not, contrary to what the response assumes, depend upon the *identification* of the propositions expressed by (1) and (2), or of those expressed by (3) and (4). It is enough for the objection that both of the propositions expressed by (1) and (2) are necessary, and that their truth has the *same source*. Given that the source of the truth of the proposition expressed by (2) is purely logical, it follows that that of the proposition expressed by (1) must be so as well. Thus a better response is called for.

The Two Dimensionalist response assumes that we should agree that, while it is necessary that if $8 > 7$, then Numerus > 7, and necessary that if Hesperus/Phosphorus exists, then Hesperus = Phosphorus, these necessities are *not logical.* It does so because it further assumes that logical necessities are, by their very nature, always knowable *a priori.* A radical response to the alleged difficulty is to reject these assumptions—i.e., to deny that logical necessities are, as such, knowable *a priori,* and to claim that the necessities expressed by (1) and (3) *are,* after all, *logical.* Similarly, one might deny that mathematical necessities are, by their very nature, knowable *a priori,* and claim that the necessity expressed by (5) is no less mathematical than that expressed by (6).

Obviously more needs to be said, lest this response should seem no more than a desperate exercise in bullet-chewing. In particular, something needs to be done to accommodate, or neutralise, the objection that now clamours for attention: Surely while it is a logical truth that if $8 > 7$, then $8 > 7$, it is no truth of logic that if $8 > 7$, then Numerus> 7. Likewise, it is logically true that if Hesperus exists, Hesperus = Hesperus, but not a logical truth that if Hesperus and Phosphorus exist, Hesperus = Phosphorus. And similarly, while it is a mathematical truth that $8 > 7$, it is no mathematical truth that Numerus > 7.

There is, it seems to me, no denying these last claims. (1), (3) do not express logical truths, and (5) does not express a truth of mathematics. But this gives trouble only if we make the further assumptions that only logical truths are logically necessary, and that only mathematical truths are mathematically necessary. And it seems to me that we can quite sensibly and reasonably reject these assumptions. Certainly all true logical propositions are logically necessary, and all true mathematical propositions are mathematically necessary. But it does not follow—and I think we should deny—that all logically necessary propositions are logical truths, and that all mathematically necessary propositions are truths of mathematics. A proposition may be *true for purely logical reasons,* or true on purely logical grounds (because true just in virtue of the nature of certain logical entities), without being a proposition *of logic.* Similarly, a proposition may be *true* on purely *mathematical grounds* (because true just in virtue of the nature of certain logical and mathematical entities), without being a proposition *of mathematics.* Putting the point another way, there is more to being a logical truth than being logically necessary, and more to being a mathematical truth than being mathematically necessary. What more there is can be seen by considering Quine's well-known characterisation of logical truths as those statements which are true and remain so under all uniform replacements of non-logical vocabulary. It is clear from this that Quine is thinking of the truths of logic as a class of *statements* (or interpreted sentences). By contrast, I am viewing logical truths as a certain class of true *propositions.* Further, unlike Quine, I think we should take logical truths to be a subclass of *necessary* truths. Allowing for these divergences, we can characterise the logical truths as those logically necessary propositions which may be expressed using only logical vocabulary. Given the essentialist theory, this means that the logical truths are just those true propositions which are true solely in virtue of the nature of logical entities, and which can be expressed in purely logical terms. Similarly, the mathematical truths are those true propositions which are true solely in virtue of the nature of logical and mathematical entities, and which can be expressed in purely mathematical terms. On this account, being knowable *a priori* is indeed a mark of logical and mathematical truths, but it is *not* required for logical or mathematical necessity. The fact that expressibility in purely logical or mathematical terms is required for logical or mathematical truth obviously plays a part in explaining how it is that logical and mathematical truths may be known *a priori*—but that is not my present concern.

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