



Brown, L., Fernandez, C., Helliwell, T., & Llinares, S. (2019). Prospective mathematics teachers as learners in university and school contexts: From university-based activities to classroom practice. In O. Chapman, & G. Lloyd (Eds.), *International Handbook of Mathematics Teacher Education: Volume 3: Participants in Mathematics Teacher Education* (2 ed., Vol. 3, pp. 434-366). Brill Academic Publishers. https://doi.org/10.1163/9789004419230_014

Peer reviewed version

Link to published version (if available):
[10.1163/9789004419230_014](https://doi.org/10.1163/9789004419230_014)

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1. PROSPECTIVE MATHEMATICS TEACHERS AS LEARNERS IN UNIVERSITY AND SCHOOL CONTEXTS

From University-based Activities to Classroom Practice

This chapter explores different ways in which prospective teachers are considered to be active constructors of their knowledge embedded in a variety of social environments. A dialogue between two theoretical perspectives, enactivism and professional noticing, is used to try to understand prospective teachers' learning about teaching mathematics and, in so doing, illustrates the learning of university mathematics teacher educators. Prospective teachers' learning (discourse and actions) and different types of practice from both theoretical perspectives are linked considering different forms of attention and labelling.

INTRODUCTION

In contributing to Volume 3, *Participants in Mathematics Teacher Education*, of the *International Handbook of Mathematics Teacher Education*, we are focusing on both prospective teachers of mathematics and ourselves, all university mathematics teacher educators (MTEs), as learners. For both the prospective teachers and ourselves, discussion and dialogue are central to learning. Salvador and Ceneida are from the University of Alicante, Spain and Laurinda and Tracy are from the University of Bristol, UK. Our contexts are different. In Alicante, the prospective teachers only work with their MTEs in the university - their MTEs do not visit them in schools. In Bristol, for 24 out of the 36 weeks of the one-year course, the prospective teachers are in schools and the university MTEs visit their prospective teachers in their classrooms. We have developed perspectives (professional noticing in Alicante and enactivism in Bristol) that support our work as university MTEs in these different contexts. We will say more about these perspectives later.

At first, in working together, we were considering the complementarity of these perspectives, that is, how two, separate, different perspectives could “improve or emphasize each other's qualities” (<https://en.oxforddictionaries.com/definition/complementarity>). However, over time, as, for instance, with Scribner and Bruner, reported in chapter 8 on *Knowing as doing in The Culture of Education* (Bruner, 1996) dialogue can lead to perspectives shifting. Our own perspectives develop.

Over time, we have come to see the process of our dialogue as developing both our perspectives, as we become aware of each other's practices. Two questions are raised through our discussions about similarities and differences in what we do and how we think about what we do: To what extent can the perspectives of an "enactive view of cognition" and the development of "professional noticing in prospective teachers" be in dialogue with each other to explain prospective teachers' learning? and, What have we learned as mathematics teacher educators by establishing a dialogue between two theoretical perspectives on prospective teacher learning?

In the next section, divided into three parts, we consider the wider literature on prospective mathematics teachers' learning, before we introduce the practical implications of our own initial perspectives. We then illustrate how two perspectives can be in dialogue to introduce new insights for these questions.

PROSPECTIVE MATHEMATICS TEACHERS AS LEARNERS: REASONING PROCESSES, ACTIVITIES AND CONTEXTS

We are aware of the multidimensional character of teachers' learning when we try to understand prospective teachers as individual learners and as learners in social groups and teams. Describing and explaining the shifts from being a novice to being an expert are linked to understanding teachers' reasoning processes in action; the ways prospective teachers work; and the contexts and activities in teacher education programs. New lenses for looking at learning and knowing, developed in educational research, support us in coming to understand better the shift from novice to expert (Putman & Borko, 2000), how the different practices are carried out, and what environments support the learning of prospective teachers. These lenses are used to recast the relationships between what prospective teachers know and are learning and the settings in which they learn. When knowledge is seen as social, situated and distributed, learning can be conceptualised as coming to know how to participate in the discourse and practices of a particular community. Furthermore, the physical and social contexts in which activity takes place are an integral part of what is learned, highlighting how participants are interacting with each other as well as the materials used to engage with authentic activities to solve problems.

From the situated perspectives about learning and knowing, much of what prospective teachers do and think is seen as intertwined with the particular contexts in which they act. These perspectives on teacher learning offer new approaches to the theory-practice problem and the relevance of mathematics as subject-matter in mathematics teacher education (Oonk, Verloop & Gravemeijer, 2015), underlining three relevant issues: the context situating the prospective teachers' learning experiences, the nature of discourse for teaching, and the tools in teachers' work (Putman & Borko, 2000).

This way of framing issues related to prospective mathematics teachers' learning regards prospective teachers as active constructors of their knowledge in a variety of social environments that influence and shape their learning and professional development process. Therefore, the learning process, the interaction of participants, the ways of working, and the environments in which they are placed are considered

to be mutually influencing. It is possible to identify different activities and contexts that aim to develop prospective teachers' reasoning processes, but, in this chapter, we focus on different theoretical perspectives that support better understanding of the processes and interactions involved. As researchers, we apply these perspectives to try to find answers to issues regarding the processes of learning; the conditions under which prospective mathematics teachers learn how the shifts from initial teacher education to classroom practices are supported; and how various settings for prospective teachers' learning give rise to different kinds of knowing.

We are interested in these issues about prospective teachers' learning processes within the domain of mathematics. We try to describe and explain how prospective teachers participate and collaborate within the different activities in teacher education, given the interconnectedness of content, context and activities in the learning environment. Furthermore, this learning process involves us in the use of tools (e.g., physical didactical resources and conceptual tools, ideas and knowledge from mathematics education, and the role of others in supporting reflection on our own practice) that engage prospective teachers in analysis of teaching situations. These tools enhance cognition, but also transform it, expanding the capacity of the prospective teachers in developing their understanding of teaching situations. The role of the facilitator stresses the importance of dialogue in the development of awarenesses, enabling prospective teachers to gain new perspectives on their practice and the practice of others (Coles, 2013). Conceptual tools also allow prospective teachers to explore features of classrooms such as teacher-student relationships, classroom management in mathematics teaching, and features of children's mathematical thinking. For example, the use by prospective teachers of theoretical frameworks, such as hypothetical models of students' learning progression, can be considered to be performance tools to support prospective teachers' learning when carrying out deliberate analysis of teaching situations, noticing children's thinking or the use of mathematical knowledge for teaching (MKT) to participate in professional discussions (Ball, Thames & Phelps, 2008; Wilson, Sztajn, Edgington & Myers, 2015; Wilson, Sztajn, Edgington & Confrey, 2014). We consider prospective teachers' learning to be a process requiring thoughtful deliberation interacting with different types and domains of knowledge (e.g., conceptual tools) and experience.

Practical arguments linking reasoning processes and actions

What does it mean to learn to be a mathematics teacher using knowledge and practice? Prospective teachers can be supported in developing expertise through engagement in reflective practice about their own practical teaching as novices. *Practical rationality* is the process of thought that ends in an action or an intention to act and can be understood as the relationship between the thinking and the action, providing a way to understand and explain the actions (Fenstermacher & Richardson, 1993). By examining prospective mathematics teachers' practical rationality, university MTEs can come to understand how prospective mathematics teachers learn.

Fenstermacher & Richardson (1993) focus on the practical rationality of teachers, allowing us to describe how prospective teachers reason about their teaching actions and understand what lies behind those teaching actions. An instance of practical rationality of teachers is their practical arguments (p. 102), describing and explaining teaching situations. A practical argument is articulated through the rationale, empirical support, and situational contexts that served as the basis for the teachers' instructional actions. In this approach, the action of describing and explaining the teaching can be considered to be a way to interpret it from the prospective teacher's prior experiences and personal knowledge to generate a range of alternative ways of acting in the classroom. Prospective teachers' practical arguments could be considered to be accounting for their actions that serve to explain and justify (as a way of understanding) teaching situations supporting future teaching actions. So, constructing a practical argument, accounting for practice, is an active process of categorisation in which previous experience and knowledge, conditioned by the context in which the activity is done, is used. This categorising process involves considering what is attended to and becoming aware of features of teaching situations of which the prospective teachers were previously unaware. This is a way to reconstruct the awareness of the key elements in the teaching situation that can lead to action.

Learning as a process of improving practical arguments: knowing and doing in context

Through this approach, prospective mathematics teachers' learning can be understood as a process in which practical arguments are developed. New issues are generated in order to further examine prospective teachers' reasoning as they explore teaching and learning. In teacher education, a teacher educator can work with prospective teachers to improve their practical arguments, enhancing the prospective teachers' skills to think more deeply and powerfully about teaching situations. Over time, prospective teachers can better link their actions and thinking by enhancing their skills of attending to and interpreting teaching situations. That is to say, identifying key elements in a teaching situation to endow meaning is a means to generate a purpose (a learning goal in the situation) as a pre-cursor to acting. So, a way to approximate actual practical situations to support analytical reflections by prospective teachers is using *practical registers*, whose analyses allow awarenesses to be raised prior to action. This idea is supported by the assumption that knowledge is situated in contexts and so prospective teachers' learning should be grounded in some type of teaching practice (Lampert & Ball, 2000; Llinares & Valls, 2009).

From this perspective, enhancing prospective teachers' discourse depends on how research-based ideas about learning and teaching are incorporated into the task of developing and examining practical arguments. Improving practical arguments is a form of modelling prospective teachers' discourse, using research-based knowledge (as a way to learn to think, talk and act as a teacher) focused on teaching and learning. The practical registers supporting the analytical perspectives of prospective teachers can be, for instance, video-records of classroom practice; students' written answers

to mathematical problems; instances of didactical resources from specific contexts; and excerpts of lessons following some type of learning progression model. Engaging prospective teachers in the reconstruction of their practical decision-making is a way to consider their learning as shifts from descriptive accounts of teaching situations to accounts rooted in research results from mathematics education research. Using registers from practice, such as, video records of classroom practice or students' written answers to mathematical problems, connects prospective teachers with particular practical settings that may afford reflection and critical analysis that is not possible in actual classrooms. These registers of practice allow prospective teachers to examine children's answers using multiple perspectives and frameworks. In this approach, when we aim to understand prospective mathematics teachers' learning, their conceptions and knowledge cannot be separated.

We acknowledge that when prospective teachers reconstruct a practical argument as a way to respond to a teaching situation, it is a way of showing what they know about the situation. In this sense, there is a link between what prospective teachers know and what they do when identifying (describing and attending to) and interpreting a practical register (or observing a mathematics lesson) to decide how to continue the teaching. Knowing from this perspective is linked to processes such as identifying and interpreting as instances of prospective mathematics teachers' practical reasoning. For example, a context where we can combine university and school-based experiences can lead to specific learning for prospective mathematics teachers. When prospective teachers are observing mathematics lessons during their school experience and have discussions with their tutors or write narratives about some relevant incidents in relation to mathematics teaching, they show what they know when talking or writing the narratives through deliberate analysis in relation to certain prompts. The prompts (such as, What are we talking about here? What of students' thinking can be inferred from their responses? and How could you respond to these students?) that guide the analytical reflective stance of prospective teachers are intended to focus their attention on the links between details of the situation and the use of knowledge to make sense of them. This is a way to recognise what prospective teachers see as relevant. Furthermore, the prompts help the prospective teachers to understand what to look for when observing mathematics lessons, incorporating these classroom-based observations of students' performance into their sense-making process to decide how to use children's thinking to guide instructional decisions (Ivars & Fernández, 2018). The description and interpretation of teaching situations allows the prospective teachers to define purposes, that is, learning goals with which to act in determined ways. These learning goals, generated from teaching situations, also generate actions that support their learning in the observed situation.

Reflections on the links between what is known and what is done can help prospective teachers in enhancing levels of awareness through the structure of their attention, what they notice in teaching (Mason, 1998; 2002). In this sense, prospective mathematics teachers' learning can be understood to be a shift toward awarenesses, enhancing their sensitivities to notice relevant incidents in a teaching

situation and responding adequately in terms of the purposes at play in the moment. This refinement of awarenesses is manifested in how attention is structured to organise the reasons that support an action being taken. This way of proceeding sees the deliberate analysis of teaching situations to be the kernel of activities in teacher education programmes, allowing the unpacking of reasons behind actions. This is a way of linking awarenesses to actions (Brown & Coles, 2011).

We will, in the next two sections, discuss in more detail two theoretical perspectives that support our thinking about prospective teachers as learners in a context. After the following two sections, we will then discuss our learning as university MTEs about prospective teachers' learning through dialogue between the perspectives, giving insights into our current thinking about the two questions above. The particular two perspectives are reported here because they are the lived perspectives of the university MTEs at the Universities of Alicante (professional noticing) and the University of Bristol (enactivism) that inform the teacher education practices. Generally, insights can develop through dialogue between other perspectives, but, here, insights in mathematics teacher education develop through the link to practice, working with prospective teachers.

The first theoretical approach discussed, professional noticing in prospective teachers, is used in a four-year primary teacher education degree at the University of Alicante, where the university tutors do not observe prospective teachers' instruction in schools. The second theoretical approach discussed, the enactive approach to cognition, is used in the one-year post-graduate teacher education course at the University of Bristol, School of Education. This one-year course involves time spent in sessions at the university, as well as time spent teaching in schools. The university tutors observe the prospective teachers in their school experiences. The context at Bristol affords these prospective teachers with learning through their own experience of teaching and with discussions of some observations of their teaching. Through working together, as MTEs at Alicante and Bristol, we have become aware of the possibility of using aspects of both perspectives to guide our work with prospective teachers and in our research.

PROSPECTIVE TEACHERS' PROFESSIONAL NOTICING

Professional noticing implies seeing phenomena from the area of expertise, which can be different from others who are not from the area. Although professional noticing is only one component of teaching expertise, its development is important today since there are calls for teachers to base their instruction on students' ideas and understandings to make appropriate instructional decisions while they are teaching (NCTM, 2014; Sherin, 2007). In fact, teaching effectively implies that a teacher "elicit evidence of students' current mathematical understanding and use it as the basis for making instructional decisions" (NCTM, 2014, p. 53).

The field of professional noticing started as research on what teachers were and were not identifying as noteworthy events in the classroom (Mason, 2002). Mason (2011) claimed that "noticing is a movement or shift of attention" (p. 45) so it implies an increase in sensitivity to the details of learning situations, avoiding judgements,

emotional content and generalities. Labels arise through a movement from “accounts of” to “accounting for” experiences. “Accounts of” are “free of theorizing, emotional content, justification ... [and] provide brief but vivid descriptions”, while “accounting for” refers to “theorizing, explaining, and accounting for not only what was observed but why it struck the observer sufficiently to be identified or marked” (Mason, 2011, pp. 39-40). Noticing describes teachers’ ability to direct their attention to relevant classroom situations (van Es & Sherin, 2002). Reasoning refers to the ways in which teachers think about what is noticed based on their knowledge and understanding. This reasoning process involves, for instance, using what one knows about the subject matter to make sense of an idea, making connections between specifics of the classroom and broader principles of teaching and learning.

Regarding the required facets of teacher knowledge (such as, content knowledge and pedagogical content knowledge), Stürmer *et al.* (2013) focused on general pedagogical knowledge that includes components of effective teaching that are transferable across subjects (goal clarity, teacher support and learning climate). In Stürmer *et al.*’s study, professional vision is an indicator of whether teachers are able to apply their knowledge about components of effective teaching to authentic classroom situations. Based on the findings of van Es and Sherin (2008), Stürmer *et al.* (2013) distinguished three different levels of knowledge-based reasoning: description, explanation and prediction. *Description* reflects teachers’ ability to identify and differentiate between relevant classroom events that require knowledge of the components of effective teaching (*e.g.*, goal clarity). *Explanation* refers to the ability to use knowledge about effective teaching to reason about a situation. In other words, it involves accounting for the situations according to the concepts of the teaching component involved. *Prediction* involves the ability to anticipate the consequences of the observed events for students’ learning processes by drawing on broader pedagogical knowledge in classroom practice (Seidel *et al.*, 2017).

The studies of van Es and Sherin, Jacobs, Lamb and Philipp (2010) particularise professional noticing to children’s mathematical thinking. Professional noticing of children’s mathematical thinking provides “a structure for teachers to better understand and act on their students’ mathematical conceptions and practices” (Thomas *et al.*, 2015, p. 296). Jacobs *et al.* (2011) conceptualise responsive teaching (professional noticing) as a progression through three interrelated skills: (i) attending to children’s strategies (accounts of, Mason, 2011), (ii) interpreting children’s understanding (accounting for, Mason, 2011) and (iii) deciding how to respond on the basis of students’ understanding. Attending to children’s strategies implies including in the descriptions mathematically significant details, such as how children counted or used tools or drawings to represent quantities. Interpreting children’s understanding implies that there was consistency between the details of specific children’s strategies and research on children’s mathematical development. For instance, identifying that a strategy reflects “number sense” better than other strategies, or recognising understandings the children do not demonstrate. Furthermore, decision-making in the classroom needs to be based on what they, the prospective teachers, have learned about children’s understanding from the specific situation. For example, providing a next problem that allows the use of a more

sophisticated strategy. This conceptualisation focuses on teachers’ ability to transfer, for instance, their mathematical content knowledge and their pedagogical content knowledge to their actions (attending to, interpreting and deciding) in teaching-learning situations. Therefore, professional noticing is an indicator of whether teachers are able to apply their knowledge to teaching-learning situations. Table 1 shows interrelations between the different conceptualisations of professional noticing used in research reports.

Table 1. Processes in the different conceptualisations of professional noticing

<i>Mason (2002; 2011)</i>	<i>Sherin (2007)</i>	<i>Jacobs et al. (2011)</i>	<i>Stürmer et al. (2013)</i>
Account of	Selective attention (noticing)	Attending to	Description
Accounting for	Knowledge-based reasoning (reasoning)	Interpreting	Explanation
		Deciding	Prediction

In the following, we go in more depth to the construct of professional noticing of children’s mathematical thinking (processes of attending to, interpreting and deciding) as a structure for teachers to better understand and act on their students’ mathematical practices.

Professional noticing of children’s mathematical thinking

Professional noticing of children’s mathematical thinking implies the prospective teachers’ ability to use their knowledge (subject matter knowledge and pedagogical content knowledge) to attend to, interpret and decide. Therefore, teachers’ mathematical knowledge for teaching (MKT, Ball, Thames & Phelps, 2008) and professional noticing are connected (Thomas, Jong, Fisher & Schack, 2017). In fact, “professional noticing allows for theoretically locating and analysing responsive instructional practices while MKT provides a framework for considering and investigating the varied knowledge-types required for rich mathematics teaching” (Thomas *et al.*, 2017, p. 10).

To attend to students’ strategies, teachers need to identify details that are mathematically salient. For instance, consider a teacher working on the following problem (Bufo, Llinares & Fernández, 2018) in class:

In a new building, there are lofts of three different sizes for sale:
 7.5m by 11.4m; 4.55m by 5.08m, 18.5m by 24.5m
 Which one is more square?

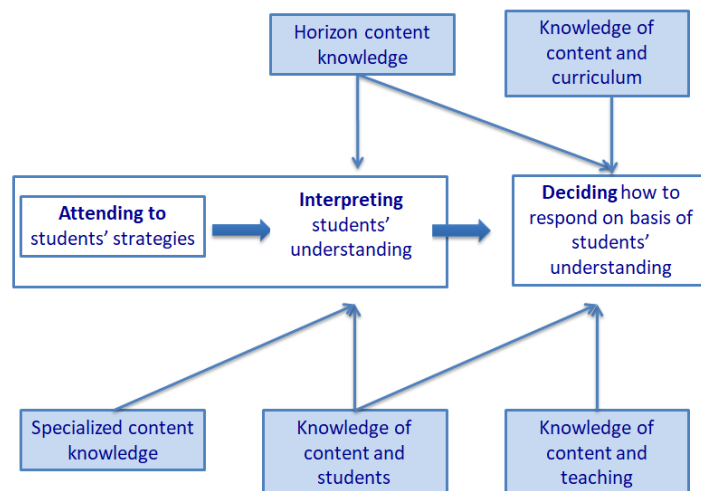
Attending implies identifying important aspects of the answers of the two students shown below. For instance, student 1 uses multiplicative reasoning, seeing the

quotient as being closer to 1 and student 2 uses additive reasoning, seeing the difference as being closer to 0:

<p>Student 1</p> $7.5 / 11.4 = 0.65$ $4.55 / 5.08 = 0.89$ $18.5 / 24.5 = 0.75$ So, the squarest loft is the second one since the quotient is close to 1	<p>Student 2</p> As the sides of the square have the same measure, the squarest loft is the loft in which the difference between sides is smaller (closer to 0). $11.4 - 7.5 = 3.9$ $5.08 - 4.55 = 0.53$ $24.5 - 18.5 = 6.0$ So, the squarest loft is the second one since the difference is smallest.
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Interpreting involves coordinating what has been identified (attending to) with what is known about mathematical development in a particular concept. This process implies going beyond knowing procedures (such as the child has done an addition, or has divided two quantities) to having the necessary knowledge to explain how the mathematical details (ideas) are involved in students' strategies. In the students' answers above, interpreting students' understanding implies identifying that, for instance, student 1 has obtained ratios between the sides of the square and has compared them. The loft whose ratio is closer to 1 is the squarest one. Therefore, this student obtains ratios and compares them in a ratio comparison problem (correctly reasoning). Student 2, for instance, has used an additive approximation obtaining differences between the sides of the square choosing the loft whose difference is closer to 0. This is incorrect reasoning in a ratio comparison problem, which requires multiplicative reasoning. Additive reasoning is the most common strategy used by students in proportional reasoning (Fernández, Llinares, van Dooren, de Bock & Verschaffel, 2012).

Figure 1. Relating Mathematical Knowledge for Teaching and Professional Noticing



Therefore, this process requires knowledge for explaining procedures used by students for: reasoning showing students' difficulties in the transition from additive to multiplicative understanding common and uncommon strategies used by students; distinguishing and interpreting their correctness; and explaining the origin of their errors (Specialized Content Knowledge and Knowledge of Content and Students; terms in brackets in this and the next paragraph are from the MKT framework, Ball, Thames & Phelps, 2008; see Figure 1 for a diagram relating MKT and professional noticing). Furthermore, knowledge about how a mathematical concept develops over time between different educational levels and making connections with other mathematical concepts (Horizon Content Knowledge) could also be useful for interpreting students' mathematical understanding.

Finally, deciding what to offer on the basis of what could be possible alternatives considering students' understanding implies using the knowledge of which aspects of the concept are the easiest or the most difficult ones for students, which are the most common errors related to the concept, and which are the strategies or representations more adequate for introducing the concept (Knowledge of Content and Students and Knowledge of Content and Teaching). Furthermore, knowledge about how students' mathematical understanding relates to the concept develops over time between different educational levels (Horizon Content Knowledge) and knowledge about curricula related to the design of materials and the use of resources (Knowledge of Content and Curriculum) are also required to propose activities that help students progress in their conceptual understanding. For instance, there follows a possible alternative to the activity in the example of the lofts that could help student 2 progress in their understanding, transitioning from additive to multiplicative reasoning:

Consider the following lofts?

4.1m by 5.1m

21m by 20m

Which loft is more square?

In this new activity, an additive approximation shows that both lofts have the same difference between the lengths of their sides ($5.1 - 4.1 = 1$ and $21 - 20 = 1$), however, using drawings, you can see that loft (b) is the more square.

As we have shown, professional noticing involves the development of knowledge-based reasoning processes (the use of different kinds of knowledge to interpret and decide, considering possible alternatives to teaching responsively (see Figure 1). Interpreting students' mathematical thinking and providing instructional decisions following such interpretations require specialised mathematical knowledge. However, such specialised mathematical knowledge is necessary but not sufficient for effective professional noticing. According to Thomas *et al.* (2017), effective professional noticing implies planning instructional practices organised

around one’s knowledge of progressions of children’s mathematical development. Therefore, effective professional noticing occurs at the intersection of Mathematics Knowledge for Teaching (MKT) and a high level of responsiveness to the mathematical activities of students (see Figure 2; Thomas *et al.*, 2017). As these authors pointed out, the figure should not be interpreted in terms of dichotomies. “Rather, teachers’ conceptions of MKT and enactment of responsive practices most likely exist upon a continuum” (p. 14).

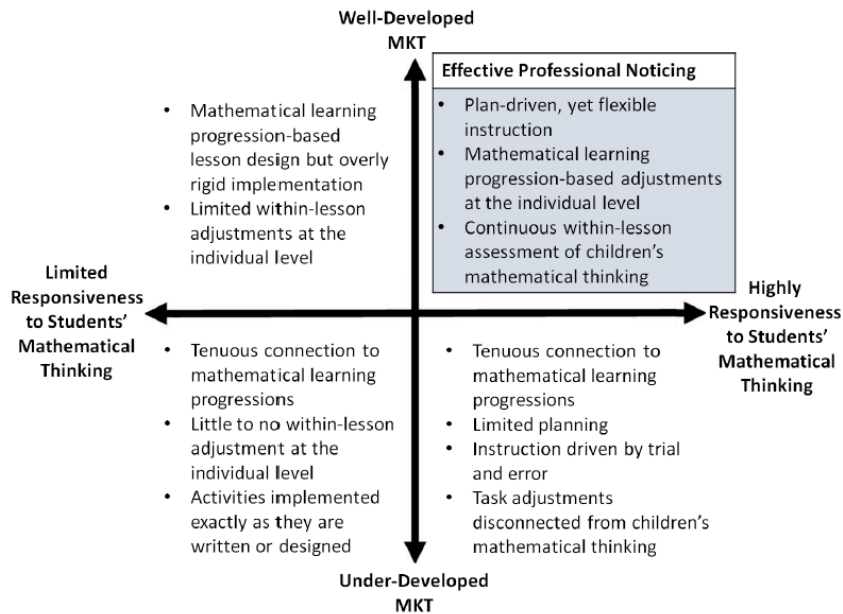


Figure 2. Enactment of professional noticing and MKT (Thomas *et al.*, 2017, p. 14)

On the other hand, professional noticing provides a structure for prospective or in-service teachers to understand and act in particular teaching-learning situations. Indeed, professional noticing establishes a structure to teach responsively. Therefore, developing prospective teachers’ professional noticing, in teacher education programs, will prepare them for classroom practice.

Developing prospective mathematics teachers’ professional noticing

Different contexts and tools such as videos, students’ written answers, narratives, online or face-to-face discussions have been used in teacher education programs to develop prospective teachers’ professional noticing (see a review in Stahnke, Schueler & Roesken-Winter, 2016). The review shows that the development of interpreting and deciding processes in teacher education programmes is challenging without a guide or framework. Structured frameworks, such as hypothetical learning

trajectories can provide prospective teachers with a guide to focus their attention on the teaching (Edgington, Wilson, Sztajn & Webb, 2016) to generate possible alternatives actions. A hypothetical learning trajectory is a construct that involves hypotheses about “the order and nature of the steps in the growth of students’ mathematical understanding, and about the nature of the instructional experiences that might support them in moving step by step toward the goals of school mathematics” (Daro, Mosher & Corcoran, 2011, p. 12). Therefore, hypothetical learning trajectories provide prospective teachers with a cognitive model for thinking about (interpreting) and acting (deciding) (Sztajn, Confrey, Wilson & Edgington, 2012; Thomas *et al.*, 2015) and with a specific language to describe mathematics teaching (Edgington *et al.*, 2016).

This language can allow prospective teachers to create a system for picking out, classifying and naming elements of students’ thinking (Wells, 1999) that can help them to identify, interpret and make instructional decisions. Prospective teachers’ discourse can inform what is noticed and how. Understanding teacher learning as “change in discourse over time” (Wilson, Sztajn, Edgington, Webb, & Myers, 2017; p. 570), in particular changes in prospective teachers’ discourse on mathematics teaching, can be interpreted as an indicator of enhanced noticing (Ivars, Fernández, Llinares & Choy, 2018). For example, Ivars *et al.* (2018) showed that the use of a hypothetical learning trajectory of the part-whole meaning of fractions as a framework to interpret students’ mathematical thinking helped prospective teachers improve their mathematical discourse in two ways: firstly, progressing from elaborating a less detailed discourse (in which they did not give evidence or they added unnecessary information from students’ answers) to entering a more detailed discourse; and secondly, considering students’ answers as examples of more general categories. In this sense, the improvement of discourse is a sign of enhanced noticing of students’ mathematical thinking. Thus, “enhancing noticing can be understood as a virtuous circle” (Figure 3; Ivars *et al.*, 2018; p.13) in which learning trajectories can help prospective teachers to begin focusing on details, avoiding generalities, judgements and emotional content. In the words of Mason, to engage teachers in practitioner research from an *account of* a phenomenon to *accounting for* this phenomenon.

ENACTIVE VIEW OF COGNITION

From an enactivist perspective, learning is the same throughout the system, whether it is children learning mathematics in classrooms, prospective mathematics teachers learning about the way they and the children act in classrooms, or mathematics teacher educators learning about how prospective teachers learn. We gain insights, our learning or knowing, through actions, our doings. In fact, as Maturana and Varela (1987) write, “All doing is knowing and all knowing is doing” (p. 26). This tenet is central to the practices of the mathematics teacher education group at the University of Bristol, School of Education. On the Bristol course for prospective teachers, the year is a journey of coming to see more (learning) related to actions. It is accepted that the prospective teachers do not take the same awarenesses with them from

university sessions, which are run so that the mathematics teacher educators' actions, what they do in sessions, are contingent upon what the prospective teachers bring with them.

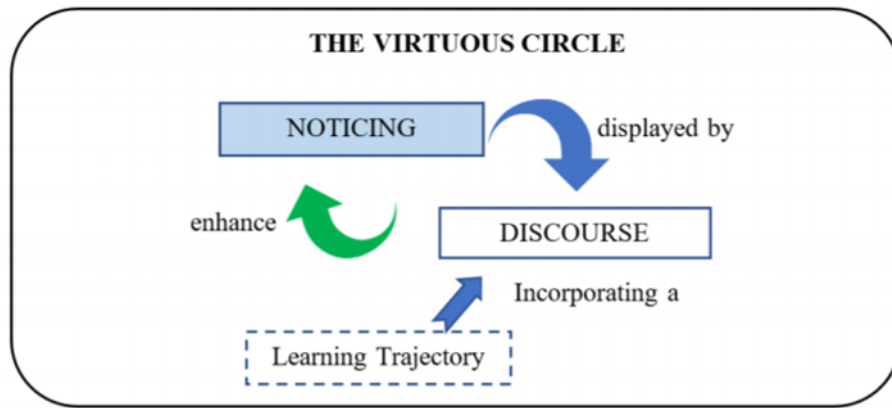


Figure 3. The virtuous circle of noticing in teacher education programs (Ivars et al., 2018, p. 13)

The prospective teachers each work in a different school, with a different set of practices that have developed over time. Sessions always have a component of the prospective teachers sharing their experiences and uncovering differences, not to find a best practice, but to expand their awarenesses of what is possible. For example, when the prospective teachers return to the university after a period of school experience, at the start of the first session they are grouped in reflective teams of three and each individual chooses a lesson or incident to work on. In turn, each of them, firstly, gets into the detail of what happened with support from the questions raised by the other two members of the group. After they have each had their time, a discussion emerges of the issues arising for them individually. For one of them, working in a context where children are grouped by achievement to be taught mathematics, it might be surprising that their peer is teaching mixed achievement groups. What teaching strategies are used in these different contexts?

Perception, to an enactivist, “consists of perceptually guided action” (Varela, 1999, p.12). This is the first of two important interrelated points. Knowing is doing is being. We literally are what we do and what we come to know is expressed by the second point, “cognitive structures emerge from the recurrent sensorimotor patterns that enable action to be perceptually guided” (Varela, 1999, p. 12). Walking into a classroom, day after day, triggers recurrent sensorimotor patterns that build cognitive structures that themselves are linked to action/perception. One example of this, common with novice teachers, is to come to be concerned about a child in the class who attracts their attention for the wrong reasons. It can seem as if, for the novice teacher, over time that child is the only one in the classroom. All the teacher’s actions and responses are focused on trying to control that child’s behaviour. The

world of significance that they bring forth is centred on only one of the many children in the room. Observation by experienced teachers supports the novice teacher to extend their cognitive structures through, perhaps, scanning the room rather than focusing on the individual or seeing what happens if they deprive the student of their attention. The expert teacher is able to become aware that a pattern is developing and act to change it if it seems not to be effective for their purposes. The focus of the Bristol course is supporting the novice prospective teachers to be able to act like experts, building cognitive structures that allow them to learn independently and with collaborative working with others. How is this done?

At the University of Bristol, the group of mathematics teacher educators works with prospective teachers at the university and also each mathematics teacher educator visits each of their tutor group to observe teaching in secondary (11-18 years of age) schools. As an enactivist mathematics teacher educator, it is important to not only work with what the prospective teachers say about their practice in schools, but there is a need to observe that practice, with a school mentor, to get a sense of the context within which the prospective teacher is working, seeing relationships between the school mentor and the prospective teacher, seeing what is the same and what is different about what the school and university mentors see when observing the prospective teacher in their classroom and, in a de-brief conversation (Brown, J., Brown, L., Coles & Helliwell, 2019), see whether the prospective teacher's attention is caught by similar or different aspects of the lesson to the mentors. This process of being supported to talk about their experiences of teaching builds cognitive structures in action, which is learning about teaching. So, when the prospective teachers work together at the university, it is through sharing perspectives from their schools in relation to issues often either suggested by the mathematics teacher educators (such as, strategies for assessment, using technology or focusing on an aspect of mathematics) or arising out of the prospective teachers' discussions, to explore incidents from each other's practice that remain with them as troubling or unresolved. This sharing of practice allows the opening up of new avenues to develop in their classrooms, with new actions to explore doing in a different context. Similarly, the group of mathematics teacher educators meet together before and after a day teaching prospective teachers and, often using the strategy of a reflecting team, support each other's learning.

Metz and Simmt (2015) discuss the importance of a "second-person empathic observer" interviewing children to support learning. The interviews support the child to stay with the detail of doing mathematics to learn. Metz and Simmt are working from an enactivist perspective, discussing, as MTEs, developing into such observers. In Alicante, working with learning trajectories with prospective teachers seems to provide a similar focus in developing the observer in the prospective teachers. The learning trajectories provide a way for the prospective teachers to be triggered by their environment into considering future actions related to classroom experiences.

Troubling or unresolved experiences provide the data through the consideration of which it is possible to act differently in the future. As Varela puts it, "It is at the moments of breakdown, that is, when we are *not* experts [...] that we deliberate and analyze, that we become like beginners, seeking to feel at ease with the task at hand"

(Varela, 1999, p. 18). This process is called deliberate analysis. Rosch *et al.* (1976) gave the label, basic-level categories, to the objects and ideas we interact with in the world. So, a chair is a “sitting-on object” and chairs have “similar overall *perceived shapes*” (Varela, 1999, p. 17). We do similar things with them. These labels linked to actions are also being created by prospective teachers in classrooms. In discussion with others, through deliberate analysis, it is possible to recognize habitual behaviours that are not effective, such as focusing on one child out of many, as described above. Staying with the detail of experience gives the possibility of new awarenesses arising. In turn, these new awarenesses, such as needing to interact with individuals, lead to thinking of a range of new behaviours that are then tried out in the classroom. Through the recurrent sensorimotor patterns that are laid down in awareness by trying out the behaviours and finding ones that seem to work, new basic-level categories, actions that do not need any attention, are created that make the complex task of teaching possible.

There is a lot of theory related to enactivism (see Reid and Mgombelo, 2015) but for the purposes of this chapter, the recursive moves between being in the classroom, working with an inside and outside observer, picking up the culture of the particular school experience and then bringing the group together at the university to offer experiences outside of the micro-environment, give a way that learning can happen throughout the system. The prospective teacher novices are learning in the same way as experts (Brown & Coles, 2011) and learning to stay with the detail of their practices to allow new awarenesses linked to actions (basic-level categories) to arise (Brown & Coles, 2012). This is a cyclical process, in which staying with the detail when describing their classroom practices, with questions arising from other observers, allows novice teachers, beginners, to “use this sort of deliberate analysis to acquire sufficient intelligent awareness to bypass deliberateness altogether and become an expert” (Varela, 1999, p. 32). Teachers’ learning and their practice is clearly linked by what is done from an enactivist perspective.

In the next two sections we illustrate the ways in which the perspectives enter into dialogue. The first section is an illustration of the perspectives interacting and the final section is where we address insights developed in answering the two questions at the start of the chapter.

ENACTIVIST PERSPECTIVE AND PROFESSIONAL NOTICING IN DIALOGUE THROUGH MASON’S “DISCIPLINE OF NOTICING”

Our focus in Alicante and Bristol is on how prospective mathematics teachers’ discourse and action is shaped by different types of activities in diverse contexts. These different activities and contexts are opportunities for making sense of aspects of teaching. We can consider that the set of meanings that are constructed in the different contexts are part of the process of becoming a teacher. In the noticing perspective, the knowledge to be used in analysing teaching situations and in the enactivism perspective, the role of the observer, the mathematics teacher educator, are key elements in understanding prospective teacher learning. We have used our interpretation of the ideas and practices from each of these perspectives within our

different contexts, however, these interpretations are developed from our histories of practices in our local contexts. From both perspectives, we support prospective teachers in generating interpretative frameworks that can guide effective and responsible action (as a way of expanding awarenesses linked to action). The similarities and differences have led to dialogues where words like awareness are used in common, albeit with differently nuanced meanings. As we work together for longer, our learning as university MTEs is through more complex uses of our common vocabulary. As a group of university mathematics teacher educators, we do not expect to be able to see the same, but find working together helps us in “expanding the space of the possible” (Davis, 2004, p. 184) in our own contexts. We illustrate these ideas through a dialogue leading to insight, or university MTE learning, as our languaging co-emerges. The insight is related to the relationship between *attention to* and *labelling*, now categories for us both that can link to action with our prospective teachers in our different contexts.

Dialogue leading to insight into attending to and labelling

Tracy, a mathematics teacher educator from Bristol who can visit prospective teachers in the classroom, and who has been using both Mason's “discipline of noticing” and the enactivist view of cognition in her research, talks with Salvador from Alicante, who works with prospective teachers in the university on various activities, does not visit them in the classroom, and uses professional noticing:

Salvador: Noticing as a construct comprises a diverse set of conceptualisations. As in the common sense of the word, noticing in mathematics education places an emphasis on teachers *attending to* particular classroom events, interpreting them with a disposition to act. Those classroom events might be examples of students’ work or excerpts from videos, say.

Tracy: An evocative image used by Mason (*e.g.*, 2002; 2011; 2017) is that of two birds, one eating, the other looking on without eating. One interpretation he offers of this image is that the two birds represent different forms of *attention*. The bird eating, caught up in the moment of doing, represents the form of attention that gets caught up in-the-moment, automatically and habitually. The bird looking on, however, represents the inner witness, a form of attention that allows us to be awake in the moment.

Salvador: Noticing has been vastly elaborated on since its original conception by Mason, the acknowledged pioneer in the area of noticing within mathematics education.

Tracy: Mason acknowledges that the discipline of noticing stems from a phenomenological underpinning, which “values lived experience [...] as a method both of enquiry and of reporting that enquiry” (p. 1). He makes a strong case for working with lived experience as an alternative to much research that has become a task of refining and using existing theoretical frameworks. So, the discipline of noticing is less valid for questions about what has happened

but is more about what might be possible and the process of enabling further possibilities into action. This fits with my work with prospective teachers in schools.

Salvador: Professional noticing supports my prospective teachers in using practical registers and arguments with language from the mathematics education literature as they work together in the university context.

Tracy: Through a post-spective process of reflecting on incidents, it is possible to consider alternative ways of acting in the moment. *Labelling* particular phenomena that have occurred opens up the possibility of recognising that phenomena again in the future and triggering a different response. By communicating accounts of this process, it can become useful for others to try out new ways of acting for themselves.

Salvador: We both use the word *labelling*. For my prospective teachers, through engaging with students' work (see the example above), say, having read articles from the literature, they become able to *label* what students have done through analysing different responses to tasks.

Tracy: There seems to be some commonality here although with different contexts. From an enactivist perspective, in the moment we act bringing our habits and patterns of behaviour (the bird eating) from our history of all our lived experience. The second bird comes through engaging in a process of "deliberate analysis" (Varela, 1999; Brown & Coles, 2012), allowing "reconstruct[ion of] the intelligent awareness that justifies the action" (Varela, 1999, p. 32). As in the discipline of noticing, from an enactivist perspective it is possible, through post-spective reflecting on accounts-of incidents and through *labelling* particular phenomena that have occurred, to expand possibilities for acting differently in the future. The language from enactivism, the biological basis of being, is related to habitual ways of categorising the world being basic-level categories. Prospective teachers often find the ways of acting in classrooms at the start of the year to be ineffective, in that their students do not do what they expected them to! They need to develop new basic-level categories that can become linked to new, more effective actions. Dwelling in the detail of the experience awakens their attention away from ineffective action so that there is the possibility that new *labels* can emerge. From an enactivist perspective, learning entails a change in the ways we categorise the world that we bring forth through perception/action. For a novice prospective teacher, the classroom is, for many, a new and strange environment without any habits.

Salvador: Wells (1999) contends that individuals make sense of observed phenomena by experience, information, knowledge building and understanding from a socio-cultural perspective. I am beginning to think again about what we do in Alicante. During the activities prospective teachers *describe* the events first, creating focal points of what they are *attending to* and

around which the negotiation of meaning is organised. This is a generating process of interpretation, *labelling* events as particular cases of general phenomena. There is then the exploration of how to act if they could continue the teaching.

Tracy: We have a new concept of *labelling* that, in enactivist terms, is a superordinate category spanning both our contexts. It allows us both to see our interactions with prospective teachers in new ways, like the idea in mathematics of directed numbers say, that becomes more complex as new situations are met where they are applied. Bateson (2000, first edition 1972) calls ideas like *labelling* explanatory principles (see the *Metatogue*, p. 38, *What is an instinct?*) You cannot really say what explanatory principles are but you can see what they do. In both our contexts, the prospective teachers are learning through doing, experiencing practical registers and practical teaching. I have expanded my space of possibilities to include access to the mathematics education literature, which we focus on in the summer term, as action.

LABELLING AND ATTENDING TO AS ANSWERS TO THE QUESTIONS

In this section, we address the two questions from earlier in the chapter in turn. *Labelling* and *attending to* were insights developed from our dialogues illustrated in the previous section.

To what extent can the perspectives of an “enactive view of cognition” and the development of “professional noticing in prospective teachers” be in dialogue with each other to explain prospective teachers’ learning?

What the dialogue has done is created *labelling* and *attending to* as insights that allow us to talk together about prospective teacher learning. The word *explain* in our question is now replaced by some more complex construct. For instance, *labelling* resonates with both our perspectives and has built a superordinate category that is more complex than each of our separate initial ideas. We use the new extended explanatory principles of *labelling* and *attending to* in our discussions about prospective teacher learning in our local contexts.

Each theoretical perspective underlines different facets of prospective teachers’ learning and knowing, considering different forms of attention to how prospective teachers integrate new knowledge about teaching situations into their worlds. From both perspectives, the prospective teachers’ experiences need to be extended and reinterpreted using new information through engaging in specific discourse activities as a way to develop a shared repertoire of resources (experience, stories and ways of addressing recurrent situations). We think that these perspectives allow us to locate the process of becoming a teacher in the actions of novice prospective teachers, provoked by their activities in different contexts. These practices are different forms of *attending to* and *labelling*, which are connected to what has been developed in former sections, like instances of enactivism and noticing. The two perspectives in

teacher education help prospective teachers make concise representations of complex teaching situations, such as, facets of knowledge management. In this process the reification is understood as “ways of doing things” shared by the community (Llinares, 2002). Professional noticing perspectives emphasize how prospective teachers can extend their cognitive structures by managing knowledge, while an enactivist perspective put its focus on linking knowing and doing. While from an enactivism perspective prospective teachers’ cognitive structures emerge from recurrent sensorimotor patterns, recognising these actions and talking about different ways to act, from a professional noticing perspective, a teacher educator can provide prospective teachers with a scaffold to notice (e.g. information about how students come to understand a specific mathematical topic; or different ways what can adopt alternative mathematical discussions in the classroom).

The scaffold, from both theoretical perspectives, is considered to be a way to help prospective teachers to recognize similarities and differences among practical registers that can support the generation of cognitive structures to expand their awareness of what is possible (as a way to stress the relationships between knowing and doing). That is to say, prospective teachers generate different interpretations of the situation and become aware of the range of alternatives to act.

Professional noticing is understood as a knowledge-based reasoning process helping prospective teachers to set out reasons for what teachers are trying to accomplish (or for what students are trying to do), why they choose to act this way, and how alternative actions can fit the purpose or learning goal sought. Learning, therefore, entails a change in the discourse in the mathematics teaching situations. While from enactivism, learning entails a change in the ways prospective teachers categorise the world that they bring forth through perception/action.

Through enactivism and noticing, becoming confident in relation to ones’ knowledge and experiences, ones’ participation in professional activities, is through the conscious appreciation of relevant events in a teaching situation. The focus of the development of an awareness of attention that allows prospective teachers to become expert is key in both theoretical perspectives through meanings endowed to attending to and labelling. Each of them, as they have been developed in Bristol and Alicante, puts the emphasis on different aspects: enactivism about the relation between doing and knowing, and noticing about knowledge management. The discipline of noticing feels like a methodology that teachers could be confident to apply to their own practice, analysing the mathematics learning of their students in similar ways to how they analysed situations at the university before teaching. The analysis, as they become practising teachers, becomes more like the deliberate analysis of enactivism as they stay with the detail of what has happened in their teaching, looking at their students’ work and attending to what happened so that they develop a new repertoire of actions in the face of, say, student misconceptions. It is in the move to classroom practice that the synergy of the two perspectives becomes powerful, where, as teachers, over time they are more able to think and act “without awareness” having learned from their histories of experiences.

What have we learned as mathematics teacher educators by establishing a dialogue between two theoretical perspectives on prospective teachers' learning?

From enactivism and professional noticing perspectives, mathematics teacher educators can develop different ways to help prospective teachers to make deliberative analysis of teaching practice to enable further possibilities for action. Both perspectives emphasize different ways to implicate prospective teachers in “notice, knowing and doing” that we can consider as developing “different forms of attention” (Mason, 1998, p. 250). In this sense, it is an issue about how thinking processes involved in attending to and labelling a situation can be linked to action (making prospective teachers more aware of the practical reasoning behind their actions).

Practically, the awareness that the focus is still on doing for the prospective teachers in Alicante when they are engaging with practical registers through access to the mathematics education research rather than being observed in classrooms, opens up the possibility in Bristol for more support for prospective teachers to access literature earlier in the course. For prospective teachers, exploring beyond their own learning of mathematics to attend to a range of possible methods of solutions seems important, giving an insight into the move away from seeing student solutions as simply right or wrong and more generally, the university MTEs noticing when dichotomies are present.

Different institutional contexts generate teacher educators' different practical decisions that we have tried to describe and explain in this chapter. From an enactivist perspective, how prospective teachers talk, listen to and discuss their own practices and the practices of others (emphasising the relevance of how teaching appears in different contexts), sharing their experiences and uncovering differences. From professional noticing, how prospective teachers get involved in becoming more aware of similarities and differences between teaching situations is shown through different types of practical registers to generate alternative possibilities for future action.

Linking descriptions, interpretations and alternative decisions to act, support how prospective teachers' labelling of particular phenomena opens up possibilities of recognising them again. So, the way in which discourse on teaching situations is improved, links knowing to action. In this sense, we are learning that prospective teachers' learning (discourse and actions) and different types of practice from both theoretical perspectives are linked considering “different forms of attention”.

ACKNOWLEDGEMENTS

The participation of Salvador Llinares and Ceneida Fernández in developing these ideas has been supported in part by the grant no EDU2017-87411-R, AEI-MINECO, Spain. Salvador Llinares also developed these ideas working with the mathematics

teacher education team at the University of Bristol, School of Education as a Benjamin Meaker Fast-Track Visiting Professor, Autumn Term, 2018.

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PROSPECTIVE MATHEMATICS TEACHERS AS LEARNERS

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