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Chance and Probability in Poincaré's Epistemology

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Résumé: Hasard et probabilité sont des concepts importants dans l'épistémologie de Poincaré, malgré les difficultés qu'ils introduisent. La notion de hasard est conçue dans un scénario conceptuel où le déterminisme règne encore ; la probabilité, à son tour, est toujours basée sur un ensemble de conventions et d'hypothèses qui cherchent à surmonter l'incertitude qui menace la connaissance scientifique. L'article consiste en une approche philosophique qui vise à clarifier ces notions à partir du point de vue de l'épistémologie de Poincaré et de montrer que le hasard trouve sa place dans les constructions théoriques lorsqu'il est instancié par les ressources du calcul des probabilités.

Abstract: Chance and probability are important concepts in Poincaré's epistemology, despite the difficulties they introduce. The notion of chance is conceived in a conceptual scenario where determinism always prevails while probability, in turn, is always based on a set of conventions and hypotheses that seek to overcome the uncertainty that surrounds scientific knowledge. The paper consists of a philosophical approach which intends to clarify these notions from the point of view of Poincaré's epistemology and to show that chance only finds its place in theoretical constructions when instantiated by the resources of the calculus of probabilities.

1 Introduction

The notions of probability and chance play important roles in Poincaré's epistemology. The author has devoted specific studies to both. Regarding

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probability, Poincaré wrote the book *Calcul des probabilités* which had two editions (1896 and 1912) as well as the article *Réflexions sur le calcul de probabilités*, originally published in the *Revue générale des sciences pures et appliquées* in 1899 and again later as the eleventh chapter of *La Science et l'Hypothèse*, in 1902. Concerning the notion of chance, in 1907 Poincaré published a paper called "Le hasard" [Poincaré 1907] in the *Revue du mois* which was republished with minor changes in 1908 as a chapter of *Science et Méthode* [Poincaré 2011], and again in 1912 as the introduction of the second edition of the *Calcul des probabilités* [Poincaré 1912]. These texts offer an interesting perspective regarding the epistemic relevance of the notions of chance and probability wherein the author explicitly assumes a deterministic standpoint related to the causality which is always an integral part of understanding the phenomena of nature [Poincaré 1907, 257].

However "understanding the phenomena of nature" is far from being simple and immediate, according to Poincaré. He argues that the transition from human experience to its scientific expression requires the data from our senses to be adapted in order to be assimilated rationally [Poincaré 1905, 251]. As a natural consequence of this view, Poincaré believes that the relations expressed by science make up the entire objective scientific knowledge when he denies any objective relevance to contents that are outside the relations expressed by the linguistic structure of scientific constructions [Poincaré 1902b, 288]. On the other hand, this perspective has important consequences particularly when experimental sciences are under analysis if we consider that the very understanding of nature tends to impose an ontological problem which necessarily precedes the epistemic question. This is because the limit of epistemic knowledge serves as the ultimate foundation of Poincaré's thought while also making it possible to conceive of a cumulative progress of scientific knowledge in terms of the true relations that it preserves, avoiding more serious ontological difficulties.

This conceptual schema is the cornerstone to understand what Poincaré called "scientific conception"—the belief that every scientific law is nothing but an imperfect and provisory statement to be replaced in the future by a better law [Poincaré 1902b, 282–283], and that the general sense of theories (i. e., the universal solutions introduced by them) gain precedence over their theoretical entities. In short, mathematical relations experimentally validated overlap the "useful image" given by theoretical entities despite their undeniably close ties with them [Poincaré 1900, 1169].

Considering the above, the main idea to be exposed in this article refers to the impossibility of dealing with the concept of chance properly in the so-called context of scientific theories, if Poincaré's fundamental epistemological theses are rigorously accepted. I intend to argue that, according to Poincaré, chance can only be incorporated into scientific theories with the previous assumption of conventions, hypotheses and its translation in terms of probability. This intermediation is indispensable, since this is the only way to conceive chance in scientific terms.

The scope of this paper is therefore to present the theoretical scenario that surrounds this problem from a philosophical point of view. Section 2 discusses some questions involving determinism and causality; section 3 presents the difficulties involved in the construction of notions of chance and probability and the last section offers some conclusions related to the arguments proposed.

2 Determinism and causality

The introduction of *La Science et l'Hypothèse* takes as its initial argument the distinction between a naïve and a mature perspective of science. The naïve perspective of science is shown as the strong belief in the certainty and perfection of deductions as a way of uncovering the "true nature of things" while conversely the mature conception of science is a less enthusiastic position which acknowledges mainly: 1) the important role of conventions and hypotheses in science; and 2) the mistrust of chains of pure reasoning as a sufficient resource to know about the very nature of things [Poincaré 1902a, 23–24]. Therefore, the mature conception of science irrevocably deals with some degree of indeterminacy when experience comes into play.

From this standpoint, physics becomes a great challenge due to the variety of principles, conventions and hypotheses which are either explicitly or tacitly assumed, to construct theories. These hypotheses have various functions and forms of expression which Poincaré presents in two texts in different ways [Poincaré 1902a, 24], [Poincaré 1900, 1166–1167]. These functions and forms of expression were combined and presented by Heinzmann & Stump in a list of four essential types—verifiable hypotheses, indifferent or neutral hypotheses, natural hypotheses and apparent hypotheses [Heinzmann & Stump 2013]. All of these have specific functions and at different moments of the conception of a theory are assumed to play crucial roles in the theoretical interpretation of the facts.[1] In this context, it is possible to understand why Poincaré considers that physics in fact provides a metaphor for the world. This is that, despite

1. "Poincaré makes hypotheses central to his analysis of science. He distinguishes four kinds of hypotheses, which are actually given in two lists of three each. Combining the two lists, these are: 1° Verifiable hypotheses; 2° indifferent (or neutral) hypotheses; 3° natural hypotheses; 4° apparent hypotheses. By verifiable hypotheses, Poincaré means general statements that have been confirmed experimentally. These are the backbone of natural science and can be seen non-controversially to be compatible with standard accounts of, say, the Hypothetical-Deductive method. [...] Indifferent hypotheses are ontological in nature and are, for example, mechanical models of underlying mechanism. Poincaré stresses the fact that these can frequently be exchanged without sacrificing empirical accuracy. [...] Natural hypotheses are necessary conditions for science but are experimentally inaccessible [...] Finally, apparent hypotheses are definitions or conventions rather than actual claims about the world. They therefore may not be considered to be hypotheses at all but are however often mistaken for such. Poincaré argues that (metric) geometry is the hypothesis most prone to such confusion" [Heinzmann & Stump 2013].

the central importance of predictability and the strict reference to experience, the distance which exists between the type of explanation offered by scientific theories and the real nature of the world allows a peculiar kind of approach which is able to disclose the *true relations* of the world [Poincaré 1900, 1169], [Poincaré 1902b, 289].

Principles and conventions are examples of hypotheses which determine the most basic structures of scientific knowledge but which are admittedly unverifiable. One is the belief in the unity and simplicity of nature [Poincaré 1900, 1164–1165]. Another would be the vague and elastic principle of sufficient reason which takes the form of the assumption that "l'effet est une fonction continue de sa cause" [Poincaré 1900, 1166], or even of a "croyance à la continuité, croyance qu'il serait difficile de justifier par un raisonnement apodictique, mais sans laquelle toute science serait impossible" [Poincaré 1899, 269]. The *rapprochement* between causality and continuity tacitly postulated in Poincaré's version of the principle of sufficient reason also consists of a natural hypothesis which needs to be viewed as an analytical principle rather than an empirical truth.

However, this conception brings up curious consequences. Sometimes, Poincaré seems aligned to the classical tradition of deterministic perspectives which, interestingly, are at the core of the formerly-defined naïve conception of science. When the author explicitly states that "nous sommes devenus des déterministes absolus. [...] Tout phénomène, si minime qu'il soit, a une cause, et un esprit infiniment puissant, infiniment bien informé des lois de nature, aurait pu le prévoir dès le commencement des siècles" [Poincaré 1907, 257], his ideas seem very close to the thinking of Pierre-Simon Laplace. This excerpt from Laplace's work below gained iconic importance in debates about determinism in physics towards the 20th century:

> Nous devons donc envisager l'état présent de l'univers, comme l'effet de son état antérieur et comme la cause de celui qui va suivre. Une intelligence qui, pour un instant donné, connaîtrait toutes les forces dont la nature est animée, et la situation respective des êtres qui la composent, si d'ailleurs elle était assez vaste pour soumettre ces données à l'analyse, embrasserait dans la même formule les mouvements des plus grands corps de l'univers et ceux du plus léger atome : rien ne serait incertain pour elle, et l'avenir comme le passé, serait présent à ses yeux. [Laplace 1840, 3–4]

Of course I would not claim that Laplace's entire conception of probability is summed up in these few lines. However this excerpt is emblematic due to the wide influence achieved on the posterior scientific tradition, stigmatized, for instance, by Maxwell's demon, a well-known argument that problematizes the general validity of the second law of thermodynamics.[2] To sum up, the concep-

2. Maxwell's argument can be summarized as follows: the second law of thermodynamics is only true because we are solely able to conceive of bodies as masses

tual difficulty involved in this question is as follows. The deterministic position aligned to the naïve conception of science can no longer be corroborated at the beginning of the 20th century, as Poincaré argues explicitly [Poincaré 1902a, 24], [Poincaré 1892, v] and yet some kind of determinism must follow on from the acceptance of the principle of sufficient reason which is a central hypothesis in Poincaré's epistemology.

In this scenario, the relevance of discussing the concept of probability takes place.

Poincaré's standpoint on probability appears to be closely linked to the development of ideas concerning the subject at that time considering that the role of probability and the deterministic perspective were intrinsically related notions. The very decline of the Laplacian point of view on determinism during the second half of 19th century may be considered as very closely linked to the important changes on the notion of probability [Kamlah 1987]. Jan von Plato [von Plato 1994] points out the most important factors of this change in a very precise way. We shall now summarize the most relevant arguments concerning the introduction of mathematical complexity in the fields of probability and the new challenges for physics.

As far as mathematics is concerned, the fall of the classic (Laplacian) perspective of probability derives from the development of more complex resources when compared with those formerly available. The classical perspective was centred on a very simple attribution of mathematical quantities and rules to "real" circumstances. Composite events, for example, were thought to be compositions of several elementary events whose probability was the sum of the probabilities of these elementary events. This is the main idea that led to the invention of the calculus of combinatorics, based on the hypothesis of the existence of symmetry of equipossible cases [von Plato 1994, 4, 6]. The insufficiency of this perspective led mathematicians towards a conceptual change which overwhelmed the finitary scope of probability thus entering the realm of pure mathematics and abandoning the idea that probability must just be a field of the empirical application of mathematics.

The development of physics also led to new ideas on probability, due to the development of statistic physics in the middle of 19th century as a by-product of theories devoted to atomistic structure of matter. These theories gradually began to be treated from the standpoint of the probability of causes mainly by means of a Bayesian approach. Specifically, the most important difficulties

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and not as separated molecules. In theory though, the latter could be possible for a being whose faculties are much more sharpened than ours, but still finite. In this case, his capability to deal with molecules should make it possible to invalidate the second law of thermodynamics [Maxwell 1872, 308]. The determinism that surrounds this argument is centered in the idea that the complete comprehension of nature is avoided due to the limits of our knowledge alone. This is why the problem deserves attention, according to Maxwell—the stability or instability of a system derives from the particular conditions through which we experience the world. See [Maxwell 1873, 440].

involved the explanation of irreversibility, the problem of specific types of heat and the behaviour of radiant heat [von Plato 1994, 10].

In which sense then, does Poincaré assume a deterministic point of view? Any satisfactory answer to this question needs to consider the fact that, despite any similarities, it is not possible to consider Poincaré to be aligned with the Laplace's ideas. The determinism that Poincaré professes is essentially different from, say, naïve determinism, if we accept as a given that it plays the role of a valid principle in the context of his structural point of view. In other words, Laplace's determinism is ontic and epistemic but Poincaré's is only epistemic and there is no way to acknowledge truth in anything which surpasses the relations set out in good theories. This *status quo* imposes an unavoidable degree of idealization on the facts made evident by science. In Poincaré's writings, the notion of causality must be understood as necessarily bonded to this context.

From this new standpoint, Poincaré conceives causality in terms of former and latter states of experience in which an effect consists of a continuous function of its cause and where the notion of time is required to build this relationship. The connection between earlier and later states of nature must be validated with reference to experience but these relations are mediated by translations of facts in terms of their theoretical entities and properties assumed hypothetically.

Bertrand Russell's claim concerning the dispensability of the notion of causality may be seen as a frontal attack on this conception. According to the English philosopher, the notion of causality is dispensable in physics [Russell 1913, 1] and only pervades the books of philosophers simply because "the idea of a function is unfamiliar to most of them" [Russell 1913, 14]. Russell argues that the superficial and imprecise notion of cause is unable to respond to the demands of contemporary science and should be replaced by the concept of the determinant function which has two advantages—firstly, the "problematic" temporal perspective is no longer of great importance when functions are under the spotlight and secondly, the idea of a function ensures a more precise approach to the phenomena under study [Russell 1913, 26].

Poincaré passed away in 1912 but there are good reasons to believe that he would not have accepted the Russellian thesis of the dispensability of the notion of cause formulated in 1913. The reason for this is that, unlike Russell, Poincaré thinks that the *meaning* of a function which describes an empirical phenomenon is given with reference to causality and shielded by conventions and hypotheses. Intuition leads this process by means of its tool set, where we can find, for instance, analogies afforded by the similarity of the structures that allow an "overall view" of science, represented mainly by the sense of mathematical order [Poincaré 2011, 31]. Thus, the notion of cause is essential, especially for the construction and invention of science. An argument in favour of this perspective is introduced by Max Kistler namely the fact that the notion of cause is not fundamental. In particular physical theories do not illegitimate the philosophical analysis of this notion because the objects of physics are distinct of causal judgments. Physics deals with values of variables characterizing properties of substances whereas causal judgments pay attention to concrete events [Kistler 2011, 108–109].

Russell's argument, *mutatis mutandis*, is part of a problem which is analogous to the problem raised by his debate against Poincaré concerning the foundations of mathematics. At that time, his refusal to recognize the role of intuition in the principle of complete induction led him, according to Poincaré, to an incomplete view of what should be the development of mathematics [Poincaré 2011, 105]. The paradoxes on set theory were interpreted by the French mathematician as consequences of foundational problems and as a sort of deviation from what is essential in mathematics—the "true" mathematics in fact [Poincaré 2011, 92]. Similarly, even if we take for granted that any formalized theory as a whole dismisses the notion of cause (something that still may appear too controversial), the notion of causality identified as the principle of sufficient reason is a reference to intuition in an attempt to seek the establishment of differential equations binding the former and latter states of the phenomena under analysis [Poincaré 1900, 1163]. This is a theoretical schema which has been used since the advent of modern science [Madrid 2013, 37].

Poincaré considers differential equations to be at the core of scientific theories because they are strictly linked to causality and continuity. They enable causal relations to be considered in terms of domain and the image of functions and also incarnate the expected continuity of phenomena, representing them as mathematical relations. Naturally, this essential interaction defines the very spirit of mathematical physics from its first steps as a science. Rather than being a mere "application" of mathematical relationships to the results of physical measurements, mathematics and physics are deeply and inevitably interconnected because physics can only properly define its own subjects and relations with the assistance of mathematics.

Hence, the intention is to assign general validity to the pertinence of the notion of cause in the range of contemporary scientific knowledge, discussions concerning how causality and determinism can be understood and validated gained greater significance—and, mostly, new solutions—after the advent of quantum mechanics (for obvious reasons). The struggles between neo-Kantians and neo-positivists concerning the difficulties raised by Heisenberg's principle of uncertainty and the probabilistic interpretation of Max Born are good examples of this state of affairs [Leite 2012, 166]. Even Poincaré was dismayed by the advent of quantum mechanics, considering that this revolution would result in a deep change insofar as differential equations would no longer be able to represent the essence of scientific truth (cited in [Madrid 2013, 36]).

The contributions of Grete Hermann regarding this subject are particularly instructive because she suggests a distinction between causality and determinism as unbounded notions. According to Hermann, the undeniable decline of the so-called Laplace demon does not call the validity of the notion of causality into question since there should be a distinction between "the concept of predictability of natural events given by calculus, and on the other hand, the causal chain of natural processes" [Hermann 1996, 97–98]. The similarity of Hermann's and Kistler's arguments is quite evident and does not require a detailed exposition.

From Hermann's point of view, strict determinism finally does not have a place in contemporary physics because uncertainty and probabilistic features which have a permanent place in the quantum theory need to be taken for granted. However, causality is not seen as a problem from this theoretical point of view. Since the advent of quantum theory, we have observed the recognition of a limit concerning the possibility of predictions involving phenomena of nature. This in general also looks like a reformulation in the spirit of the basic arguments invoked by Poincaré as far as predictability was concerned.

The gap between science and nature, which turns the former into a kind of metaphor for the world (mainly due to the recognition of the limits of our intellect in apprehending and understanding phenomena) ultimately justifies the postulated determinism—even where the causes cannot be seen, we are compelled to believe that they do exist. As Jean Cavaillès pointed out, "les probabilités apparaissent comme la seule voie d'accès envisageable au chemin de l'avenir dans un monde qui n'est plus doté des arêtes vives de la certitude mais se présente désormais comme le royaume flou des approximations" [Cavaillès 1940, 154].

This belief and the conditions for the realization of science both argue in favour of a deterministic point of view which is of relevance when attempting to understand how Poincaré conceives chance and probability since physics takes place in the tension between the requirements imposed by rationality and the fragmentary character of our experiences. Precisely the mismatch between the perfection of mathematics and the nebulosity of experience is exactly why the calculus of probabilities in physics gains importance.

3 Probability and chance

The central role of probability on Poincaré's thought is well known since the author expressly wrote that "condamner ce calcul [des probabilités], ce serait condamner la science tout entière" [Poincaré 1899, 262]. However one little-noticed aspect (the central argument of this article) is that, according to Poincaré, in physics the calculus of probabilities plays the role of a kind of instantiation of chance. I shall now try to develop this idea further.

3.1 The emergence of probability on Poincaré's works

Poincaré's interest in the field of probability and other subjects that surround it may be considered as something that "penetrated Poincaré's work almost by force", as Laurent Mazliak stresses mainly as a consequence of his work devoted to two different theoretical branches of knowledge, namely, celestial mechanics (particularly, the solution of the well-known three-body problem) and thermodynamics (above all the kinetic theory of gas) [Mazliak 2015, 176].

Considering the three-body problem, probability comes into play when Poincaré is dealing with the use of integral invariants. After the introduction of the first theorem of the eighth paragraph,[3] he is compelled to appeal to probability in order to satisfactorily develop the corollary which follows from this first theorem, which refers, in turn, to the exceptionality of certain trajectories which may exist in an ideal space [Poincaré 1890, 71].

Poincaré alleges that the construction of this problem has no precise sense by itself and that it needs to be completed by the introduction of three kinds of probability—the initial probability of a point P belongs to a region r_0, the probability that some trajectories do not cross r_0 more than k times (a condition that establishes the exceptionality of the trajectory) and finally the probability that this trajectory does not cross r_0 in a settled period of time [Poincaré 1890, 71]. This construction is enlightening in order to show how the essential nature of the hypotheses introduced to define probability in the general schema of solution of this problem according to Poincaré.

In turn, Jeremy Gray argues that the very interest of Poincaré's work on the calculus of probabilities emerged due to the difficulties found in thermodynamics [Gray 2013, 518]. It is important to underline the fact that Poincaré credited this branch of knowledge with great relevance even suggesting that it would be possible to "raise exclusively on thermodynamics the entire edifice of mathematical physics", due to the solidity of its principles (mainly Meyer's principle of conservation of energy and Clausius' principle of dissipation of entropy) [Poincaré 1892, v]. The ideality of several theoretical entities involved in the discussions of thermodynamics—let us stress, for instance, how Poincaré declares the hypothetical character of perfect gases [Poincaré 1892, 153],—as well as the central role played by hypotheses and conventions—something verified, for example, when the constructions of curves which determine the isothermal expansion of gases are established *if* the validity of the laws of Mariotte and Gay-Lussac are taken for granted [Poincaré 1892, 151]—make thermodynamics an outstanding example of Poincaré's ideas if we consider that these features necessarily involve a kind of probabilistic approach.

3. "Théorème I. Supposons que le point P reste à distance finie, et que le volume $\int dx_1 dx_2 dx_3$ soit un invariant intégral ; si l'on considère une région r_0 quelconque, quelque petite que soit cette région, il y aura des trajectoires qui la traverseront une infinité de fois. En effet le point P restant à distance finie, ne sortira jamais d'une région limitée R. J'appelle V le volume de cette région R" [Poincaré 1890, 69].

However, perhaps the most relevant aspect of thermodynamics referring to chance and probability is indicated in the following excerpt, where the author aims to foresee the future of physics:

> Dans quel sens allons-nous nous étendre [dans la physique mathématique], nous ne pouvons le prévoir ; peut-être est-ce la théorie cinétique des gaz qui va prendre du développement et servir de modèle aux autres. [...] La loi physique alors prendrait un aspect entièrement nouveau ; ce ne serait plus seulement une équation différentielle, elle prendrait le caractère d'une loi statistique. [Poincaré 1905, 231]

In the kinetic theory of gases, the impossibility of determining the behaviour of each molecule compels us to interpret the system as a whole from a simplified perspective, i.e., from the average behaviour of gas molecules, which ultimately provides a description similar to that obtained by experimental verification [Poincaré 1900, 1163]. According to the author,

> Dans la théorie cinétique des gaz, on suppose que les molécules gazeuses suivent des trajectoires rectilignes et obéissent aux lois du choc des corps élastiques ; mais, comme on ne sait rien de leurs vitesses initiales, on ne sait rien de leurs vitesses actuelles. Seul, le calcul des probabilités permet de prévoir les phénomènes moyens qui résulteront de la combinaison de ces vitesses. [Poincaré 1899, 263–264]

Thereby, despite the admittedly artificiality of the solution (which is also the best on offer), the kinetic theory of gases provides a kind of solution which can fulfil the lack of a scientific model for statistical equilibrium [Poincaré 1906, 164–165]. Thus, due to its capital importance, defining probability became a central task for any interpretation of Poincaré's epistemology.

3.2 Chance in mathematics?

Given Poincaré's deterministic conception, chance has a specific meaning which the author derives from two "negative" limits. Firstly, the notion of chance as such cannot be understood as something with no link at all to causal relations. Instead, chance must be considered a state of affairs wherein it is beyond our capability to grasp the constituents that interact in order to determine rigorously the causal changes involved in the process with the necessary degree of detail and accuracy [Poincaré 1907, 259–260].

Secondly, the distinction between "what we know" and "what we do not know"—or the knowledge-ignorance duality—is at the core of the notion of chance. From the author's point of view, it is not correct to consider chance simply as a synonym for our ignorance about the world. In this sense, one can see an implied critique of Laplace's concept which assigns any circumstances whose true causes are unknown and which have no characteristic of regularity to chance [Laplace 1840, 2–3]. This conception led Laplace to consider chance and probability as contrary notions, when he distinguished "rapports des probabilités" and "des anomalies dues au hasard" [Laplace 1840, 90]. These two conceptions have the fact that they impose negative limits on knowledge in common but they are quite different in their essence. The former necessarily involves an inherent rationality while the latter is conceived as a set of anomalies which are beyond any kind of known law.

The shift proposed by Poincaré regarding this position is based on the fact that he also considers chance from a standpoint which is not disconnected from the notion of law and therefore, cannot just refer to the measure of our ignorance or lack of regularity. Chance and probability are thus conceived as linked in their essence and this idea is supported by the distinction introduced by Poincaré between phenomena "sur lesquels le calcul des probabilités nous renseignera provisoirement" and "ceux qui ne sont pas fortuits et sur lesquels nous ne pouvons rien dire tant que nous n'aurons pas déterminé les lois qui les régissent" [Poincaré 1907, 258]. The "laws of chance", a term that he borrowed from Joseph Bertrand, expresses the probability of events, and not the absence of regularity:

> Une question de probabilités ne se pose que par suite de notre ignorance : il n'y aurait place que pour la certitude si nous connaissions toutes les données du problème. D'autre part, notre ignorance ne doit pas être complète, sans quoi nous ne pourrions rien évaluer. Une classification s'opérerait donc suivant le plus ou moins de profondeur de notre ignorance. [Poincaré 1912, 30–31]

An apparent contradiction is embodied by considering how mathematical problems can be understood in this context. Poincaré offers several examples of the calculus of probabilities applied to the range of pure mathematics. The perfection required in mathematical constructions does not allow any consideration of the possibility that the same indeterminacy which surrounds, for instance, the constructions of physics can exist. Considering that Poincaré tends, in the long run, to adopt a constructivist perspective of mathematics, the claim of any kind of indeterminacy in the constructions and operations of mathematical objects is even more unacceptable.

In order to solve this question, my suggestion is that Poincaré does not conceive of mathematical problems as authentic probabilistic questions. The way of presentation and the relevance of the techniques of solution are factors which approximate certain mathematical problems by considering these *in terms* of probability. This is a typical approach given the creative and inventive virtues of the human intellect and an example would be when we are dealing with the π number:

> Il semble que le nombre π nous paraît choisi au hasard, parce que sa genèse est compliquée et que nous raisonnons inconsciemment

> sur lui, comme nous avons coutume de le faire sur les effets produits par un ensemble de causes compliquées. [Poincaré 1912, 23]

The very sense that Poincaré assigns to the term “probability” in his study of the three-body problem also seems to corroborate this view [Poincaré 1890, 71–73].

3.3 Modelling chance

Therefore, if we consider the arguments exposed in section 3.2 to be true (above all, the idea that the way of presentation and the relevance of the techniques of solution work for similar mathematical problems and authentic probabilistic problems), clearly the author believes that probability exists in the strict sense of the word when dealing with empirical phenomena. These, in turn, involve a necessary step toward idealization and abstraction, in order to make the experiences and the perfection of mathematical forms compatible as already explained. This process of idealization/abstraction of the physical phenomena occurs on three closely related levels: 1) a preliminary act of choosing the most relevant facts to help understand causal series; 2) the translation of crude facts into scientific facts; and 3) the assimilation of these facts in terms of mathematics. Only after this process has been worked through can chance be treated as probability, when it becomes fitted to be expressed in mathematical terms.

The first level brings together the choices which lead us to identify the relevant events to “connaître autre chose qu’un fait isolé” which is achieved through a process of induction and generalization [Poincaré 1900, 1163]. Taking for granted the hypotheses of the unity of nature and the determinism of causal relations, facts must be chosen with a view to increasing understanding, to “augmenter le rendement de la machine scientifique” [Poincaré 1900, 1164]. This process requires making a selection from innumerable available facts those which can contribute to increasing the impression of the world’s harmony [Poincaré 2011, 12].

However, one must keep in mind the fragile nature of the choices involved in this first level of abstraction and idealization of the facts. The determinism and the monism of nature professed by Poincaré presupposes both great complexity and interdependence of the causal relations as well as the inability of our intellect in order to establish a precise distinction for each of them. Given this circumstance and adopting the maxim that “mieux vaut prévoir sans certitude que de ne pas prévoir du tout” [Poincaré 1900, 1164], intuition guides the choices of the intellect in order to decide which are the leading facts and leads to a generalization of the laws governing the regularity of phenomena, slicing and combining them in the way which seems the most justifiable and the least artificial possible [Poincaré 1907, 265–266]. In that spirit, when introducing the laws of chance, Poincaré pays less attention to what he calls "the coincidence of causes"—those circumstances which, although apparently dissociated, contribute to an unexpected result being obtained. The factors are linked and interdependent but our need to choose only a few determinants may lead us to neglect data that may prove to be decisive in the resulting consequences although initially insignificant [Poincaré 1907, 265].

With regard to the second level—i.e., the translation of crude facts into scientific facts—it is noteworthy that our perspective of the world derives from our experiences, which, as indicated above, have a remarkable fragmentary character and depend ultimately on the way in which crude facts are translated into scientific facts. This translation, in turn, indicates the facts which are the most significant to elucidate the valid relations reported by phenomena and also, in each fact, which are the most important attributes to be emphasized, and how it is possible to separate the wheat from the chaff in order to define and develop true relations.

Poincaré is explicit when he argues that the intention of building a science without use of preconceptions is an unattainable ideal. He supports his opinion with two main reasons. Firstly, the existence of a foregone conception of the world from which one cannot easily break away and secondly the fact that the very language employed in the translation of crude facts into scientific facts is saturated with "des idées préconçues inconscientes, mille fois plus dangereuses que les autres" [Poincaré 1900, 1164]. Thus, the translation of facts sounds like a reformulation whose scope is the simplification or restriction of experiences to make them expressible in mathematical terms but whose final product cannot be taken as completely immune or aseptic and must intransigently keep its reference to experience. This is the challenge involved in the translation of the world through science—keeping what is essential (and, thus, showing what can be the object of valid relations empirically testified) while a rough cut is made when we consider the fullness of its nature. Ultimately, a certain experimental phenomenon is translated in terms of a point on a curve which expresses the regularity of a set of relations and their correlated phenomena [Poincaré 1900, 1168].

The third level specifically deals with the assimilation of the fact with regard to mathematical theory and thus we realize that intuition is the faculty which guides the process as a whole. At this point, probability is the most important factor, for if theories are ways to shed light on valid relations between former and latter states of affairs and if these theoretical constructions are made under the restrictions mentioned above, some degree of uncertainty will always be part of any predictions. There is therefore no previous rule for the assimilation of scientific facts in mathematical terms, something that, according to Poincaré, is made up of two distinct moments:

> [...] tout problème de probabilité offre deux périodes d'étude : la première, metaphysique pour ainsi dire, qui légitime telle ou telle convention ; la seconde, mathématique, qui applique à ces conventions les règles du calcul. [Poincaré 1912, 29]

The application of mathematics in the calculus of probabilities (i.e., the "second step" explicated by Poincaré) does not involve any significant challenges with regard to the epistemology underpinning the process. The great difficulty inherent to this issue precedes the presentation of the problem in mathematical terms and is centred in the instance that Poincaré calls metaphysical, perhaps for lack of a better term. It is directly related to, say, the way the theoretical entities that compound scientific facts correspond and are associated with the mathematical data of the problem, and the only guide for this process is the intuition that leads the choices to be made. The following excerpt from Poincaré's work is interesting in this regard:

> [La loi des erreurs (ou courbe normale)] ne s'obtient pas par des déductions rigoreuses ; plus d'une démonstration qu'on a voulu en donné est grossière, entre autres celle qui s'appuie sur l'affirmation que la probabilité des écarts est proportionnelle aux écarts. Tout le monde y croit cependant, me disait un jour M. Lippmann, car les expérimentateurs s'imaginent que c'est un théorème de mathématiques, et les mathématiciens que c'est un fait expérimental. [Poincaré 1912, 170–171]

The problem involved in this process can be summed up as follows—the premise assumed ("the probability of variations is proportional to the variations") is neither mathematical, experimental nor even demonstrable. It is a hypothesis assumed to be true because it sounds "reasonable" and nothing more than that.

Poincaré offers the example of two methods that can be chosen to solve a simple problem of probability [Poincaré 1899, 262]. If two dice are thrown, what is the probability that at least one of them results in the number 6? Two different forms of reasoning are proposed and lead to different results (the correct solution, 11/36, and the incorrect solution, 6/21).[4] In a sense, the methods are equivalent, as they are supported by rational justifications concerning the choices involved, in order to provide a mathematical structure supposedly able to apprehend the realm of the world. However, these two different ways of reasoning end up giving different results—although approximate—and therefore, cannot be considered equivalent for obvious epistemic reasons.

However, what makes the first convention more plausible than the second? Ultimately, the "right" structure is the one which provides a solution which is more compatible with the empirical data obtained. These are the touchstones for the right choices made at the "metaphysical" level just because this degree

4. "Je jette deux dés ; quelle est la probabilité pour que l'un des deux dés au moins amène un six ? Chaque dé peut amener six points différents : le nombre des cas possibles est $6 \times 6 = 36$; le nombre des cas favorables est 11 ; la probabilité est 11/36. C'est la solution correcte. Mais ne pourrais-je pas dire tout aussi bien : Les points amenés par les deux dés peuvent former $(6 \times 7)/2 = 21$ combinaisons différentes ? Parmi ces combinaisons, 6 sont favorables ; la probabilité est 6/21" [Poincaré 1899, 262].

of dependence on presupposed hypotheses cannot be banished from the step that associates facts to mathematical frames. Poincaré's arguments aimed at identifying conditions which are "too simple" from the point of view of the calculus of probabilities shows this clearly:

> [Conditions trop simples] ce sont celles qui conservent quelque chose, qui laissent subsister un invariant. [...] Elles sont trop simples, si elles conservent quelque chose, si elles admettent une intégrale uniforme ; si quelque chose des conditions initiales demeure inaltéré, il est clair que la situation finale ne pourra plus être indépendante de la situation initiale. [Poincaré 1907, 269]

The excerpt above is exemplary of how empirical data assume a form that can be attained by the language of science, by means of this "metaphysical" assimilation. This "something that remains unchanged", the invariant, becomes the main reference able to guarantee the constancy of the causal relations. In some circumstances, it is even possible to put empirical data in the background in the name of the validity of the mathematical framework—as Poincaré postulates when discussing the chance that is involved with playing roulette:

> Les données de la question, c'est la fonction analytique qui représente la probabilité d'une impulsion initiale déterminée. Mais le théorème reste vrai, quelle que soit cette donnée, parce qu'il dépend d'une propriété commune à toutes les fonctions analytiques. Il en résulte que finalement nous n'avons plus aucun besoin de la donnée. [Poincaré 1907, 267]

To sum up, the causal relations which exist in the world (metaphysical relations, in his terms) are transformed by the process described above into mathematical equations. Experience continually provides a basis for truth-value propositions assumed as valid, but intense intellectual work is also needed to establish conventions and regulatory hypotheses which enable the modelling of scientific facts and their alignment with theories that underpin them.

It therefore seems clear that theories are constructed in function of the hypotheses assumed previously. However a particular kind of hypotheses is required to build theories and, as Poincaré stresses, effective uses of the calculus of probabilities are those where different hypotheses give the same conclusions [Poincaré 1899, 269]. This condition leads to an important consequence, in harmony with the epistemological thought of Poincaré—the artificiality of nominalism is avoided because, in the long run, assumed hypotheses shall fade in order to give evidence to true relations, considering that these hypotheses become indifferent when they lead to analogous conclusions, no matter how essential they initially seem.

3.4 Chance instantiated: probability

What then does Poincaré call chance? It is not a set of phenomena which escapes all causal relations (once causal relations are postulated in all circumstances, even when not perceived) and it is not simply “a measure of our ignorance” (considering that chance, once subjected to the “laws of chance” cannot have only a negative sense). Chance, as conceived by Poincaré, denotes situations that, despite being determined by causal relations, are beyond our ability to dissect them in detail, a circumstance which forces us to adopt approximate behaviour related to them which occurs through the use of mathematical tools available for the calculus of probabilities. When chance escapes from any possibility of mathematical expression, one shall consider that we are dealing with Poincaré’s third degree of ignorance, where “généralement, on ne peut plus rien affirmer du tout au sujet de la probabilité d’un phénomène.” [Poincaré 1899, 264]. Thus, all the instances of chance that can be brought to scientific treatment (and, as consequence, those which do matter) must be expressed in terms of probability.

Schematically, if we consider that scientific theories consist of a set of propositions in a given language $\mathcal{L}$, which gathers scientific facts (i.e., seeing exclusively the level of object-language), we realize that there is no space, in compliance with a deterministic point of view, to the chance $\mathcal{H}$ (from *hasard* in French) in any possible instance of $\mathcal{L}$. The concept of probability $\mathcal{P}$, in turn, is encompassed by $\mathcal{L}$ and can even play an important role in it. Accordingly, the encompassment of $\mathcal{H}$ into $\mathcal{L}$ should occur only through the possibility of translation $\mathcal{T}$ of $\mathcal{H}$ into $\mathcal{P}$, such that $\mathcal{H} \to \mathcal{T}(\mathcal{H}) = \mathcal{P}$. Considering that, the notion of chance must not be taken as typical in $\mathcal{L}$ itself but exclusively in the metalanguage of $\mathcal{L}$. That is why one can consider that probability is a resource to the instantiation of chance into the range of scientific knowledge expressed by $\mathcal{L}$. These relations must assume a mathematical form, something that is offered by the resources of the calculus of probabilities.

In the article “Le hasard”, Poincaré offers two examples regarding the explanation of how one may understand the notion of chance properly. These are the minimum causes which produce great effects (as in the case of unstable equilibrium of an inverted cone) and the variable degree of complexity of causes which involve a high number of variables, like in the case of the distribution of grains of dust in the air (which depends on the weight of each particle, the variation of air currents, the interaction of the particles with each other, and so on) [Poincaré 1907, 259–260, 263–264]. Still, in *La Science et l’Hypothèse*, when treating on the calculus of probabilities, Poincaré argues that the classification of the problems concerning probability can be carried out in various ways. He proposes a division of these problems with reference to their degree of generality, as well as the degree of ignorance related to data which must increase the amount of strategies with relation to the solution of the problems [Poincaré 1899, 263–264].

Taking for granted these arguments, the common point with regard to the classification of the problems of probability and chance are, in general, the variability of the knowledge of the data that matter to the solution of the problem introduced, as well as the veiled or explicit assumptions that drive the solutions. When Poincaré exemplifies the chance in the excerpts quoted above, he does so from properties that would be variable in terms of their degree of generality, but are rooted in the second degree of ignorance—i.e., when the scientific laws are established but the initial or final states of affairs (depending on if we're dealing with probability of effects or with probability of causes) are not well known [Poincaré 1899, 263–264].

In this sense, the proposed models are, as a rule, simplifications of a set of much richer and more extensive of complex variables and/or imperceptible causes. In some circumstances, we are even compelled to adopt models that we know do not correspond to reality, due to the fact that our ignorance of all the data of the problem leaves us no choice [Poincaré 1902a, 203]. Newtonian mechanics and the kinetic theory of gases are recurrent examples in Poincaré's work concerning this subject. The troublesome three-body problem gives an idea of the degree of increasing complexity found in circumstances that are still far from the "real" interplay of forces which govern the equilibrium of orbits in the solar system [Poincaré 1890, 263].

Poincaré did not hide his dissatisfaction with explanations regarding the difficulties that mainly involve the highest degrees of generality, of ignorance and the theory of errors [Poincaré 1899, 269]. Ultimately, he postulated the principle of sufficient reason as the basis for this process, when it assumes the form of a belief in the continuity of the relations expressed by mathematical regularity:

> Pourquoi donc est-ce que je cherche à tracer une courbe sans sinuosités ? C'est parce que je considère a priori une loi représentée par une fonction continue (ou par une fonction dont les dérivées d'ordre élevé sont petites), comme plus probable qu'une loi ne satisfaisant pas à ces conditions. Sans cette croyance, le problème dont nous parlons [la prescription d'une loi expérimentale] n'aurait aucun sens ; l'interpolation serait impossible ; on ne pourrait déduire une loi d'un nombre fini d'observations ; la science n'existerait pas. [Poincaré 1899, 268]

This is the way in which probability instantiates chance—phenomenal chance is included in scientific study by means of the assumption of hypotheses that allow its expression in mathematical terms but these hypotheses lose their primacy in favour of the relations they bring to light.

4 Conclusion

Taking into account the relation between reason and experience in the formulation of scientific knowledge as conceived by Poincaré, it would be plausible to consider that reason appeals to experience according to the dimension of its need and while seeing its own limitations. The two extreme examples of the different ways of combining reason and experience are arithmetic, where experience is useful only in order to provide opportunities to find new problems and forms of solutions, and in physics, where experience is the very source of truth [Poincaré 1893, 32], [Poincaré 1900, 1163]. However, in both cases, in all cases, intuition guides the process, borrowing from experience what is needed to build satisfactory (rational) theories. Poincaré's structural perspective, the central role attributed to hypotheses and conventions, the arithmetic nominalism, the empiricism that sustains physics and, largely, mechanics, may all be arguments in favour of this thesis.

Once all these factors have been conditioned to Henri Poincaré's deterministic perspective, the conceptual importance of the notions of chance and probability and the way in which they are articulated are matters of fact. The idea that the notion of probability instantiates chance epistemically derives precisely from this condition. From the standpoint of Poincaré's epistemology, there is no way to conceive chance as a chain of purely circumstantial phenomena, as the antithesis of scientific laws, or as events that are given outside causality. The notion of probability is the theoretical artifice that effectively brings chance to scientific fact when it allows facts to be expressed mathematically in which a given degree of predictive uncertainty is, at least temporarily, unavoidable.

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