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Spatial Elements in Visual Awareness. Challenges for an Intrinsic "Geometry" of the Visible

Liliana Albertazzi
CIMeC (Center for Mind/Brain Sciences) and Department of Humanities, University of Trento (Italy)

Résumé : Un enjeu majeur pour les recherches actuelles dans les sciences de la vision consiste à mettre au point une approche dépendante de l'observateur – une science des apparences visuelles située au-delà de leur véridicité. L'espace dont nous faisons l'expérience subjective est en réalité hautement « illusoire », et les éléments de base du champ visuel sont des structures qualitatives, contextuelles et relationnelles, et non des indices métriques et dépendants du stimulus. Sur la base de nombreux résultats disponibles dans la littérature traitant de la manière dont fonctionnent les divers constituants de l'espace (formes, surfaces, etc.), l'article décrit les éléments qualitatifs de base d'un tel espace et pose la question de la « géométrie » des apparences visuelles. Il formule enfin un ensemble de propositions pour d'éventuelles recherches poursuivant l'examen de l'espace visuel d'un point de vue expérimental.

Abstract: A challenge for current vision science is to develop an observer-dependent science—a science of visual appearances beyond veridicalism. The space that we subjectively experience in vision is, in fact, highly "illusory", and the primitives of the visual field are qualitative, contextual, and relational patterns rather than metric or stimuli-dependent cues. Drawing on the extensive evidence that the experimental literature on visual space perception offers on the behavior of the various constituents of that space such as shapes and surfaces, the paper describes the qualitative primitives of such a space and addresses the question of the intrinsic "geometry" of visual appearances. The paper also makes suggestions for potential future developments of examining visual space from an experimental viewpoint.

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1 Introduction

The visual objects in conventional psychophysical science are mainly understood and represented in terms of Euclidean geometry, starting from primitives defined in terms of points, lines, and surfaces. It is generally assumed that there are Euclidean surfaces in the visual field, and that shapes have geometrical properties replicable in computational terms [Marr 1982]. This is generally seen to be an unproblematic issue in the current computational theory of vision. However, doubts about the Euclidean nature of visual spaces and visual objects have been raised by a broad array of so-called geometric illusions: straight lines are seen as curved [Hering 1861], [Zöllner 1860] or slightly tilted [Morgan & Moulden 1986], [Münsterberg 1897], arcs are perceived as flattened [Müller-Lyer 1889], [Tolanski 1964], vertical segments are overestimated with regard to horizontal ones [Chapanis & Mankin 1967], [Oppel 1854-1855], circles and squares are perceived with apparent size [Ebbinghaus 1902], [Coren & Enns 1993], as are areas [Wundt 1898], lines are perceived with apparent rather than actual length [Diaz & Delay 1990, 1992], [Müller-Lyer 1889], [Ponzo 1912], cubes, prisms, cones and conical shells undergo perspectival reversal [Benussi 1925], [Deręgowski 2014], [Kopfermann 1930], [Necker 1832], [Thiéry 1895], the contours of triangles are vividly present in the total absence of any stimulus indicating such lines [Kanizsa 1979], arrays of triangles can spontaneously "point" as a group in a selected direction [Attneave 1968], [Palmer & Bucher 1982], objects appear in such a manner as to seem physically "impossible" [Penrose & Penrose 1958], while objects which are impossible from a geometric viewpoint are perceived as physically plausible [Huffman 1971], [Kulpa 1987], and geometrically complete drawn parallelograms appear to be phenomenally incomplete because of 3-D interpretation of the orientation of the drawing with respect to the picture plane [Massironi & Bruno 1997]. Apparently, Wonderland is not merely a space of literary imagination! To put it in more explicit terms, the space we *see in* is intrinsically imaginative (Goethe spoke in terms of an exact sensory imagination). The main qualitative "skeletal" [Metzger 1941] characteristics of this space are convexities (the space outside) and concavities (the space inside) [Massironi 1998], and between these two characteristics, there may be an entire phenomenology of spatial variations. Perkins & Cooper, when testing the "errors" committed by subjects in the perception of surfaces inclinations, described the human perceiver as a "sloppy geometer" [Perkins & Cooper 1980]. However, this is an unfair characterization, because what the subject perceives is something intrinsically qualitative and should not be compared to a Euclidean metric viewpoint or indeed even more complex spaces defined by Cayley/Klein geometries [Koenderink, Albertazzi *et al.* 2010]. The qualitatively different perception among the observers implies that the spatial percepts of the general population may well be highly variable between observers. In other words, different individuals see in (partially) different ways; however, the differences are identifiable [Foley 1965]. These results gainsay the assumption of psychophysical and neurophysiological research based on the concept of an ideal observer. Seeing is a *process* where space, more than being a static sequence of planes, is bodily situated in a frame of basic egocentric directions—such as right/left, above/below, in front of/behind—with a biological normative aspect (above is "better" than below, in front is "better" than behind, etc.).

Examples of the "odd" behavior of shapes abound in the experimental literature on space perception [Gregory 2009], [Shapiro & Todorovic in press], [Wackermann 2010], as to so-called pictorial space illusions, see [Dunning 1991]. Those experiments make manifest the role of the perceiver in *shaping* appearances in visual awareness, so that the boundary between what is usually supposed to be real and what is a product of imagination is very labile. Perhaps the most insightful explanation of what occurs in seeing is still the Aristotelian idea of *phantasia*, i.e., the mind's organization of qualities in subjective space and time [Aristotle 1986]. What occurs in seeing, in fact, is a question more of a psychic organization of qualitative contents in a multidimensional simultaneity than of a set of relations and operations following the rules of formal geometry. A good example of how shapes behave in awareness is Michotte's parallelepiped, an anamorphic line drawing of a parallelepiped on a sheet of paper which is viewed with one eye at a high slant [Albertazzi 2011], [Massironi & Savardi 1991], [Michotte 1948], [Vishwanath 2014]. What is seen is a truncated pyramid with a transparent strongly voluminous appearance. Michotte observed that the visual effect of solidity (the plastic effect) was just like stereopsis in that the wire frame object appears to stick out of the paper and to be so real as to be graspable. To be noted is that this 3D effect takes place in the continuity of seeing, which suggests that 3D must be an ecologically dominant feature. Another extraordinary effect is the "Corrugated Mondrian" effect, where different patches of grey (of identical stimuli) appear to be of different colors because of the (again, "illusory") perceived light in the phenomenal space [Adelson 1993].

The visual field in seeing appears as a twofold extendedness [*Extensität*] (the term in [Brentano 1995]) filled with qualities at a certain location, where also the voluminous appearance of shapes is perceived more or less as such, more or less *remote* from the perceiver him/herself.[1] From the observer's viewpoint, in fact, depth is also subjective [Koenderink 2013], [Koenderink, van Doorn *et al.* 2011], [Vishwanath 2010]. For all these reasons, the phenomenal space of awareness has important similarities with pictorial space: one may perhaps say that they are different degrees of reality of appearances [Mausfeld 2013], [Metzger 1941].

A major source of surprise is the discovery that we are not the only living beings that perceive "illusions". So-called geometrical illusions such as the

1. The German term *Extensität* is translated with extendedness because it refers to the qualitative nature of perceptual space: it does not refer to the classical mathematical concept of extension [*Ausdehnung*].

Ponzo, the Müller-Lyer and the horizontal-vertical illusion, seem to be perceived by species as diverse as apes (chimpanzees [Fujita 1997]), monkeys, e.g., [Barbet & Fagot 2002], [Bayne & Davis 1983], [Suganuma, Filgueiras Pessoa *et al.* 2007], [Tudusciuc & Nieder 2010], see also [Fujita 1996], ungulates, e.g., horses, [Timney & Keil 1996], birds (domestic chickens, [Winslow 1933], [Nakamura, Watanabe *et al.* 2014], [Rosa Salva, Rugani *et al.* 2013]), ring doves [Warden & Baar 1929], pigeons [Fujita, Blough *et al.* 1991, 1993], [Nakamura, Fujita *et al.* 2006], [Nakamura, Watanabe *et al.* 2008, 2009], grey parrots [Pepperberg, Vicinay *et al.* 2008], and fish [Sovrano, Albertazzi *et al.* 2015]. The evidence of similar perceptual scaffoldings in species so different suggests either the presence of homologous traits inherited from a common ancestor, or the rediscovery of analogous frames by otherwise very different species. The space that we see in, besides being anisotropic, is essentially a space for the action of "invisible" forces and vectors [Leyton 1999], where the visual shapes change according to the configuration, the context, and the temporal extension in which they momentarily appear or remain visible. The δ movement [Kenkel 1913], [Harrower 1930] is an excellent example of what occurs in the unfolding of the acts of presentation. The phenomenon is visible when, in the tachistoscopic exposure of a cardboard disc for ca 100 ms, there is an expanding movement of the internal part of the figure, i.e., one sees an appearance that unfolds together with a *distortion of the margins* even in brief processes. The δ movement is also a phenomenal expression of the *arousal* and *disappearance* of visual forms, for example when we light up a dark room and the objects seem to expand. The same happens with lines [Eagleman & Sejnowski 2007], [Fröhlich 1923], [Harrower 1930], [Hubbard 2014], at least in the case of European subjects.

Spatial appearances are also cross-modally perceived by the general population (i.e., men and women of different expertise, and non-synaethetes) as naturally associated with color, sound, taste, touch and olfaction.

Experimental data show that circles are associated with red, triangles with yellow, acute angles with the range of red-yellow, and obtuse angles with the range of blue-green [Albertazzi, Da Pos *et al.* 2012]. This holds also for biological shapes, in which the pattern "round" and "elongated" appear to be matched respectively by reddish colors and bluish colors [Albertazzi, Malfatti *et al.* 2015]. These collections of data raise the issue of a "geometry" of *the purely visible*, i.e., of the formal representation and modeling of what a subject perceives in actual seeing, where s/he is *directly* presented with appearances, not with physical objects. The expression "pure visibility [*Sichtbarkeit*]" is reminiscent of Reid's idea of space [Reid 2000], but more specifically refers to Fiedler's theory of pure visibility [Fiedler 1991] and the idea that the mind produces the building block materials of appearances. I do not endorse Fiedler's separation between vision and the other senses, apart from touch, but I largely agree with his view of what *seeing* is (see also [Hildebrand 1893]). More than being illusory, the space of vision requires primitives and laws of organization different from the physical ones. The primitives of a geometry of the visible, in fact, more often than not are imbued with connotative dimensions making them *meaningful* for the perceiver.

In considering the main characteristics of a geometry of visual space, one has to be aware of a twofold possible approach: one addresses the *genesis* of visual shape (dynamically unfolding in the microgenesis of a presentation); the other addresses the *appearances as such* and their primitives.

In light of the foregoing considerations, I shall proceed as follows in this paper. I shall briefly list some issues concerning what can be viewed as a sort of radical change of perspective in vision science [Albertazzi 2013a], thereby dealing with the issue of the primitives of visual space. In other words, I shall conduct the *pars destruens* of a viewpoint well established in vision science. Hopefully, this will promote the development of a *pars construens* of a "geometry" of visual space, which necessarily requires collaborative work among experimental phenomenologists, philosophers, and mathematicians. Given the exploratory nature of the present inquiry, it is necessarily merely descriptive in Brentano's terms [Brentano 1995]. However, suggestions are given as to potential future developments of the framework from an experimental viewpoint (see for example [Albertazzi, Canal *et al.* 2015b]).

2 Appearances and physical objects

The rationale underlying the reversal of viewpoint proposed above is well exemplified by the contraposition of the Galilean and Humean points of view on the question of whether qualities (the *matter* of appearances) can or cannot be objects of scientific inquiry [Albertazzi 2013b]. As well known, qualities are classically distinguished between primary or physical properties (spatiality, solidity, hardness, weight, shape, size, position, motion), and secondary ones (colors, tastes, smells, sounds), the latter classically thought to be a product of British empiricism. However, it was Theofrastus already in the third Century BC, who pointed out that in perception

> [...] one could develop these inquiries better [...] so as to show what qualities are proper to the sentient subject. [Theofrastus 1976, vi 2 1]

Galileo's opinion on the point was very clear:

> [...] I think that tastes, odors, colors, and so on are no more than mere names so far as the object in which we place them is concerned, and that they reside only in consciousness. [Galilei 1623, 274]

A correct statement indeed! Visual appearances (also in dreams) *are* given in perceptual awareness, have a qualitative nature and, strictly speaking, are psychic facts, i.e., they pertain to consciousness. However, as Hume remarked:

> It is universally allowed by modern enquirers, that all the sensible qualities of objects, such as hard, soft, hot, cold, white, black, &c. are merely secondary, and exist not in the objects themselves, but are perceptions of the mind, without any external archetype or model, which they represent. If this be allowed, with regard to secondary qualities, it must also follow, with regard to the supposed primary qualities of extension and solidity; nor can the latter be any more entitled to that denomination than the former. The idea of extension is entirely acquired from the senses of sight and feeling; and if all the qualities, perceived by the senses, be in the mind, not in the object, the same conclusion must reach the idea of extension, which is wholly dependent on the sensible ideas or the ideas of secondary qualities. [Hume 1975, 154]

The difference between physical and phenomenal objects has been shown since the first Gestalt studies. In current science, however, it is widely believed that Gestalt descriptions function mainly as heuristic cues for the presence of physical objects. This conception is broadly endorsed today by most scientists of perception when they assume that phenomenology *describes* (in first person accounts), while psychophysical and/or neurophysiological research, whose observables are metric quantities, *explains* (in third person accounts) [Albertazzi 2013a], [Spillmann 2009].

What is entirely missing in a Galilean framework, however, is a place for *meaning* (a sort of Faustian "stone guest" in current science) and *value* [Köhler 1939]. Current vision science is mainly syntax oriented. For example, in vision, algorithmic processes are applied to optical data, i.e., what happens is essentially the transformation of (meaningless) structures into (meaningless) structures [Koenderink 2013]. But a giraffe is not simply an aggregate of similar patches of black and white on a surface!

Meaningful appearances are incontrovertible *primary ecological facts* for humans and non-human living beings, i.e., they are *met* [Kanizsa 1979, 1991], [Metzger 1941, 2009]. To cite Goethe, one should not look for anything *behind* phenomena, because phenomena themselves *are* the explanation. Certainly, evolution has changed the external phenomenal world [*Umwelten*] [Uexküll 1934], [Koenderink 2012] by consolidating certain expectations, so that perceived qualities include anticipatory components necessary for the behavior and survival of living beings [Rosen 1991, 2012]. However, there are no pre-established, guaranteed-to-be-successful templates (understood as computational file formats) for every possible experience. What we have developed (or inherited) are schemes for actions like those offered primarily by the expressive configurations and affordances that we encounter in the environment and provide us with immediate information on how to behave [Gibson 1979]. One of these templates, for example, is the capacity to see multiplicity in unity, a "template for thinking", to use Bohm's expression [Bohm 1994]. To remain in the field of color perception, a particularly striking case concerns the so-called semantic dimensions of color. Belonging to the perceptive dimensions of color are qualities such as serious, mighty, serene, melancholic, already described in Goethe's triangle [Goethe 1982] (though in fact first developed by Schreiber), but also qualities such as touchy-feely [Chirimuuta 2011], dull [Hering 1920], shrill [Kandinsky 1911], etc.; and there is evidence that any of bright-dark qualities is not unrelated to a scaling of light-heavy relationships, and these lead easily to soft-hard comparisons. Likewise, warm-cool relates to wet-dry dimensions [Albers 1963, 59]. There seems to be nothing more intermodal and interdisciplinary than the semantic dimensions of color, which are of concern to color science and color theory, art and design, and psychology. Two well-known cases in this field are the difference between cold and warm colors, that can be considered as one of the basic invariants of perceived color [Da Pos & Green-Armytage 2007], [Da Pos & Valenti 2007], [Ou, Luo *et al.* 2004], [Xin, Cheng *et al.* 2004], and the difference between light and heavy colors [Arnheim 1954], [Itten 1961]. Neither of these differences derives from either physical properties of radiation or physiological processes, and classical colorimetry has nothing to say in this regard [Boynton 1979], [Brainard 1995], [Koenderink & van Doorn 2003]. For example, the basic polarity of the semantic attributes of color, warm-cold, is not represented in colorimetry space.

3 Qualities

Qualities need to be reconsidered in light of a thorough taxonomy of qualities, which is currently lacking. The best analyses are still those by Metzger and Rausch [Metzger 1941], [Rausch 1966] (see also [Albertazzi 2010], [Pont 2013]). *Prima facie* one distinguishes among qualities of the skeletal structure (such as shape, movement, change, stability and instability); qualities of material (such as phenomena of brightness, transparency, hardness/softness); and tertiary qualities (the ways of being of qualities, in Metzger's terms) such as expressive and physiognomic qualities (friendly, aggressive, masculine, appealing) and affordances [Gibson 1979]. The difficulty of composing a proper taxonomy of qualities, however, is caused by their very nature, because qualities are intrinsically relational. Some examples will help.

There is a figure/ground organization allowing whatever appearance to be visible, and the rules governing the emergence of a figure on a ground follow a precise hierarchy of conditions [Rubin 1949], [Peterson & Gibson 1993]. Visual qualities are always in the eye of the beholder, and they are given and reported in first person account, i.e., in subjective judgments based on visual awareness; qualities always appear in a mereological relation of whole (awareness) and parts (configurations of qualities as appearances); the configurations of qualities always have different degrees of *Prägnanz* [Koffka 1935], i.e., they follow an order of appropriateness; qualities are intrinsically cross-modal (for example, we perceive bright, thick, matt tunes). There are also qualities of qualities, such as the coloratura in music, or the φ movement (the objectless movement consisting in a perception of motion to which does not correspond a concomitant perception of objects in motion) and the β movement (when two fixed lights are lit with an inter-stimulus interval of ca 50 msec, only one light is seen in motion from left to right (see [Wertheimer 1912]). Psychophysical methods and measurements do not apply here; whence derives the difficulty of designing methods to test qualities. Qualities are also given in a series of polarities (such as small/large, high/slow, sweet/sour, coarse/fine), and of spatial directions (left/right, below/above, in front of/behind). Issues therefore arise as to whether, for example, when perceiving something hot, one perceives cold as its amodal background; whether certain basic couples of qualities exist in all the modality domains, such as the hot/cold pair; or whether there are overlapping couples with the same semantic space, such as hot/cold, dry/wet, and sweet/sour. These are not idle questions, because answering them may induce us to reconsider the Aristotelian idea of a science of the sensible qualities based on couples of contraries such as hot and cold, and dry and wet [Aristotle 1906, 420a 25-420b], [Theofrastus 1976, ivi, on Democritus, 442b 11], [Aristotle 1980, 8001a, 10 ff.], [Aristotle 1986, 420a25-420b], [Theofrastus 1976, on Democritus, vi, 1, 6]. This notion engenders a very different conception of the human perception of the environment, and of nature, which in our awareness appears to be intrinsically and naturally multimodal, qualitative, and scaled: for example, physically there is no object which is "more or less long" or "more or less short".

4 Aisthesis

If one analyses vision as a whole of appearances, it is difficult to maintain a sharp distinction between what pertains to science and what to aesthetics; or to be more precise, among perceptual, pictorial, and mental spaces, all of which are highly imaginative [Albertazzi 2006a,b]. In current science, subjective appearances are translated into third person account and re-presented in mathematical models, through metrics which tear off their qualitative character. This is one of the reasons why artistic products have hitherto often furnished the best *re-presentations* of appearances, much more than metrical and computational models. The writings of Alberti, Da Vinci, von Hildebrand, De Chirico, Marinetti, Klee, Kandinsky, *et al.*, and also handbooks on how to draw [Ruskin 1857], or to paint [Hogarth 2002], are real sources of visual theory, showing what points, lines, surfaces and volumes are from a perceptual viewpoint; i.e., what the primitives of spatial appearances in visual awareness are. Observations relevant for the study of spatial primitives are to be found on the aesthetics of space [Lipps 1897], the already mentioned nature of the pure visible or appearance [*Sichtbarkeit, sichtbare Erscheinung*], (see [Fiedler 1991]), the difference between close and distant images [Hildebrand 1893], abstract shapes (animal and inorganic as well) [Riegl 1966], the relation between abstraction and empathy [Worringer 1908], and the perception of shape organization (lines to planes, to shapes, etc.) [Wölfflin 1950], [Kandinsky 1926].

Art is in fact an instrument with which to generate knowledge about natural processes, rendering manifest *the conditions to make appearances visible*: consider, for example, drawing, painting, and sculpting techniques, or procedures to generate metallic luster in ceramics. The visual analyses, specifically developed by artists straddling two centuries (the eighteenth and the nineteenth) are major explorations of the phenomenology of vision. Potentially, they could help to rewrite entire bodies of literature in vision studies: consider, for example, studies on shape from shading, where the concept of the shading cue may be replaced by that of the cue for relief articulation [Koenderink, van Doorn *et al.* 2015].

Currently some of these issues are being examined by both experimental aesthetics and cross-modality studies, which are undergoing a rapid scientific development. However, in such research the (ontological) dichotomy between so-called objective features and subjective aspects of experience still dominates, and the methodologies adopted are usually those of psychophysics. As varied as they may be, what these studies analyse are usually very simple stimuli: say, a grapheme and a color, a color and a sound, a smell and an olfaction (for a review see [Spence 2011]). What we cross-modally and generally perceive in the environment, however, are high-level, cross-modal, interlocked, and most of all meaningful, configurations of qualities [Albertazzi, Canal *et al.* 2015a]. What is missing in many contemporary studies is analysis of the characteristic of *pure visibility*, i.e., the purely qualitative aspect of visual objects, and the specific nature of the *spatial extension* of visual objects, of which artists, instead, have always been aware [Albertazzi 2011]. The study of artworks thus becomes a "laboratory" for the analysis of the laws of seeing and visual appearances [Metzger 2009, chap. 12, 2]. Indeed, the artist does nothing but test, exemplify and reshape the construction of appearances based on laws that are also active in the natural perception of objects. This point of view has also been expressed to some extent by, among others, Da Vinci see [da Vinci & McCurdy 2002], Klee [Klee 1961] and Kandinsky [Kandinsky 1926].

5 Spatial primitives

Our visual field, in the case of shapes, appears to be partitioned into spatial entities that can be described in terms of *seen* points (or punctoids), *seen* lines (or lineoids), *seen* surfaces (or surfaceoids), and *seen* volumes (or volumoids).[2]

2. The coined new terms punctoids, lineoids, etc. refer to how the visual points, the visual lines, etc., qualitatively appear in the visual field. The use of such terminology should avoid the danger to take these patterns as Platonic ideas or abstract concepts, which would be the case if one adopted terms such as "pointness", "liness", etc.

What are these perceptual entities? Are they geometric entities? Are they coextensive with what we cognitively ascribe to the physical world? Alternatively, are they dependent on how our mind generates and externalizes these entities—i.e., *psycho-geometric* entities? What are the conditions of their appearance, their linkages and interdependencies? Are they meaning-bearers, and if so how? As mentioned, the arts are a rich field of research into these issues whose results should be further tested from an experimental phenomenological viewpoint.

It is necessary for experimental phenomenology [Albertazzi 2013a], [Vishwanath 2013]:

> To start with the perceptual phenomena and strictly avoid mixing up perceptual phenomena with their physical or physiological causes or to derive from the latter any principle of classification. [Hering 1920, 24]

The main tenets of experimental phenomenology pose problems of method, units of measurement, metrics (if any), and a formal theory of the spatial appearances [Kubovy 1999].

In order to identify the nature of the visual primitives, for example, one has to consider them in the space in which they appear, a space embedded with forces, raising, lowering, pushing appearances in different directions, and at different perceived velocities (see the concept of *teleiosis* in [Brentano 1988]). Embedded in this space are the laws of growth of form appearances [Thompson 1961], [Goethe 1978], [Haeckel 1904], [Ostwald 1922]. Consider a subjective *visual* square: in perceiving certain configurations as "squared", we perceive not a Euclidean figure but a specific quality (i.e., "squaroid") that covers a variable but always small number of variations. "Squaroid" has the character of a zone of particularly pronounced potential transformations which is merely perceptual. A visual square is also "sitting" on one of its sides, and therefore perceived as stable. When rotated by 45 degrees [Chen & Freeman 1984], [Schumann 1900], notwithstanding any change in its metrical size and dimensions, the square presents a different appearance: that of a diamond, which is certainly more similar to a rhombus than to a square, and in unstable equilibrium. Visual squares can appear differently in tachistoscopic presentation [Sander 1926], void [Bressan 1985], [Woodhouse & Taylor 1987], textured so as to have an inky, spongy, metallic, etc. quality; of different size if filled with lines [Helmholtz 1867], [Mather, O'Halloran *et al.* 1991], and endowed with connotative dimensions (good/bad, pleasant/unpleasant, appealing/disgusting) [Köhler 1939]. One can measure how square forms differ from Euclidean squares. In so doing, one sees that the kind of units used for its measurement are not the *JND* of psychophysics, because one is measuring qualitative dimensions as parts of perceived space [Albertazzi 2012]. Testing the nature of a square appearance means relying on subjective judgments in first-person account. There is no other way to proceed, because what has to be tested and measured is *how* a square appearance appears to subjects, in the space of mental visual awareness. What is needed is a geometry of visual forms based on subjective qualities as perceived by humans, and in some respects also by “other species of mind” [Allen & Bekoff 1997]. More than *imposing* a metrics, one must rely on degrees of (qualitative) similarity as perceived by subjects, and on what is the order of appropriateness for that specific type of shape (for example, the *best* clue of the category squaroid). The complexity of the judgment of similarity and order is immediately evident when one consider that, depending on dimension, color, texture, etc., from time to time, in different contexts, one shape can be perceived as more square than another.

The first step in proceeding towards a geometry of the purely visual is to develop a taxonomy of primitives’ appearances which is as complete as possible, something similar to what Katz did for colors and their ways of appearing [Katz 1935]: for example, classifying the properties, the modes of appearances and the conditions for a figure to look like a square, and the extent to which it does so. It is necessary to abandon the idea that spatial primitives are stimuli re-presentations and in fact a product of information recovery and sensory completion: i.e., estimating properties of “objective” quantitative reality and having a direct access to its statistics captured in the mathematics of Bayesian estimation [Albertazzi, van Tonder *et al.* 2010], [Koenderink 2010, 2012], [Hoffman 2013], [Lappin, Norman *et al.* 2011], [Vishwanath 2005].

6 Visual points, lines, surfaces and volumes

The questions to start with in order to frame a “geometry” of visual space are the following: What are the visual spatial primitives? e.g., what is a visual point as to the “what” and the “where”? What are the relations among visual primitives, their visual extendedness, and space? What are their properties? (dimensionality, modality/amodality, direction, color, etc.). What are the conditions of their phenomenal appearance? (E.g., when *is* a visual point visible? when does a visual point *become* a visual linelet, a disc, a volume?) How can one classify, measure and model the subjective appearance of visual primitives? (E.g., in the way in which one does so with the subjective experience of colors.)

Visual spatial primitives include *seen* points, *seen* lines, and *seen* surfaces. What we perceive in the environment, in fact, are not Euclidean dimensions, but fine threads, smooth wooden tables, water surfaces, gelatins, fogs: we do not perceive points with neither dimensions nor color, or one-dimensional lines. It is usually assumed that in 3D perception the customary order of perceived objects is from points to lines to surfaces and volumes. However, we perceive the depth, solidity and graspability of visual objects also in monocular vision, i.e., in the visual field, where lower-order primitives are usually seen as boundaries of higher-order elements (i.e., surfaces are boundaries of volumes, lines of surfaces, and points of lines). I shall distinguish different forms of dependence of lower-order elements on higher-order ones, so that the *in*dependence of the individual primitives as such is to be viewed as the limiting case of dependence.

Dimensionality is one of the characteristics of spatial appearances, but dimensions cannot be understood in the physical sense. Take the case of visual points (punctoids). These are not zero-dimensional, but may be one- or bi-dimensional (consider a hyper-compressed line reduced to its minimum, or imperceptibly circular or even oval). Do visual points have internal structure, i.e., parts? Can they be viewed as Euclidean points? Do they have borders of various types (knurled or frayed)? Very rarely do they have perfectly circular borders. When does a point in a 3D space (a star in the sky) become a sphere? When does a point becomes a dash line? Do points have orientation? If so, at what dimension? Do color and texture influence their appearance? Do points change spatial attitude (depth, meeting of lines, position) in configurations? Are there expressive points?

Consider a visual line (lineoid). It is one-dimensional, but has a marginal second dimension: to be phenomenally visible, in fact, it has to possess a certain width. Does the presentation of a line in a 3D space or contextually with other lines change its appearances? Do lines have visual location, direction, weight (think of chiaroscuro), depth, density (are there free lines)?

Consider a visual surface (surfaceoid): bidimensionality is a limit case (such as an A4 sheet of paper). However the bidimensionality of a fronto-parallel surface changes if seen at 90 degrees and becomes a visual line, the line being its non-detachable boundary. Do lines have visual location, direction, weight, depth, density (are there free surfaces)?

Consider a visual volume (volumoid). Is "being voluminous" also perceived in monocular vision? Is it necessarily related to grasping (motor action)? Do visual volumes have location, direction, weight, depth, density (think of clouds or fog)?

Another important consideration is that dimensions in presentation may not be homogeneous, because visual space itself, besides being finite, is qualitative and non-homogeneous [Mach 1984, chap. 6–10]. Visual primitives are also rarely detachable one from another: for example, how can one detach the visual surface boundary of a volume, such as the red quality spread on the surface of an apple? or the striated rose-yellow of a voluminous appearance of clouds?

Because pictorial space is a bidimensional space, it is a good "laboratory" for the analysis and the explanation of the forms in visual space and their genesis, i.e., of the conditions that allow them to be visible. When we draw or paint on a canvas, what we are in fact doing is represent, i.e., objectivize, the laws governing visual seeing. Artists are true to nature [Ruskin 1857], because from a point of view of pure visibility nature itself is an indefinite potential of morphological figurations localized in a space of pulling forces.

7 Experimental phenomenology of a visual point

There is lack of wide-ranging experimental evidence, based on subjective judgments on the perception of points, lines and surfaces, on which to build a theory of the primitives of visual awareness. Experimental evidence on the presence or absence of parts in visual points, for example, would be essential information on which to construct a theory of these and other visual spatial primitives. A point can be a marked part of some higher-order object, such as the origin of a line; or it can be unmarked, like an undifferentiated part of a line, or of a crystal clear blue sky without discontinuities of any sort, as in the *Ganzfeld* [Metzger 1930]. A point is also a position in space signalled by the arrangement or the structure of other objects, such as the target of an arrow.

Here I shall consider *seen* points (punctoids) and their two main modes of appearance: a point as an independent object (an object so small that it can be seen even when it does not have phenomenally significant dimensions); and a point as a dependent object. In both cases, the phenomenal visual points have a form (contrarily to [Koffka 1935, chap. 4]).

It is difficult to define a point in phenomenal space as an independent object. In fact, points may be independent from any higher-order object, but not from the ground, which influences their appearance (a black point on a white ground and a white point of the same size on a black ground look different); points require color, and specific dimension to be visually distinguished and distinguishable in the field in which they appear.

A point as an independent object may be defined as having a minimal extension, with tolerance; as being localized; and as having a qualitative width and color (and color connotative characteristics as well, such as cold/warm, and merry/sad); as having figural and border characteristics (thorny, hooked, frayed margins as they appear in natural forms, such as stars or bacteria); and as being expressive (appearing good or bad, according to their position on a square frame, for example). Briefly, for a point to be a point there may be an organization of size, texture, border, color, and orientation (relatively to their borders, relatively to the planes on which they appear, such as right/left, above/below, in front of/behind); and also expressive characteristics, such as appearing stable or instable in space. Any model considering a point without dimensions, would, under this conceptualization, be wrong. Once these characteristics, have been carefully described, they must be tested experimentally before proceeding to define punctoid as a primitive.

The second mode of appearance of a point is as a dependent object. Consider the intersection point between lines, the extremes of a line, a point induced by convergent segments separated by a spatial interval; or the tip of an arrow, a cone, an edge, etc. Examples of dependent points in high-level configurations are also Hermann's grid (1870), Ehrenstein's figures and phenomena of neon color spreading [van Tuijl & Leeuwenberg 1979]. Points can lie on a ground, a line, a surface or inside or outside a volume: in a cone, for example, a point on the vertex is seen as external, a point on the surface is seen as external, while a point in the lower part is seen either on the base surface of the cone, slightly raised because of transparency, or on the inner surface of the cone. A point may amodally overlap with another point or with a line (both externally or internally), and there may be coincidence of boundaries of many overlapping points (consider the cone, above). In stereokinetic cones (or ellipses) points also overlap in time, changing their localization either in surface or voluminous appearances. There may be amodal points dependent on a texture of points, or induced ones dependent on four converging segments. Points may have directions depending on their irregular borders or on their potentiality to move within the same higher-order object on which they depend, without changing their type (i.e., being marked or unmarked). There are different degrees of freedom of directions in points as dependent objects. In fact, they can be marked points (the end point of the arrow); imaginary points (the internal points in a line or arranged in loose configurations); unmarked points (when the point is merely the "target" of the arrow). Consider then the midpoint of a disc and its degree of freedom: its centre has full potential movement (complete freedom of direction). However, if one removes a quarter of the disc, then half of the disc, its centre has fewer possible movements (50% degree of direction) (see the concept of *plerosis* in [Brentano 1988]). Finally, the direction of points may have degrees of punctoid (i.e., of *Prägnanz*).

As to the color of points as independent objects, they have their own color, although influenced by the ground on which they appear. The color of points as dependent objects, on the other hand, depends on the objects on which they depend, such as the color of the lines to which they belong, or it is influenced by the surface on which they appear.

The question of what a point is—or in other words, what the meaning of a point in the visual field is—can be answered only by attentive description of the multifarious modes of its appearances, and by a series of experimental tests based on subjective judgments in first-person account. The visual appearance, location, and meaning of points in space, in fact, depends only on *seeing.* Only when one has the whole visual grammar of a point is one entitled to look for the definition of a visual point.

8 Experimental phenomenology of a visual line

A line's appearance is multifarious as well, and the initial questions are similar. What is a line? When is a line visible? When does a line cease to appear as a line? When does a line become a solid/transparent surface or a hole? Do lines have orientation? If so, at what dimension? Do color and texture influence their appearance? Do lines change spatial attitude (curvature, position of shape) in configurations? Are there expressive lines? "How" and "when" do we see lines: for example straight ones or curved ones? [Lipps 1897], [Platt 1960].

Here I shall consider the nature of seen lines (i.e., lineoids) and their two main modes of appearance: a line as an independent object (a unidimensional thin line that can be seen even when it does not have phenomenally significant dimensions), and a line seen as a dependent object.

It is difficult to think of a line in phenomenal space as a totally independent object. In fact, visual lines may be independent from any higher-order object; but like points, lines are not independent from the ground, which influences their appearance (a black line on a white ground and a white line on a black ground look different); lines require some other difference in color, and dimension, for example, to be visually distinguishable. A visual line is an elongated object that must have some minimal thickness to be visible (for example, threads, or a differently colored horizon of the sea), and a thick line behaves differently from a thin line. A line can be perceived as such although not drawn, as the continuous prolongation of two converging segments, or as two crossing lines of two converging segments.

A line as an independent object may be defined as having a minimal extension, with tolerance: its end-points are marked and have no degree of possible movement (whereas its internal points are not marked and have two degrees of possible movement); it is localized and has a qualitative width; it has color and connotative chromatic characteristics such as cold/warm, merry/sad; it has figural characteristics such as parts, potentially overlapping, so that a line is only a formal limit of a surface; it has border characteristics, such as thick, curved, zigzagged, blurred margins; and here the phenomenology is wide, because of the existence of smooth margins of a zigzagged line or zigzagged margins of a curved line, etc. Lines have expressive characteristics (appearing good or bad, cold or warm, aggressive or calm, quick or slow), relatively to their thickness, density, direction, degrees of freedom in a configuration. For a line to be a visual object there must be an organization of size, texture, border, color, and orientation (also in their "skeletal" characteristics, for example appearing stable or instable in space).

The second mode of appearance of a line is as a dependent object. In this case, a line can be a marked (or unmarked) part of some higher-order object (e.g., an undifferentiated part of a surface such as imaginary lines, lying as non-marked on a surface, or its marked boundary). A line can be a position in space signalled by a suitable arrangement of other objects (depending on their distance, context and relative position).

A growing set of visual lines extends in a surface, and the differently pronounced lines in the set are seen as parts of space (consider a row of trees seen by a perceiver in motion), their density being given by the different regular or irregular directions that they have (to the right/to the left, toward/below, toward/above, centripetal/centrifugal). In the visual context lines are boundaries of both external surfaces and an inner area. They can also be boundaries of transparent volumes and depth cues as in chiaroscuro. Lines can have an overlapping of boundaries and parts in higher-order objects, such as surfaces and volumes: in the case of a ball, for example, the circular line that one sees is not a boundary of surfaces but of volumes, and one has virtual lines dependent on the position of the viewer (the same holds for an oval disc, or for a small ball moving on a bigger one). The visual sides of a volumetric cone are boundaries of the solid; the same holds for a stereokinetic cone, although with a higher degree of *Pregnänz* of the appearance of the voluminous solid shape and a temporal shifting [Albertazzi 2004].

As the boundary of a surface-object (figure), the non-independent line has the color of the surface to which it belongs; unmarked internal lines have the color of the surface as well. In the case of lines as boundaries in higher-order objects with different colors, such as a few quadrants of a disc, one may wonder what the color of the common boundary is, because the figure-ground effect may not apply here: one might see two side-to-side boundaries, or one boundary only, having the color of the visually more salient figure. Only experiments can give an answer to these preliminary questions.

9 Experimental phenomenology of visual surfaces and visual volumes

From the standpoint of seeing, surfaces and volumes appear to be strictly interrelated primitives. When surfaces are perceived lying on the same plane, the margin assumes a double function—as a contour enclosing both the surfaces—and the stratification of the two surfaces goes in parallel with the function of the dividing border. Color is a determinant of both surface stratification and the plastic effect in space. If a colored surface overlaps with and partially occludes another differently colored surface, the visual margin separating the two belongs to the former and its contour, while the surface to the rear is perceived amodally as continuing behind the former. If, on the contrary, the two surfaces are perceived as lying on the same plane, the margin line where the two abut assumes a double function. In other words, the stratification of the two surfaces, due to color, goes in parallel with the function of the dividing border [Da Pos & Albertazzi 2015].

In perceiving, at near distance, and in stereoscopic images we obtain both a 3D shape and an egocentric scale. The difference between stereoscopic and pictorial space resides in the fact that what we perceive in pictures is a 3D structure (3D shape and space) but that structure is not scaled to the ego-center [Vishwanath 2014]. When one has a perception of stereopsis (either through a stereoscope or a physical object), it is because things are scaled to the egocenter. Therefore one explanation for why, when one looks at a picture with one eye through an aperture [Vishwanath & Hibbard 2013] or with a synopter [Koenderink, Wijntjes *et al.* 2013], pictures appear as if they are "stereoscopic" is because there is some presentation of egocentric scale [Vishwanath & Hibbard 2013]. Vice versa, when we look into the distance in real scenes, the perception of 3D space/shape again becomes more "pictorial". This can be explained on the basis of the fact that one no longer has the presentation of egocentric scale beyond a certain distance, because egocentric distance becomes very difficult to assess farther and farther away [Vishwanath 2014].

Because color and light conditions, and chiaroscuro as well, influence the appearance of distance, stratification, texture, transparency, pictorial relief, they are the conditions for volumes to appear in the visual field. Surfaces themselves are transparent boundaries of volume appearances, like a quality spread all over them [Alberti 1972]. The depth quality and plastic effect of the objects in a scene are also fundamental to our perceived quality of *realness* [Mausfeld 2013], [Michotte 1948], [Vishwanath 2010]. Both paintings and natural scenes show how the realness of visual objects derives to a great extent from appearances made of secondary qualities in extended visual space, such as flat, round, volumetric, soft, luminous, remote, close, graspable, etc.; and how the space of vision is essentially a space constructed by the observers.

10 Preliminary conclusions

It emerges from the foregoing discussion that qualitative primitives in visual space awareness—such as punctoids, lineoids, surfaceoids and volumoids—lie at the basis of the perceived visual configurations such as trees, plants, living beings, landscapes, and art works. They have specific and general properties, and can be individually analysed only as limit cases of independence. They form a whole/part hierarchy, may have coincident boundaries, present degrees of *Prägnanz*, and bear expressive characteristics. If the "elements" of a geometry of the visible are to be identified, they have the nature of *psycho-geometric* primitives, i.e., *qualitative patterns, non-detachable from seeing.* A geometry of the visible *is* the geometry of an act of visual presentation.

Complete identification of primitives like punctoids, lineoids, surfaceoids and volumoids can be achieved only after thorough empirical testing of their characteristics based on subjective judgments in first person account. Standard mathematical geometry is inadequate to account for them, because the primitives of visual space are not merely geometrical projections of retinal stimulation. Indeed, most aspects of the space-time in which those primitives appear are still far from being clearly understood, such as the dynamics of their morphogenesis in presentation, and the kind of continuum to which they pertain [Albertazzi 2002]. Other preliminary aspects that should be considered are the laws of organization and interrelation among the primitives: for example, one may expand the classic Gestalt laws by introducing the law of meaning [Pinna & Albertazzi 2010].

From a biological and ecological viewpoint, when the various shape configurations in nature and their coming into being are considered, the rules underlying the development of visual spatial primitives seem to apply also to the organic world of nature: consider punctoids (pointed formation from seeds to microorganism), lineoids (linear formation of crystals or of lightning), surfaceoids (connective biological tissue), and volumoids (of clouds, dunes or water masses) as analyzed in Haeckel and Thompson [Haeckel 1904], [Thompson 1961]. In a few recent experimentaI studies on organic shapes [Dadam, Albertazzi *et al.* 2012], [Albertazzi, Canal *et al.* 2014], for example, it has been shown that when the images of organic forms are grouped on the basis of certain characteristics (such as shape, margin, texture, dimensionality), rounded forms are perceived as harmonic and dynamic, while elongated forms are perceived to some extent as disharmonious and static. Furthermore, it has been shown that these spatial patterns are naturally associated with colors [Dadam, Albertazzi *et al.* 2012], and that round shapes are naturally matched by reddish colours (in the orange-red interval of the Hue Circle) and elongated shapes are matched by bluish colour (in the blue-green interval of the Hue Circle). From these experiments one may conclude that the perception of a “round” attribute is generally associated with “reddish”, “harmonic”, “dynamic” and “warm”, while the perception of an “elongated” attribute is associated with “greenish-bluish”, “disharmonious”, “static”, and “cold”. Adopting a qualitative approach to visual shapes might then result in extraordinary findings, ones such that it is possible to think in terms of a natural history of forms evolved from a few basic spatial primitives imbued with meaning—the idea at the core of Goethe’s *Ur-formen* [Goethe 1978]. Visible in the “book of nature”, in fact, is the tendency towards qualitative patterns and the repetition of patterns showing the same principles at work in different taxonomic realms: a challenge for a science of visual appearances and their primitives, and for an intrinsic geometry of visual space.

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