**Belgeo**
Revue belge de géographie

**1-2-3-4 | 2000**
**Special issue: 29th International Geographical Congress**

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**Electronic version**
URL: http://journals.openedition.org/belgeo/13883
DOI: 10.4000/belgeo.13883
ISSN: 2294-9135

**Publisher:**
National Committee of Geography of Belgium, Société Royale Belge de Géographie

**Printed version**
Date of publication: 30 December 2000
Number of pages: 47-64
ISSN: 1377-2368

**Electronic reference**
Frank Canters, « Advances in small-scale map projection research », *Belgeo* [Online], 1-2-3-4 | 2000, Online since 12 July 2015, connection on 30 April 2019. URL : http://journals.openedition.org/belgeo/13883 ; DOI : 10.4000/belgeo.13883

# *Advances in small-scale map projection research*

Frank Canters

1 From the early days of map making to the present time, the challenge of representing the round Earth or part of it on a flat piece of paper without introducing excessive distortion has attracted the attention of many geographers, physicists, astronomers, mathematicians, and mapmakers. Putting aside the Medieval period, when representations of the Earth's surface were influenced by religious ideology and loaded with symbolic meaning (e.g. the famous East-oriented T-O maps that depict the Earth as a round disk, subdivided in three continents by a T-shaped sea and surrounded by an O-shaped ocean), the accurate representation of the Earth as it is known at a particular time has always been an important objective of contemporary map making. This does not imply that the subject of map projection has received an equal amount of interest from the ancient Greek period, when Claudius Ptolemy wrote his famous Geography, all the way up to the twentieth century. Major breakthroughs in the history of map projections were prompted by various external factors like, for example, the increasing geographical knowledge during the Renaissance, which led to modifications of earlier map projections and the development of a whole series of new map projections suitable for displaying the entire globe, or the development of the calculus (first applied to map projections by Johann Heinrich Lambert in the late eighteenth century), which gave the cartographer or mathematician the necessary tools for the development of new map projections that fulfil certain general conditions, the most important of these being the preservation of angles and area.

2 The introduction of modern computers marks a new era in the evolution of map projection science. Being dominated for almost two centuries by the formulation of analytical solutions to increasingly complex map projection problems, the computer cleared the way for a numerical treatment of map projection. Numerical approaches have been used to solve various practical problems related to map projection, including the efficient transfer of data from one map projection to another (Doytsher and Shmutter,

1981; Wu and Yang, 1981; Snyder, 1985; Kaltsikis, 1989; Canters, 1992) and the automated identification of map projection type and/or map projection parameters for maps for which this information is not known (Snyder, 1985). The computer has also been used to evaluate and compare distortion on various map projections (Tobler, 1964; Francula, 1971; Peters, 1975; Canters and Decleir, 1989; Laskowski, 1998; Canters, 1999) and to automatically determine the value of the parameters of standard map projections so that overall scale error is reduced (Snyder, 1978; Snyder, 1985). One of the most stimulating outcomes of computer-assisted map projection research so far has been the development of new map projections with distortion patterns that are closely adapted to the shape of the area to be mapped, guaranteeing minimum overall scale error within the approximate boundaries of the area (Reilly, 1973; Lee, 1974; Snyder, 1984, 1986, 1988; Canters and De Genst, 1997; González-López, 1997). Although the theoretical foundations for the development of these projections, and the first simple applications of the minimum-error principle, date from the pre-computer age (Laborde,1928; Miller, 1953), a more complicated use of the technique requires extensive numerical processing, practical only in a digital setting.

3 The digital revolution also had a large impact on the everyday use of map projections. As computer-based mapping tools make it possible for an ever growing group of people without any formal cartographic background to make their own maps, it also gives them the freedom to become more creative with map projections. Complicated map projections that are difficult to draw manually, and for that reason were seldom used in the past, can now be plotted quickly using map projection software libraries or map projection tools that come with standard GIS software. Projection parameters can easily be changed, making it possible to experiment with alternative views of the same geographical area with very little effort. Although this may lead to a greater awareness of the advantages and disadvantages of using a particular map projection for a given purpose, it also creates much opportunity for misuse. This author is not aware of the existence of any commercially available map projection tool that offers the user some basic guidance in choosing a proper map projection for a particular mapping task. The lack of such tools is very unfortunate, especially in small-scale mapping where the user often has a large number of potential map projections at his or her disposal, and the choice of projection may determine to a large extent if data are portrayed adequately in relation to the purpose of the map and, accordingly, if the map fulfils its role as a communicative device or not (see e.g. American Cartographic Association, 1991). Examples of bad map design are indeed quite common, in print journalism as well as in the electronic media (Gilmartin, 1985; Monmonier, 1989). Offering the mapmaker some assistance in selecting a proper map projection for a small-scale map is therefore recommended. A few attempts have been made to automate (or partly automate) the map projection selection process (Jankowski and Nyerges, 1989; Mekenkamp, 1990; De Genst and Canters, 1996). They will be discussed in the last part of this paper.

4 The main objective of this paper is to review previous and current studies in small-scale map projection research, and to discuss the research that has been done by this author in relation to similar work by others. It is by no means this author's intention to provide a comprehensive review of all the work that has been done in the different research areas that will be mentioned. Only some of the most significant contributions in each sub-area of research will be discussed, in order to define a suitable framework for discussion. It is hoped that this paper will give the reader a good impression of the marked evolution of

map projection science over the last forty years, of changes in the way map projections are used (or will be used in the near future), and of the important role the rapidly increasing, widespread use of computers has played in this process.

## Evaluation of map projection distortion

The projection of a curved surface on a plane map always introduces distortion. As a result of this distortion map scale varies from one location to another and is generally different in every direction. When trying to represent the Earth on a plane our primary concern is to choose a map projection on which distortion is a minimum. Accomplishing this necessitates an understanding of how map projection distortion takes place and how it is distributed over the entire map area. Tissot (1881) demonstrated that at an infinitesimal scale every map projection is an affine transformation. When applying a map projection, each infinitesimal circle, centred around a point *p* on the sphere, is transformed into an ellipse, which is called the indicatrix of Tissot (figure 1). The indicatrix offers a complete description of the distortion characteristics of the projection for a particular location on the map. Its semi-diameters *a* and *b* represent the maximum and minimum value of the local scale factor in the immediate vicinity of the point. A scale factor equal to one means no distortion, a scale factor smaller or larger than one points to linear compression or linear stretching respectively. Once the semi-diameters *a* and *b* of the ellipse and its orientation are known, various distortion measures can be calculated, including the distortion of angles, the distortion of area and the local scale distortion in an arbitrary direction. Distortion characteristics of a projection can be visualised by mapping Tissot's indicatrix for selected positions on the graticule (the framework of meridians and parallels that determines the appearance of the projection), or by calculating distortion measures for a large number of graticule intersections and constructing isocols (lines of equal distortion) (figure 2).

**Figure 1. Elementary circle in the tangent plane on the sphere (a), and corresponding ellipse of distortion in the mapping plane (b).**

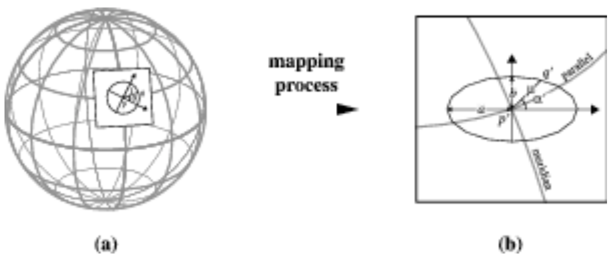
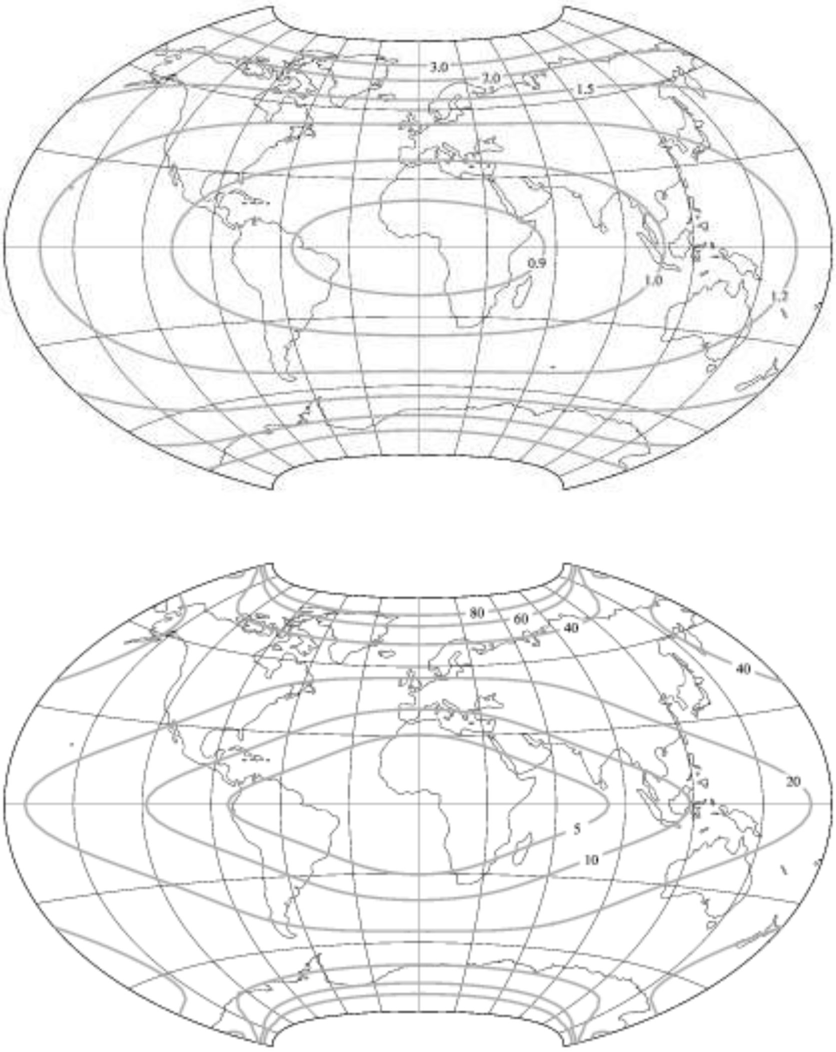


Figure 2. Low-error polyconic projection with two-fold symmetry, equally spaced parallels and a correct ratio of the axes *(Canters projection)* with lines of constant area scale (a), and lines of constant maximum angular distortion (b).




6 Although graphic representations of local distortion characteristics, as shown in figure 2, offer a clear and detailed insight in the spatial distribution of map projection distortion, they do not permit a quick comparative evaluation of distortion for a large set of map projections. They also do not give us information about the optimal choice of parameter values (e.g. the position of the centre of the projection, the latitude of the standard parallel(s)) for a particular map. Both comparative evaluation and optimisation of map projection parameter values require that distortion for the entire area of the map (or at least one aspect of it, e.g. angular, area of distance distortion) is characterised by a single value. While some authors have suggested to characterise the distortion of a map projection by the maximum amount of distortion attained over the mapped area (Tissot, 1881; Bludau, 1891; Wagner, 1962), most studies are based on the use of average distortion values. A large part of the work on the comparative evaluation of map projection distortion concentrates on the mapping of the whole planisphere (see e.g. Francula, 1971; Peters, 1975; Canters and Decleir, 1989; Laskowksi, 1998; Canters, 1999). The large variety of map projections that is available for world maps has indeed tempted many

cartographers to find an objective means of deciding which one of these projections can be considered the best. Of course, the outcome of any study of this kind will depend on how the average distortion value for a projection is calculated. Over the years a multitude of distortion measures and algorithms for calculating average distortion values have been proposed. A clear distinction should be made between two fundamentally different ways of defining and characterising map projection distortion, i.e. at the local scale or at the finite scale.

7 The most traditional way of calculating an average distortion value for a projection is to divide the area to be mapped into a large number of small plots, calculate the value of a local distortion measure for the midpoint of each plot, and average the obtained values, using the area of each plot as a weight factor. This approach is entirely based upon the classical theory of map projections, and the concept of Tissot's indicatrix, which enables us to describe the distortion characteristics of a projection for each individual location on the map. One of the best-known early examples of the use of this technique is the study by Behrmann (1909), in which he compares map projection distortion for different equal-area projections by calculating the maximum distortion of angles for equal increments of longitude and latitude, and deriving a mean angular distortion value for each projection. For obvious reasons, however, it was only after the invention of modern computers that the technique reached its full potential, and could be applied in a more systematic way to analyse and compare distortion for a large number of map projections (see e.g. Francula, 1971; Canters and Decleir, 1989). The traditional method of calculating map projection distortion in a point-wise manner, however, has its limitations when it comes to characterising distortion for larger areas. This is especially true for world maps. A critical evaluation of map projection rankings based on average local distortion values indicates that, although in general map projections with well-balanced distortion patterns have the lowest average distortion values, there are some notable examples of map projections with less favourable distortion characteristics that score remarkably well (Canters, 1999). This is particularly the case for cylindrical projections, which are known for their extreme and visually disturbing rate of distortion away from the standard parallels, and are usually not recommended for world maps (American Cartographic Association, 1989).

8 The major reason why an evaluation that is based on the point-wise calculation of local distortion values does not produce map projection rankings as expected lies in the fact that a simple averaging of distortion values for a discrete set of locations does not take account of the spatial variation of distortion typical of each projection. Obviously it is the cumulative effect of scale distortion over finite distances that will determine how well large-size geographical features are represented on a small-scale map. One may therefore expect to be more successful in evaluating the departure of a feature's representation on the map from its original representation on the globe by measuring map projection distortion at the finite scale. One of the first attempts to quantify map distortion in the large was by Fisher and Miller (1944). By defining twenty equilateral spherical triangles on the globe and comparing these with their mapped versions, obtained by connecting the projected vertices of the original triangles with straight lines, they calculated the linear scale ratio (map distance over spherical distance) for each side, the areal scale ratio (plane area over spherical area) for each triangle, and the angular difference (plane value minus spherical value) for each pair of sides. From the calculated ratios the maximum, minimum and mean value was derived, as well as the standard deviation. With the advent of modern computers finite distortion measures could be calculated for much larger

samples. In a study of 1964, Waldo Tobler, a pioneer in the development of numerical solutions to map projection problems, adopted the approach described by Fisher and Miller. However, instead of working with a small number of a priori defined spherical triangles, Tobler calculated mean linear scale ratio, mean areal scale ratio and mean angular difference for a variety of map projections and for areas of different size, using samples of 300 to 500 randomly selected triplets of latitude and longitude, depending on the size of the area (Tobler, 1964).

9 Other studies on finite map projection distortion that follow deal with the definition of suitable measures and algorithms to quantify the average distortion of finite distance on a map projection (Gilbert, 1974; Peters, 1975; Albinus, 1981; Canters, 1989). The idea behind these studies is that all types of distortion that occur on a map can be considered the result of a distortion of scale that changes continuously with location and direction. As such the average distortion of finite distance will give a good indication of the extent to which the plane map differs from the surface that is projected. In all four studies that are listed above the average distortion of distance is calculated for a large number of randomly selected pairs of points. Alternative solutions that are proposed differ in two ways: (a) the definition of the index that is used to quantify the average distortion, (b) the method (algorithm) that is used for the selection of the random pairs of points. Recently Canters (1999) presented a new method to measure the deviation from true area proportions and the distortion of shape on small-scale maps, and applied it to quantify finite distortion for a large set of map projections that are commonly used for world maps. The proposed method measures relative differences in area and shape for a randomly selected set of spherical circles that are homogeneously distributed over the area of interest, by comparing each circle on the globe with its representation on the map, and averaging the obtained distortion values. The study indicates an inverse relationship between the mean deviation from true area proportions and the mean distortion of shape. Projections with a low distortion of finite distances prove to have favourable distortion characteristics (a moderate distortion of shape and proportions), which confirms that the average distortion of finite distance is an appropriate indicator for the joint effect of the two most important aspects of map projection distortion at the small scale, i.e. the distortion of correct area proportions and the distortion of shape.

10 Acknowledging that map projection distortion can present itself in many different ways (local distortion of scale in a particular direction, distortion of distance, angles, azimuth, shape, absolute area, relative area proportions,...), some authors have also proposed the use of combined distortion measures, which provide a general assessment of the combined impact of two, three or even more different aspects of distortion. Some of these measures, especially those referring to various aspects of distortion at the local scale, already have a long history (Airy, 1861; Jordan, 1896; Kavrayskiy, 1958), others have been proposed more recently (Canters and Decleir, 1989; Laskowski, 1991; Bugayevskiy and Snyder, 1995). One of the major difficulties in the definition of combined distortion measures is that not all aspects of distortion are expressed in the same units, and therefore cannot be directly compared. Overcoming this problem requires that measurements of different aspects of distortion are properly calibrated. Only recently Laskowski (1998) proposed a method for the standardisation of different distortion measurements that offers a partial solution to the problem of unlike units. However, the more fundamental problem of weighting different aspects of distortion equally is far from solved and remains an interesting area for future research.

## Low-error map projections

11 Distortion measures that allow us to quantify one aspect of distortion (or the combined effect of different types of distortion, see above) for a complete area, are not only useful to compare distortion on different map projections. They can also be used to optimise the parameters of one particular projection, so that minimum overall distortion is guaranteed within the boundaries of the area to be mapped. In most studies on map projection optimisation, optimal values for the parameters of the projection of interest are obtained by minimising the sum of the squares of the errors in local scale along the two axes of Tissot's indicatrix (corresponding with the two directions along which local scale reaches its maximum and minimum value respectively, see above). Projections that are derived in this manner are usually referred to as *minimum-error projections*. The majority of papers on minimum-error projections has been published between 1850 and 1950, before modern computers were available. In these studies the problem of optimisation is necessarily limited to a small number of map projection parameters (mostly one), and to map projections with simple mathematics (azimuthal, cylindrical and conical projections) (Airy, 1861; Young, 1920; Tsinger, 1916; Kavrayskiy, 1934).

12 The introduction of computers created new opportunities for research into minimum-error projections that before then had been impossible to think of. A good example of this is the work that has been done on the development of new low-error conformal map projections. In 1856 Chebyshev stated that a conformal projection has the least possible overall distortion if the sum of the squares of local scale errors over the region is a minimum, and that this results if the region is bounded by a line of constant scale. Mathematical proof of this statement was given by Grave in 1896. Inspired by Chebyshev's theorem several authors have developed new low-error conformal graticules, with sometimes very complicated patterns of isocols (lines of equal distortion), following the outline of the region to be mapped. All these graticules are obtained by applying the following complex-algebra polynomial transformation to the *x, y* - co-ordinates of a standard conformal map projection, and optimising the value of the polynomial coefficients so that overall scale error is reduced:

$$X + iY = \sum_{j=1}^{n} (A_j + iB_j)\,(x + iy)^j \qquad (1)$$

13 One of the best known examples of the use of the technique is the study by Miller (1953), in which he presents a new low-error conformal map projection for Eurafrica. Applying (1) with n=3 he transformed an oblique stereographic projection of the area, changing the lines of constant scale from circles to ovals with the major axis lying along the central meridian. Later he adapted the transformation to derive new conformal projections with oblique oval-shaped isocols for Central Asia and Australia (Miller, 1955; see also Sprinsky and Snyder, 1986). While Miller applied a third-order polynomial to obtain projections with oval isocols, other authors have used higher-order transformations to derive map projections with isocols of more irregular shapes. Reilly (1973) used a sixth-order polynomial for the development of a new conformal map projection for the topographic mapping of New Zealand. Starting from the regular Mercator, he developed a graticule

with lines of constant scale roughly following the outlines of the two main islands. Snyder (1984) took a similar approach to develop a new low-error conformal projection for a 50-state map of the United States, using a tenth-order polynomial. Optimal coefficients for these more complex transformations are obtained by minimising overall scale error using the method of least-squares. This requires iterative solution of a set of simultaneous equations, the number of equations depending on the order of the polynomial (see Snyder, 1985, pp. 86-90).

14 Instead of minimising the sum of the squares of the errors in local scale, some authors preferred to work with finite distortion measures. In 1977 Waldo Tobler presented a new low-error projection for the United States. The projection was obtained by minimising the root-mean-square error for all great-circle distances between selected points on a regular grid covering the area. It has no formulas in the usual sense, but is defined by rectangular co-ordinates for the points used in the analysis. Peters (1978), also a pioneer in the work on finite distortion, optimised parameter values for various world map projections by minimising average distortion for 30,000 distances, connecting randomly chosen pairs of points with uniform probability distribution over the continental surface. In 1989 Canters presented a new method for the development of world map projections with low distortion. Expressing the relationship between the co-ordinates in the map plane and the co-ordinates on the globe by the following two polynomials:

$$x = \sum_{i=0}^{n} \sum_{j=0}^{n-i} C_{ij} \lambda^{i} \phi^{j} \qquad (2)$$

$$y = \sum_{i=0}^{n} \sum_{j=0}^{n-i} C'_{ij} \lambda^{i} \phi^{j} \qquad (3)$$

15 with *x, y* the map projection co-ordinates λ and ϕ the geographical longitude and latitude, and *Cij* and *C'ij* the polynomial coefficients defining the properties of the graticule, he presented a whole set of new projections with intermediate distortion characteristics (neither conformal, nor equal-area) that are suited for global mapping purposes (see also Canters, 1999). All graticules were obtained by minimising the average distortion of finite distances, using a modified version of Peters' distortion measure, and a well-considered strategy for the selection of distances. The attractiveness of the method lies in its use of polynomial type equations, which makes it very easy to impose useful restrictions on the geometry of the graticule (shape of the parallels and the meridians, spacing of the parallels, ratio of the axes, length of the pole line,....) Figure 2 shows an example of a low-error polyconic projection with two-fold symmetry, equally spaced parallels and a correct ratio of the axes, also referred to as the Canters projection, with lines of constant area scale and lines of constant maximum angular distortion superimposed. The Canters projection is used in Belgian geography textbooks and atlases, and has also been adopted for the production of wall maps by Belgian organisations that are active in development co-operation (ABOS, NCOS).

16 Just like the complex-algebra transformation described above (eq. (1)) is used to produce low-error conformal projections, transformation functions can be defined that allow us to

modify the distortion pattern of a standard equal-area projection so that it better fits the shape and orientation of the area to be mapped. The easiest way to accomplish this is by a simple linear stretch of the graticule along one of the co-ordinate axes, followed by a compression of equal magnitude along the perpendicular axis to maintain the equal-area property. Applied to Lambert's oblique azimuthal equal-area projection it changes the circular isocols into ovals, which can be given any arbitrary orientation by a simple rotation of the co-ordinate axes prior to the transformation. A good choice of the affine coefficient and a proper orientation of the axes will lead to less variation of scale and angular distortion for elongated areas. Tobler (1974) presented examples of this simple equal-area transformation for areas of different size. A more complicated transformation of the oblique azimuthal equal-area projection that uses sine functions to alter the spacing of the original graticule in the x- and y direction was proposed by Snyder (1988). It allows the mapmaker to create equal-area graticules with oval, rectangular or rhombic isocols that have less distortion of angles and scale for non-circular regions. Snyder presented examples with oval isocols of different eccentricity for the mapping of the Atlantic Ocean as well as a map of the conterminous United States with rectangular isocols. Canters (1991) proposed two simple polynomial transformations, with appropriate constraints on the value of the polynomial coefficients to make sure that the general condition for an equal-area transformation in the plane is satisfied. Applied separately or in combination these transformations make it possible to derive a variety of low-error equal-area graticules with alternative geometry, starting from any standard equal-area projection. The proposed transformations were used for the development of various new equal-area graticules, useful for maps at the global as well as at the continental scale. Special attention was paid to the development of a new low-error equal-area projection for the fifteen member states of the European Union (Canters and De Genst, 1997; Canters, 1999). Except for Rodos and the easternmost part of Finland, scale error on this new projection is well below 1% for the entire EU (figure 3).

**Figure 3. Fifth-order low-error transformation of the oblique azimuthal equal-area projection for the European Union, with lines of constant maximum scale error (%).**

## Map projection selection

17 The selection of a suitable map projection is an important issue in small-scale map design, especially for the mapping of large areas, where map projection distortion exceeds the threshold of visibility. Over the years a great number of map projections have been proposed. Even for the skilled cartographer, who has a good knowledge of map projection principles, choosing among this variety of existing projections is not an easy task. Map projection selection interferes with several other variables in map design. As such it is not possible to compile a magic table telling us unambiguously what type of map projection is best for a given application. With the increasing use of geographical information systems and the development of new mapping software for personal computers cartographic tools are coming within the reach of an ever growing group of users that are unfamiliar with the principles of cartographic design. This has led some cartographers to determine what is currently known about map design from years of theoretical and practical research and to try to translate this knowledge into a set of rules that can be built into microcomputer-based software. A few researchers have addressed the problem of automated (or semi-automated) map projection selection, taking into account the limitations imposed by the digital medium, as well as the new opportunities for map projection development that are offered by the computer (see above).

18 The choice of a map projection is strongly related to the purpose of the map. The area that will be covered, the ways in which the map is going to be used and the intended audience for whom the map is targeted are all major elements in the decision process. Hence selection should start with the definition of a unique set of requirements that best suits the purpose of the mapping, and that can be translated into a set of map projection properties. Based on these properties a proper choice should be made. In spite of the practical importance of map projection selection only very few authors have treated the subject in any detail. Knowledge on map projection selection mostly appears as a heterogeneous and partly inconsistent collection of rules, repeatedly found in general textbooks on cartography or map projection. Many authors only provide a summary of the map projections that are most frequently used for the mapping of various areas and for different map purposes. These listings are of limited use to a novice cartographer, since they do not provide any insight in the reasoning that brings the cartographer to choose for a particular projection.

19 Snyder (1987, p. 34) is one of the few authors who treat the problem of map projection selection in a systematic way. To facilitate the selection process, he presents a decision tree that is based on: (a) size, shape, orientation and location of the region to be mapped, (b) special distortion properties (conformal, equal-area, equidistant, correct scale along a chosen great-circle), and (c) application-specific considerations (e.g. straight rhumb lines, straight great-circle routes, interrupted designs). Snyder also recognises that world maps cannot be satisfactorily represented by means of conic type projections, and that other selection criteria may be involved in global mapping than in continental or regional mapping (e.g. the decision to interrupt the graticule or not). He therefore makes a clear distinction between world maps and maps of smaller areas. For world maps many types of projections are listed, depending on the type of application. For smaller areas he recommends the use of conic type projections (azimuthal, conical or cylindrical

projections). Maps of a hemisphere, which usually have a circular outline, are also treated as a separate class. For these maps Snyder recommends the use of azimuthal projections.

20 Nyerges and Jankowski (1989) adopted Snyder's scheme to develop a knowledge base for map projection selection. The formalised knowledge was implemented in a prototype expert system for map projection selection, known as MaPKBS or *Map Projection Knowledge-Based System* (Jankowski and Nyerges 1989). MaPKBS, as it was presented in 1989, cannot be regarded as a definite solution to the problem of automated map projection selection. It takes into account only a fraction of the criteria that may be involved in the selection process. However, the development of the system proves that some of the trade-offs involved in map projection selection can be managed by an automated system if appropriate heuristics are defined, with or without the use of AI techniques. Two other attempts to develop expert systems for map projection selection have been reported, one by Smith and Snyder (1989) and one by Kessler (1991). The first attempt has not been described in sufficient detail to review it, the second has never been officially published. According to Snyder (1993, p. 276) MaPKBS, as well as the other two attempts, have been aborted in the research stage because the principals became involved in other projects.

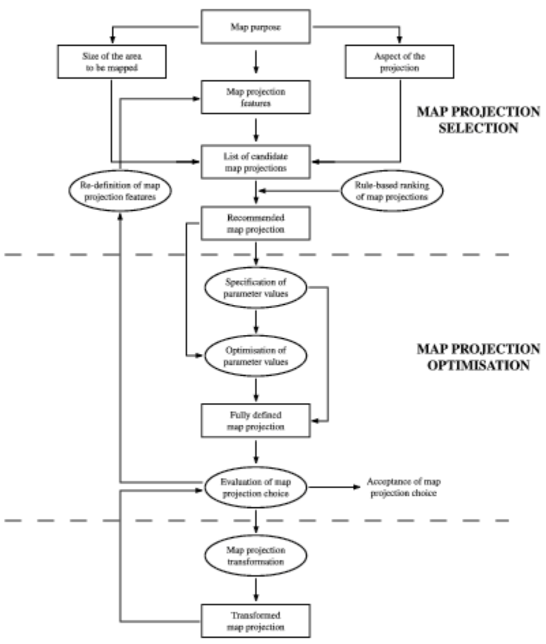
21 Mekenkamp (1990) presents a very simple procedure for automated map projection selection that is based on a set of no more than eleven projections, all belonging to the conic group. For Mekenkamp the selection process consists of answering two fundamental questions: (a) what is the shape of the region to be mapped?, and (b) what is the purpose of the map? He distinguishes between *one-point* (round), *two-point* (rectangular) and *three-point* (triangular) regions, leading to the choice of an azimuthal, a cylindrical and a conical projection respectively. Mekenkamp only considers oblique aspects, since these produce the least distortion for a given area. By letting the position of the meta-pole move without any restriction the general location of the area (near the pole, near the equator, at mid-latitude), which is one of the main criteria in Snyder's scheme, becomes irrelevant to the selection. This strongly simplifies the selection process. Mekenkamp also considers a functional argument in deciding on the type of region. If attention has to be focused on one point (e.g. the location of an airport) or if the relation between two points is to be emphasised (e.g. the traffic flow between two cities), then the region to be mapped will be designated as a one-point region or a two-point region respectively, and the projection class will be chosen accordingly.

22 Next to the type of region the user must select the map property that best suits the purpose of the map (conformal, equal-area, equidistant, straight great-circles, orthographic view). Mekenkamp's procedure is straightforward, it can be easily implemented, and it always produces a unique solution, which is not so for the decision model proposed by Snyder. Still the method has some important restrictions. First, it is not always easy to characterise the shape of a region as round, rectangular or triangular, especially not for large and/or fragmented areas (e.g. different landmasses). Second, the restriction to conic type projections makes that Mekenkamp's approach cannot be used for global mapping purposes without introducing excessive distortion. Finally, since the choice is restricted to oblique aspects of conic projections, the user is not allowed to specify geometric properties that dictate the position of the meta-pole and/or the type of projection, e.g. a straight central meridian, a straight equator, straight parallels... While geometric properties are seldom mentioned in connection with map projection selection, they prove to be very important criteria that are often applied in cartographic practice,

although mostly the mapmaker is not conscious of the fact that he or she is actually applying them (see Canters, 1999, pp. 225-228).

23 Canters and De Genst propose a semi-automated strategy that allows the mapmaker to interactively change the geometry and the properties of the projection until a satisfying solution is obtained (Canters, 1995; De Genst and Canters, 1996; Canters, 1999). In their view the selection process consists of two parts (figure 4). In the first part of the selection process map projection requirements are formulated, and a list of candidate projections is defined. To reduce the complexity of the decision process the total number of map projections from which a choice can be made is kept to a strict minimum. Projections included in the selection procedure are chosen in such a way that each possible combination of map projection properties that can be set, is satisfied by at least one projection. This guarantees that, once properties have been stated, the list of candidate projections will be limited to one or maximally a few projections. If the list contains more than one projection, all candidate projections are ranked by applying a simple set of decision rules. The projection that gets the highest score is selected. In the second part of the process the parameters of the projection that are free to choose are optimised, using an appropriate technique for error reduction. Once the optimised projection is obtained the result is evaluated by the mapmaker, who ultimately decides if the projection is accepted or not. A quick visualisation of the graticule, with isocols showing the distribution of distortion, as well as a brief error report, is essential to allow the mapmaker to perform this evaluation. If parameter optimisation does not produce the expected result, the mapmaker can decide to apply a polynomial transformation to the obtained graticule to achieve a further reduction of distortion (see also section on low-error projections). If this does not lead to a major improvement the original map projection requirements are re-defined and the process is resumed. It is clear that in this approach map projection selection is seen as a dynamic process that can only be partly automated and still requires a great deal of human intervention.

Figure 4. Semi-automated map projection selection (Canters, 1999).




24 Typical for all approaches that have been presented is that the specification of map projection features forms the entry to the selection process. This is not always an easy task. In an ideal situation, a procedure for map projection selection should be able to identify the required features automatically from the type of application, or at least assist the mapmaker in selecting the appropriate features. Unfortunately, knowledge about the relationship between map application and map projection properties is sparse and very often inconsistent. On the other hand no detailed taxonomy of map applications (map function, map use) is available that could serve as a reference for a systematic study of map projection requirements. Hence the few selection strategies that have been proposed so far leave all the responsibility to the user, who is supposed to identify those features that he or she thinks are most relevant to the application. More research on the relationship between map application and map projection properties, including the integration of already existing knowledge, is highly needed in order to develop methods that can assist the non-experienced user in choosing a projection that fits the purpose of the map.

## Conclusion

25 While the importance of map projection is widely recognised in large-scale mapping, its meaning in small-scale map design is less well understood. Most literature about map projection is restricted to a purely mathematical treatment of the subject. In the majority of textbooks on map projection the selection and use of map projections is given little or no consideration. Even skilled cartographers often have limited notion of the role the projection plays in map design and communication, and do not consider it as important

as the other elements in map design. With this knowledge it should not surprise the reader that one regularly comes across examples of non-appropriate use of map projections in print journalism, in the electronic media, and even in scientific publications. Present-day map projection software, with its virtually unlimited possibilities for the selection and modification of map projections, merely increases the risk of making improper decisions, especially if handled by people that lack a basic understanding of map projections.

26 This paper discusses part of the work that has been carried out over the last forty years in three important areas of small-scale map projection research: (a) evaluation of map projection distortion, (b) development of low-error map projections, (c) automated (and semi-automated) map projection selection. The first part of the paper concentrates on the problems associated with the measurement of map projection distortion. Different approaches for characterising map projection distortion at the local scale, and at the finite scale, are reviewed and criticised. The second part focuses on the different methods that have been proposed for the development of low-error map projections. Special attention is paid to polynomial transformation methods, which allow the mapmaker to derive low-error graticules from standard map projections without loss of the properties of the latter. The last part of the paper deals with the selection of map projections, and reviews recent work on the development of methods that assist the mapmaker in choosing a projection that optimally fits the purpose of the map.

27 More efforts should be made to assure that cartographic practice benefits more from the achievements of modern map projection research. Today it is still true to say that there is a wide gap between theoretical research on map projections, which is seldom promoted beyond scientific publication, and map projection use, which far too often relies on the application of simple rules-of-thumb. Most of the topics that have been discussed in this paper have direct implications for everyday map projection use. By embedding techniques for the measurement of map projection distortion and for map projection optimisation in practical tools for map projection selection, the objectivity of the selection process will be increased. This will certainly lead to a more creative and well-considered use of map projections. It is therefore hoped that future developers of map projection tools will include techniques for distortion assessment and for map projection optimisation in their products, and will offer the mapmaker at least some elementary assistance in making a proper map projection choice.

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## ABSTRACTS

The main objective of this paper is to review previous and current trends in small-scale map projection research, and to discuss the work that has been done by this author in relation to similar work by others. Attention is focused on three issues: (a) evaluation of map projection distortion, (b) development of low-error map projections, (c) automated (and semi-automated) map projection selection. It is hoped that this paper will give the reader a good impression of the marked evolution of map projection science over the last forty years, of changes in the way map projections are used (or will be used in the near future), and of the important role the rapidly increasing, widespread use of computers has played in this process.

Dit artikel geeft een overzicht van vroeger en recent onderzoek rond kaartprojecties in de kleinschalige cartografie, en situeert het werk van de auteur binnen het geheel aan onderzoek dat binnen dit domein plaatsvindt. De aandacht is toegespitst op drie thema's : (a) evaluatie van vervorming op kaartprojecties, (b) ontwikkeling van nieuwe projecties met een geringe vervorming, (c) geautomatiseerde (en semi-geautomatiseerde) methoden voor het selecteren van kaartprojecties. Hopelijk geeft dit artikel de lezer een goed beeld van de markante evolutie die het onderzoek rond kaartprojecties in de laatste veertig jaar heeft doorgemaakt, van veranderingen in de wijze waarop kaartprojecties gebruikt, of in de toekomst gebruikt zullen worden, en van de belangrijke rol die het snel toegenomen gebruik van computers in deze evolutie gespeeld heeft.

## INDEX

**Keywords:** map projection research, distortion measures, low-error map projections, map projection selection

**motsclesnl** onderzoek rond kaartprojecties, vervormingsmaten, projecties met een geringe vervorming, selectie van kaartprojecties

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