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### Truth as Contextual Correspondence in Quantum Mechanics

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**Résumé** : La sémantique sous-jacente à la structure propositionnelle de la mécanique quantique des espaces de Hilbert implique une ambiguïté intrinsèque concernant l'impossibilité d'assigner des valeurs de vérité définies à toutes les propositions avant trait à un système quantique sans générer de contradiction de type Kochen-Specker. Bien que ledit résultat de Kochen-Specker interdise une assignation globale et absolue des valeurs de vérité aux propositions de la mécanique quantique, il n'exclut pas les assignations contextuelles. À cet égard, le « théorème d'unicité » de Bub-Clifton est utilisé pour montrer que le caractère défini des valeurs de vérité est restauré de façon cohérente pour un sous-ensemble déterminé de propositions, défini par l'état du système quantique considéré et une observable « préférée » appropriée. Il est suggéré que le choix le plus naturel pour cette dernière, en particulier vis-à-vis du problème de l'assignation de valeurs de vérité en mécanique quantique, est que ce soit l'observable sur le point d'être mesurée qui ait une valeur déterminée. On fournit ainsi une conception de la vérité de la correspondance contextuelle qui est appropriée au domaine quantique du discours. La conception de la vérité qui en résulte est compatible avec une conception réaliste de la vérité, qui nie qu'il puisse exister un contexte de référence universel ou un point d'appui archimédien à partir duquel la totalité des faits de la nature puisse être évaluée logiquement.

**Abstract**: The semantics underlying the propositional structure of Hilbert space quantum mechanics involves an inherent ambiguity concerning the impossibility of assigning definite truth values to all propositions pertaining to a quantum system without generating a Kochen-Specker contradiction. Although the preceding Kochen-Specker result forbids a global, absolute assignment of truth values to quantum mechanical propositions, it does not exclude ones that are contextual. In this respect, the Bub-Clifton "uniqueness theorem" is utilized for arguing that truth-value definiteness is consistently restored with respect to a determinate sublattice of propositions defined by the state of the quantum system concerned and a suitable "preferred" observable. It is suggested that the most natural choice of the latter, especially for confronting the problem of truth valuation in quantum mechanics, results in the determinateness of the observable to be measured. An account of truth of contextual correspondence is thereby provided that is appropriate to the quantum domain of discourse. The resulting account of truth is compatible with a realist conception of truth. Such an account essentially denies that there can be a universal context of reference or an Archimedean standpoint from which to evaluate logically the totality of facts of nature.

### 1 Introduction

In investigations concerning the problem of truth in the physical sciences, the correspondence theory of truth has frequently been thought of as the most eminent. Although the correspondence theory admits various different formulations, the core of any correspondence theory is the idea that a proposition is true if and only if it corresponds to or matches reality. The classical version of the theory describes this relationship as a correspondence to the facts about the world, e.g., [Burgess & Burgess 2011, 70–72]. If so, then adopting a correspondence theory of truth amounts to endorsing instances of the following scheme:

[CF] The proposition that P is true if and only if P corresponds to a fact.

Alternatively, if one construes the notion of a "fact" in terms of the weaker notion of an obtaining "state of affairs", as in an Austin-type theory, then, [CF] is re-expressed as follows:

[CS] The proposition that P is true if and only if there is a state of affairs X such that P corresponds to X and X obtains.

The useful feature of states of affairs is that they refer to something that can be said to obtain or fail to obtain, to be the case or not to be the case, to be a fact or fail to be a fact, that is, they exist even when they are not concretely manifested or realized.

Regardless of the exact formulation of a correspondence account of truth, correspondence theorists normally conceive of truth as a non-epistemic notion; that is, a proposition cannot be claimed true or false in virtue of its knowability or provability, e.g., [Devitt 2001, 606]. Any proposition is either determinately true or determinately false independently of our power to establish which value it is. Even if it is impossible to produce a basis on which we may ascertain the truth value of a proposition this does not imply that it does not possess any such value. It always has one. The possession of truth values is therefore entirely independent of our means of warranting their assignment. In this sense, the truth of a proposition is also supposed to transcend our possible knowledge of it, or its verification. I shall argue immediately below that the propositional structure of classical mechanics allows truth-value assignments in conformity with such a traditional conception of a correspondence account of truth.<sup>1</sup>

### 2 Truth-value assignment in classical mechanics

In classical mechanics a system S with n degrees of freedom is described by a phase space  $\Omega_S$  with 2n coordinates  $\{q_i, p_i\}$  which correspond to generalized position and momentum coordinates. The state of S at any temporal moment t is represented by a point  $X_t = \{q_i(t), p_i(t)\}$  of  $\Omega_S$ . Physical quantities are represented by real-valued functions on the phase space, e.g., the position q of a mass point is a function  $q: \Omega_S \to R^3$ . Physical properties—namely, values of various physical quantities of the system—are represented by Borel subspaces  $\Omega_S^A, \Omega_S^B, \ldots$  of  $\Omega_S$  and will be denoted by  $P(A), P(B), \ldots$ , respectively. Hence, a property is represented by a characteristic function  $P(A): \Omega_S \to \{0, 1\}$  with P(A)(X) = 1 if  $X \in \Omega_S^A$  and P(A)(X) = 0 if  $X \notin \Omega_S^A$ . We say that the characteristic function takes the value 1 or the property P(A) pertains to system S at time t if the state of S is represented by a point lying in the corresponding subset  $(X_t \in \Omega_S^A)$ , and that P(A) does not pertain to S if the state of the system is represented by a point outside this subset  $(X_t \notin \Omega_S^A)$ . In terms of propositions  $P_A, P_B, \ldots$  this means that a proposition  $P_A$  is true if the property P(A) pertains to S, and false otherwise. That is, the proposition  $P_A$  asserting that "system S acquires the property P(A)", or equivalently, that "the value a of some physical quantity A of S lies in a certain range of values  $\Delta^{n}$  (" $a \in \Delta^{n}$ ), is true if and only if the associated property P(A)obtains. In the propositional structure of classical mechanics, each point in phase space, representing a classical state of a given system S, defines a truthvalue assignment to the subsets representing the propositions. Each subset to which the point belongs represents a true proposition or a property that is instantiated by the system. Likewise, each subset to which the point does not belong represents a false proposition or a property that is not instantiated by the system. Thus, every possible property of S is selected as either occurring

<sup>1.</sup> My goal in the present paper is not to rehearse from a purely philosophical viewpoint the usual objections to a correspondence theory of truth as, for instance, the so-called "comparison objection", e.g., [McDermid 1998], but to investigate the applicability of the core intuitive notion of a correspondence account of truth within the propositional language of fundamental physics.

or not; equivalently, every corresponding proposition pertaining to S is either true or false.

Hence, for present purposes, the really essential thing about the mode of representation of classical systems is that the algebra of properties or propositions of a classical mechanical system is isomorphic to the lattice of subsets of phase space, a Boolean lattice,  $L_B$ , that can be interpreted semantically by a 2-valued truth function. This means that to every proposition  $P \in L_B$  one of the two possible truth values 1 (true) and 0 (false) can be assigned through the associated characteristic function; equivalently, any proposition is either true or false (tertium non datur), e.g., [Dalla Chiara, Giuntini et al. 2004, 21]. Thus, the propositions of a classical system are semantically decidable. They are either determinately true or determinately false independently of any perceptual evidence or cognitive means by which we may verify or falsify them. Classical mechanical propositions are endowed with a determinate truth value.

From a physical point of view this is immediately linked to the fact that classical physics views objects-systems as bearers of determinate properties. Specifically, classical physical systems are taken to obey a so-called "possessed values" or "definite values" principle that may be succinctly formulated as follows:<sup>2</sup>

*Definite values principle:* Any classical system is characterized, at each instant of time, by definite values for *all* physical quantities pertaining to the system in question.

That is, classical properties (values of physical quantities) are considered as being *intrinsic* to the system, as being *possessed* by the system itself. They are independent of whether or not any measurement is attempted on them and their definite values are independent of one another as far as measurement is concerned. Successive measurements of physical quantities, like position and momentum that define the state of a classical system, can be performed to any degree of accuracy and the results combined can completely determine the state of the system before and after the measurement interaction, since its effect, if not eliminable, takes place continuously in the system's phase space and is therefore predictable in principle. Hence, during the act of measurement a classical system conserves its identity; measurement does not induce any qualitative changes on the state of the measured system. The process of measurement in classical physics is passive; it simply reveals a fact which has already occurred. Thus, the principle of value-definiteness implicitly incorporates the following assumption of non-contextuality:

*Non-contextuality:* If a classical system possesses a property (value of a physical quantity), then it does so independently of any measurement context, i.e., independently of *how* that value is eventually measured.

<sup>2.</sup> The principle of value-definiteness has variously been called in the literature as, for instance, "the determined value assumption" in [Auletta 2001, 21, 105].

This means that the properties possessed by a classical system depend in no way on the relations obtaining between it and a possible experimental or measurement context used to bring these properties about. If a classical system possesses a given property, it does so independently of possessing other values pertaining to other experimental arrangements. All properties pertaining to a classical system are simultaneously determinate, regardless of our means of exploring and warranting their assignment. Accordingly, the propositions of a classical system are considered as possessing determinate truth values—they are either determinately true or determinately false—prior to and independent of any actual investigation of the states of affairs the propositions denote; that is, classical mechanical propositions possess investigation-independent truth values, thus capturing the radically non-epistemic character of a traditional correspondence account of truth. Consequently, the propositions of a classical system are considered as being either true or false in virtue of a stable and welldefined reality which serves as the implicit referent of every proposition. All propositions are therefore meant to have *determinate truth conditions*, so that it does no harm to avoid specifying the exact domain of reference. Thus, in a classical universe of discourse, it is supposed to exist implicitly an Archimedean standpoint from which the totality of facts may be logically evaluated.

## 3 Truth-value assignment in quantum mechanics

On the standard (Dirac-von Neumann) codification of quantum theory, the elementary propositions pertaining to a quantum mechanical system form a non-Boolean lattice,  $L_H$ , isomorphic to the lattice of closed linear subspaces or corresponding projection operators of a Hilbert space. Thus, a proposition pertaining to a quantum system is represented by a projection operator P on the system's Hilbert space H or, equivalently, it is represented by the linear subspace  $H_P$  of H upon which the projection operator P projects. Since each projection operator P on H acquires two eigenvalues 1 and 0, where the value 1 can be read as "true" and 0 as "false", the proposition "a system S in state  $|\psi\rangle$ has the property P(A)" is said to be true if and only if the corresponding projection operator  $P_A$  obtains the value 1, that is, if and only if  $P_A|\psi\rangle = |\psi\rangle$ . Accordingly, the state  $|\psi\rangle$  of the system lies in the associated subspace  $H_A$ which is the range of the operator  $P_A$ , i.e.,  $|\psi\rangle \in H_A$ . In such a circumstance, the property P(A) pertains to the quantum system S. Otherwise, if  $P_A |\psi\rangle = 0$ and, hence,  $|\psi\rangle \in \perp H_A$  (subspace completely orthogonal to  $H_A$ ), the counter property  $\neg P(A)$  pertains to S, and the proposition is said to be false. It might appear, therefore, that propositions of this kind have a well-defined truth value in a sense analogous to the truth-value assignment in classical mechanics.

There is, however, a significant difference between the two situations. Unlike the case in classical mechanics, for a given quantum system, the propositions represented by projection operators or Hilbert space subspaces are not partitioned into two mutually exclusive and collectively exhaustive sets representing either true or false propositions. As already pointed out, only propositions represented by subspaces that contain the system's state are assigned the value "true" (propositions assigned probability 1 by  $|\psi\rangle$ ), and only propositions represented by spaces orthogonal to the state are assigned the value "false" (propositions assigned probability 0 by  $|\psi\rangle$ ) [Dirac 1958, 46–47], [von Neumann 1955, 213–217]. Hence, propositions represented by subspaces that are at some non-zero or non-orthogonal angle to the unit vector  $|\psi\rangle$  or, more appropriately, to the ray representing the quantum state are not assigned any truth value in  $|\psi\rangle$ . These propositions are neither true nor false; they are assigned by  $|\psi\rangle$  a probability value different from 1 and 0; thus, they are undecidable or indeterminate for the system in state  $|\psi\rangle$  and the corresponding properties are taken as indefinite. This kind of semantic indeterminacy imposes an inherent ambiguity with respect to the classical binary true/false value assignments, rigorously expressed, for the first time, by Kochen-Specker's theorem. According to this, for any quantum system associated to a Hilbert space of dimension higher than two, there does not exist a 2-valued, truthfunctional assignment  $h: L_H \to \{0, 1\}$  on the set of closed linear subspaces,  $L_{H}$ , interpretable as quantum mechanical propositions, preserving the lattice operations and the orthocomplement. In other words, the gist of the theorem, when interpreted semantically, asserts the impossibility of assigning definite truth values to all propositions pertaining to a physical system at any one time, for any of its quantum states, without generating a contradiction. What are, therefore, the maximal sets of subspaces of  $L_H$  or the maximal subsets of propositions that can be taken as simultaneously determinate, that is, as being assigned determinate (but perhaps unknown) truth values in an overall consistent manner?

### 3.1 Maximal sets of simultaneously determinate propositions

In this respect, we employ the Bub-Clifton so-called "uniqueness theorem" [Bub 2009], [Bub & Clifton 1996]. Consider, to this end, a quantum system Srepresented by an *n*-dimensional Hilbert space whose state is represented by a ray or one-dimensional projection operator  $D = |\psi\rangle\langle\psi|$  spanned by the unit vector  $|\psi\rangle$  on H. Let A be an observable of S with  $m \leq n$  distinct eigenspaces  $A_i$ , while the rays  $D_{A_i} = (D \vee A_i^{\perp}) \wedge A_i, i = 1, \ldots, k \leq m$ , denote the non-zero projections of the state D onto these eigenspaces. Then, according to the Bub-Clifton theorem, the unique maximal sublattice of the lattice of projection operators or subspaces,  $L_H$ , representing the propositions that can be determinately true or false of the system S, is given by

$$L_H(\{D_{A_i}\}) = \{P \in L_H : D_{A_i} \le P \text{ or } D_{A_i} \le P^{\perp}, \forall_i, i = 1, \dots, k\}.$$

The sublattice  $L_H(\{D_{A_i}\}) \subset L_H$  is generated by (i) the rays  $D_{A_i}$ , the nonzero projections of D onto the k eigenspaces of A, and (ii) all the rays in the subspace  $(D_{A_1} \vee D_{A_2} \vee \ldots \vee D_{A_k})^{\perp} = (\vee D_{A_i})^{\perp}$  orthogonal to the subspace spanned by the  $D_{A_i}$ , for  $i = 1, \ldots, k$ . The set of maximal (nondegenerate) observables associated with  $L_H(\{D_{A_k}\})$  includes any maximal observable with k eigenvectors in the directions  $D_{A_i}$ ,  $i = 1, \ldots, k$ . The set of nonmaximal observables includes any non-maximal observable that is a function of one of these maximal observables. Thus, all the observables whose eigenspaces are spanned by rays in  $L_H(\{D_{A_k}\})$  are determinate, given the system's state D and A.

Identifying such maximal determinate sets of observables amounts, in effect, to a consistent assignment of truth values to the associated propositions in  $L_H(\{D_{A_k}\})$  of  $L_H$ , not to all propositions in  $L_H$ .  $L_H(\{D_{A_k}\})$  represents the maximal subsets of propositions pertaining to a quantum system that can be taken as having simultaneously determinate truth values, where a truth-value assignment is defined by a 2-valued (or Boolean) homomorphism,  $h: L_H(\{D_{A_k}\}) \to \{0, 1\}$ . If the system's Hilbert space H is more than 2-dimensional, there are exactly k 2-valued homomorphisms on  $L_H(\{D_{A_k}\})$ , where the  $i^{th}$  homomorphism assigns to proposition  $D_{A_i}$  the value 1 (i.e., true) and the remaining propositions in  $L_H(\{D_{A_k}\})$ ,  $i = 1, \ldots, k$ , the value 0 (i.e., false). The determinate sublattice  $L_H(\{D_{A_k}\})$  is maximal, in the sense that, if we add anything to it, lattice closure generates the lattice  $L_H$  of all subspaces of H, and there are no 2-valued homomorphisms on  $L_H$  [Bub 2009].

In fact, the Bub-Clifton determinate sublattice  $L_H(\{D_{A_i}\})$  constitutes a generalization of the usual Dirac-von Neumann codification of quantum mechanics. On this standard position, an observable has a determinate value if and only if the state D of the system is an eigenstate of the observable. Equivalently, the propositions that are determinately true or false of a system are the propositions represented by subspaces that either include the ray denoting the state D of the system, or are orthogonal to D. Thus, the Dirac-von Neumann determinate sublattice can be formulated as

$$L_H(D) = \{ P \in L_H : D \le P \text{ or } D \le P^\perp \}.$$

It is simply generated by the state D and all the rays in the subspace orthogonal to D. If the system's Hilbert space H is more than 2-dimensional, there is one and only one 2-valued homomorphism on  $L_H(D)$ : the homomorphism induced by mapping the state D onto 1 and every other ray orthogonal to D onto 0. Apparently, the sublattice  $L_H(D)$  for a particular choice of an observable A in state D forms a subset of Bub-Clifton's proposal  $L_H(\{D_{A_i}\})$ . The latter will only agree with  $L_H(D)$  if D is an eigenstate of A, for then the set  $\{D_{A_i}\}$ consists of only D itself. In general, the sublattice  $L_H(\{D_{A_i}\})$  contains all the propositions in  $L_H(D)$  that it makes sense to talk about consistently with A-propositions, namely propositions that are strictly correlated to the spectral projections of some suitable preferred observable A. From this perspective, the Dirac-von Neumann sublattice is obtained by taking A as the unit (or identity) observable I. As [Bub & Clifton 1996] rightly observe, however, there is nothing in the mathematical structure of Hilbert space quantum mechanics that necessitates the selection of the preferred determinate observable A as the unit observable I, whilst, in addition, this choice leads to von Neumann's account of quantum measurement resulting in a sequential regress of observing observers.

Then, the following question arises. What specifies the choice of a particular preferred observable A as determinate if  $A \neq I$ ? The Bub-Clifton proposal allows, in effect, different choices for A corresponding to various different "no collapse" interpretations of quantum mechanics, as for instance Bohm's hidden variable theory, if the privileged observable A is fixed as position in configuration space, or modal interpretations that exploit the bi-orthogonal decomposition theorem, e.g., [Dieks 1989]. In them the preferred determinate observable is not always fixed but varies with the quantum state.

### 4 Context-dependence of truth valuation in quantum mechanics

In our view, if one wishes to stay within the framework of Hilbert space quantum mechanics and refrains from introducing additional structural elements, the most natural and immediate choice of a suitable preferred observable, especially, for confronting the problem of truth-value assignments, results in the determinateness of the observable to be measured. This is physically motivated by the fact that in the quantum domain one cannot assign, in a consistent manner, definite sharp values to all quantum mechanical observables pertaining to a microphysical object, in particular to pairs of incompatible observables, independently of the measurement context actually specified. In terms of the structural component of quantum theory, this is due to functional relationship constraints that govern the algebra of quantum mechanical observables, as revealed by the Kochen-Specker theorem alluded to above and its recent investigations, e.g., [Cabello 2006], [Kirchmair, Zähringer et al. 2009]. In view of them, it is not possible, not even in principle, to assign to a quantum system definite non-contextual properties corresponding to all possible measurements. This means that it is not possible to assign a definite unique truth value to every single yes-no proposition, represented by a projection operator, independent of which subset of mutually commuting projection operators one may consider it to be a member. Hence, by means of a generalized example, if A, B and E denote observables of the same quantum system, so that the corresponding projection operator A commutes with operators B and E([A, B] = 0 = [A, E]), not however the operators B and E with each other  $([B, E] \neq 0)$ , then the result of a measurement of A depends on whether the system had previously been subjected to a measurement of the observable B

or a measurement of the observable E or in none of them. Thus, the value of the observable A depends upon the set of mutually commuting observables one may consider it with, that is, the value of A depends upon the selected set of measurements. In other words, the value of the observable A cannot be thought of as pre-fixed, as being independent of the experimental context actually chosen, as specified, in our example, by the  $\{A, B\}$  or  $\{A, E\}$  frame of mutually compatible observables. Accordingly, the truth value assigned to the associated proposition " $a \in \Delta$ "—i.e., "the value a of the observable A of system S lies in a certain range of values  $\Delta$ "—should be contextual as it depends on whether A is thought of in the context of simultaneously ascribing a truth value to propositions about B, or to propositions about E.

This state of affairs reflects most clearly the unreliability of the so-called "definite values" principle of classical physics of section 2, according to which, values of physical quantities are regarded as being possessed by an object independently of any measurement context. The classical underpinning of such an assumption is conclusively shown to be incompatible with the structure of the algebra of quantum mechanical observables. Whereas in classical physics, nothing prevented one from considering *as if* the phenomena reflected intrinsic properties, in quantum physics, even the *as if* is restricted. Indeed, quantum phenomena are not stable enough across series of measurements of non-commuting observables in order to be treated as direct reflections of invariable properties; the microphysical world seems to be sensitive to our experimental intervention.

Now, the selection of a particular observable to be measured necessitates also the selection of an appropriate experimental or measurement context with respect to which the measuring conditions remain intact. Formally, a measurement context  $C_A(D)$  can be defined by a pair (D, A), where, as previously,  $D = |\psi\rangle\langle\psi|$  is an idempotent projection operator denoting the general initial state of system S and  $A = \sum_i a_i P_i$  is a self-adjoint operator denoting the measured observable. Of course,  $C_A(D)$  is naturally extended to all commuting, compatible observables which, at least in principle, are co-measurable alongside of A. Then, in accordance with the Bub-Clifton theorem, given the state D of S, D restricted to the set of all propositions concerning A is *necessarily* expressed as a weighted mixture  $D_A = \sum_{i=1}^{k} |c_i|^2 |a_i\rangle\langle a_i|$  of determinate truth-value assignments, where each  $|a_i\rangle$  is an eigenvector of A and  $|c_i| = |\langle \psi, a_i \rangle|, i = 1, \ldots, k$ . Since  $D_A$  is defined with respect to the selected context  $C_A(D)$ ,  $D_A$  may be called a representative contextual state.<sup>3</sup> In other

<sup>3.</sup> In justifying the aforementioned term, it is worthy to note that the state  $D_A$ , which results as a listing of well-defined properties or equivalently determinate truthvalue assignments selected by a 2-valued homomorphism on  $L_H(\{D_{A_i}\})$ , may naturally be viewed as constituting a *state preparation* of system S in the context of the preferred observable A to be measured. This is intimately related to the fact that both states D and  $D_A$  represent the *same* object system S, albeit in different ways. Whereas D refers to a general initial state of S independently of the specification of any particular observable, and hence, regardless of the determination of any mea-

words,  $D_A$  is a mixed state over a set of basis states that are eigenstates of the measured observable A, and it reproduces the probability distribution that D assigns to the values of A. Thus, with respect to the representative contextual state  $D_A$  the following conditions are satisfied:

- (i) Each |a<sub>i</sub>⟩ is an eigenvector of A. Thus, each quantum mechanical proposition D<sub>A<sub>i</sub></sub> ≡ P<sub>|a<sub>i</sub>⟩</sub> = |a<sub>i</sub>⟩⟨a<sub>i</sub>|, i = 1,...,k, assigns in relation to C<sub>A</sub>(D) some well-defined value to A (i.e., the eigenvalue α<sub>i</sub> satisfying A|a<sub>i</sub>⟩ = α<sub>i</sub>|a<sub>i</sub>⟩).
- (ii) Any eigenvectors |a<sub>i</sub>⟩, |a<sub>j</sub>⟩, i ≠ j, of A are orthogonal. Thus, the various possible propositions {P<sub>|a<sub>i</sub>⟩</sub>}, i = 1,..., k, are mutually exclusive within C<sub>A</sub>(D). In this sense, the different orthogonal eigenstates {|a<sub>i</sub>⟩}, i = 1,...,k, correspond to different values of the measured observable A or to different settings of the apparatus situated in the context C<sub>A</sub>(D).
- (iii) Each |a<sub>i</sub>⟩ is non-orthogonal to D = |ψ⟩⟨ψ|. Thus, each proposition P<sub>|a<sub>i</sub>⟩</sub> whose truth value is not predicted with certainty is possible with respect to C<sub>A</sub>(D).

It is evident, therefore, that the contextual state  $D_A$  represents the set of all probabilities of events corresponding to quantum mechanical propositions  $P_{|a_i\rangle}$  that are associated with the measurement context  $C_A(D)$ . In it the propositions  $P_{|a_i\rangle}$  correspond in a one-to-one manner with disjoint subsets of the spectrum of the observable A and hence generate a Boolean lattice of propositions. Thus, the  $P_{|a_i\rangle}$ -propositions are assigned determinate truth values, in the standard Kolmogorov sense, by the state  $D_A$ . Accordingly, by freeing symbols P and C from their specific preceding denotations, the following instance of the correspondence scheme is valid:

[CC] The proposition that P-in-C is true if and only if there is a state of affairs X such that (1) P expresses X in C and (2) X obtains,

where C denotes, in general, the context of discourse, and specifically, in relation to the aforementioned quantum mechanical considerations, the experimental context  $C_A(D)$  linked to the proposition  $P \in L_H(\{D_{A_i}\})$  under investigation.

### 4.1 Philosophical implications

If, however, truth-value assignments to quantum mechanical propositions are context-dependent in some way as the scheme [CC] implies, it would appear

surement context, the state  $D_A$  constitutes a conditionalization state preparation of S with respect to the observable to be measured, while dropping all 'unrelated' reference to observables that are incompatible with such a preparation procedure. The importance of the state preparation procedure in quantum mechanics, functioning as a contextual disentanglement process, is analyzed in a detailed manner in [Karakostas 2007, sect. 4].

that one is committed to antirealism about truth. In our opinion, this assumption is mistaken. The contextual account of truth suggested here is compatible with a realist conception of truth; as I shall argue in the sequel, it subscribes neither to an epistemic nor to a relative notion of truth. Such an account essentially denies that there can be a "God's-eve view" or an absolute Archimedean standpoint from which to state the totality of facts of nature. For, in relation to the microphysical world, there isn't a context-independent way of interacting with it. Any microphysical fact or event that "happens" is raised at the empirical level only in conjunction with the specification of an experimental context that conforms to a set of observables co-measurable by that context [Karakostas 2004], [Svozil 2009]. In this respect, empirical access to the non-Boolean quantum world can only be gained by adopting a particular perspective, which is defined by a determinate sublattice  $L_H(\{D_{A_i}\})$ , or, in a more concrete sense, by the specification of an experimental context  $C_A(D)$  that, in effect, selects a particular observable A as determinate. Within the context  $C_A(D)$ , the A-properties we attribute to the object under investigation have determinate values, but the values of incompatible observables, associated with incompatible (mutually exclusive) experimental arrangements, are indeterminate. Hence, at any temporal moment, there is no universal context that allows either an independent variation of the properties of a quantum object or a unique description of the object in terms of determinate properties. And this yields furthermore an explicit

algebraic interpretation of the Bohrian notion of complementarity (a non-Copenhagean, of course), in so far as quantum mechanical properties obtain effectively determinate values—alternately, the associated propositions acquire determinate truth-value assignments—within a particular quasi-Boolean substructure  $L_H(\{D_{A_i}\})$ , whereas the underlying source of quantum mechanical "strangeness" is located in the fact that they cannot be simultaneously realized or embedded within a single Boolean logical structure. Furthermore, the proposed account of truth, as encapsulated by the scheme

Furthermore, the proposed account of truth, as encapsulated by the scheme [CC] of contextual correspondence, ought to be disassociated from an epistemic notion of truth. The reference to an experimental context in quantum mechanical considerations should not be viewed primarily as offering the evidential or verificationist basis for the truth of a proposition; it does not aim to equate truth to verification. Nor should it be associated with practices of instrumentalism, operationalism, and the like; it does not aim to reduce theoretical terms to products of operational procedures. It rather provides the appropriate *conditions* under which it is possible for a proposition to receive consistently a truth value. Whereas in classical mechanics the conditions under which elementary propositions are claimed to be true or false are determinate independently of the context in which they are expressed, in contradistinction, the truth-conditions of quantum mechanical propositions are determinate within a context. On account of the Kochen-Specker theorem, there simply does not exist, within a quantum mechanical discourse, a consistent binary assignment of determinately true or determinately false propositions independent of the appeal to a context; propositional content seems to be linked to a context. This connection between referential context and propositional content means that a descriptive elementary proposition in the domain of quantum mechanics is, in a sense, incomplete unless it is accompanied by the specified conditions of an experimental context under which the proposition becomes effectively truthvalued (see, in addition, [Karakostas 2012]). In other words, the specification of the context is part and parcel of the truth-conditions that should obtain for a proposition in order for the latter to be invested with a determinate (albeit unknown) truth value. In the quantum description, therefore, the introduction of the experimental context is to select at any time t a specific sublattice  $L_H(\{D_{A_i}\})$  in the total non-Boolean lattice  $L_H$  of propositions of a quantum system as co-definite; that is, each proposition in  $L_H(\{D_{A_i}\})$  is assigned at time t a definite truth value, "true" or "false", or equivalently, each corresponding property of the system either obtains or does not obtain. In effect, the specification of the context provides the necessary conditions whereby bivalent assignment of truth values to quantum mechanical propositions is in principle applicable. This marks the fundamental difference between conditions for well-defined attribution of truth values to propositions and mere verification conditions. In the quantum description, therefore, the specification of the experimental context forms a *pre-condition* of quantum physical experience, which is necessary if quantum mechanics is to grasp empirical reality at all. In this respect, the specification of the context constitutes a methodological act preceding any empirical truth in the quantum domain and making it possible.

Nor the proposed contextual conception of truth is a relative notion; the propositions to which it applies are relative. They are relative to a specific maximal sublattice  $L_H(\{D_{A_i}\})$  of propositions which are determinately true or false of a system at any particular time. For, as already argued, a quantum mechanical proposition is not true or false *simpliciter*, but acquires a determinate truth value with respect to a well-defined context of discourse as specified by the state of the quantum system concerned and a particular observable to be measured. Thus, the conditions under which a proposition is true are *jointly* determined by the context in which the proposition is expressed and the actual microphysical state of affairs as projected into the specified context. What makes a proposition true, therefore, is not that is relative to the context (as an alethic relativist must hold, see, for instance, [MacFarlane 2005]) but whether or not the conditions in question obtain. The obtainment of the conditions implies that it is possible for us to make, in an overall consistent manner, meaningful statements that the properties attributed to quantum objects are part of physical reality. In our approach, the reason that a proposition is true is because it designates an objectively existing state of affairs, albeit of a contextual nature.

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