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General Equilibrium and Welfare in International Trade

John S. Chipman*

General equilibrium is investigated in the tradition of Cournot, Mill, and Marshall, as applied to countries rather than individuals, on the assumption that individual preferences can be aggregated. This includes the competitive equilibrium of free trade, as well as of restricted trade resulting from tariffs, analyzed by Marshall as well as by Johnson. Johnson's theory of tariff wars is analyzed as an example of Cournot's theory of duopoly, leading to a Nash equilibrium in contrast to the competitive equilibrium of free trade. Finally, the effect of unstable equilibrium is discussed as presented in recent work by Wan and Zhou, suggesting a new concept of negotiated equilibrium.

Keywords : international trade, tariffs, tariff wars, multiple equilibria, negotiated equilibrium

Equilibre général et bien-être dans la théorie du commerce international

Suivant la tradition de Cournot, Mill et Marshall, on étudie un modèle d'équilibre général s'appliquant à des pays et non à des individus, sous l'hypothèse que les préférences individuelles peuvent être agrégées. Ce modèle couvre à la fois le cas d'équilibre concurrentiel avec libre échange et le cas de commerce restreint par des droits de douanes, analysé par Marshall aussi bien que par Johnson. La théorie de la guerre des droits de douane de Johnson est analysée comme un exemple de théorie du duopole de Cournot, conduisant à un équilibre de Nash par opposition à l'équilibre concurrentiel de libre échange. Pour terminer, on discute le cas d'un équilibre instable, tel que Wan et Zhou l'ont présenté récemment, suggérant ainsi un nouveau concept d'équilibre négocié.

Mots clés : commerce international, droits de douane, guerre tarifaire, équilibres multiples, équilibre négocié.

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In this paper I consider the general equilibrium of countries in the tradition of Mill (1844; 1852), Mangoldt (1863), Marshall (1879; 1923), Edgeworth (1894), Pareto (1895), and Viner (1937). Attention is not limited to competitive equilibrium, but is also devoted to what is now known as Nash equilibrium, in particular the equilibrium of countries imposing tariffs against each other. The key reference is that of Johnson (1954), whose model turns out to be analytically equivalent to a model of duopoly going back to Cournot (1838).

I will discuss two different models which can be used to compare the differential effects of free trade and tariff-encumbered trade on the welfares of the citizens of two countries. In each case, in order to simplify, I abstract from processes of production, and assume that in each country there is a constant rate of production of two goods per period. Thus the models are all of simple exchange equilibrium. In each case I examine a tariff-encumbered equilibrium in which the two countries engage in a tariff war along the lines of Johnson's (1954) seminal contribution. The outcome of the tariff war, which is a Nash equilibrium, is compared with that (or those) of free trade, which are competitive equilibria. A third concept of negotiated equilibrium is suggested.

In section 1, I assume that consumers within and as between countries have the same homothetic (hence aggregable) preferences of the Millian or "Cobb-Douglas" type, in which the ratio of expenditure on each of the two commodities to income is constant (in fact, in this application, one-half). While I consider a general model of fixed production, in order to make efficient comparisons I assume that each country produces 1 unit of its import good and $\omega > 1$ units of its export good per period. In a tariff war between them it turns out that for each country the equilibrium tariff factor is $\sqrt{\omega}$, so that the equilibrium ad valorem tariff rate is $\sqrt{\omega} - 1$. For example, if $\omega = 2$, the equilibrium tariff rate will be 41.4%. This equilibrium, based on successive applications of the optimal-tariff reaction function, is approximately achieved in about four stages of the tariff war. Under these assumptions it is shown that both countries are worse off after the tariff war than under free trade (that is, under the Nash equilibrium than under the competitive equilibrium), but better off under the Nash equilibrium than under autarky. It turns out that the welfare levels of the countries at the end of the tariff war are slightly more than midway between those under autarky and those under free trade.

In section 2, I examine a fascinating recent contribution by Wan and Zhou (2008), in which the free-trade competitive equilibrium is not unique, but consists of two stable equilibria and one unstable equilibrium as in Marshall (1879), the equitable ("fair") equilibrium being unstable and the inequitable ("unfair") ones stable. This possibility is achieved by assuming a zero endowment (rate of production) of each country's import good, and a unit endowment of its export good, combined with preferences that are parallel (hence aggregable) with respect to each country's export good, i.e., exhibiting constant marginal utility of the export good, leading to each country having a strong relative preference for its own import good. What they show is that, for each country, while one of the two stable (unfair) competitive equilibria is better than the (unique) tariff-encumbered equilibrium, the other one is (about equally) worse. The cautious approach for each country is then to choose the Nash equilibrium rather than run the risk of falling into the worse competitive one, or else to negotiate a tariff rate in between.

1. Symmetric endowments and identical Millian preferences

I shall first take up the case of Millian preferences, in which each country acts as though it maximizes a utility function of the form $U(x_{1k}, x_{2k}) = x_{1k}x_{2k}$, where x_{ik} denotes country k's consumption of commodity i.[1] Letting the first subscript denote the commodity and the second the country, country k's demand function for commodity i is

$$h_{ik}(p_{1k}, p_{2k}, Y_k) = \frac{Y_k}{2p_{ik}} \quad \left(\text{i.e.,} \quad p_{ik}h_{ik}(p_{1k}, p_{2k}, Y_k) = \frac{Y_k}{2}\right), \qquad (1.1)$$

which is equal to its disposable income divided by twice the price of commodity i on country k's markets. On the supply side, I shall assume that country k produces constant amounts ω_{1k} and ω_{2k} of commodities 1 and 2 in each period, so that its earned income is $p_{1k}\omega_{1k} + p_{2k}\omega_{2k}$. Its disposable income will be this amount plus the deficit in its balance of payments on current account, D_k (denominated in its own prices), which in this application will consist of its tariff revenues.[2]

Let us now consider a situation of trade between countries 1 and 2, in which the parameters ω_{ik} are such that country 1 exports

1 Cf. Mill (1852, Vol. II, Book III, Ch. XVIII, 122–151; 1865, 124–153). See also Chipman (1965, 483–491; 1979), Melvin (1969), Harwitz (1972), and Appleyard and Ingram (1979). The given utility functions constitute a special case of identical homothetic preferences, allowing one to aggregate preferences within each country (cf. Chipman 1974).

2 Note that in the absence of saving and capital movements, if country 1 imposes a tariff on its import of commodity 2, it will have a deficit in its balance of trade when reckoned in its own domestic prices, even though its trade will be balanced when reckoned in world prices. This is because the tariff revenues (which we assume to be distributed in lump-sum fashion by the government to the population) bring about and thus correspond to an excess of domestic consumption over domestic production at domestic prices, hence they constitute a deficit in country 1's balance

$-z_{11} \equiv \omega_{11} - x_{11}$ units of commodity 1 to and imports $z_{21} \equiv x_{21} - \omega_{21}$ units of commodity 2 from country 2, and likewise country 2 exports $-z_{22} \equiv \omega_{22} - x_{22}$ units of commodity 2 to and imports $z_{12} \equiv x_{12} - \omega_{12}$ units of commodity 1 from country 1, where in equilibrium the material-balance conditions $z_{i1} + z_{i2} = 0$ necessarily hold for commodities $i = 1,2$.

1.1. Derivation of tariff-modified offer functions

Country 1's trade-demand function $\hat{h}_{21}$ for its import good (commodity 2), which is expressed as a function of its domestic prices p_{11}, p_{21} and the deficit in its balance of trade D_1, is defined in terms of its ordinary demand function h_{21} by

$$z_{21} = \hat{h}_{21}(p_{11},p_{21},D_1;\omega_1) = h_{21}(p_{11},p_{21},p_{11}\omega_{11} + p_{21}\omega_{21} + D_1) - \omega_{21} = \frac{p_{11}\omega_{11} + p_{21}\omega_{21} + D_1}{2p_{21}} - \omega_{21}, \tag{1.2}$$

where $\omega_k = (\omega_{1k},\omega_{2k})$.

For any tariff factor $T_2 = 1 + \tau_2$ imposed by country 1 on its import of commodity 2 (where τ_2 is the ad valorem tariff rate), country 1's tariff-modified excess-demand function for this import good (which is expressed as a function of world prices p_1,p_2) is defined implicitly by

$$\begin{aligned}\hat{z}_{21}(p_1,p_2,T_2;\omega_1) &= \hat{h}_{21}(p_1,T_2p_2,(T_2-1)p_2\hat{z}_{21}(p_1,p_2,T_2;\omega_1);\omega_1) \\ &= \frac{p_1\omega_{11} + T_2p_2\omega_{21} + (T_2-1)p_2\hat{z}_{21}(p_1,p_2,T_2;\omega_1)}{2T_2p_2} - \omega_{21},\end{aligned}$$

where we have used $D_1 = (T_2-1)p_2\hat{z}_{21}(p_1,p_2,T_2;\omega_1)$ for the tariff revenues. Collecting terms, this leads to the explicit formula

$$z_{21} = \hat{z}_{21}(p_1,p_2,T_2;\omega_1) = \frac{p_1\omega_{11} - T_2p_2\omega_{21}}{(T_2+1)p_2} = \frac{p_1\omega_{11} + p_2\omega_{21}}{(T_2+1)p_2} - \omega_{21}. \tag{1.3}$$

Normalizing world prices to $(1,r_2)$ where $r_2 = p_2/p_1$, country 1's tariff-modified inverse excess-demand function $\hat{r}_2(z_{21};T_2)$ for

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of trade when expressed in its domestic prices; thus, country 1's budget (balance-of-trade) equation

$$p_{11}z_{11} + T_2p_{21}z_{21} = (T_2-1)p_{21}z_{21} = D_1,$$

which states that its balance of trade denominated in domestic prices is equal to its tariff revenues, immediately implies that $p_{11}z_{11} + p_{21}z_{21} = 0$, i.e., that its trade is balanced when expressed in external prices.

its import good 2 is now defined as the solution of the equation

$$\omega_{21} + z_{21} = \frac{\omega_{11} + r_2\omega_{21}}{(T_2 + 1)r_2}, \tag{1.4}$$

which is

$$\hat{r}_2(z_{21};T_2) = \frac{\omega_{11}}{z_{21} + (\omega_{21} + z_{21})T_2}. \tag{1.5}$$

Country 1′s tariff-modified offer function is then defined as

$$-z_{11} = F_1(z_{21};T_2) \equiv z_{21}\hat{r}_2(z_{21};T_2) = \frac{\omega_{11}}{1 + (\omega_{21}/z_{21} + 1)T_2}. \tag{1.6}$$

An exactly similar formula holds for country 2. Thus we have a system of Marshallian offer functions

$$-z_{11} = \frac{\omega_{11}}{1 + (1 + \omega_{21}/z_{21})T_2} \quad \text{and} \quad -z_{22} = \frac{\omega_{22}}{1 + (1 + \omega_{12}/z_{12})T_1}. \tag{1.7}$$

1.2. General equilibrium of the tariff-modified system

We note from (1.7) that this system is linear in the reciprocals of the trades. Then, using the material-balance conditions $z_{i1} + z_{i2} = 0$ for $i = 1,2$, (1.7) may be written in the matrix form

$$\begin{bmatrix} \omega_{11} & -\omega_{21}T_2 \\ -\omega_{12}T_1 & \omega_{22} \end{bmatrix} \begin{bmatrix} 1/z_{12} \\ 1/z_{21} \end{bmatrix} = \begin{bmatrix} 1 + T_2 \\ 1 + T_1 \end{bmatrix}. \tag{1.8}$$

A sufficient condition for its solution is that $T_1T_2 < \omega_{11}\omega_{22}/\omega_{12}\omega_{21}$.

The solution to (1.8) is

$$\begin{bmatrix} 1/z_{12} \\ 1/z_{21} \end{bmatrix} = \frac{1}{\omega_{11}\omega_{22} - \omega_{12}\omega_{21}T_1T_2} \begin{bmatrix} \omega_{22}(1 + T_2) + \omega_{21}T_2(1 + T_1) \\ \omega_{12}T_1(1 + T_2) + \omega_{11}(1 + T_1) \end{bmatrix}, \tag{1.9}$$

or equivalently,

$$z_{12} = \frac{\omega_{11}\omega_{22} - \omega_{12}\omega_{21}T_1T_2}{\omega_{22}(1+T_2) + \omega_{21}T_2(1+T_1)} \text{ and} \qquad (1.10)$$
$$z_{21} = \frac{\omega_{11}\omega_{22} - \omega_{12}\omega_{21}T_1T_2}{\omega_{12}T_1(1+T_2) + \omega_{11}(1+T_1)}.$$

For the consumption levels, we have for country 1

$$x_{11} = \omega_{11} - z_{12} = \frac{\omega_{11}(\omega_{21}+\omega_{22})T_2 + \omega_{21}(\omega_{11}+\omega_{12})T_1T_2}{\omega_{22} + (\omega_{22}+\omega_{21})T_2 + \omega_{21}T_2T_1} \qquad (1.11)$$

and

$$x_{21} = \omega_{21} + z_{21} = \frac{\omega_{11}(\omega_{21}+\omega_{22}) + \omega_{21}(\omega_{11}+\omega_{12})T_1}{\omega_{11} + (\omega_{11}+\omega_{12})T_1 + \omega_{12}T_1T_2}, \qquad (1.12)$$

with analogous formulas for country 2.

1.3. Optimal tariffs and reaction functions in a leading special case

At this point, in order to simplify I introduce the assumption that $\omega_{12} = \omega_{21} = 1$ and $\omega_{11} = \omega_{22} = \omega$, i.e., each country produces ω units of its export good and 1 unit of its import-competing good, where $\omega > 1$.

Now if T_1 (country 2's tariff factor on its import of commodity 1) is given initially, country 1's problem is to maximize its utility $U(x_{11}, x_{21}) = x_{11}x_{21}$ with respect to its tariff factor T_2 (subject to the given value of country 2's tariff factor T_1). From (1.11) and (1.12) the problem is then to maximize

$$x_{11}x_{21} = \frac{(\omega+1)(\omega+T_1)T_2}{\omega + (\omega+1)T_2 + T_1T_2} \cdot \frac{(\omega+1)(\omega+T_1)}{\omega + (\omega+1)T_1 + T_1T_2} \qquad (1.13)$$

with respect to T_2. Differentiating (1.13) with respect to T_2 and equating it to zero we obtain

$$\frac{1}{T_2} - \frac{\omega+1+T_1}{\omega + (\omega+1)T_2 + T_1T_2} - \frac{T_1}{\omega + (\omega+1)T_1 + T_1T_2} = 0. \qquad (1.14)$$

After some elaborate calculations this reduces to the equation

$$[(\omega+1)T_1 + T_1^2]T_2^2 = \omega^2 + \omega(\omega+1)T_1, \qquad (1.15)$$

from which we obtain country 1's *reaction function*[3]

$$T_2 = \sqrt{\frac{\omega^2 + \omega(\omega+1)T_1}{(\omega+1)T_1 + T_1^2}} \equiv \Phi(T_1),$$

expressing its tariff factor T_2 on commodity 2 as a function Φ of country 2's tariff factor T_1 on commodity 1. Because of the complete symmetry of the situation, by interchanging T_1 and T_2 in (1.16) we obtain country 2's reaction function $T_1 = \Phi(T_2)$. In the special case in which country 2 does not impose a tariff on its import of commodity 1, i.e., $T_1 = 1$, the formula for country 1's "optimal tariff" factor on its import of commodity 2 reduces to $T_2 = \sqrt{\omega(2\omega+1)/(\omega+2)}$.[4]

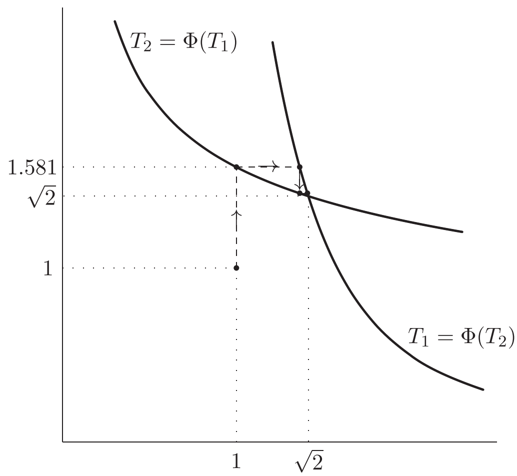

Figure 1: Tariff-war reaction functions and dynamic approach to equilibrium. The case $\omega = 2$

3 The idea of a reaction function was originated by Cournot (1838, Ch. VII, 90 and Fig. 2; 1897, 81). The terminology appears to be due to Fellner (1949, p. 59), who provided an excellent exposition. He also referred to it there (pp. 64, 68n) as a "Cournot function".

4 Following Torrens (1844, esp. 331–372), the first systematic discussion of the benefit to a country of a tariff appears to be that of Mill (1844), who held (p. 27) that it "almost always falls in part upon the foreigners who consume our goods." The first formula for an optimal tariff rate appearing in the literature appears to be that of Launhardt (1885, §17, 85; 1993, Ch. 17, 86), which is based on a quadratic utility function. The next is one due to Auspitz and Lieben (1889, Book VI, Ch. 80–82, 408–429; 1890; 1914, 267–280), who provided a geometric treatment on the assumption that the marginal utility of the country's export good is constant, followed (1889, 442; 1914, 288) by a formula $x \cdot V''(x)$ for the optimal tariff rate (see Niehans and Jäggi 1995, 382), where $V(x)$ is their curve (Fig. 77) of the "cost

These two reaction functions are displayed in Figure 1 for the case $\omega = 2$, and may be compared with those depicted by Cournot (1838) in his Figure 2. They are hyperbolae.

We may now proceed with Johnson's (1954) dynamic approach to equilibrium.[5] Starting from free trade, if country 1 begins the tariff war, it imposes a tariff factor T_2 given by (1.16) for $T_1 = 1$. Given this tariff factor, country 2 retaliates by following the same formula (with T_1 and T_2 interchanged) for its new tariff factor $T_1 > 1$. We now show that there is an equilibrium in this game; in fact it is a *Nash equilibrium*.[6] If there is an equilibrium, then by the symmetry of the model both countries must impose the same tariff factor. Then, setting $T_1 = T_2 = T$ in (1.16) and squaring both sides, we obtain the polynomial equation

$$T^4 + (\omega + 1)T^3 - \omega(\omega + 1)T - \omega^2 = 0, \tag{1.17}$$

which has one positive root $\sqrt{\omega}$ and three negative roots $-1, -\omega, -\sqrt{\omega}$. Thus an equilibrium exists, and the equilibrium tariff factor is $T = \sqrt{\omega}$.

Now we show that this Nash equilibrium is *stable*. If country 1 starts the tariff war under conditions of free trade ($T_2 = 1$), it will clearly set its tariff factor $T_2 > 1$. If country 2 had started the tariff war, it would of course have done the same. But now that country 1's tariff factor T_2 has increased, country 2 will choose a *lower* tariff factor T_1 than it would have done if it had started the tariff war, contradicting the intuition that a worse attack warrants a greater retaliation. This is because, from (1.16),

$$\frac{dT_2}{dT_1} = \Phi'(T_1) = \frac{-\omega(\omega+1)T_1^2 - 2\omega^2 T_1 - \omega^2(\omega+1)}{2[(\omega+1)T_1 + T_1^2]^{3/2}} < 0. \tag{1.18}$$

Now that country 2 has increased its tariff factor T_1, country 1 will *reduce* its tariff factor T_2 in the next round. And then country 2 will *increase* it tariff factor T_1. The two tariff factors will thus converge.

An illustration is given in the table below, where $\omega = 2$, hence the optimal tariff factor is $\sqrt{2} = 1.41421356237$. The dynamic process is illustrated in Figure 1, exhibiting the successive pairs

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of exports and benefit of imports''. The third is that of Bickerdike (1907), followed by Edgeworth (1908), who provided formulas based on the assumption of vanishing cross-elasticities (for a discussion see Chipman 1993). The final and definitive one was that of Johnson (1951, 29), who acknowledged the assistance of J. de V. Graaff.

5 A slightly different approach is followed by Gibbons (1992, 75–79).

6 Cf. Nash (1951). See also Gibbons (1992, 14–21). The concept was derived independently by Johnson (1954), and of course goes back to Cournot (1838, Ch. VII, 88–100; 1897, 79–89, and Figures 2 and 3). However, Johnson (1954, 148–9) showed that in general there need not be an equilibrium; there could instead be a limit cycle.

(1,1),(1,1.581),(1.365,1.581),(1.365,1.431), (1.409,1.431), etc., showing that the equilibrium is virtually reached in four iterations. Note that in seven iterations the equilibrium tariff is reached with an accuracy of six decimal points.

T_1	T_2
1	1.58113883008
1.36453054220	1.43051123097
1.40903150706	1.41587780352
1.41368079121	1.41438429260
1.41415886865	1.41423108545
1.41420794843	1.41421536096
1.41421298615	1.41421374698

1.4. Welfare properties of the free-trade, optimal-tariff, and autarky equilibria

If $T_1 = T_2 = T$, the above formula (1.13) provides an indicator of each country's welfare in an equilibrium with a common tariff factor T, which we may denote

$$V(T) = \left(\frac{(\omega+1)(\omega+T)}{\omega+(\omega+1)T+T^2} \right)^2 T.$$

It is of interest to compare this welfare level under three situations: free trade ($T = 1$), a common Cournot-Johnson-Nash equilibrium tariff ($T = \sqrt{\omega}$), and autarky ($T = \infty$). It is not hard to see that the optimal equilibrium with equal tariff factors $0 < T < \infty$ (which includes subsidies for the case $0 < T < 1$) is the free-trade equilibrium $T = 1$. This is seen by differentiating $V(T)$ with respect to T, leading to the cubic equation

$$T^3 + 3T^2 - 4 = (T-1)(T+2)^2 = 0$$

which has the unique positive root $T = 1$.

The following table provides the welfare level of each country for three values of ω, under the three alternative equilibria.

It may be noted that in each case, the welfare level under the optimal-tariff equilibrium is slightly more than midway between those of autarky and free trade.

ω	$V(\infty)$	$V(\sqrt{\omega})$	$V(1)$
2	2	2.1838	2.25
3	3	3.7128	4.00
4	4	5.5556	6.25

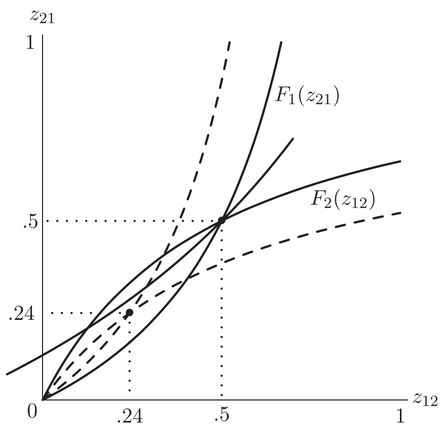

Figure 2: The effect of optimal tariffs on trade and welfare under Millian ("Cobb-Douglas") preferences. The case $\omega_{kk} = 2$, $\omega_{jk} = 1$ ($j \neq k$), $\hat{T} = \sqrt{2}$

An illustration is provided in Figure 2 displaying the Marshallian offer curves for the case $\omega = 2$, where from (1.7) the offer functions have the form

$$-z_{kk} = F_k(z_{jk};1) \;=\; \frac{2}{2 + 1/z_{jk}} \quad \textit{for } k = 1{,}2,\; j \neq k$$

under free trade, and (as indicated by the dashed curves)

$$-z_{kk} = F_k(z_{jk};\sqrt{2}) \;=\; \frac{2}{1 + \sqrt{2}(1 + 1/z_{jk})} \quad \textit{for } k = 1{,}2,\; j \neq k$$

under the common Nash-equilibrium tariff factor $\hat{T} = \sqrt{2}$. A trade-indifference curve for country 1 is drawn through the competitive-equilibrium point (.5,.5); this passes above the Nash-equilibrium point (.24,.24) (where from (1.10) we have $z_{12} = z_{21} = 2/(4+3\sqrt{2}) = .24264$), showing that country 1 is better off under free trade than under the Cournot-Johnson-Nash optimal-tariff equilibrium (and similarly for country 2). It should be emphasized that this conclusion follows from the strong symmetry assumptions we have made as between the countries; otherwise, as Johnson (1954) showed, one of the countries could be better off under the tariff-war equilibrium than under free trade.

2. Free trade and optimal tariffs under symmetrically disparate preferences and multiple equilibrium

In this section I discuss an ingenious paper by Wan and Zhou (2008).[7] They assume that there are aggregate preferences in each of two countries which can be represented by the quasi-linear, quadratic, and mirror-image utility functions

$$\begin{aligned} U_1(x_{11},x_{21}) &= x_{11} + 4x_{21} - 2x_{21}^2 \\ U_2(x_{12},x_{22}) &= x_{22} + 4x_{12} - 2x_{12}^2, \end{aligned} \tag{2.1}$$

where x_{kk} is country k's consumption of its own export good and x_{jk} its consumption of its import good $j \neq k$. Note that for marginal utilities to be positive, these functions are well-defined only for $x_{21} < 1$ and $x_{12} < 1$, which we hereby assume. Note also that they are each linear in the country's export good. Thus, individual preferences are "parallel" with respect to the country's export good (which plays the role of money: cf. Boulding 1945, 857) permitting their aggregation into preferences for the respective countries (but of course not for the world).[8]

Wan and Zhou assume further that country k produces exactly one unit of its export good (commodity k), and none of its import good (commodity $j \neq k$). In the notation of section 1 we have $\omega_{kk} = 1$ and $\omega_{ik} = 0$ for $i \neq k$.

7 Also the unpublished paper by Yinggang Zhou, "A Tariff Reduces Unpredictability: A Solution to the Tariff Reduction Paradox" (2006), Cornell University, Ithaca, NY.

8 If one replaces x_{11}, x_{21}, and Y_1 in (2.2) below by the demands $x_{11}^{\nu}, x_{21}^{\nu}$ and income Y_1^{ν} of the νth individual in country 1, and then averages the two demands over the N individuals, one obtains exactly the same formulas, where x_{i1} and Y_1 are respectively replaced by the averages $\mathbf{x}_{i1} = \sum_{\nu=1}^{N} x_{i1}^{\nu}/N$ $(i = 1,2)$ and $\mathbf{Y}_1 = \sum_{\nu=1}^{N} Y_1^{\nu}/N$. This shows that the "parallel preferences" (with respect to the export good) can be aggregated over the whole economy, and thus generated by utility functions (2.1) expressed in terms of these averages.

Maximizing country 1's Wan-Zhou utility function U_1 in (2.1) subject to the budget constraint $p_{11}x_{11}+p_{21}x_{21}=Y_1$, we obtain the demand functions

$$x_{11}=\frac{Y_1}{p_{11}}-\frac{p_{21}}{p_{11}}+\frac{1}{4}\left(\frac{p_{21}}{p_{11}}\right)^2 \quad\text{and}\quad x_{21}=1-\frac{p_{21}}{4p_{11}}, \tag{2.2}$$

where Y_1 is country 1's disposable national income, equal to the sum of its domestic product, which is simply the domestic (and world) price p_1 of its export good (commodity 1), and its tariff revenues $(T_2-1)p_2z_{21}$, where p_2 is the world price of its import good (commodity 2) and $z_{21}=-z_{22}$ is the negative of its import of good 2. Since from (2.2) its demand for imports is independent of its income, its excess-demand function for its import good, expressed as a function of world prices, is simply

$$\hat{z}_{21}(p_1,p_2,D_1;\omega_1)=1-\frac{T_2p_2}{4p_1}. \tag{2.3}$$

Proceeding somewhat as in section 1, we combine (2.3) with the balanced-trade condition $p_1z_{11}+p_2z_{21}=0$ and the material-balance condition $z_{11}=-z_{12}$ to get

$$p_1z_{12}=p_2z_{21} \tag{2.4}$$

(expressed in world prices). Substituting (2.4) into (2.3) we obtain

$$z_{21}=1-\frac{T_2}{4}\frac{z_{12}}{z_{21}}, \quad\text{hence}\quad z_{21}^2=z_{21}-\frac{T_2}{4}z_{12},$$

whence country 1's tariff-inclusive offer function is

$$-z_{11}=z_{12}=4z_{21}(1-z_{21})/T_2. \tag{2.5}$$

By symmetry, a similar formula obtains for country 2:

$$-z_{22}=z_{21}=4z_{12}(1-z_{12})/T_1. \tag{2.6}$$

Wan and Zhou compute the optimal tariff factors to be $T_1=T_2=2.5$. Under free trade ($T_1=T_2=1$), there are three solutions to (2.5) and (2.6), namely

$$(z_{12},z_{21})=\left(\frac{5-\sqrt{5}}{8},\frac{5+\sqrt{5}}{8}\right),\ \left(\frac{3}{4},\frac{3}{4}\right),\ \left(\frac{5+\sqrt{5}}{8},\frac{5-\sqrt{5}}{8}\right), \tag{2.7}$$

which compute to

$$(z_{12},z_{21})\ =\ (.34549,\ .90451),\ (.75,\ .75)\ (.90451,\ .34549),$$

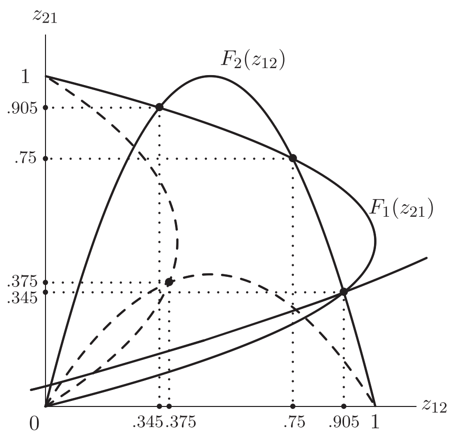

Figure 3: Optimal tariffs versus free trade under Wan-Zhou preferences

(see Figure 3).[9] As has been well known since the time of Marshall (1879), the first and third of these equilibria are dynamically stable, but the second is unstable.[10] However, under the optimal tariffs ($T_1 = T_2 = 5/2$), there is a single equilibrium

$$(z_{12}, z_{21}) = (3/8, 3/8) = (.375, .375), \tag{2.8}$$

which is stable (again see Figure 3).

Figure 3 is the Marshallian diagram (Marshall 1879, Figure 8; 1923, 353, Figure 20)—still current in the literature on international trade—which is similar to the "Edgeworth box" but with country 1's origin at the southeast corner $(z_{12}, z_{21}) = (1,0)$ where $z_{12} = -z_{11} = 1 - x_{11}$ and $z_{21} = x_{21}$, hence $(x_{11}, x_{21}) = (0,0)$; thus country 1's direction of preference is northwest. Likewise, country 2's origin is at the northwest corner (0,1), and its preference direction is southeast. It is clear from the diagram that the stable free-trade equilibrium (.905, .345) is inferior for country 1 to the optimal-tariff equilibrium (.375, .375), being dominated in both components. Of course, the latter

9 The possibility of multiple equilibrium in economics seems to have been first noticed by Mill ([1852], 1865: Vol. II, Book III, Ch. XVIII, §6), in a passage referred to by Marshall (1879, 15) who pointed out that Mill "has seen that under certain curcumstances there may be several different positions of equilibrium of trade"; but Marshall objected that "it appears to me that the special example which he has chosen does not illustrate the general problem in question." (Mill's analysis in his *Principles* built upon his own earlier 1844 work, from which he quoted.) Marshall proceeded to use his own diagrammatical analysis to illustrate the problem and demonstrate the possibility of multiple equilibrium.

10 For a phase diagram of this case, see Figure 12 in Chipman (1987, 936; 2008, 216).

is in turn also inferior for country 1 to (because dominated by) the *other* stable equilibrium (.345, .905).

The following table provides information about the welfare levels of country 1 and country 2 at the three stable equilibrium points shown in Figure 3, where $W_1(z_{12},z_{21}) = U_1(1 - x_{11},x_{21})$ and $W_2(z_{12},z_{21}) = U_2(x_{12},1 - x_{22})$.

Equilibrium	(z_{12},z_{21})	$W_1(z_{12},z_{21})$	$W_2(z_{12},z_{21})$
SE free trade	(.90451, .34549)	1.23872	2.63627
Johnson-Nash	(.375, .375)	1.84375	1.84375
NW free trade	(.34549, .90451)	2.63627	1.23872

Starting from the stable optimal-tariff equilibrium (.375, .375), which affords welfare levels of 1.84375 for both countries, and faced with the option of free trade, policy makers in country 1 are confronted with the risk of moving to the stable free-trade equilibrium (.90451, .34549) with an inferior welfare level of 1.23872, or moving to the other stable free-trade equilibrium (.34549, .90451) with a superior welfare level of 2.63627. If these policy-makers are minimaxers, they will choose to remain in their Johnson-Nash equilibrium (.375, .375) (see Figure 3). If they are Bayesians, and if they consider the two stable free-trade equlibria to be equally likely a priori, then since the expected utility is 1.937495 they will take a chance on free trade, provided the given numerical utility function correctly reflects their risk preferences. However, if instead one replaces $W_1(z_{12},z_{22})$ by its (natural) logarithm, the expected utility of free trade will be less than the utility of protection. And it is not as if the policy-makers could choose free trade one year and optimal tariffs the next; they must consider a succession of years under the same free-trade equilibrium versus a succession of years under the same optimal-tariff equilibrium.

An interesting policy proposal made by Wan and Zhou is for the countries to negotiate a common tariff that is less than that of the Cournot-Johnson-Nash equilibrium (and thus closer to free trade), but still large enough to rule out multiple equilibria.

All in all, this contribution may be considered as a fascinating and impressive achievement. It would provide an ideal basis for international negotiation of tariff reduction, provided the model fits the data precisely. But this question of its realism and relevance requires some discussion.

I start with the question of the basis for international trade, which has long been considered to be comparative advantage in some sense. That doctrine is based on the premise that the main basis for trade is relative differences in resources and factor endowments, as opposed to differences in tastes. Houthakker (1957), in examining the reasons for differing consumption patterns in different countries, came to the conclusion that these differences could be explained largely by differences in relative prices, i.e., the higher consumption of pork relative to beef in Germany in comparison with France could be explained by differences in their relative prices in the two countries in that period, rather than by differences in tastes.[11]

On the other hand, both the Ricardian and Heckscher-Ohlin explanations of trade patterns are based entirely on differences either in "climate" or in resource endowments. True, for the Heckscher-Ohlin theorem to hold, one must make stringent assumptions about preferences, namely that they are identical and homothetic within and between countries (Chipman 1987, 938; 2008, 218). These assumptions are undoubtedly unrealistic; indeed, homotheticity of preferences violates Engel's law. And there are convincing examples of changes in trade patterns that reflect differences in relative demands resulting from different income levels and Engel's law (Minabe 1966, 1205).

But in the Wan-Zhou model, one must have *both* different endowments *and* different preferences, acting in concert. Given the demand functions (2.2), at unitary relative prices we have $x_{11} = 1/4$ and $x_{21} = 3/4$ in country 1, and likewise $x_{12} = 3/4$ and $x_{22} = 1/4$ in country 2, indicating very different consumption patterns, with each country exhibiting a strong relative preference for its import good.[12]

And the model has each country producing only its export good and no import-competing goods. This appears to remove the main political force behind tariff protection. This latter assumption has been somewhat relaxed in Zhou (2006) (see footnote 7 above), but the combined force of the two assumptions remains.

In the literature on international trade, there has been much attention devoted to comparison of free trade and autarky, but very little until recently to comparison of free trade and restricted or tariff-encumbered trade. The importance of multiple equilibrium to the latter has now been demonstrated by Wan and Zhou. But remarkably, in the literature on general equilibrium there has been very little attention paid to it. The standard result that one finds in the literature is that of Arrow, Block, and Hurwicz (1959, 89–90), who proved that a *sufficient* condition for general exchange equilibrium to be unique is that commodities be gross substitutes in consumption

11 Of course, this assumes that either transport costs or trade impediments are present to cause these differences in relative prices, contrary to the model being presented here.

12 This contrasts with the situation under CES preferences (cf. Chipman 2010) in which multiple equilibrium requires each country to have a relative preference for its own export good. The Wan-Zhou result is also an exception to Mas-Colell's rule (1991, 285) that "uniqueness is more likely in models that generate a large volume of trade."

(Metzler 1945): that is, that a rise in the price of one commodity should lead to a rise in the excess demands for all the other commodities.[13] This result followed from the proof of uniqueness in Wald (1936, 652–6; 1961, 383–7), which I discussed in Chipman (1965, 725–7; 2008, 86–8) in relation to the application of Wald's condition to the case in which utility functions have the CES (constant elasticity of substitution) form $\left[\sum_{i=1}^{n} \alpha_i x_i^{1-1/\sigma}\right]^{1/(1-1/\sigma)}$, where $\sigma > 0$ is the constant elasticity of substitution. Wald's sufficient condition for uniqueness in this case reduces to $\sigma > 1$. This is a very stringent condition! But it is by no means necessary, since it turns out that in a two-agent mirror-image pure-exchange economy in which both agents have CES utility functions with $\sigma > \frac{1}{2}$, competitive equilibrium is necessarily unique; and even if $\sigma \leqq \frac{1}{2}$, multiple equilibrium is extremely rare, requiring consumers in each country to have a very high relative preference for its export good. I discuss this in detail in Chipman (2010, Theorem 2, 136).

There are many ways in which the model presented here could be improved. In particular, it would be desirable to allow for variable production as represented by supply ("Rybczynski") functions $y_{ij} = \hat{y}_{ij}(p_1, p_2; L_j, K_j)$ expressing the output of commodity i in country j as a function of the world prices and the endowments of labor L_j and capital K_j in country j; cf. Rybczynski (1955), Chipman (1987, 931–2; 2008, 203–6). Still a further relaxation of assumptions could be provided by allowing separate preferences for labor and capital in each country, as in Johnson (1959; 1960). Such a model would undoubtedly provide wide scope for multiple world equilibrium, yet still retain enough structure to give rise to interesting results.

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13 An alternative but hard-to-interpret sufficient condition has been proposed by Arrow and Hahn (1971, 211).

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