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Baingio Pinna, Jan Koenderink and Andrea van Doorn



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The Phenomenology of the Invisible: From Visual Syntax to "Shape from Shapes"

Baingio Pinna Department of Humanities and Social Sciences, University of Sassari (Italy)

> Jan Koenderink Laboratory of Experimental Psychology, University of Leuven (Belgium)

Andrea van Doorn Department of Experimental Psychology, Utrecht University (The Netherlands)

Résumé : Nous abordons ici l'étude de la phénoménologie des objets visuels à partir de comptes rendus verbaux, de réponses à des questions ou de descriptions spontanées, ainsi que d'associations libres. Nous demandons même parfois aux sujets de réaliser de simples croquis. Cet éventail de méthodes permet de sonder la structure profonde de la conscience visuelle. Celle-ci est avant tout révélée par ce qui n'est pas spontanément mentionné par des qualités accidentelles ou encore par des changements induits lors de variations ou ajouts mineurs. Nous avons trouvé une concordance remarquable parmi un grand nombre de participants. Ceci démonte que les comptes-rendus verbaux permettent d'explorer puissamment la structure de la conscience visuelle. Tout comme le langage, les « objets visuels simples » ont des racines profondes dans la conscience. Ces fondations « invisibles » servent à définir des figures simples, un « carré » par exemple, comme des objets prototypiques et uniques. La nature des « objets visuels » implique ainsi l'existence d'un « vaste domaine

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invisible ». Sans une telle prise en compte, la compréhension correcte de la nature de l'objet visuel n'est pas possible. Cette découverte devrait permettre de mener à une théorie de la syntaxe formelle des formes visuelles fondamentales.

Abstract: Here, we address the phenomenology of visual objects, using verbal reports, spontaneous descriptions, answers to questions and responses to suggestions, as well as free associations. We occasionally ask for simple sketches. Such methods allow probing of the deep structure of visual awareness. This is above all revealed by what is not spontaneously mentioned, or is only mentioned by way of incidental properties. It is also disclosed by changes induced by minor variations or additions. We find remarkable agreement over a large number of participants. Thus verbal reports are a powerful probe of visual structure. Similarly to language, "simple visual objects" have deep roots in subsidiary awareness. These "invisible" foundations serve to define simple figures, like a "square", as unique, prototypical objects. The nature of "visual objects" thus involves an extensive "invisible" realm. Without taking this into account, a proper understanding of the nature of the visual object is not possible. Finally, this should lead to a formal syntactical theory of basic visual shapes.

1 Introduction. From vision to description: what do we see?

The starting question preceding every visual phenomenological and psychophysical experiment is "what do you see?" This question is expected to reveal the simplest, immediate answer about what is perceived and, above all, the most "correct", "current" and "true" one. The terms written in quotation marks are usually considered to be isomorphic to the perceptual outcomes and can also be considered as synonyms on the basis of the linguistic reports made by the subjects. Briefly, what emerges through the spontaneous and immediate descriptions and responses is commonly accepted during visual phenomenology and psychophysical experiments as the closest result to actual perception. In fact, this kind of description is more pure, as it is less influenced by later unpredictable mediations, so it offers results that are free from thoughts, doubts or other uncontrolled top-down feedback that can corrupt or distort the primary vision process [Kanizsa 1979], [Koffka 1935], [Metzger 1963], [Titchener 1905b].

For example, the pattern illustrated in Fig. 1a (p. 131) was spontaneously described as "a square" (for a full detailed description about how these data were collected see the "General methods" section). This outcome is

complete and does not require anything else. Nobody casts doubts about its correctness and completeness, since this description is spontaneous, immediate, unanimous, and referred to the unique "thing" seen in Fig. 1a. The word "square" appears to describe fully and isomorphically the object illustrated. Furthermore, this word subsumes all the visual properties related to the object, so it appears to stand for the perceived thing. Therefore, the thing is its name and the name is the thing. The two components appear to be bijectively related.

The soundness, appropriateness and likelihood of the linguistic descriptions are tested and corroborated in everyday life, where people use the spoken language in a quasi-total agreement to talk and communicate about visual objects. It is rare to refute the innate trueness of the descriptions, but this may apply to life's situations that are more complicated than looking at a single square. In fact, this only occurs under special conditions, for example when the linguistic renditions diverge due to specific and antithetical interests of the interlocutors (e.g., in the case of attractiveness or beauty and other tertiary or expressive qualities).

In general, the common and shared use of linguistic descriptions in everyday life is the best demonstration of isomorphism, correctness and completeness in relation to the visual object. It can be stated that if the linguistic description had not had these properties, it would not have been used as it is. Furthermore, in sciences like physics, chemistry or even mathematics, the linguistic descriptions commonly used in everyday life are not completely denied. Instead, they continue to be incorporated within the corpus of specialist words of the science considered.

Given this basic assumption that is shared with epistemological foundations, in the next sections the meaning of the linguistic description in relation to what is perceived will be analyzed and reconsidered. This will not only be on the basis of what is seen and then reported, but mainly on the basis of what is not perceived at first sight, or better still, of what is invisible, implicit or simply omitted in the subjects' descriptions.

The main thesis of this work suggests that what is not perceived at first sight and what is invisible can tell us much more about vision than what is recognized. Briefly, the main purpose of this analysis is to demonstrate that phenomenologically a visual object is like a floating iceberg, with most of its mass below the surface of the water. In fact, it is of major interest that this work does not only address what can easily and immediately be observed "above the surface of the water", but also focuses mostly on the rest of it, the hidden, unseen part. The scientific consequences of this approach will first be deduced and then be considered within vision science.

2 General methods

Subjects Unless otherwise specified, different groups of 15 undergraduate students (both male and female) of linguistics, literature, human sciences, architecture and design, participated in each experiment described in the following sections. The subjects had some elementary knowledge of Gestalt psychology and visual illusions, but they were naive both to the phenomena studied and to the purpose of the experiments. They all had normal or corrected-to-normal vision.

Stimuli The stimuli were those illustrated in the next section. They were displayed on a 33 cm color CRT monitor (Sony GDM-F520 1600x1200 pixels, refresh rate 100 Hz), driven by a MacBook computer with an NVIDIA GeForce 8600M GT, in an ambient room illuminated by an Osram Daylight fluorescent light (250 lux, 5600° K). All the conditions were displayed in the fronto-parallel plane at a distance of 50 cm from the subject. A chin rest stabilized the head position of the subjects who used binocular vision.

Procedure A phenomenological free-report method was adopted. Different groups, each consisting of 15 subjects, only described one stimulus. This was to avoid interactions and contamination between stimuli. The descriptions reported throughout the paper use similar phrases and words to those provided by the spontaneous descriptions of at least 12 of the 15 subjects in each group, but were edited for brevity and representativeness. To provide a fair representation of the descriptions given by the subjects and to avoid contamination by the interpretations of the authors, the descriptions were judged by three independent graduate students of linguistics, naive with respect to the hypotheses. The descriptions are incorporated into the text to help the reader in the stream of argumentation. All the reports were quite spontaneous and the presentations were stopped when the subjects finished their reports. They viewed the stimuli during the time of the report. Observation time was unlimited.

During the experiment, subjects were allowed to add afterthoughts while watching and to see in different ways. The subjects could also receive suggestions and questions of any kind from the experimenter (see the following sections). All possible variations occurring during free exploration were written down by the experimenter. This was necessary to define the best conditions for the occurrence of emerging phenomena.

3 The binding description-perception

When Fig. 1b was shown to a new group of subjects, the most spontaneous and common answer given was "a rotated square". The rotation is a new emerging attribute not needed in Fig. 1a. By increasing the geometrical rotation of the square shown in Fig. 1 up to 45 degrees, a diamond, rhombus or lozenge were the terms used as a description (see Fig. 1c). Furthermore, here the term "square" was barely mentioned. Indeed, these new outcomes were apparently different, as they replaced the use of the term "square" by the subjects. The question arises as to why these synonyms only emerge in the 45 degree tilted square and not in the square shown in Fig. 1a? This question is not trivial. Phenomenologically, Fig. 1c is not univocally defined and needs more words to be described, mainly because it can be perceived in different, although very similar, ways. This is not the case of the square of Fig. 1a, whose unique and univocal nature is immediate and strong. This outcome further suggests that the linguistic descriptions trace, maybe like a carbon paper, the emergence of visual things from the stimulus pattern. They are reported and mentioned through their name and syntax relations as they emerge visually. The next stimulus, shown in Fig. 1d, was most often described as "a black square" (80%), or less often simply as "a square" (20%). Again a new attribute emerges, the achromatic color, definitely implicit in the previous conditions. According to these last remarks, since the color was not mentioned in the previous conditions, if the linguistic description traces the visual emergence of attributes we can draw the conclusion that color is not as visually marked in those conditions as it is in Fig. 1d.



FIGURE 1: Squares and their attributes

By presenting Fig. 1e to a new group of naive subjects, the most spontaneous and common answer was "a distorted or deformed square". Under these conditions, the new visual attribute required corresponds to the loss of regularity clearly traced in the description. However, when the next handmade pattern of Fig. 1f was shown, the main outcome was simple and "a square" was the only description given. Under a condition such as that shown in Fig. 1f, the loss of regularity was not mentioned, but it was discharged or discounted, although the geometrical conditions might require it. In contrast, Fig. 1g appears totally different, being described as "a square made of crunched up paper". Thus its material attributes are revealed, a detail not mentioned in the previous figures. Finally, the patterns of Figs. 1h-1j were perceived as "a bevelled square", "a broken square" and "a strange square" respectively.

At this stage, the following questions emerge spontaneously. Why are some attributes mentioned and required and others are not? Conversely, why are some attributes omitted in some conditions and reported in others? These questions are the main topic of the next section.

4 Logical and phenomenological issues about perceptions and descriptions

Given these preliminary immediate descriptions, some basic remarks are worth highlighting. All the previous figures, except Fig. 1c, are phenomenologically perceived as squares although only a few of them are squares in the true geometrical acceptation of the term. In contrast, although the pattern of Fig. 1c is a square, it is perceived as a new figure with a new name. The assignment of a new name can be considered as the main process of perceiving an emerging "thing" that from now on becomes a distinct visual object.

Moreover, going back to Fig. 1a, the observed figure, the square, stands alone: it is defined without any other attribute. It is a square and nothing else, i.e., a square *tout court*, a square *par excellence*, a true singularity and a one-word object [Koenderink 1999], [Koenderink, van Doorn *et al.* 1992], [Johnston & Passmore 1994], [Rogers & Cagenello 1989]. At this stage, a question arises spontaneously: why is this figure a singularity, a one-word description neither revealing nor accompanied by other emerging properties?

The first answer could be that other possible attributes are left out because they are less important than the ones reported. Why then, in the other conditions, such as those illustrated in Fig. 1, do other attributes emerge? Should "the most important" attribute be the only one that continues to be highlighted? How can the term "major" be defined? Furthermore, why has only one specific attribute been selected instead of another and, why is only one mentioned, while sometimes two instead of three are reported, or maybe four or even five? The notion of "importance" can open intricate issues placed beyond the very simple figures studied here. This might suggest a different and simpler alternative way.

A second answer could be found if we consider that other possible attributes are "invisible" or unnoticeable. Hence, they are not traced nor picked up by the linguistic description. As a matter of fact, phenomenologically, not all attributes appear to be placed at the same level (gradient) of perceptibility. Some can be totally invisible or only perceptible after attention is focused on them. Some others remain totally invisible in spite of clear suggestions. This is the case, for example, which is quite common in nature, where animals and plants jeopardize or save their lives by playing the visible-invisible antinomydilemma [Pinna & Reeves 2015]. The advantage of this second answer is that it remains within the phenomenological domain without introducing further notions that could be transferred to other more cognitive domains. However, its limit is based on the question: why are some attributes immediately in the foreground or partially visible, while others are totally invisible? Does this reflect a natural organization of visual attributes?

A third answer, related to the previous one, may be highlighted. This suggests that other attributes are not invisible, just omitted. Therefore, it results that this is mostly a linguistic issue and not a visual one. In other words, although further attributes of the square are linguistically implicit, they can still be clearly perceptible but are simply not mentioned. In addition, related to this point, further attributes may be omitted simply because they are considered as superfluous. Therefore, the word "square" contains most of the information needed to complete the description. As a consequence, a square does not need any further attribute to be defined as such.

A fourth possible answer maintains that this square stands alone since it is the best example, the model or the prototype of other squares. Being a prototype [Rosch 1973, 1975, 1977], it does not need any other phenomenological attributes or qualifying words.

This sample of possible answers, expressed in an acceptation regarded as the closest to the phenomenal one, can be considered as a basic step that is useful to reach higher and higher levels of abstraction, starting from the phenomenal base. Briefly, they can be considered like a *gradus ad Parnassum*, a step towards the highest complexity of the organization of visual attributes.

The previous answers can be better understood if they are reconsidered in the light of the following questions: why are some attributes more important than others or more visible than others? Why are some attributes superfluous and how can they be rebuilt on the basis of the main one-word object? Why are some other attributes implicit and why are these mentioned only, or mostly, under specific conditions? What is a prototype and how is it created? How many prototypes exist, one for each object? What is an object?

These questions are useful to reveal the status of the theoretical complexity of each answer in relation to others. In other words, they can test whether each answer is plausible and useful to solve its related issues, or whether it leaves open and triggers more questions than the ones it can answer.

These remarks are persuasive if we think about the way the square of Fig. 1a could be seen after its comparison with all the other figures. A new group of subjects barely reported the attributes previously omitted. Only when they were asked to make a direct comparison with the other figures, did they describe Fig. 1a as an un-rotated square, with the sides oriented along the vertical-horizontal directions, white, regular, intact, unblemished and made of a rigid and polished material.

Therefore, in spite of all the attributes introduced, the square was still mostly perceived as a one-word object. In fact, the most common outcomes were those given with the description: "this is a square" (Fig. 1a) and "this is a rotated square" (Fig. 1b). In more formal terms, the phenomenal salience of those attributes was considered neither necessary nor sufficient in the double implication (if and only if). Phenomenally, this result makes the visibility of those attributes superfluous and implicit. As such, they can be omitted. This is not the case for the other conditions illustrated in Fig. 1, where the previous attributes were judged necessary and sufficient to define and qualify the object, i.e., the square. Among all the possible attributes, only those specifically reported were considered necessary and sufficient, while the others were not. So, only the proper and specific attributes were mentioned and this was with the phenomenal meaning of being the specific qualifier of the square, the right and unique qualifier of something that needs to be qualified in that specific way.

If this is true, the following phenomenal consequences can be immediately deduced. Not all the word-attributes used and introduced by the subjects manifest the same phenomenal status. Some are one-word objects, like "square", while others are qualifiers or attributes. Going back to the descriptions of the patterns concerning Fig. 1, only "square" and the terms used for describing Fig. 1c are objects; all the others are attributes and qualifiers, adding a distinctive property or a specific characteristic to the object.

If vision is the domain and description is its co-domain, then description traces vision and both domain and co-domain could be ruled by a common and possibly isomorphic syntax. In a stronger version of this phrase, we could say that both *should* be ruled by a common and possibly isomorphic syntax. As a consequence, the related syntax should be based on the distinction between the object and its attributes within the domain and on the difference between the noun and the adjective within the co-domain. The noun, like the object, is the content word that serves as the subject or the object of a preposition. This is the way the square was perceived. Differently, the adjective is a word that expresses an attribute of something and, is then the word class that qualifies the noun. These are the ways the previous squares were defined and qualified.

Although at first sight this consequence could appear trivial or tautological, in the sense that one defines circularly the other and *vice versa*, this is not the case if we consider the fact that, by saying "square", we perceive and talk about one among many possible attributes. In fact, the shape is an attribute, which apparently and logically should share the same epistemological status as color, orientation, material and so on. However, the status is logically but not phenomenologically the same. As we stated above, under our conditions, the shape is clearly perceived as the figure *tout court*, while the other terms appear like its direct attributes, i.e., adjectives. More clearly, since the square does not appear as an attribute, but as the figure. This syllogistic argument is based on a strong phenomenological foundation, but it requires further deductions.

First, not all object properties are placed at the same hierarchical level. The shape can be considered as primary relative to the other attributes and, under our conditions, these are so primary they coincide with the object itself, the object *per se.* Second, a hierarchy is expected among the other attributes. Therefore, the attributes belonging to different domains are expected to appear with a different visual saliency and primacy, as if they were placed in a phenomenal position closer and closer or eventually farther from the foreground of the object. If this is true, the grammatical arrangement of adjectives might reflect the visual hierarchical organization of attributes within a visual object. Besides, if the spoken language reflects bijectively the visual language, then a similar syntax organization is expected at a phenomenal level. As a final consequence, the syntax, both in vision and in linguistics, represents the set of rules for forming admissible objects and, thus, sentences.

A further deduction suggests that the dilemma related to the previous questions, "Why does Fig. 1a represent a singularity, a one-word description neither revealing nor accompanied by other emerging properties?" and "why do other properties appear like their attributes or adjectives?", can be reconsidered in the following biological and evolutionary terms. This idea can be briefly summarized in terms of adaptive fitness, according to which the visual properties are organized as they are because they evolved as they did. Therefore, the syntax organization of visual attributes is biologically suitable and reflects the ability of organisms or species to survive and reproduce in the environment in which they find themselves. The consequence of this survival and reproduction is that organisms contribute genes to the next generation. Within this approach, all the previous answers based on importance, visibility, implicitness and proto-typicality assume a new and synergistic meaning.

These phenomenological remarks suggest the following further issue. Most of the patterns of Fig. 1 are perceived as squares with attributes, i.e., with something further. These "somethings" can also be perceived as happenings [Pinna 2010, 2011, 2012] changing the basic one-word condition of being only a square. For example, as demonstrated above, a rotated square is not just a square but a square with something else; with the attribute and the happening of being rotated. In other words, these happenings are like both attributes and verbs denoting action, occurrence, or state of existence and, at the same time, they are emerging attributes that one at a time introduce and define different and possibly independent dimensions (orientation, color, etc.) or domains of their being squares. All these attributes clearly assume different biological meanings and salience that can change among species.

A second important consequence is that omissions are important pieces of information, useful to understand the phenomenology of the object perception and its inner syntactic organization. More specifically, the more attributes are omitted (invisible attributes), the more likely the chance of the object being a one-word thing, or, the less the described attributes, the better the singularity of the perceived object. Briefly, the "phenomenal singularity" can be defined as the limit of the sequence of attributes that tends to zero, or as the object that does not require further attributes. It follows that the phenomenal singularity is the best instance for a specific one-word object.

The logical and phenomenological issues suggested in this section support the connection between vision and linguistics. However, these issues also suggest that vision and linguistics can be considered as a floating iceberg with a significant proportion of its mass below the surface and placed at a different depth elevation between successive contour lines (iso-lines). This is the main topic of the next section.

5 The gradient of visibility

Typically, every experimental and psychophysical task mostly focuses on specific attributes defined *a priori*. The free exploration and manipulation of a phenomenon is mainly restricted to an informal group of subjects that naively describe and define the visible and most prominent effect to be measured. These free outcomes are not noted down because they are considered to be preliminary and mainly aimed to define the experimental variables and their *a priori* description. However, during the preliminary phenomenological exploration, it is quite common that things hidden at first sight emerge and sometimes become even more evident than other attributes or "things" previously placed in the foreground. This suggests that together with what we perceive here and now as the most emerging and prominent piece, or tip of the visual "iceberg", it could be necessary to define what we can see within the multiplicity of possible visual outcomes.

Why is this important? Firstly, new hidden phenomena, possibly more interesting from a scientific point of view than the ones under investigation, could be discovered. Secondly, the hidden phenomena can reveal an inner self-organization of attributes and things, i.e., the inner structure within a visual object and the organization to create the visual world. By exploring only the contour line of the object's function, namely the curve connecting the components where the function has the same value, the nature of the function's gradient is lost. The vector of the gradient is always perpendicular to the contour lines. More generally, the perception and phenomenology of the thing to view may not be enough to predict its behavior and understand a target phenomenon in the full etymology of the term (from Ancient Greek $\varphi \alpha \nu \delta \mu \epsilon \nu \sigma \nu$ —*phainómenon*, "thing appearing to view"). What emerges could depend on what is submerged, underground or in the background. As Gestalt psychologists clearly demonstrated, to understand the emergence of a figure, it is also necessary to study the phenomenal nature of the background and their reciprocal relationship [Koffka 1935], [Rubin 1915, 1921], [Wertheimer 1922, 1923]. By comparing the squares of Fig. 1, this entails that what is submerged has to be brought to the surface, similarly to what is emerged.

If a visual object can be considered as being similar to an iceberg, with hidden and structured attributes supporting it, then the question "what is a visual object?", derived from the most famous of Koffka's questions "why do things look as they do?", becomes an intriguing starting issue and its results are useful to understand the full set of perceptual attributes that are strictly linked and embedded within the term "object". As discussed in the previous sections, this term can be phenomenally considered like a structured holder [Koffka 1935], [Metzger 1941, 1963, 1975a,b, 1982], [Palmer 1999], [Pomerantz & Kubovy 1986] containing an organized set of multiple properties, some of which are placed in the foreground and then made explicit, while others are implicit and yet more can become explicit or, on the contrary, implicit or invisible after a while [Pinna 2012]. Just on the basis of the question "what is a visual object?" the meaning of the term "object" is phenomenologically ambiguous, poorly defined at first sight and not completely immediate in its element components. Moreover, not all possible object properties define the phenomenology of the "object" with equal strength. As previously stated, not all the object properties and meanings pop up perceptually with the same salience. In fact, some are more prominent than others [Rosch 1977]. This is the gradient of visibility. It is interesting that within this phenomenological notion some attributes, as they emerge from the implicit background, could be very different from what is expected and taken for granted, as it will be demonstrated in the next set of conditions.

Going back to Fig.1a, the interesting implicit outcome of the previous descriptions was that none of the subjects (and this might also be the case for the readers) spontaneously reported the color of the square. The color seems to be taken for granted and then left, implicit. When a new group of subjects was asked: what color is the square? The most popular answer obtained was "white". In fact, just asking the question was sufficient to show different phenomenal alternatives not previously considered, so, now the square was also described as black or transparent. When these possible results were highlighted, or directly suggested by the experimenter, the outcome revealed mainly that "the square is white". Nevertheless, these possible results became visible and accepted as such with a visible salience, although less strong than the main one.

It also became obvious that they can all be seen, compete to be perceived and especially to be the unique recognized result. In greater detail, by noticing the phenomenal plausibility of these outcomes, the competition among them becomes visible and, at the same time, it becomes visible that they were unnoticed at first sight. The color of the square, which appeared composed by the color of the figure's inner surface, or by the color of its boundary contours also popped out from invisibility. However, it was mostly the former that assumed the status of color [Pinna & Ehrenstein 2013], [Pinna & Deiana 2015]. A further implicit or invisible attribute was the color of the background, which was never reported during the experiment or during these reorganizations, revisions or shaking ups. In fact, when it was asked: "what is the color around the square?", the answer of the subjects was "white", "empty" or both. These outcomes demonstrate that, by asking new and unexpected questions, phenomenal organizations start to emerge.

The shape and color organization just described, can also be approached in a totally different way; apparently far away from vision, but supporting the previous results, i.e., by asking the subject to "draw a square". All of us could do something like taking a black pastel and drawing a square shape as illustrated in Fig. 1a. Among 100 subjects interviewed, 99 of them drew the square as described. One of them reproduced the square shown in Fig. 1c, rotated through 45 degrees. Instead, upon asking "draw a rotated square" and "draw a black square", the outcomes achieved were similar to those of Figs. 1b and 1d. The task "draw a white square" produced the same results as the task "draw a square". Finally, by reversing the background, all the results were symmetrically reversed. In conclusion, these results suggested that the reproduction task can be considered as a further way to perceive an object, to pop out salient attributes and to hide others. At the same time, going from description back to vision is a way to demonstrate the strong connection between them.

In summary, as well as what emerges through the phenomenal description, there is also its complement, i.e., what is not described, implicit or taken for granted. By asking further questions and then delving into the hidden part of the visual object, unexpected issues can emerge. Therefore, what is taken for granted could cause surprise by revealing new components that influenced the final result. The question is then: why is something taken for granted? Phenomenally this may happen because it does not come to the fore, but takes second place with respect to some other attributes that are perceptually stronger and more salient. To be more precise, this is because the object attributes are hierarchically organized according to their strength, creating the visibility gradient. This implies that to understand what a visual object is, it is necessary to understand, not only what is perceived at first sight and with the maximum strength, but also what is seen as secondary, in the background and even what is totally invisible.

The notion of the visibility gradient is demonstrated here in a phenomenological acceptation prior to and epistemologically independent from Helmholtz's likelihood principle [Helmholtz 1867], according to which the sensory input is organized into the most probable distal object or event consistent with the sensory data (the proximal stimulus). As a consequence, the visibility gradient is also prior to the Bayesian approach based on statistical inference [Mamassian, Landy *et al.* 2002], assuming perception as a visual inference problem [Helmholtz 1867] according to which the posterior probability is proportional to the product of prior probability and likelihood.

In the next sections a further step will be taken by showing also that the shape, which is the attribute mostly taken for granted and identified with the object *tout court*, is not as it appears to be but can be decomposed in a multiplicity of different and sometimes antinomic hidden subcomponents.

6 The visibility problem: a further gradus ad Parnassum

In the previous section we stated that to understand the meaning of a visual object it is necessary to study the phenomenal nature of the invisible and hidden components. This statement will now be tested under conditions placed a step forward with respect to the one illustrated in Fig. 1a, although starting from it. The main purpose is to study the square and its invisible components.

Together with the circle, the square might be considered as one of the most fundamental of all shapes. Geometrically and phenomenally, the square is entirely based on the balanced proportion between its edges/sides and angles/vertices. A square is a flat shape with equal sides and equal right angles. A square can easily fit the definition of a rectangle with all angles at 90° and a rhombus with all sides of equal length. Moreover, given its special geometrical attributes, the square is used as a reference for all the other shapes. The square represents the basic shape used to measure any kind of object, shape or space. Every shape, either regular or irregular, is measured in squares (m^2) or in the 3-D version of the square cubes (m^3) . The square is the unit and, more generally, the "brick" of all other shapes, see [Pinna 2011].

By comparing Fig. 1a and 1c, the meaning of the square appears to be related to the orientation of the sides along the main directions of space. This entails that the phenomenal squareness cannot only be considered as the absolute proportion between sides and angles but it should also be seen in terms of dynamic prominence of the sides upon the angles. This is probably related to the roles assumed by the sides of the square in relation to the lower horizontal side (the base of the square) that elicits phenomenal stability and flatness. In addition, the word "square" appears to be related to visual attributes like straight, level, parallel, steady, equal, smooth, unchanging, plane, constant, stable, uniform and flat. The name itself, i.e., "base", demonstrates its role as the foundation of the square, the component where the square rests. The relevance and high visibility of the base in assigning meaning to the whole shape is supported by the fact that the upper horizontal side does not have any particular name and that the two vertical sides appear indiscriminately as the height of the square, reinforcing the unique and singular role of the base.

The comparisons between the square of Fig. 1a and the diamond of Fig. 1c can be reconsidered in terms of a comparison between squareness and diamondness, which makes visible the inner properties of the square and consequently its meaning. This also highlights what part of a square is below the immediately visible surface [Mach 1914], [Rock 1973], [Schumann 1900]. By rotating the square by 45 degrees, it appears as a diamond, namely a new shape different from the square. The two figures are similar and opposite at the same time. While the square is perceived as flat and stable, the diamond is seen as sharp and unstable. The two apparent shapes manifest opposite inner meanings. If this is true, a counterintuitive and unexpected property of the square emerges through this in depth analysis. In fact, both the square and the diamond show two main components, sides and angles, but in their appearance they are not balanced, as the sides are asymmetrically prominent in the square while the angles dominate in the diamond. They are antinomic shapes, showing the paradoxical inner dynamics of the basic components of what we call "square" that is apparently linear and right without antinomies and inner conflicts.

This implies that the emergence of the sides within the square, i.e., the "sidedness" shape attribute, mostly reveals the squareness. In contrast, the emergence of the angles within the diamond, i.e., the "pointedness", manifests the diamondness, in antithesis with the squareness. The perpendicular orientation of the same shape along the main direction of the space only enhances one of the two opposite sub-attributes of the shape, i.e., the sidedness or the pointedness.

Briefly, the square of Fig. 1a that appeared as a singularity, a one-word object, revealed its inner components through a deeper phenomenological investigation that changed its appearance and manifested the inner asymmetry between two more elementary attributes: sidedness and pointedness. This is the problem of visibility that can hide and even make invisible an antinomic dynamic such as the one described here. Moreover, if the apparent regularity, saliency and singularity of the square reveal an inner dynamics between opposite more elementary shape components, what happens if different inner components of the square are accentuated and brought to visibility?

The next conditions are aimed to support the previous analysis. By changing the equilibrium between the two attributes and accentuating the sidedness or pointedness, the whole meaning of the figure is expected to change accordingly. Therefore, although the geometry of the figure is kept constant, the square is expected to appear as a diamond or *vice versa*.

Visibilia ex invisibilibus: the visible from the invisible

Pinna & Sirigu proposed a principle of grouping, called the accentuation principle, stating that, all else being equal, elements tend to group in the same oriented direction of the discontinuous element placed within a whole set of continuous or homogeneous components [Pinna & Sirigu 2011]. The discontinuous element behaves similarly to an accent, or a visual emphasis, within the multiplicity of phenomenal components.

The role of the accentuation principle in shape perception can also be demonstrated by alternately highlighting sidedness or pointedness of the same geometrical figure, as shown in Fig.2. Under these conditions, despite the vertical and horizontal configurable orientation effects (namely, the perception of local spatial orientation determined by global spatial directional structure, according to which the elements of Fig. 2 should all appear as diamonds), [Attneave 1954], [Attneave & Frost 1969], [Palmer 1980, 1999] alternate rows of figures are perceived as rotated squares or as diamonds on the basis of the position of the small circle (either empty or filled) placed near (either inside or outside) the angles or the sides of each figure [Pinna 2010, 2011], [Pinna & Albertazzi 2011]. By perceiving rotated squares instead of diamonds, the subjects are discerning the same geometrical figure, but different phenomenal figures, i.e., squares with their sidedness and diamonds with their pointedness. Geometrically they are exactly the same since the 45 degree rotated square coincides with the diamond. Phenomenally they are different since the two figures appear different, are given different names and have opposite inner shape attributes.

The sidedness and pointedness shape attributes could be phenomenally considered as shapes within a shape. This wording is not incorrect if we compare the conditions where the small circles are placed, namely in different positions inside and outside of the square, but still close to sides or angles. Indeed, this choice of words can be further confirmed by the new shapes triggered by the small circle placed in different random spatial positions inside or outside a geometrical diamond. In reality, in Fig. 3 several instances out of all the infinite possible variations are illustrated. In this picture, the possible shapes within a shape emerge only after careful observation. Under these conditions, the accentuation reveals the multiplicity of hidden and invisible shapes within the geometrical diamond. This entails that underneath the surface of the visible square there are many other possible shapes, literally hidden and invisible. Therefore, what is visible, the square, diamond or something in between, also depends on these invisible shapes.

The role and place of the invisible over visible can be better appreciated when irregular figures are used instead of regular ones. In Fig. 4a, the same geometrical figures, accentuated by black circles placed in different spatial positions within each shape, are perceived as irregular quadrangular shapes that appear different from one another. In Fig. 4b, a control without small



FIGURE 2: Rotated squares or diamonds?



FIGURE 3: Accentuation reveals the multiplicity of hidden and invisible shapes within the geometrical diamond

circles is illustrated. However, although all the irregular shapes of the control appear mostly equal, they can be perceived at the same time as different. As the different shapes emerged from invisibility by becoming visible, for example after the perception of Fig. 4a, they also contributed to the perception of each shape in the control condition. Therefore, the shape seen and selected at the end, can be considered as the final product deriving from the interaction among the multiplicity of inner invisible or hidden shapes, first perceived in Fig. 4a and then transposed to Fig. 4b.



FIGURE 4: Irregular quadrangular shapes different one from the other (a). Controls (b and c)

These results are corroborated by others obtained with a new group of subjects that first examined the control (Fig. 4b) and after this were confronted with the condition illustrated in Fig. 4a. Then the elements of the control were perceived as all being the same, without any shape difference among them.

The quadrangles of Fig. 4a not only are they different shapes, but also they appear as shapes pointing in different directions, oriented differently and each moving in the direction indicated by the accentuation. This outcome can be better appreciated in Fig. 4c, where these phenomenological attributes of the shape due to the inner shapes, represent a true syntactic organization similar to the one described in Fig. 1. However, differently from Fig. 1, under these conditions the syntax only occurs within the shape domain and among inner shapes attributes. The syntax organization now emerges through the pushpull hidden interaction among the multitude of shapes playing within the same geometrical quadrangle.

The syntax organization of inner hidden shapes creating a visible whole shape is analyzed in more depth in the next section.

7 Shape from shapes

The shape resulting from the dynamics of inner shapes can be better appreciated by making the figure even more irregular, adding alternate concavities and convexities as illustrated in Fig. 5. In this figure, each position of the small circle is crucial to reveal, for example, a specific angular object or face, a sharp thing oriented, moving fast or slowly in one specific direction instead of another, or with different tertiary and living attributes.

The shape from shapes organization is more salient in Fig. 6. By comparing the controls illustrated in the first row of Fig. 6, they appear like two identical juxtaposed wiggly figures. The repetition and juxtaposition of the same figure and the configuration-orientation effect [Attneave 1968], [Palmer 1980]



FIGURE 5: Shape from shapes due to the accentuation principle

together determine the specific orientation of the figures that appears polarized into one or the opposite horizontal direction. Overall, both objects could be perceived as two living humanoid organisms moving in opposite directions, from left to right or from right to left.

In the second row of Fig. 6, the same two figures with small circles inside them, are mostly perceived as two different humanoid organisms, male and female (as spontaneously reported by naive subjects), facing each other and walking in opposite directions [Heider & Simmel 1944]. In spite of the reiteration, juxtaposition and the configured orientation playing on the same figure, the accentuation principle reorganizes the two geometrically equal objects and turns them into two dissimilar ones.



FIGURE 6: Examples of organic-shape emergence

The third row increasingly demonstrates the strong effect due to the accentuation imparted by the filled circle. Three different organisms walking in different directions (left, central to frontal, right) are now perceived. These results might suggest that they depend on the similarity of the filled circle to an eye. As a whole, this presupposes that past experience and higher cognitive processes are likely to be involved in the organic segmentation, weakening the organic early segmentation highlighted by the accentuation principle. The counter-examples for this argument are illustrated in Fig. 7 where the small shape used to induce accentuation and its position weakens this counterargument in favor of the organic shape due to the accentuation of one of the many inner invisible shapes.

Going back to the first row of Fig. 6, it is evident that amoeboid shapes can be perceived at the same time and very easily appear as made up of a multitude of possible inner shapes that are only accentuated by the small circle. In reality, the accentuation can now be made through the focus of attention. Under these circumstances, attention operates within a set of multiple and reversible possible inner local shapes that were previously invisible, while they can now be perceived and switched on.

Fig. 7 demonstrates that by replacing the circles of the second row of Fig. 6 with small squares, the resulting effects of the accentuation previously described do not change at all. In fact, the square has a different shape from the eye, therefore, there is no need to invoke the role of past experience. However, it can be counter-argued that, although the square is dissimilar to the eye, it can appear as such in the two walking organisms perceived in the first two rows of Fig. 6. This is indeed what can be ascertained.

To confute this further past experience objection, the last two conditions shown in Fig. 7 can be considered. The circle is now placed outside the wiggly shape, thus further weakening the cognitive relation to the eye. Through this change, the outside shape accentuates and polarizes the direction of the two objects that appear to be moving respectively left and right, and, more importantly, have a different shape.

Differently from what was suggested by the eye argument, on the basis of the accentuation principle it can be stated that it is not the perception of the eye to outline that of the shape as a kind of humanoid organism. On the contrary, it is the organic organization due to accentuation that favors either the perception of the circle and of the square as an eye placed on something that has been previously segregated as a face or a head due to the circle and the square. In other words, the circle and the square accentuate the inner local shape of the head that feeds back to the circle and the square to be perceived as an eye. This reversed argument can explain the fact that the square could be perceived as an eye, although no-one has ever experienced a square eye.

In summary, every possible shape, even the most stable and regular one like a square, when observed more closely appears to be composed of multiple local shapes and is then unstable and reversible. Each figure *per se* manifests



FIGURE 7: Counter-arguments against the role played by past experience

a particular shape and a specific set of visual meanings that are useful to define and change the whole final organization. This suggests that the spatial locations of the circle within a figure and within irregular shapes activate a competition among inner local shapes. This makes the overall emerging hierarchical organization and the roles of its components ephemeral and reversible. Therefore, through different sights, for example the protrusions and convexities of the previous figures, objects could be perceived alternately as the head of some kind of living being or as its limbs.

The organic segmentation is irreversible, more stable and easily controlled by accentuating one protrusion against the others. More saliently, it is accentuated and more stable and the final shape appears unique. This is the case of the square shown in Fig. 1a, accentuated in its sidedness by the horizontal or vertical directions of space and synergistically enhanced if one or, better, more circles are introduced along the sides.

More generally, this implies that a shape could be considered similar to some kind of holder containing a multiplicity of inner sub-shapes. These are invisible under most circumstances and organize reciprocally to create the whole emerging meaningful percepts that we call for example "square", "diamond", and so on. Some of the sub-shapes clearly emerge in the foreground, while others are confined in limits or placed below the threshold of visibility and last, but not least, certain remain in the hidden part of the iceberg of the gradient of visibility. However, although invisible, these shapes continue to play implicit roles and influence the emergent perceived shape. Finally and more importantly, the previous results suggest that all subshapes visible with different saliency are not randomly related, but appear organized according to a syntax structure. This determines the emergence, for instance, of an organic Gestalt showing attributes of "headedness" and "bodiness". These attributes clearly depend on a syntax organization, in the sense that without a true syntax they could not be formed. If syntax is the arrangement of words and phrases to create well-formed sentences in a language, the visual syntax of shape from shapes could be considered as the arrangement of these inner invisible local shapes that are crucial to the creation of the well-formed visible shape of the perceived world.

8 Conclusions

In the previous sections, the meaning of linguistic descriptions in relation to what is perceived has been analyzed and reconsidered, not only on the basis of what emerges perceptually and linguistically, but also on the basis of what is not immediately perceived; of what is hidden, invisible, implicit or simply omitted from the subjects' reports.

It was assumed that the invisible attributes and visual components elicited a better understanding of vision than what is actually perceived. Through several effects, it was demonstrated that a visual object might be considered phenomenally similar to a floating iceberg with the most significant proportion of its mass below the water's surface.

We demonstrated that next to what emerges through the phenomenal description, there is also what is not described, what is implicit or taken for granted. Through a phenomenological investigation of the invisible attributes and shapes, unexpected components emerged and these were demonstrated to be necessary to understand more deeply the visual nature of the perceived object. This is the case of several sub-shape components, like sidedness and pointedness, which are related to the sides and angles in the case of squares.

Through the phenomenology of the invisible, the binding of the object attributes appears to be organized according to visual syntax, thus creating a gradient of visibility. This implies that to understand what a visual object is, it is necessary to understand not only what is perceived at first sight and with maximum strength, but also what is seen as secondary, in the background and even what is totally invisible.

It is worth highlighting that the gradient of visibility could be understood only on the basis of a syntactical organization, which determines unequivocally the position of each visual attribute along the gradient of visibility. As linguistic syntax is the arrangement of words and phrases to create wellformed sentences, visual syntax can be considered as the arrangement of inner attributes and shape components to create well-formed visible objects with attributes placed in different hierarchical positions. This approach suggests answers to complex issues like the gradient of visibility and the complexity of the organization of visual attributes, but at the same time it opens new, complex, issues. One of these issues confronts the problem that if a shape emerges from inner sub-shapes, how and where do the inner local shapes form [Lewin 1933]. This is not a trivial, but a real issue, aimed to avoid postponing indefinitely the problem of shape formation. A possible answer could be based on the grouping principles that precede shape formation. To perceive the pointedness, two perpendicular sides should first group together to create an angle. If this is true, then visual perception could be considered as a set of different kinds and levels of organization, where one is based and emerges on the previous one and where all together determine increasing nested sets of more and more complex organizations.

The main deduction derived by the previous outcomes and theoretical notions is related to the binding problem and, in greater detail, to the fact of syntax organization of visual attributes in a well-formed visual object and in the formation of shape from shapes. This latter could be considered a potential candidate to explain how attributes belonging to different domains and processed in parallel neural pathways, or different sub-components belonging to the same domain (as in the case of the shape), are bound to create a wellformed visual singularity and wholeness.

A final and more general remark is related to the fact that evolution did not prepare our visual systems for reduced cue stimuli (like flat squares and so on), but rather, for natural scenes. From our results, it is not clear whether our principles and deductions apply when cues to the real world are complete and the stimulus is therefore "adequate" for vision. Our hypothesis, which is now an open issue, is that the accentuation could be reasonably extended from simple drawings like squares to biological conditions, where the appearance and the evolutionary success of a living organism depend on the accentuation of single parts of the body aimed to hide, show, deceive, attract, repel other organisms of the same or of different species. This is for example the case of make-up in fashion, likely aimed to accentuate part components (eyes, lips and so on) to enhance attractiveness, to change the shape and the meaning, to highlight inner shapes within the shape, to modify the gradient of visibility, to show something and hide something else at the same time. This promising extension requires further work.

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