Decoherence and its Role in Interpretations of Quantum Physics

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Submitted in Accordance with the Requirements for the
Degree of Doctor of Philosophy

University of Leeds

School of Philosophy, Religion and History of Science

February 2019

The candidate confirms that the work submitted is his/her/their own and that appropriate credit has been given where reference has been made to the work of others.

Acknowledgements

I would like to thank my supervisors Juha Saatsi and Stephen French for their patience, support and guidance over the last four years. I am grateful to the Arts and Humanities Research Council for funding this project. I am also grateful for helpful and interesting conversations I have had over the last four years with Callum Duguid, Guido Bacciagaluppi, Karim Thébault, Richard Dawid, and Simon Saunders.

Abstract

The purpose of this thesis is to examine different conceptions of decoherence and their significance within interpretations of quantum mechanics. I set out three different conceptions of decoherence found in the literature and examine the relations between them. I argue that only the weakest of these conceptions is empirically well supported, and that the other two rely on claims about the structure of the histories (robust patterns within the wavefunction) which we occupy which require justification.

I also examine the ways in which conceptions of decoherence are used to solve aspects of the quantum measurement problem and support modern interpretations of quantum mechanics. I focus particularly on Wallace's Everettian interpretation of quantum mechanics. I argue that while decoherence is generally successful in supporting this interpretation in a variety of ways, the very strong conception of decoherence on which he relies is itself difficult to justify.

I consider a variety of possible approaches to justifying the use of this strong conception of decoherence and argue that many of them are either unconvincing, or rely on controversial cosmological claims. Finally, I suggest that the best way to justify the use of this strong conception of decoherence is by appealing directly to its indispensability to an otherwise very attractive interpretation of quantum mechanics.

Contents

1 Introduction

The aim of this thesis is to carefully consider what is meant by decoherence as the term is used within the existing literature concerning the foundations and philosophy of quantum mechanics, and to examine the role played by different conceptions of decoherence within specific interpretations of quantum mechanics.

Chapters 2 and 3 will carefully develop and examine three different conceptions of decoherence found in the existing literature, and why it is thought to be useful in attempts to solve the quantum measurement problem. Chapters 4 and 5 will look at a number of concerns relating to the use of decoherence for this purpose and argue that these concerns, at least as commonly presented, need not worry us particularly.

In chapter 6 I will set out what I believe to be a serious problem with the conception of decoherence which is commonly used within particular interpretations of quantum mechanics, which is called medium decoherence. I will argue that this conception (which is the strongest conception I will consider in this thesis) is stronger than our empirical evidence can support, and it is far from clear that any other form of justification for this conception can be provided.

Where possible in this thesis, I endeavour to provide an account of the significance of different conceptions of decoherence and the problems which they solve and produce, while remaining neutral between all interpretations which do not introduce a collapse postulate. Where this is not possible, I focus on Wallace's formulation of the Everett interpretation of quantum mechanics as set out at length in Wallace 2012. I focus on this interpretation because it is a conceptually well-developed and widely respected interpretation which very clearly and explicitly relies on, and defends, a clear conception of decoherence, unlike many others. In chapters 7 and 8, I abandon all attempts at interpretation neutrality, and focus purely on the support for this strong conception of decoherence which can be offered by Wallace's Everettian interpretation. In chapter 7, I consider a proposal by Wallace that the assumptions which underlie medium decoherence could be treated as Humean laws of nature. I will argue that this approach is undermined by the difficulty of reconciling a Humean account of laws with the metaphysics of Wallace’s project. In chapter 8, I look at some of the major problems produced for Wallace’s project if the medium decoherence assumption is dropped, and suggest that this could offer an argument from explanatory indispensability capable of supporting this assumption.

The concluding chapter will return to other interpretations, and argue that more research is needed to understand how these interpretations can best respond to the issue for decoherence presented in chapter 6.

Before turning to decoherence and the technical issues related to it, however, this chapter will provide a very brief introduction to textbook quantum mechanics, as well as the quantum measurement problem, and three popular modern interpretations which aim to solve it. This is intended to make clear the broader nature of the philosophical problems which decoherence is thought to help with, and to motivate interest in these problems. If the reader is interested in a more extended presentation of textbook quantum mechanics and its technical aspects, then I recommend Rae 2008 as a clear and direct textbook. For a clear and extended presentation of the philosophical issues associated with quantum mechanics and a range of interpretive responses I recommend Lewis 2016.

1.1 A Short Introduction to Linear Quantum Mechanics

The fundamental mathematics of text book quantum mechanics can be expressed quite simply. The central equation responsible for the normal dynamical evolution of a particle is Schrödinger's equation:

$$i\hbar\frac{\partial}{\partial t}\psi(\boldsymbol{r},t) = \hat{H}\psi(\boldsymbol{r},t)$$

Where $\hat{H}$ is the Hamiltonian energy operator, $\hbar$ is a constant, i is the square root of -1, and $\psi(\boldsymbol{r},t)$ is the wavefunction of the particle in terms of particle position and time.

The wavefunction is a function which represents all possible information about the results of measurements which could be made on the particle. In the basis of a particular observable, such as position, the wavefunction can be represented as a sum of the wavefunctions which would represent the state corresponding to different possible measurement outcomes multiplied by a complex coefficient[1]. In a position basis:

$$\psi(\boldsymbol{r},t) = \sum_n \alpha_n \phi_n$$

Where α_n is the complex coefficient, and ϕ_n is the nth position eigenfunction. Given that ϕ_n is a position eigenfunction, the position operator $\hat{R}$, applied to this function, will yield:

$$\hat{R}\phi_n = \boldsymbol{r}_n\phi_n$$

Where $\boldsymbol{r}_n$ is the nth position vector of the particle. Thus, the wavefunction can be thought of as a sum of states each of which corresponds to a particular measured value. There are many observables in terms of which this decomposition into eigenfunctions can be made, and they will not all share the same eigenfunctions.

A distinctive feature of quantum mechanics is that the wavefunction representing a state of the particle will very often be a sum of multiple position eigenfunctions, each of which corresponds to a different position eigenvalue. This does not mean that there are multiple particles at different positions. Rather, the wavefunction which describes the state of a single particle includes components which correspond to multiple

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[1] For it to be possible to decompose a wavefunction like this, the measurement involved must have eigenstates which form a complete orthonormal basis.

classically incompatible positions. This distinctively quantum state of affairs is known as a *superposition. T*he real physical state of affairs which a superposition wavefunction represents is a matter of much disagreement, as will be seen later in this chapter when I present a range of different interpretative strategies.

It is important to note that a superposition is defined by reference to a particular set of basis eigenfunctions. A particle may for example be in a superposition of position eigenstates, while occupying a single energy eigenstate.

The Hamiltonian $\widehat{H}$ is the energy operator, and has energy eigenfunctions and eigenvalues in a similar way. Unlike other measurement operators, however, it appears within Schrödinger's equation, and is of central importance to the evolution of the wavefunction over time, as given by that equation. This equation is linear and deterministic.

If the Hamiltonian of the particle is known between times j and k, then the progressive changes to the wavefunction as a result of that Hamiltonian can be encapsulated in a single operator $\widehat{U_{jk}}$ such that:

$$\psi(\boldsymbol{r}, t = k) = \widehat{U_{jk}}\, \psi(\boldsymbol{r}, t = j)$$

Where:

$$\widehat{U_{jk}} = \exp\left(\frac{-i\left(t_k - t_j\right)\widehat{H}}{\hbar}\right)$$

Clearly, given the one-to-one relationship between the state of the wavefunction at different times given in these equations, the dynamics of the Schrödinger equation which gives rise to these operators is entirely deterministic without any innately probabilistic dynamics.

A common way of representing this form of deterministic wavefunction evolution is with an arrow e.g.:

$$\psi(\boldsymbol{r}, t = k) \Rightarrow \psi(\boldsymbol{r}, t = j)$$

The linearity of these Schrödinger dynamics means that the time evolution operators have the following property:

$$\hat{U}\psi(\boldsymbol{r},t) = \sum_n \alpha_n \hat{U}\phi_n$$

Where:

$$\psi(\boldsymbol{r},t) = \sum_n \alpha_n \phi_n$$

In other words, the effect of the time evolution operator on a wavefunction is the same as the sum of the effect of that operator on the components of that wavefunction, each multiplied by its complex coefficient. E.g.:

$$\frac{1}{\sqrt{2}}\psi_1(t=j) + \frac{1}{\sqrt{2}}\psi_2(t=j) \Rightarrow \frac{1}{\sqrt{2}}\psi_1(t=k) + \frac{1}{\sqrt{2}}\psi_2(t=k)$$

In the case of a particle in a superposition of two position states at time j, this means that the state of that system at a later time k is the sum of the state to be expected at k, given the two initial positions, each multiplied by their respective coefficients. It is important to note that both the coefficients and the functions themselves may be complex, as this is what gives rise to distinctively quantum interference phenomena, in which components in the final wavefunction arising from the two initial positions when added together may subtract from one another, rather than adding together. This phenomenon will be discussed in detail in the next chapter. For now, though, it is just important to understand that interference is a distinctively quantum phenomenon, which cannot be understood or modelled without the linear evolution of complex wavefunctions.

So far, then, all the dynamics I have presented are linear and deterministic. Superposition states evolve and interfere with one another to produce other states at other times, all of which can be decomposed in terms of bases of measurement eigenfunctions which (in some sense) correspond to particular measurement outcomes. What is missing however is any rule to tell us which of these measurement outcomes we are to expect when we come to actually measure the system. The answer lies in the Born rule and the projection postulate, which describe dynamics fundamentally different to those given by Schrödinger's equation.

1.2 Born's rule and the projection postulate

The probability of a particular measurement outcome is given by the Born rule. For a position measurement with the eigenstates given above, the probability of measuring a particular position is as follows:

$$P(\boldsymbol{r}_n) = |\alpha_n|^{\mathbf{2}}$$

That is, the probability of a particular measurement eigenvalue is equal to the modulus squared of the coefficient of the eigenfunction which corresponds to that eigenvalue.

The Born rule offers a means of obtaining measurement probabilities from a linearly evolving wavefunction. Performing such measurements, however, does not leave that wavefunction undisturbed. The projection postulate (also sometimes referred to as the collapse postulate, for reasons which will become clear later in this chapter) says that, after the measurement is made, the wavefunction of the system will become equal to the eigenfunction associated with the eigenvalue which was measured. This postulate is of great importance to the empirical adequacy of standard textbook quantum mechanics. Without it, there is no reason to suppose that a measured property of a system will be the measured property of that system if the measurement is immediately repeated. Nonetheless, this postulate is at the heart of much controversy about the fundamental nature of quantum mechanics as a theory.

The projection postulate is radically different not only to the Schrödinger dynamics presented in the previous section, but to all our other fundamental physical theories. It is fundamentally stochastic, unlike the deterministic unitary Schrödinger dynamics. It is also strikingly abrupt and discontinuous, so much so that Dirac describes it as "…a jump in the state of the dynamical system." (1935, pp. 36), as the wavefunction suddenly transitions from the superposition of many eigenstates to the single measured state. Most troubling of all, however, is the question of just when this abrupt quantum jump takes place.

The projection postulate is tied to Born's rule for the probabilities of measurement outcomes. In consequence, the projection postulate seems to come into play just in cases where observable features of the system are measured. This raises two concerns:

Firstly, if measurement is taken here in the usual sense of just the process of a human being examining the system and recording information about it, then the situations in which the projection postulate apply seem to be determined by the presence or absence of a human observer. This anthropocentrism seems out of place in one of our most empirically successful scientific theories. Whether or not this anthropocentric conception of measurement is really what was intended by authors such as Dirac 1935 and von Neumann 1932 when they introduced this postulate, and what other conception they might have preferred, is somewhat unclear (Myrvold 2018).

Second, then, is the question of just when measurement should be considered as having taken place. Again, the lack of clarity as to what constitutes a measurement makes the answer unclear. This leads to the troubling situation that the evolution of quantum systems appears to be governed by two radically different dynamical rules, but we have no clear answer to the question of which of these rules applies in which cases.

One possible solution to this problem is to abandon the idea that the wavefunction really describes the state of the system at all, and instead to treat it as a representation of our knowledge of the system rather than the system itself. This would give some justification for measurements having such a direct and drastic effect on the state of the wavefunction. The basic question that it would leave unanswered, however, is just what the physical reality underlying the wavefunction could be.

It seems, then, there is something unsatisfactory about adding the projection postulate. To clarify this point further, I will now turn to look at what happens if we consider a measurement apparatus as a quantum mechanical system purely subject to the linear deterministic Schrödinger dynamics without any discontinuous quantum jumps. Before doing that, however, it will be necessary to introduce the notion of quantum entanglement, and a different way of representing quantum states known as Dirac notation. Both of these will be of great importance throughout the rest of this thesis.

1.3 Dirac Notation

The central equations of quantum mechanics can be represented in two main ways. The first is to treat the state of the system as a wavefunction, as I have done so far in this chapter. The wavefunction is a sinusoidal function of the variables of the system such as particle position and time. As discussed, applying operators to the wavefunction can evolve it in time or obtain probabilities of particular measurement outcomes.

The second way in which quantum mechanics can be expressed is in terms of a state vector rather than a wavefunction. This formalism will be explained in chapter 2, but in essence the different eigenfunctions of a particular measurement are each assigned an element within a vector, and the complex coefficients of different eigenfunctions form the contents of the vector. This produces a vector of the type:

$$[\psi] = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{pmatrix}$$

Matrices are then used to represent operators.

Both of these formalisms are mathematically equivalent, but significantly different in their structures and representation. Dirac notation is designed as a means of specifying system states and operations performed on them, which can be applied equivalently using either the state vector or wavefunction formalism. In the same eigenbasis used above, the quantum state of a system would then be represented as:

$$|\psi\rangle = \alpha_1|\phi_1\rangle + \alpha_2|\phi_2\rangle + \cdots \alpha_n|\phi_n\rangle$$

The bracket with which $|\psi\rangle$ is written is called *ket*, and indicates that this term is a wavefunction/state-vector. Here, $|\psi\rangle$ is represented as a sum of other eigenbasis kets, but it could also perfectly well be treated either as a state vector within vector formalism, or as a sum of eigenfunctions within a wave formalism.

The second aspect that must be understood regarding these representations is how they deal with probabilities, and the measure of overlap between different states. This quickly becomes a more complicated and technical issue than can be clearly explained within this introduction. Rae 2008, chapter 6, gives a clear account of the relations between operations presented in the different formalisms. Here I will simply give a very brief account of the significance of an inverted ket, such as $\langle\psi|$, which is known as a *bra*. This denotes the complex conjugate of a wavefunction, or the Hermitian conjugate of a state vector:

$$\langle\psi| = \psi^*$$

$$\langle\psi| = [\psi^\dagger] = (\alpha_1^* \quad \alpha_2^* \quad \ldots \quad \alpha_n^*)$$

This conjugate form of a system state is important in a variety of operations a few of which will be seen in chapters 2 and 3.

Dirac notation will be the default notation for quantum states throughout the remainder of this thesis.

1.4 Entanglement

As I have already implied, quantum mechanics can be applied not just to single particles, but to a vast range of systems (any systems at all on some interpretations). Two systems, with state vectors independent of one another, jointly compose a composite system of which they are both parts, which will have its own state vector:

$$|\Psi\rangle = |\Phi\rangle|\Theta\rangle = [\Phi]\otimes[\Theta]$$

Where, $|\Phi\rangle$ and $|\Theta\rangle$ are the state vectors of the two independent subsystems, and $|\Psi\rangle$ is the composite system which jointly compose.

In quantum mechanics, however, the states of two different systems are not always independent of one another. In terms of basis vectors of the subsystem $\{|\phi_i\rangle\}$ and $\{|\theta_i\rangle\}$ respectively, the state of the composite system might for example be:

$$|\Psi\rangle = \alpha|\phi_1\rangle|\theta_1\rangle + \beta|\phi_2\rangle|\theta_2\rangle$$

If the state of the composite system is of this form, then there will be direct correlations between measurements made on the two subsystems. In this case, if the eigenvalue associated with $|\phi_1\rangle$ is measured in the first system, then the eigenvalue associated with $|\theta_1\rangle$ will always be measured in the second. If, on the other hand, the eigenvalue associated with $|\phi_2\rangle$ is measured in the first system, then the eigenvalue associated with $|\theta_2\rangle$ will always be measured in the second. This interdependence of system states is known as entanglement.

Entanglement is possible between spatially separated systems. The correlations between spatially separated measurement results in entangled systems are what give rise to claims that quantum mechanics is incompatible with special relativity (For more on these claims, and various possible responses see Berkovitz 2017.) Special relativity relies on the principle of locality, which says that no signal can travel faster than the speed of light. With a carefully designed experimental apparatus, it is possible to produce correlations which are difficult to explain without some form of superluminal signal between parts of the system. I will not go into details of these experiments, originally conceived by Einstein, Podolsky & Rosen 1935 and extensively developed by Bell 1964. For an overview of these experiments see Shimony 2017.

If two systems are entangled, then it will generally not be possible to express the state of the individual systems as a wavefunction. The composite system will have a wavefunction but, as with $|\Psi\rangle$, it will not be a state which can be expressed as a product state of separate wavefunctions for the two subsystems. In the next chapter I will present a way in which we can attempt to express quantum states for the two individual subsystems using a piece of mathematics called a reduced density matrix. As I will discuss, this is effective for understanding the behaviour of subsystems in many cases, but is still very different to the individual subsystems possessing their own independent wavefunctions.

Before moving on, I will give a very brief, very superficial, characterisation of what is meant by the word decoherence. The next two chapters will be taken up with setting out and discussing three different technical conceptions of decoherence. For this chapter though, a far more basic and general conception will suffice.

Decoherence is the process of the state of a quantum system becoming entangled within its environment by a particular system property. After this has occurred, operations performed just on the original system will display little or no interference phenomena under operations affecting the entangled system property.

A great deal more will be said about this process, but a basic conception of what the word decoherence means will be useful for understanding the way different interpretations respond to the quantum measurement problem.

1.5 Von Neumann Measurement

Now I turn to look at the effect of quantum measurement if we drop the projection postulate, and assume that the linear quantum formalism can be applied to any system or any process (including the state of a human observer). Von Neumann 1932 developed a scheme for modelling measurements in this way, taking measurement in the most generic sense possible.

Von Neumann characterises a measurement apparatus $|a\rangle$ as one that fulfils the following requirement:

$$|a_{ready}\rangle|\phi_i\rangle \Rightarrow |a_i\rangle|\phi_i\rangle$$

That is, for a measurement apparatus designed to measure the state of a physical system, where the set of eigenstates of the measurement being performed is $\{|\phi_i\rangle\}$, the apparatus will, if brought into contact with that system in one of those eigenstates, when in a state ready for measurement, then evolve to a state $|a_i\rangle$ corresponding to the eigenstate of the measured system. In other words, if the system to be measured initially occupies eigenstate 1, then when it is brought into contact with the system, a measurement apparatus will transition to a state of its own which corresponds to state 1 of the measured system.

If a putative measurement apparatus failed to behave in this way, then it does not seem that it could reasonably be described as being a measurement apparatus.

Now, though, let us look at what happens when an apparatus of this kind comes into contact with a system initially in a superposition of measurement eigenstates $\alpha|\phi_1\rangle + \beta|\phi_2\rangle$. In this case, due to the linearity of quantum dynamics in the absence of the projection postulate, the effect of the measurement will be:

$$|a_{ready}\rangle(\alpha|\phi_1\rangle + \beta|\phi_2\rangle) \Rightarrow \alpha|a_1\rangle|\phi_1\rangle + \beta|a_2\rangle|\phi_2\rangle$$

That is, rather than reaching a single determinate measurement outcome state, the measurement apparatus has become entangled to the quantum superposition state of the measured system. The composite system of measurement apparatus and measured system is now in a superposition state. This is clearly not what we expect by way of a measurement outcome.

The problem with this outcome is sometimes said to be that it contradicts our observations, as we do not observe macroscopic measuring apparatuses in such entangled superposition states. Myrvold 2018 points out that this way of phrasing the problem is misleading. As there is no obviously sensible way to interpret a measurement apparatus occupying a state like this, it does not seem that we could establish what it would be like to see such a state, or that our experiences are really contrary to such predictions.

Nonetheless, we are left with a clear interpretational problem of how to reconcile the linear quantum mechanics, which predicts such bizarre superposition states on the macroscopic scale, with our observed everyday experience. This tension between linear quantum mechanics and our everyday experience is referred to as the *quantum measurement problem*. Quantum mechanics is an extremely well confirmed theory, and relies on notions like superposition to account for interference phenomena. If this theory is treated as a complete description of reality and applied universally, however, then superposition states will arise for the macroscopic world around us, and it is very unclear how we should interpret these.

There are many different formulations and subdivisions of the precise nature of this problem, and the next chapter will introduce a more technical presentation of the problem, and a subdivision which is useful for understanding the importance of decoherence. For now, though, I wish to give an overview of four responses to this problem, and the interpretational strategies which arise from them.

1. Antirealist interpretations. Wavefunction states, though predictively powerful, do not really represent the unobservable world, and so we should not expect to directly observe such states.
2. Collapse interpretations. Wavefunction states subject to the linear Schrödinger dynamics are not universal, and cannot reasonably be extended to the macroscopic world of everyday experience. These approaches rely on some form of collapse dynamics such as I have already touched on.
3. Hidden variables interpretations. The wavefunction is real and universal, but is not a complete description of the world. This response adds additional variables to the physical theory which give rise to particular measurement outcomes for measured properties on macroscopic scales, which are what we see in the macroscopic world.
4. Everettian interpretations. The wavefunction is real, universal and complete. On these views, our belief that macroscopic objects such as the ones we interact with in day-to-day life always have determinate states, and do not enter superpositions, is dropped.

The next four sections will examine each of these interpretational approaches in turn, and give a brief overview of how they work and the issues which they face.

1.6 Antirealist Interpretations

The main antirealist interpretations of quantum mechanics are the QBist or quantum Bayesian approach of Fuchs, Mermin & Schack 2014, and the pragmatist approaches of Healey 2012 and Friederich 2015. All of these approaches agree that the wavefunction does not represent any physical part of the world. They differ however in what they take the significance of the wavefunction to be.

The quantum Bayesian approach regards the wavefunction as having a purely subjective epistemic role. On this view, the wavefunction is simply a representation of an agent’s degrees of belief regarding the outcomes of measurements made on a particular system. The degrees of belief regarding a system, and consequently the wavefunction attributed to the system, may differ between two agents without either agent being either irrational or mistaken.

On a pragmatist view, on the other hand, the wavefunction is objective but not representational. On this view, the ascription of quantum states is subject to particular rules. To ascribe a wavefunction in a way that breaks these rules is to be mistaken in one’s state ascription. If the rules of quantum mechanics are followed, both in the ascription of quantum states and their subsequent evolution, then the result will be that we are able to ascribe reliable probabilities to non-quantum measurement claims.

A significant question for this interpretation is that of just when nonrepresentational distinctively quantum states turn into representational non-quantum measurement claims. This form of problem, of identifying just when the quantum mechanical gives way to the macroscopic world of our everyday experience, is a challenge faced by all interpretations which regard the dynamics of the quantum mechanical world as fundamentally different to the world of our everyday experience.

In order to distinguish nonrepresentational claims about quantum states from non-quantum measurement claims which are seen as representational, pragmatist interpretations appeal to decoherence. This distinction allows pragmatist interpretations to take an antirealist view of claims about quantum states while allowing for the possibility of a realist view regarding scientific theories of the macroscopic world around us. I think there is work to be done making clear precisely the sense of decoherence being used here, and understanding precisely what it achieves. This work however is beyond the scope of this thesis.

1.7 Collapse Interpretations

It has already been mentioned that the projection postulate by which the wavefunction of the system changes to the measured eigenstate following a measurement is also known as the collapse postulate. *Collapse* is a word used to describe this physical transition from a wavefunction that is a superposition of many measurement eigenstates to a wavefunction which is aligned to just one of these eigenstates.

I noted that there are significant difficulties in identifying the point at which measurement counts as having occurred, and consequently a significant difficulty saying just when this change to the system's wavefunction takes place. Since the first appearance of wavefunction collapse within the literature on quantum mechanics, a number of interpretations have arisen which attempt to give a more precise account of the process of collapse and so solve this problem. For an overview of these approaches and some of the issues which they face see Ghirardi 2018.

Modern collapse views all stem from ideas set out in Ghirardi, Rimini & Weber 1986, and consequently are generally referred to as GRW interpretations. These interpretations differ substantially from the original notion of wavefunction collapse brought about by measurement. In particular, measurement as such is no longer thought to be a direct cause of wavefunction collapse. Thus, the tricky questions of how to characterise measurement, and what its physical significance could be, are avoided.

Instead, the form of collapse used is random and spontaneous. Ghirardi 2018 writes “The key assumption of [Quantum Mechanics with Spontaneous Localizations] is the following: each elementary constituent of any physical system is subjected, at random times, to random and spontaneous localization processes (which we will call hittings) around appropriate positions.” It should be noted that the word localisation here means localisation in a position basis. GRW interpretations treat position as fundamentally different to other observables in quantum mechanics, and it is particle position alone which is subject to collapse.

Collapse takes the form of multiplying the existing wavefunction in a position basis by a Gaussian function centred on a particular point. The wavefunction is not replaced by an entirely determinate position, but by a new wavefunction tightly centred on a particular position. The position to which the wavefunction localises is a fundamentally probabilistic function of the initial wavefunction.

The typical time periods for such spontaneous localisations, and the parameters of the Gaussian function of the localisation, are parameters which on the GRW view have yet to be empirically identified, though there are various estimates. In principle, it should also be possible to empirically test the GRW interpretation as its dynamics differ from standard quantum mechanics in (in principle) measurable ways. Such tests, however, remain beyond our technical capabilities. These issues, as well as conceptual concerns relating to the primitive ontology of this interpretation, continue to be debated amongst its advocates.

On this interpretation, though wavefunctions and the rules governing them are universally applicable, the linear Schrödinger dynamics are not. These dynamics apply only for short periods of time in between spontaneous particle localisations.

1.8 Hidden Variables Interpretations

Hidden variables interpretations seek to solve the quantum measurement problem by adding additional variables to the Schrödinger wavefunction dynamics. By far the most popular interpretation of this kind is the de Broglie-Bohm interpretation, also known as Bohmian mechanics, which was developed independently by de Broglie 1927 and Bohm 1952. For an overview of this interpretation see Goldstein 2013.

Like the GRW interpretations just discussed, Bohmian mechanics regards particle position as a fundamentally different observable to other observables within quantum mechanics. As Bell puts it:

> “[I]n physics the only observations we must consider are position observations, if only the positions of instrument pointers. It is a great merit of the de Broglie-Bohm picture to force us to consider this fact. If you make axioms, rather than definitions and theorems, about the "measurement" of anything else, then you commit redundancy and risk inconsistency.” (Bell 1982 pp. 166.)

For reasons which will become clear in the next chapter, it is not possible to have a hidden variables theory of quantum mechanics in which all observable variables have definite values at all times. This appeal to the primary role of measurement, within the physics experiments which underlie quantum mechanics, is intended as justification for taking particle position as a preferred observable with its own additional dynamics.

The fundamental Bohmian ontology is made up of particles which always have definite (but generally unknown) positions. These particle positions are in addition to linear Schrödinger quantum mechanics which applies universally in Bohmian quantum mechanics. Bohmian mechanics resolves the tension between our everyday experience of objects occupying single particular local positions, and linear quantum mechanics which predicts that they should enter superposition states of many positions, by identifying the objects of our everyday experience with arranged collections of Bohmian particles.

Superficially, this interpretation may appear very traditional in its ontology, as we are used to the idea of the world around us being made up of particles. The particles of Bohmian mechanics, however, are profoundly different to our general understanding of the particles which make up our world. As already mentioned, it is only position which is treated as a preferred variable in Bohmian mechanics. This means that the only definitive properties which Bohmian particles possess are position and mass. All other properties such as energy, electrostatic charge, and spin (a property which will be explained further in the next chapter) are not intrinsic properties of Bohmian particles. Instead, all these properties are contained within the wavefunction, and its influence on the trajectories of the Bohmian particles. It is also the wavefunction, and its direct influence on the trajectories of Bohmian particles, which gives rise to interference phenomena on the Bohmian interpretation.

The wavefunction is entirely responsible for determining the trajectories of all Bohmian particles (though not their initial positions). The influence of the wavefunction on Bohmian particles can be expressed either in terms of a guiding equation, or quantum potential. These representations are mathematically equivalent, but give different impressions of the nature of the influence of the wavefunction on particle trajectories.

The Bohmian guiding equation gives the changes of position of a particle k initially in position $\boldsymbol{R}_k$ is expressed in the wave formalism as follows:

$$\frac{d\boldsymbol{R}_k}{dt} = \frac{\hbar}{m_k} Im\left(\frac{\psi^* \boldsymbol{\nabla}_{\mathrm{k}} \psi}{\psi^* \psi}\right)$$

Where $\boldsymbol{\nabla}_{\mathrm{k}}$ is a spatial differential of the position of particle k, m_k is the mass of particle k, and the function Im returns just the imaginary part.

It should be noted that this equation gives the velocity of particle K rather than its position. The positions of Bohmian particles are not directly determined by the wavefunction. In Bohmian mechanics, particles begin in definite but unknown positions. It is this ignorance of the initial positions of Bohmian particles which is the origin of quantum mechanical probabilities within the Bohmian interpretation. Both the evolution of the wavefunction, and the trajectories of Bohmian particles, are entirely deterministic.

These dynamics are applied universally so that there is not fundamentally any split between the quantum world of the very small, and the world of our everyday experience. The question therefore arises of why the distinctively quantum interference phenomena, seen in isolated interferometer experiments, cease to be observed once a measurement of the system is made.

The answer to this question is typically expressed (e.g. Dürr, Goldstein, & Zanghì 1992) in terms of a local *effective wavefunction* which governs the behaviour of particles within a particular subsystem. Goldstein 2013, however, points out that the justification for this use of an effective wavefunction (presented originally by Bohm 1952) amounts to what is now commonly known as decoherence. Interference suppression from environmental decoherence is a direct consequence of the linear Schrödinger dynamics, and as these dynamics are a universally applicable feature of Bohmian mechanics, it accounts for interference suppression in these cases entirely independently of any specific features of the Bohmian interpretation.

1.9 Everettian Interpretations

I now turn to the family of interpretations about which I will have most to say in this thesis. These interpretations are characterised by the view that the evolution of the wavefunction described by linear Schrödinger quantum mechanics is a universally appropriate and complete description of the world. These interpretations, of which there are many variants, all stem from the doctoral work of Everett 1957. For a clear presentation of many of these variants see Barrett 1999.

At first glance it may seem unclear how an interpretation of quantum mechanics which simply endorses the universality of linear Schrödinger dynamics could possibly solve the measurement problem, and be reconciled with our everyday experience.

To understand Everett's key insight into this question, consider the final state of the measurement process modelled by linear quantum mechanics which was considered earlier:

$$|a_{ready}\rangle(\alpha|\phi_1\rangle + \beta|\phi_2\rangle) \Rightarrow \alpha|a_1\rangle|\phi_1\rangle + \beta|a_2\rangle|\phi_2\rangle$$

Now consider a further measurement operation, this one to discover whether or not the measurement apparatus has made a successful measurement:

$$|Measurement?\rangle|a_1\rangle \Rightarrow |Yes\rangle|a_1\rangle$$

$$|Measurement?\rangle|a_2\rangle \Rightarrow |Yes\rangle|a_2\rangle$$

When this operation is applied the result is as follows:

$$|Measurement?\rangle(\alpha|a_1\rangle|\phi_1\rangle + \beta|a_2\rangle|\phi_2\rangle) \Rightarrow \alpha|Yes\rangle|a_1\rangle|\phi_1\rangle + \beta|Yes\rangle|a_2\rangle|\phi_2\rangle$$

$$\alpha|Yes\rangle|a_1\rangle\left|\phi_1\right\rangle + \beta|Yes\rangle|a_2\rangle\left|\phi_2\right\rangle = |Yes\rangle\left(\alpha|a_1\rangle\left|\phi_1\right\rangle + \beta|a_2\rangle\left|\phi_2\right\rangle\right)$$

That is, when applied to the superposition state which is reached as the final stage of the original quantum measurement, our measurement of whether or not a determinate measurement has taken place determinately yields the answer yes. One way of thinking of this measurement would be for a scientist, after having made their original quantum measurement, to record on a piece of paper whether or not they had made a determinate measurement. What this result shows is that, as the scientist writing the word “yes” is the product of the linear evolution of both of each of the superposed states which the scientist occupies, they will determinately write the word “yes”. Indeed, under the vast majority of operations that could be performed on the scientist, they will behave just as if they had indeed made a determinate measurement, rather than entering a peculiar superposition state.

This result is central to undermining the assumption that linear quantum mechanics is really at odds with our everyday experience in a way which requires some alteration or suspension of linear quantum mechanics in the case of this experience. This opens the door to interpretations of quantum mechanics which feature only the universal wavefunction as their fundamental ontology.

Interpretations of this type face a variety of challenges, to which they must respond. These include the problem of the preferred basis (which will be discussed further in the next chapter) and the problem of the origin of probability (which will be discussed further in chapters 4 and 8). In response to these challenges a wide variety of approaches have been developed. These include the many minds view set out by Albert and Loewer 1988, the many histories view originally developed by Griffiths 1984, and the many worlds interpretation, the first example of which does not seem to be widely agreed upon. The situation is further complicated as, even within these broad groupings, particular authors differ considerably in their positions. An example of this will be seen in chapter 3, where I will set out some of the detailed technical disagreements between the different developers of the many histories interpretation.

I cannot do justice to these many, varied and technical approaches to Everettian quantum mechanics in this introduction. Instead, I will just outline the basic points of the form of the many worlds interpretation recently developed in Wallace 2012, as this is the Everettian interpretation which will receive most attention in this thesis.

Like all Everettian interpretations, Wallace’s interpretation takes, as its representation of the fundamental ontology of the universe, the universal wavefunction. All other ontology on Wallace's view is made up of robustly persistent patterns within the universal wavefunction. *Worlds* are defined as persistent patterns within that wavefunction which fulfil certain technical criteria. Within the portion of the universal wavefunction which constitutes a particular world, particular objects are also identified with patterns.

On Wallace's view, these patterns can be identified as the objects of our everyday experience, because they approximate the classical dynamics of medium-sized dry goods subject to classical equations of motion with such a high level of accuracy. For this reason, these objects and their dynamics are often referred to as quasi-classical.

Wallace regards these patterns as emergent objects, in a very weak sense of emergence[2], which he attributes to Dennett:

> "**Dennett's Criterion:** a macro object is a pattern and the existence of a pattern as a real thing depends on the usefulness -- in particular the explanatory power and predictive reliability -- of theories which admit that pattern in their ontology." (Wallace 2012, pp. 50).

Thus, apples feature in Wallace's emergent ontology because there are robust patterns within the linearly evolving wavefunction which play the functional role of apples, as described in our theories of biology and Newtonian mechanics, and these theories prove useful to us for their explanatory power and predictive reliability. All ontology of Wallace's interpretation, other than the wavefunction itself, is emergent in this sense. Worlds, and the quasi-classical entities within them, are nothing more than robustly persistent patterns within the wavefunction.

The existence of patterns which robustly fulfil these criteria is directly related to decoherence. As this phenomenon is (at least in its simplest form) just a product of linear Schrödinger dynamics for particular types of system, it does not constitute any addition to, or change of, linear quantum mechanics. Understanding just how this works and the issues it raises requires a far more technical examination of this connection which will be undertaken over the next two chapters.

As a result of this direct connection to linear Schrödinger quantum mechanics, Wallace often describes his interpretation as being nothing

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[2] There are many different conceptions of emergence within the philosophical literature. For an overview see O'Connor & Wong 2015. In this thesis, the only conception with which I will be concerned is that used by Wallace.

more than the theory of quantum mechanics itself taken seriously. He writes:

> “This, in short, is the Everett interpretation. It consists of two very different parts: a contingent physical postulate, that the state of the universe is faithfully represented by a unitarily evolving quantum state; and an a priori claim about that quantum state, that if it is interpreted realistically it must be understood as describing a multiplicity of approximately classical, approximately non-interacting regions which look very much like the ‘classical-world’.” (Wallace 2012 pp. 38).

As will become clear later in this thesis, I believe that Wallace is profoundly mistaken in believing this to be an a priori claim. In essence this is because, while the simplest conception of environment induced decoherence can be seen as a direct consequence of the unitarily evolving quantum state, a stronger conception is needed in order to produce the approximately non-interacting worlds of Wallace's interpretation. This direct reading of linear quantum mechanics is definitely the intention of Wallace’s project, however. In chapter 3 I will identify an implicit premise in Wallace's characterisation of worlds which does not seem as though it can possibly be established a priori. First, though, it will be necessary to look at a more technical formulation of decoherence and the quantum measurement problem.

1.10 Conclusion

In this chapter I have presented a generic understanding of the basic form of the quantum measurement problem, and four types of interpretational responses which have been given to it.

The purpose of this thesis is to investigate the role of decoherence within interpretations of quantum mechanics. Environment induced decoherence is a feature of linear Schrödinger quantum mechanics. It will arise, therefore, in any interpretation which uses these dynamics. All interpretations that I have discussed use these dynamics in some form at least some of the time. They vary however in when and how they are used.

In this thesis I limit my focus to those interpretations which regard the linearly evolving wavefunction as describing features of the world and universal. That is, I will limit my focus to Everettian and Bohmian interpretations. As I noted in this chapter, the role of decoherence in interpretations of quantum mechanics is not limited to just Bohmian and Everettian interpretations. Other interpretations, however, take a significantly different view of decoherence, as a consequence of the different view they take of the linear Schrödinger dynamics. As a result, I have decided to leave consideration of them out of this thesis.

The next two chapters will look at conceptions of decoherence which are applicable to all Everettian and Bohmian interpretations. Chapter 6 will also focus on generic features of the unitarily and continuously evolving wavefunction, and so should be applicable to all Everettian and Bohmian interpretations.

Chapters 4 and 5 respond to criticisms of decoherence-based approaches to the interpretation of quantum mechanics, which are primarily intended as objections to Everettian many worlds interpretations. As a result, they are principally relevant to Everettian many worlds interpretations.

In chapters 7 and 8, I turn to focus purely on Wallace's Everettian interpretation of quantum mechanics, and look at how it could respond to problems regarding the way decoherence is used within this interpretation, which are developed over the course of the thesis.

In the next chapter I will develop a more technical conception of the measurement problem and a definition of environment-induced decoherence. These will be essential ground work for two more technical conceptions of decoherence developed in chapter 3. I will argue over the course of this thesis that the reduced density matrix conception of decoherence, which I set out in the next chapter, has the clearest and strongest connection with direct empirical evidence, while also being the weakest. Medium decoherence, on the other hand, is a far stronger conception, of significant use in interpretations of quantum mechanics, but far more difficult to connect to any direct empirical evidence.

2 Decoherence and the Quantum Measurement Problem

In the last chapter, I presented a general overview of the quantum measurement problem, and a number of interpretations which have been developed in response to it. I noted that the suppression of interference phenomena as a result of interaction with the environment played a role in several of these interpretations, and that this phenomenon was known as decoherence.

The main purpose of this chapter is to introduce a more mathematically rigorous conception of this phenomenon, and examine its significance in solving aspects of the quantum measurement problem. In order to do this, a more mathematically sophisticated conception of quantum measurement, and a useful subdivision of the quantum measurement problem will first be introduced.

2.1 Quantum Measurement

In this section I will examine the process of quantum measurement and seek to set out the main difficulties of understanding probabilities within quantum mechanics. To make the problems posed as clear as possible, they will be related to the behaviour of particles passed through a series of Stern-Gerlach (S-G) devices. I will then outline a useful subdivision of these problems introduced by Schlosshauer 2007.

2.1.1 The Stern-Gerlach Apparatus

A Stern-Gerlach device uses an inhomogeneous magnetic field to separate a beam of particles according to a physical property called spin, as shown in figure 1. The particles are deflected from the path according to their spin property and the direction and gradient of the magnetic field. Spin is a quantised property, and when measured (e.g. by adding particle detectors to see where particles arrive after passing through the Stern-Gerlach device), the particles will always be found to possess a spin value of either $+\frac{1}{2}$, or $-\frac{1}{2}$. (Feynman 1964 Vol II, 35-2)

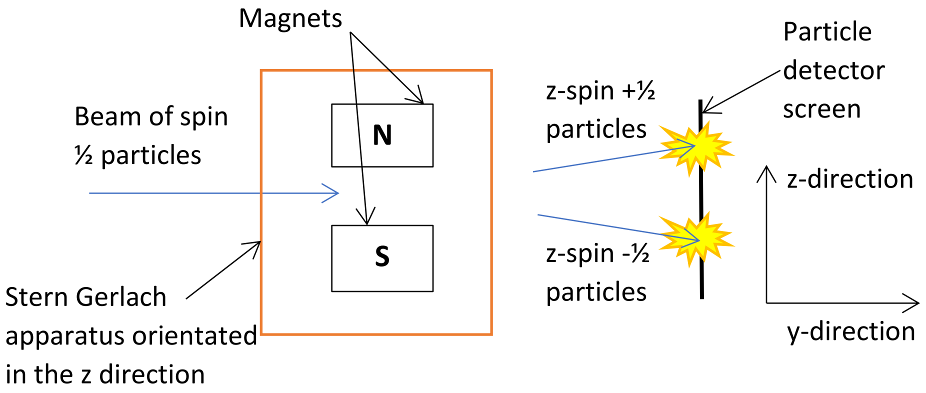

Figure 1. The practical arrangement of a Stern-Gerlach apparatus, and its effect on spin ½ particles

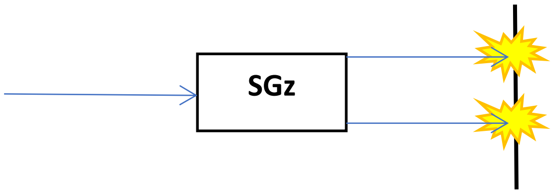

Figure 2. The abbreviated representation of a Stern-Gerlach apparatus used hereafter.

For ease of representation, the details of the apparatus will be omitted hereafter, and instead represented as shown in figure 2.

These two beams will have intensities which vary depending on the state of the particles in the beam incident to the apparatus. This can be most easily expressed using Dirac notation. In this notation, the spin properties of the particles incident to the apparatus can be expressed as:

$$|\psi\rangle = \alpha|\uparrow_z\rangle + \beta|\downarrow_z\rangle$$

Where α and β are complex numbers such that $|\alpha|^2 + |\beta|^2 = 1$. $|\uparrow_z\rangle$, and $|\downarrow_z\rangle$ represent the states of particles with properties spin up and spin down in the z direction. These are known as z-spin eigenstates. The values that these states yield, when measured by a z-spin measuring apparatus ($+\frac{1}{2}$, and $-\frac{1}{2}$ respectively), are called eigenvalues.

The relative intensities of the two beams will then be:

$$I_\uparrow \propto |\alpha|^2$$

$$I_\downarrow \propto |\beta|^2$$

These relative intensities become probabilities when the beam is reduced so as to allow only 1 particle to pass through the apparatus at a time. The probabilities of the particle being measured to have each spin value are then given by Born's rule: $P_\uparrow = |\alpha|^2$, $P_\downarrow = |\beta|^2$.

This is the simplest case of probabilities in quantum mechanics. The probabilities examined here do not appear to present any particular difficulties of interpretation. A simple epistemic interpretation of these probabilities is adequate. That is, we have no reason to think that these probabilities represent anything more than our ignorance of the particles true state. There is no reason, so far, to doubt that there is a measurement independent z-spin property, the probabilities of which, given our knowledge of the system, are given by our theory of quantum mechanics. This would closely resemble the understanding of probability as used in many special sciences. The main problem with taking α and β to simply represent our epistemic uncertainties about the particle in this way is interference.

2.1.2 Non- Commuting Variables

To understand interference we first need to understand non-commuting variables. The most famous case of non-commuting variables is that of position and momentum, summarised in Heisenberg's famous uncertainty principle: $\Delta x \Delta p \geq \frac{\hbar}{2}$. That is, that the uncertainty about the position of a particle multiplied by the uncertainty about its momentum must always be greater than a particular (very small) value.

Though sobering about our epistemic position within the world, it might be thought that these limitations on our knowledge do not present conceptual difficulties. Measurement uncertainties have always plagued the empirical sciences, and it is not unreasonable to suppose that there might be limits to how far these could, even in principle, be reduced. The only ways in which this differs from usual measurement uncertainties, are that it seems extremely likely that this is an in principle limitation, which technological advances could not overcome, and that this uncertainty relates to two quantities rather than one. In short it could reasonably be supposed that this kind of uncertainty principle arises simply from the process of accurately measuring one variable, necessarily having a random effect on the other variable, meaning that no two measurements could ever identify two such variables beyond a certain level of accuracy. In fact, though, as we shall see by examining further arrangements of S-G devices, this interpretation is quite inadequate to account for the appearance (and non-appearance) of interference phenomena observed empirically. Non-commuting variables bear no such straightforward interpretation, and understanding these relationships is a major challenge for any interpretation of quantum mechanics.

Returning to our Stern-Gerlach apparatus we find that here too our particle has observables the measurement of any one of which will disrupt the value of another. So far, we have measured the z-spin of a beam of particles by passing them through a magnetic field orientated in the z-direction, as shown in figure 1. We will now introduce a new observable, x-spin, which can be found, similarly to z-spin, by passing the particles through a magnetic field orientated in the x-direction.

We will now consider the surprising results obtained when we pass our particles through a series of x-spin, and z-spin measurements.

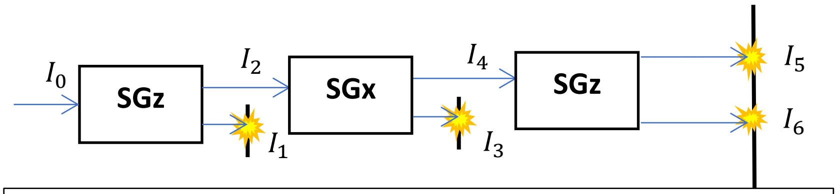

Figure 3. A series of Stern-Gerlach measurement arrangements, orientated in the z-direction and x-direction, with the intensities of the different beams labelled for further consideration.

If we assume that the intensity of the particle beam entering the first apparatus I_0 is known, we can easily calculate the intensity of the unobserved beams, I_2, and I_4, by simply deducting the intensity of the beams which have separated off, I_1, and I_3. We therefore know the intensity of all the labelled beams, either by direct measurement or by deduction.

The experimental results obtained for such an apparatus are that exactly half of the z-spin up particles contained in beam 2, are found to have x-spin down, and are detected when beam 3 is measured. More surprisingly, half of the x-spin up particles contained in beam 4 are measured to have z-spin down following the final S-G apparatus. That is, half of the particles which were in beam 2, having been measured to have z-spin up, now have z-spin down in beam 6. Seemingly something in the intervening process has caused half of these particles to change their z-spin value. As mentioned, though, we shall see that this intuitively appealing understanding is not ultimately viable.

One issue which this result highlights is that quantum mechanics appears to be unavoidably probabilistic. As in the previous case, the intensities here become probabilities when the particle flow rate is slowed to allow just 1 particle to pass through the system at a time. The probability of a particle from beam 2 ending up in beam 6 does not appear amenable to an epistemic interpretation of the kind usually attributed to probabilities. It seems to be an inherently unpredictable event, as particles, which have apparently undergone an identical process, seem then to behave in significantly differing ways. Though certainly surprising given the deterministic nature of other physical laws thought to be fundamental, this is not necessarily a problem. There is, after all, no obvious guarantee that the fundamental laws of nature must all be deterministic.

Now, however, we come to the truly peculiar results, the understanding of which has been the main goal of the philosophy of quantum physics.

To do this a new component will be needed. This component is a beam recombiner. It uses magnetic fields to recombine beams of particles of known spin state into a single beam.

Figure 4. The schematic representation of a beam recombiner.

Using a recombiner to merge the beams of particles separated by the SGx apparatus we find that the resultant beam of particles, beam 5, retain the spin-z properties they were measured to have prior to the SGx gate. This is a very strange result. We have seen that the process of measuring a particle’s x-spin state disrupts its z-spin value. The most obvious cause of this change is the magnetic field used to split these particles according to their x-spin state. Here though, particles which have passed through this field seem, nonetheless, to have retained their z-spin value.

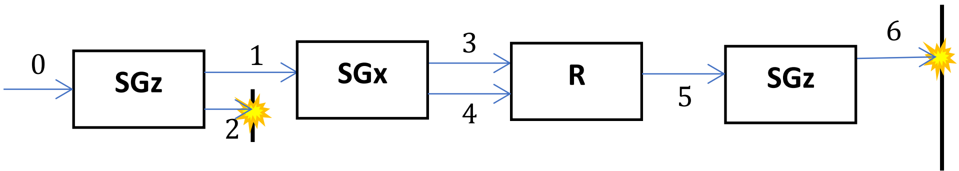

Figure 5. After passing through a SGx apparatus to split the particles into 2 beams according to x-spin value, these beams are recombined, with the result that no effective measurement has taken place. Surprisingly the z-spin value is retained in this case.

The obvious solution to this problem is to suppose that the magnetic fields used to recombine the beams have in some way reversed the effect of the SGx apparatus’ magnetic field. This would not be without problems as a solution, as it raises serious questions about how the z-spin particle state is recorded as the particle moves from SGx to R, given that any particle in beams 3 and 4 would, if measured, have a 50:50 chance of having either observed z-spin value, but this problem might not be insurmountable.

In fact however, this solution is not workable. As shown in figure 6, inserting a particle detector to count particles in beam 4 results in a return to the 50:50 mix z-spin measurements seen in figure 3. This apparently means that a single particle, initially z-spin up in beam 1, which leaves the SGx gate in beam 3 with an x-spin up state, has its trajectory altered by another magnetic field, and is then subjected to a z-spin measurement, will either certainly be measured to have z-spin up, or have a 50:50 chance of either outcome depending on whether there is an obstruction on a path which this particle did not in fact take!

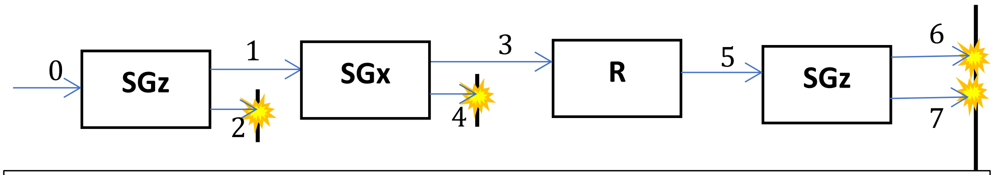

Figure 6. When a detector is inserted into beam 4, effectively measuring the x-spin state of particles in beams 3 and 4, the z-spin property of particles in beam 3 is lost between beams 1 and 6/7, as in figure 4. This is difficult to understand given that the path of particles which pass through beam 3 is identical to that in figure 5, which behave very differently.

This is such peculiar behaviour that a radically new conceptual framework is needed to account for it. In particular the usual assumptions, accepted until now, that particles have determinate trajectories independent of measurement, and that they have determinate spin properties independent of measurement, need careful examination.

In fact, as noted in the previous chapter, at least one of these premises is now rejected by the three main modern interpretations of quantum mechanics. We will now return to the formalism needed to account for this behaviour (introduced earlier in the Dirac notation). The predictive reliability of this formalism is recognised by adherents of all interpretations, though Bohmian and modal interpretations believe it to be incomplete as a description of quantum processes.

2.1.3 The Dirac Notation

Earlier we saw how the action of an SGx apparatus on a beam of particles is represented using Dirac notation. There the beam was represented as a superposition of z-spin state components:

$$|\psi\rangle = \alpha|\uparrow_z\rangle + \beta|\downarrow_z\rangle$$

The reader may have wondered why there were no terms referring to x-spin states, and how measurement outcome probabilities could be calculated for x-spin measurements. The answer can be deduced from what happens when a beam of particles measured to be z-spin up is subjected to an x-spin measurement. There is an equal probability of obtaining each x-spin measurement. This is represented in Dirac notation by identifying z-spin states with equal superpositions of x-spin states.

$$|\uparrow_z\rangle = \frac{1}{\sqrt{2}}(|\uparrow_x\rangle + |\downarrow_x\rangle)$$

$$|\downarrow_z\rangle = \frac{1}{\sqrt{2}}(|\uparrow_x\rangle - |\downarrow_x\rangle)$$

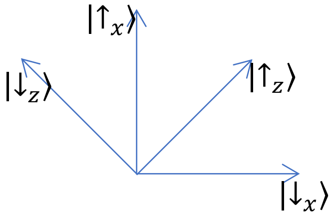

Figure 7. A graphical representation of the relationships between x-spin and z-spin eigenstates.

With this relationship in mind, let us consider the states as represented in Dirac notation of particles in different beams in figure 5.

The beam entering our first apparatus has not been measured, so we attribute a general state:

$$|\psi_0\rangle = \alpha|\uparrow_z\rangle + \beta|\downarrow_z\rangle$$

This will always hold true for any coherent beam, for some values of α, and β.

After passing through the SGz apparatus this beam is split into 1, and 2 as follows:

$$|\psi_1\rangle = \alpha|\uparrow_z\rangle$$

$$|\psi_2\rangle = \beta|\downarrow_z\rangle$$

(For reasons of clarity the beam states will not be renormalized.)

To understand the effect of the SGx gate we use the relationships between x-spin and z-spin states mentioned earlier:

$$|\psi_1\rangle = \alpha|\uparrow_z\rangle = \frac{\alpha}{\sqrt{2}}(|\uparrow_x\rangle + |\downarrow_x\rangle)$$

Applying the SGx apparatus to this superposition of x-spin states yields two beams:

$$|\psi_3\rangle = \frac{\alpha}{\sqrt{2}}|\uparrow_x\rangle$$

$$|\psi_4\rangle = \frac{\alpha}{\sqrt{2}}|\downarrow_x\rangle$$

These beams are then recombined to recover the former state:

$$|\psi_5\rangle = |\psi_3\rangle + |\psi_4\rangle = \frac{\alpha}{\sqrt{2}}(|\uparrow_x\rangle + |\downarrow_x\rangle) = \alpha|\uparrow_z\rangle$$

Which of course passes straight through the final SGz apparatus without any state change:

$$|\psi_6\rangle = |\psi_5\rangle = \alpha|\uparrow_z\rangle$$

Meaning that all particles not detected in beam 2, will be detected in beam 6.

It should be noted that the above analysis applies as much for a single particle passing through this system, as for a beam. This seems to imply that a single particle travels simultaneously along both path 3, and path 4. This may seem an incredible assertion, but all interpretations of quantum mechanics, except for Bohmian and some modal interpretations, would accept it, and even these would acknowledge that there is something associated with the particles, which follows both paths. Moreover, we should remember the truly extraordinary experimental results which led to formation of this view.

To emphasise this point, now let us consider the change produced if we introduce a detector to path 4 as seen in figure 6.

Just as before, the states of particles on paths 3 and 4 are:

$$|\psi_3\rangle = \frac{\alpha}{\sqrt{2}}|\uparrow_x\rangle$$

$$|\psi_4\rangle = \frac{\alpha}{\sqrt{2}}|\downarrow_x\rangle$$

Now, though, path 4 is blocked by a detector. As a result only path 3 leads to the recombiner. The state of beam 5 is therefore different:

$$|\psi_5\rangle = |\psi_3\rangle = \frac{\alpha}{\sqrt{2}}|\uparrow_x\rangle$$

Again, using the relations between x-spin and z-spin, this becomes:

$$|\psi_5\rangle = \frac{\alpha}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}(|\uparrow_z\rangle + |\downarrow_z\rangle)\right) = \frac{\alpha}{2}(|\uparrow_z\rangle + |\downarrow_z\rangle)$$

This, of course, yields 2 beams when passed through the final SGz apparatus, rather than the 1 seen for the figure 5 arrangement.

$$|\psi_6\rangle = \frac{\alpha}{2}|\uparrow_z\rangle$$

$$|\psi_7\rangle = \frac{\alpha}{2}|\downarrow_z\rangle$$

For a particle entering the figure 6 arrangement therefore the measurement probabilities are as follows:

The probability of being detected in beam 2 is $|\beta|^2$.

The probability of being detected in beam 4 is $\frac{|\alpha|^2}{2}$.

The probability of being detected in beam 6 is $\frac{|\alpha|^2}{4}$.

The probability of being detected in beam 7 is $\frac{|\alpha|^2}{4}$

Whereas for a particle entering the arrangement shown in figure 5:

The probability of being detected in beam 2 is $|\beta|^2$.

The probability of being detected in beam 6 is $|\alpha|^2$

It is the ability of this quantum mechanical formalism to make predictions in such intuitively baffling cases, and the exceptional predictive reliability of these predictions, which has led to a conceptually very peculiar formalism becoming widely accepted.

For all its predictive accuracy, however, this formalism does little or nothing to resolve the conceptual puzzles presented by quantum mechanics. The most obvious of these in the present context is “But which path(s) does the particle actually follow?”.

It might be acceptable to believe that particles can follow 2 different paths simultaneously, but if this were the case you would expect to see a fraction of a particle detected on path 4, every time a particle passes through the system in figure 6. In fact, however, the experimental evidence indicates that an entire particle will be detected on path 4 with a particular probability. Even if we are willing to accept that particles can be in two places at once, we might baulk at ‘particles can be in two places at once but you’ll only find them in 1 place if you look’.

We have seen that it is only the interference between beam 3, and beam 4 when recombined, that accounts for the preservation of z-spin state in figure 5. But, if a particle passing through this system must follow both paths, why don’t we see it on both paths when we insert detectors? On the other hand, if the particle only follows 1 path, but must have information as to whether there is a detector on the other, to determine its future behaviour, how is that information communicated?

This is an example of the range of conceptual difficulties, generally referred to as ‘The Measurement Problem’. Before we can properly examine possible solutions to these problems, we need a more complete set of the problems in question. To understand these problems as simply as possible, we will move to a more idealised Von Neumann measurement framework. But before we do this, we need to introduce one more concept, which has been obscured so far by the mechanics of the measurement system used.

2.1.4 The Projection Postulate

Put simply, the projection postulate says that following the measurement of a particle, the quantum state describing the measured property becomes the state guaranteed to produce the result that was observed (the measurement eigenstate). As a result performing the same measurement successively for a particle will produce the same result, as shown in figure 8.

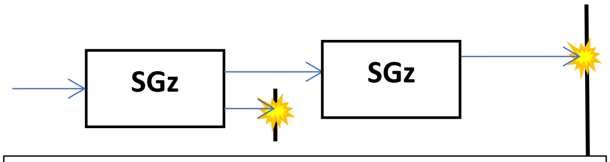

Figure 8. Successive measurements of the same property yield the same measured result.

In the case of the S-G systems considered in this chapter, this is ensured by the way in which measurements are performed by splitting beams according to particular properties. In fact, though, this is a very general phenomenon which seems to apply for all quantum measurements. As such, it will be a feature of the maximally general von Neumann characterisation of measurement presented in the next section.

2.1.5 Von Neumann Measurements

Given that, as we have seen, particles seem to follow 1 path when measured, but behave as if they have followed both when the path they follow is left unmeasured, it seems as if there must be something important about the process of measurement, which in some way profoundly influences the behaviour of the particles. It therefore seems reasonable to examine in detail every stage of the measurement processes that lead us to possess evidence for this apparently inconsistent particle behaviour, focussing in particular on how records of behaviour are created.

Up until this point we have looked at measured particles, and the basic structure of the apparatus through which they are measured. We have not so far paid any attention to the process of recording measurement results. We have noted that the particles we have discussed end up at a receptor plate where they produce a detectable reaction, and presumably this reaction is recorded, by an experimental physicist, or a recording computer system or similar. But no consideration has been given to the quantum mechanical state of the observer, or the results record.

Of course it is not usual to think of a macroscopic results record as having a quantum state, but as quantum mechanics is supposed to be a theory capable of adequately modelling the microscopic world (and it is extremely successful in this) then it should be capable of modelling the macroscopic world which those many microscopic systems constitute.

To examine this process von Neumann came up with an idealised measurement model, for the connections formed between measured particle and measurement apparatus. Because of the extreme generality of this model, the measurement apparatus can be taken to be anything from the state of a small area of receptor plate, to an entire team of experimental physicists and supercomputers. The only assumption, made by von Neumann, was that the linear unitary dynamics encapsulated in Schrödinger's equation obtain throughout, reasoning, as mentioned, that even human beings should be describable in quantum mechanical terms.

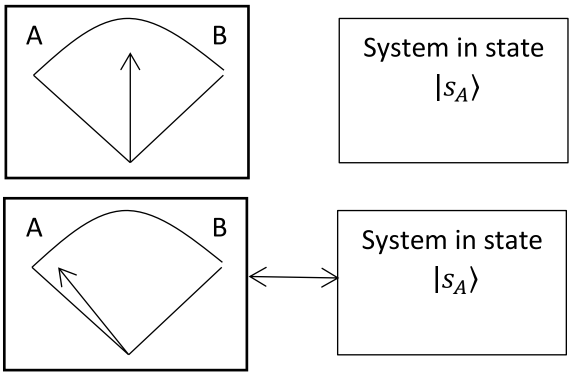

Figure 9. A simple diagrammatic representation of the idealised measurement process which von Neumann sought to model.

In order for our idealised measurement apparatus to be a functional measurement apparatus, it must have at least as many possible states as the possible distinguishable states of the system it seeks to measure, in order to be able to record any possible system state. These record states can be defined as the set $\{|a_i\rangle\}$, each state of which will correspond to a state in the set of possible system states $\{|s_i\rangle\}$; finally a state is needed for the apparatus in which it is not recording a result, but ready to record one: we define this state to be $|a_r\rangle$. (Schlosshauer 2007, p51)

With these states defined, we can, using von Neumann's scheme, represent quantum measurements amazingly simply:

$$|a_r\rangle|s_i\rangle \Rightarrow |a_i\rangle|s_i\rangle$$

Or in other words, a prepared measurement apparatus coming into contact with a system in its ith eigenstate, will transition to the ith record state, thereby recording the state of the system, while the state of the system remains constant.

It seems reasonable to assert that the process described here must apply for any measurement apparatus of any kind. A measurement apparatus which failed to follow this pattern would have failed to make an adequate record of the system state, and would in fact, therefore, not be a measurement apparatus at all.

Though this may all seem obvious and trivial, it leads to far less intuitive conclusions when the von Neumann scheme is applied to the measurement of a system whose state is a superposition of the eigenstates measurable by the apparatus. Consider for example, a von Neumann x-spin measurement for a particle initially in state z-spin up.

$$|\uparrow_z\rangle = \frac{1}{\sqrt{2}}(|\uparrow_x\rangle + |\downarrow_x\rangle)$$

$$|a_r\rangle\left(\frac{1}{\sqrt{2}}(|\uparrow_x\rangle + |\downarrow_x\rangle)\right) \Rightarrow \frac{1}{\sqrt{2}}(|a_\uparrow\rangle|\uparrow_x\rangle + |a_\downarrow\rangle|\downarrow_x\rangle)$$

This appears to show that the measurement apparatus itself enters a superposition state matching that of the measured particle, so that the composite system of particle and apparatus ends up in a superposition of accurate x-spin up measurement, and accurate x-spin down measurement.

Of course this is a deeply peculiar result, but that is not in itself a major problem. Quantum mechanics is intended to predict deeply peculiar results, such as the baffling behaviour of particles in an S-G apparatus discussed earlier. The problem with this analysis is far more fundamental; it simply appears to be wrong. When a scientist looks at the receptor in figure 3, they don’t see a plate in a superposition of two apparently contradictory measurement results (what indeed would that look like?). The scientist sees only 1 state of the receptor plate, which we cannot predict in advance.

The situation then is this:

- In order to account for interference phenomena we must accept a mathematical formalism according to which particles are represented as if they possessed multiple incompatible properties simultaneously, such as position in the S-G arrangements discussed.
- This theoretical framework moreover proves to be the basis for a spectacularly successful theory of microscopic phenomena.
- A mysterious (though predicted) feature of this formalism is that, while particles seem to follow multiple incompatible trajectories simultaneously, upon measurement they are only ever found on one.
- In order to try and solve this mystery, we have examined the measurement process using von Neumann’s analysis, which assumes that quantum mechanics is capable of describing the macroscopic world.
- This analysis leads to the conclusion that the macroscopic world is given to precisely the same baffling behaviour as the microscopic, and macroscopic objects routinely enter superpositions, and might consequently display interference phenomena!

Given that these behaviours are not generally observed in the macroscopic world, it seems that either our everyday experience, one of our most successful scientific theories, or this analysis must be wrong.
Before accepting this uncomfortable choice, however, we should pin down just what (aside from bafflement) the problem we seek to solve is. This will occupy the next section. Having clearly categorised the problem, we shall examine the substantial contribution decoherence makes towards solving these problems.

2.1.6 The Measurement Problem(s)

The phrase “the Quantum Measurement Problem” is frequently used in literature relating to foundations and interpretations of quantum mechanics. However, authors vary significantly in what they take the problem to be. In fact, the Quantum Measurement Problem seems to have come to refer to a set of several problems relating to the quantum measurement process, which as we have seen proves mysterious. The basic common feature of these problems is that they all relate to difficulties “in accounting for the fact that measurements have any outcomes at all” (Saunders 2001). In this section I will focus on the division of these problems presented by Schlosshauer (Schlosshauer 2007, pp. 50). The component problems identified by Schlosshauer are:

1. The Problem of the Preferred Basis
2. The Problem of the Non-Observability of Interference
3. The Problem of Outcomes

To understand these problems, each will be related to the quantum measurements discussed earlier. I will return to discuss these problems in more detail at the end of the chapter.

First let us consider (2), the non-observability of interference. This problem came to light in the last section, when it became apparent that quantum states of the kind that describe interference phenomena among microscopic particles are seemingly also applicable to macroscopic objects.

Recall the Stern-Gerlach based interference experiments already discussed. There we saw how destructive interference phenomena between combining beams of particles could lead to the number of particles in the resulting beam being far lower than in the incident beams. This is generally assumed to be a phenomenon which applies only to the microscopic. But if the linear Schrödinger dynamics really describe the world we live in, then this raises the question of why we do not observe distinctively quantum phenomena like interference in our day to day lives. More generally, the problem of the non-observability of interference is the problem of explaining when and why interference phenomena will, or will not be, displayed.

It is actually surprisingly difficult to find an intuitively natural example of a macroscopic state of affairs in which interference phenomena might be expected. This difficulty is partly due to systems with many degrees of freedom making large-scale equivalents of a recombination of beams, with sufficient precision to allow interference, seem implausible. And partly because a necessary pre-condition for interference phenomena is superposition states, and macroscopic superposition states are themselves very counterintuitive. This brings us on to (3): The problem of outcomes.

Simply put, the problem of outcomes is the problem of how we obtain the measurement outcomes we do, or indeed any at all, given that, if quantum mechanics holds for all scales, no single determinate measurement result is ever reached.

Schlosshauer argues that this problem can be separated into two parts, which might be termed the generic and the specific. The generic is the question of why we perceive a definite measurement, rather than a peculiar superposition state, when examining the measurement apparatus. In other words, why is it that a scientist examining the apparatus in figure 3 will perceive a determinate measurement rather than a superposition state? (This is sometimes referred to as the problem of the preferred basis, but for clarity I will follow Schlosshauer 2007 and Crull 2013 in distinguishing this from other aspects of that problem.)

The specific problem is why we perceive the specific result which we in fact perceive. That is, why does a scientist examining the apparatus in figure 3 perceive the measurement result spin up rather than spin down?

Finally let us consider (1) the problem of the preferred basis. The problem of the preferred basis, as set out by Schlosshauer (2007, pp. 53-55), is the question of why the measurement interaction gives rise to states in which the state of one particular property of the target system is preserved, while the state of other non-commuting variables is changed. To understand this problem, consider von Neumann's ideal measuring device performing a measurement on the x-spin state of a particle, initially in the state $\alpha|\uparrow_x\rangle + \beta|\downarrow_x\rangle$:

$$|a_r\rangle(\alpha\,|\uparrow_x\rangle + \beta|\downarrow_x\rangle) \Rightarrow \alpha|a_{x\uparrow}\rangle|\uparrow_x\rangle + \beta|a_{x\downarrow}\rangle|\downarrow_x\rangle$$

The concern is that by a simple basis transformation it is possible to obtain from this final state a description of the system state in a basis of non-commuting variables, such as z spin state:

$$\begin{aligned} &\alpha|a_{x\uparrow}\rangle|\uparrow_x\rangle + \beta|a_{x\downarrow}\rangle|\downarrow_x\rangle \\ &\qquad = \alpha(|a_{z\uparrow}\rangle|\uparrow_z\rangle + |a_{z\downarrow}\rangle|\downarrow_z\rangle) + \beta(|a_{\uparrow}\rangle|\uparrow_z\rangle - |a_{z\downarrow}\rangle|\downarrow_z\rangle) \\ &\qquad = (\alpha + \beta)|a_{z\uparrow}\rangle|\uparrow_z\rangle + (\alpha - \beta)|a_{z\downarrow}\rangle|\downarrow_z\rangle \end{aligned}$$

(Eigenvector transformations from Rae 2008 pp. 126)

The measurement device therefore seems to have entered a state simultaneously representing both the x-spin and the z-spin states simultaneously. Given that these observables are non-commuting, this appears to violate the uncertainty principle, by which two non-commuting observables cannot be measured simultaneously with precision, and more importantly fails to correspond to observed measurements, in which the measurement of x spin renders both z-spin states equi-probable.

These are problems which have occupied the attentions of those seeking to understand quantum mechanics for most of the last century. Recently however significant progress has been made in resolving some of these problems. The breakthrough which has allowed this progress is the realisation that decoherence has wide implications. We are now in a position to understand how decoherence contributes to the resolution of these problems, and this will be the aim of the next few sections. First, though, a more mathematically rigorous formulation of quantum mechanics is needed.

2.2 Decoherence

2.2.1 Matrix Formalism

Matrix mechanics is essentially equivalent to the Dirac formalism used up to this point. It is often used by physicists because it is easier to use for mathematical, and particularly computational, purposes. In the matrix formalism, all possible states (superposition or otherwise) are represented as vectors of unit length. To understand how this can be done consider the relationships between x-spin and z-spin eigenstates introduced earlier.

$$|\uparrow_z\rangle = \frac{1}{\sqrt{2}}(|\uparrow_x\rangle + |\downarrow_x\rangle)$$

$$|\downarrow_z\rangle = \frac{1}{\sqrt{2}}(|\uparrow_x\rangle - |\downarrow_x\rangle)$$

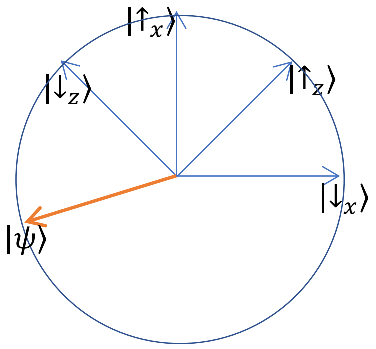

Figure 10. A graphical representation of state quantum states as vectors of unit length.

Figure 10 shows the relations between the x-spin and z-spin eigenstates, and another arbitrary state $|\psi\rangle$. Represented like this, it is easy to see that $|\psi\rangle$ can be represented as a combination of the vectors assigned to either set of eigenstates:

$$|\psi\rangle = a|\uparrow_x\rangle + b|\downarrow_x\rangle$$

$$|\psi\rangle = c|\uparrow_z\rangle + d|\downarrow_z\rangle$$

Once these relations have been established, it is easy to see how $|\psi\rangle$ can be represented as a vector in terms of an eigenstate basis. So that in the x-spin basis:

$$|\psi\rangle = a|\uparrow_x\rangle + b|\downarrow_x\rangle = \binom{a}{b}_x$$

And in the z-spin basis:

$$|\psi\rangle = c|\uparrow_z\rangle + d|\downarrow_z\rangle = \binom{c}{d}_z$$

Note that these 2 vectors express precisely the same state, just in terms of different bases.

The reader may have noticed that the a, b, c, d can be either positive or negative. In fact, for reasons that will now be explained, these numbers can also be complex (include imaginary components). That this is necessary becomes clear if it is considered that there is a third direction in which a particle's spin can be measured. The relation of these spin eigenstates to x-spin and z-spin eigenstates is exactly like that which x-spin and z-spin eigenstates bear to one another. This cannot be represented on the circle of real unit vectors in figure 10, as it requires the incorporation of complex numbers.

$$|\uparrow_y\rangle = \frac{1}{\sqrt{2}}(|\uparrow_z\rangle + i|\downarrow_z\rangle)$$

$$|\downarrow_y\rangle = \frac{1}{\sqrt{2}}(|\uparrow_z\rangle - i|\downarrow_z\rangle)$$

The eigenvectors of the states discussed so far, expressed in the z-spin basis, are:

$$|\uparrow_x\rangle = \frac{1}{\sqrt{2}}\binom{1}{1}_z$$

$$|{\downarrow_x}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}_z$$

$$|{\uparrow_y}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix}_z$$

$$|{\downarrow_y}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}_z$$

$$|{\uparrow_z}\rangle = \begin{pmatrix}1\\0\end{pmatrix}_z$$

$$|{\downarrow_z}\rangle = \begin{pmatrix}0\\1\end{pmatrix}_z$$

2.2.2 Density Matrices

Density matrices are an extension of the usual vector state formalism, which is capable of capturing the probabilities of different measurement outcomes for a mixed state, that is, the state of a system in which the state vector itself is not known with certainty. This is important, as it is often practically difficult to ensure a pure beam of particles with the desired state vector, and no other. To understand how a density matrix does this, we will first examine a density matrix for a pure state, and then look at how this is expanded to deal with the case of mixed states.

For a system with 2 degrees of freedom, such as a particle's spin, with a state vector

$$|\psi\rangle = \begin{pmatrix}a\\b\end{pmatrix}$$

the density matrix is:

$$\rho = |\psi\rangle\langle\psi| = \begin{pmatrix}a\\b\end{pmatrix}\begin{pmatrix}a^* & b^*\end{pmatrix} = \begin{pmatrix}|a|^2 & ab^*\\a^*b & |b|^2\end{pmatrix}$$

The first thing to notice about this matrix is that the diagonal elements, $|a|^2$, and $|b|^2$, are equal to the probabilities, as specified by the Born rule, of each of the measurement outcomes which define the basis. The other two, so called off-diagonal elements, carry the information relevant for interference effects, in which as we have seen probabilities of combined beams are not simply added together. These will prove very important to the theoretical understanding of decoherence.

If there is epistemic uncertainty as to the state vector of the system (e.g. because the state preparation apparatus is unreliable), such as between the state vectors $|\psi_1\rangle = \begin{pmatrix} a \\ b \end{pmatrix}$ or $|\psi_2\rangle = \begin{pmatrix} A \\ B \end{pmatrix}$, the general definition of a density matrix is:

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i| = p_1|\psi_1\rangle\langle\psi_1| + p_2|\psi_2\rangle\langle\psi_2|$$

$$= \begin{pmatrix} p_1|a|^2 + p_2|A|^2 & p_1 ab^* + p_2 AB^* \\ p_1 a^* b + p_2 A^* B & p_1|b|^2 + p_2|B|^2 \end{pmatrix}$$

It should be noted that, considering the Born rule, the diagonal elements of this matrix remain equal to the probability of particular measurement outcomes.

To understand decoherence we will need to introduce reduced density matrices, which can be used to predict measurement outcomes for a particular subsystem of a quantum system. We will see that these matrices seem in some cases to resemble classical behaviour. This may represent substantial progress in understanding how it is possible to model the macroscopic world quantum mechanically, and thereby resolve some of the measurement problems listed above. Before we do this, however, we will examine the effects of off-diagonal matrix elements more closely, in the case of classical and quantum uncertainties.

2.2.3 Off-Diagonal Density Matrix Elements

Nonzero off-diagonal elements occur within density matrices when one or more of the constituent (single state vector) density matrices has more than one nonzero element. Consider for example a two-state system of x-direction spin states:

If there is a purely classical uncertainty as to which of two basis states the system occupies, the density matrix will be,

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i| = p_{x\uparrow}|\psi_{x\uparrow}\rangle\langle\psi_{x\uparrow}| + p_{x\downarrow}|\psi_{x\downarrow}\rangle\langle\psi_{x\downarrow}| = \begin{pmatrix} p_{x\uparrow} & 0 \\ 0 & p_{x\downarrow} \end{pmatrix}$$

Whereas, in the case of a pure state vector corresponding to a superposition of the two states, it will be,

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i| = |\psi_x\rangle\langle\psi_x| = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \begin{pmatrix} \alpha^* & \beta^* \end{pmatrix} = \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{pmatrix}$$

In both density matrices the diagonal elements of the matrix will correspond to measurement probabilities if the state of the system is measured. However, the off-diagonal elements in the second matrix may produce different results when linear operations are performed on the system. These linear operations are typically used in quantum mechanics to model the effects of a physical process acting on the system. A simple example would be the effect of passing a particle through an S-G apparatus as seen previously.

Suppose an operation is performed on the system whose effect when applied to the basis states is to produce uneven superpositions, for example:

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow \frac{1}{2}\begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix}$$

And,

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow \frac{1}{2}\begin{pmatrix} -1 \\ \sqrt{3} \end{pmatrix}$$

Which can be achieved with the matrix operator,

$$\frac{1}{2}\begin{pmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{pmatrix}$$

The effect of the unitary operator on the density matrix can be calculated as,

$$\rho' = U\rho U^{\dagger}$$

Applying this operator to the mixed density matrix yields,

$$\begin{aligned}\rho' &= \frac{1}{4}\begin{pmatrix}\sqrt{3} & -1\\ 1 & \sqrt{3}\end{pmatrix}\begin{pmatrix}p_{x\uparrow} & 0\\ 0 & p_{x\downarrow}\end{pmatrix}\begin{pmatrix}\sqrt{3} & 1\\ -1 & \sqrt{3}\end{pmatrix}\\ &= \frac{1}{4}\begin{pmatrix}3p_{x\uparrow}+p_{x\downarrow} & \sqrt{3}(p_{x\uparrow}-p_{x\downarrow})\\ \sqrt{3}(p_{x\downarrow}-p_{x\uparrow}) & 3p_{x\downarrow}+p_{x\uparrow}\end{pmatrix}\end{aligned}$$

The diagonal elements of this matrix show that a diagonalised initial density matrix will always yield a state in which the observation probabilities produced arise as a linear weighted addition of the probabilities for each pure state.

For the pure density matrix describing a superposition, however,

$$\begin{aligned}\rho' &= \frac{1}{4}\begin{pmatrix}\sqrt{3} & -1\\ 1 & \sqrt{3}\end{pmatrix}\begin{pmatrix}|\alpha|^2 & \alpha\beta^*\\ \alpha^*\beta & |\beta|^2\end{pmatrix}\begin{pmatrix}\sqrt{3} & 1\\ -1 & \sqrt{3}\end{pmatrix}\\ &= \frac{1}{4}\begin{pmatrix}3|\alpha|^2+|\beta|^2-\sqrt{3}(\alpha^*\beta+\alpha\beta^*) & \sqrt{3}(|\alpha|^2+|\beta|^2)-\alpha^*\beta+3\alpha\beta^*\\ \sqrt{3}(|\alpha|^2+|\beta|^2)+3\alpha^*\beta-\alpha\beta^* & 3|\beta|^2+|\alpha|^2+\sqrt{3}(\alpha^*\beta+\alpha\beta^*)\end{pmatrix}\end{aligned}$$

Now, in addition to the weighted addition of $|\alpha|^2$, and $|\beta|^2$, there is an additional term in the diagonal elements (highlighted). This additional term can be either positive or negative depending on the relative phases of α, and β (but will always be real) and depending on its value, will shift the probabilities probability between the possible spin values. These interference effects are a major feature that distinguishes quantum mechanical uncertainties from classical probabilities, and will not occur if the off-diagonal elements of the initial density matrix are zero.

Note that the structure of the density matrix is basis-dependent. So, if we changed our basis states from $|\psi_{x\uparrow}\rangle$, and $|\psi_{x\downarrow}\rangle$, to $|\psi_{z\uparrow}\rangle$, and $|\psi_{z\downarrow}\rangle$, the mixed state considered above would no longer be described by a diagonalised matrix. That is, if the operation on the system first considered were modelled in the z-spin basis, the starting density matrix would not be diagonalised and the resulting matrix would show interference effects in the diagonal elements.

2.2.4 Reduced Density Matrices

Consider 2 spin-half particles described by state vectors:

$$|\psi_1\rangle = \binom{a}{b}$$

$$|\psi_2\rangle = \binom{A}{B}$$

These 2 systems themselves constitute a larger quantum system:

$$|\Psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle = \begin{pmatrix} aA \\ aB \\ bA \\ bB \end{pmatrix}$$

If, however, these 2 particles do not have their own independent states, but are entangled so that they are in a superposition of e.g. both being spin-up and both being spin-down, the vector state of the composite system will have non-zero elements only for the 1st and 4th, e.g.:

$$|\Psi\rangle = \frac{1}{5}\begin{pmatrix} 3 \\ 0 \\ 0 \\ 4 \end{pmatrix}$$

This state cannot be produced by any pair of independent particle states. There is no longer a state vector capable of describing either of the particles individually, but only a vector capable of describing the entangled pair.

This would seem to rule out the possibility of there being any density matrix capable of describing particle 1 individually. After all, the definition given of a density matrix is given in terms of the system's possible state vectors, and no possible vector for the particle is consistent with the state of the total system. Strictly speaking it does indeed rule out any such matrix, but as we shall see something called a reduced density matrix has been developed specifically to try and capture many of the valuable features of a density matrix for this sort of quantum subsystem.

Consider the density matrix for the composite system $|\Psi\rangle$

$$\rho_\Psi = |\Psi\rangle\langle\Psi| = \frac{1}{25}\begin{pmatrix} 9 & 0 & 0 & 12 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 12 & 0 & 0 & 16 \end{pmatrix}$$

It is of course impossible to say what the state vector of any subsystem should be. On the other hand it is possible to make predictions about the outcomes of measurements. Looking at the matrix diagonal shows us for instance, that for particle 2 the Born rule probability of measuring the particle to be spin-up is 0.36, and the probability of measuring it to be spin down is 0.64. It therefore seems as if it should indeed be possible to provide density matrices for component subsystems. After all, for each known state of particle 2 there is a corresponding well-defined state vector for particle 1.

The density matrix of particle 1 should therefore have the diagonal elements:

$$\rho_{\Psi_1} = \begin{pmatrix} 0.36 & \\ & 0.64 \end{pmatrix}$$

This, however, is not really any help without the other elements. If we wish to know anything about the behaviour of the subsystem under subsequent operations, the off-diagonal elements are essential. The crucial clue to being able to fill in the elements is the fact that quantum systems perfectly entangled to an external system will not display interference phenomena. The absence of these phenomena has been established, both empirically and by examining the density matrix for the total system. To see this, consider the generic form of the operations discussed earlier:

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

And,

$$\begin{pmatrix}0\\1\end{pmatrix} \Rightarrow \begin{pmatrix}\beta\\\alpha\end{pmatrix}$$

With associated matrix operator:

$$U = \begin{pmatrix}\alpha & \beta^*\\\beta & \alpha\end{pmatrix}$$

Suppose this operator is to act only on particle 1. It should then be possible to find the appropriate off-diagonal elements for the reduced density matrix, by looking at the difference between the weighted sum of probable outcomes (as in the case where the eigenstate is determinate but unknown), and the outcome probabilities actually obtained.

We shall move to consider the generic state vector for these two entangled systems:

$$|\Psi\rangle = \begin{pmatrix}A\\0\\0\\B\end{pmatrix}$$

Which specifies only that the particles are entangled such that they will be found on measurement to be both spin up, or to be both spin down. The associated density matrix will be:

$$\rho_\Psi = \begin{pmatrix}|A|^2 & 0 & 0 & AB^*\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\\A^*B & 0 & 0 & |B|^2\end{pmatrix}$$

The matrix operator U in this basis will become:

$$U' = \begin{pmatrix}\alpha & 0 & \beta^* & 0\\0 & \alpha & 0 & \beta^*\\\beta & 0 & \alpha & 0\\0 & \beta & 0 & \alpha\end{pmatrix}$$

Using this, we can accurately find the predicted probabilities of each state of particle 1 following this operation. This relies on nothing beyond the standard formalism of quantum mechanics employed so far.

If no interference effects are displayed, the resulting probabilities for the state of particle 1 will be:

$$P(\uparrow) = P(\uparrow \mid \uparrow_0)P(\uparrow_0) + P(\uparrow \mid \downarrow_0)P(\downarrow_0) = |\alpha|^2|A|^2 + |\beta|^2|B|^2$$

$$P(\downarrow) = P(\downarrow \mid \uparrow_0)P(\uparrow_0) + P(\downarrow \mid \downarrow_0)P(\downarrow_0) = |\alpha|^2|B|^2 + |\beta|^2|A|^2$$

We have then only to obtain the result in the quantum mechanical case and compare the results to establish the degree to which interference phenomena are displayed.

$$\rho_{\Psi\ \mathrm{final}} = U'\rho_\Psi U'^*$$

$$\rho_{\Psi\ \mathrm{final}} = \begin{pmatrix} \alpha & 0 & \beta^* & 0 \\ 0 & \alpha & 0 & \beta^* \\ \beta & 0 & \alpha & 0 \\ 0 & \beta & 0 & \alpha \end{pmatrix} \begin{pmatrix} |A|^2 & 0 & 0 & AB^* \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ A^*B & 0 & 0 & |B|^2 \end{pmatrix} \begin{pmatrix} \alpha^* & 0 & \beta & 0 \\ 0 & \alpha^* & 0 & \beta \\ \beta^* & 0 & \alpha^* & 0 \\ 0 & \beta^* & 0 & \alpha^* \end{pmatrix}$$

$$\rho_{\Psi\ \mathrm{final}} = \begin{pmatrix} |\alpha A|^2 & \alpha\beta^*AB^* & \alpha\beta^*|A|^2 & |\alpha|^2AB^* \\ \alpha\beta^*A^*B & |\beta B|^2 & |\beta|^2A^*B & \alpha^*\beta|B|^2 \\ \alpha^*\beta|A|^2 & |\beta|^2AB^* & |\beta A|^2 & \alpha^*\beta AB^* \\ |\alpha|^2A^*B & \alpha\beta^*|B|^2 & \alpha\beta^*A^*B & |\alpha B|^2 \end{pmatrix}$$

Looking to the central diagonal, we see that for particle 1 the measurement probabilities have become:

$$P(\uparrow) = |\alpha A|^2 + |\beta B|^2$$

$$P(\downarrow) = |\beta A|^2 + |\alpha B|^2$$

These are exactly the results expected in the absence of interference, and show that for operations performed on a quantum system that is perfectly entangled to an external system, no interference effects will be displayed. These results will be obtained from the reduced density matrix only if the off-diagonal elements are taken as zero.

This phenomenon, that quantum systems entangled to an external system display diminished interference effects to an extent which is (in fact) proportional to their degree of entanglement, is what is known as decoherence. This result has been confirmed both in theory and in experiment. The reduced density matrix formalism captures this by introducing a $\langle E_i | E_j \rangle$ term in the off-diagonal elements, where the set $\{|E_i\rangle\}$ represent the states of the external system that would be brought about by particular target system states. If these have a high degree of overlap, the systems are weakly entangled and interference phenomena will be largely unaffected. If they have little or no overlap, the systems are highly entangled, and interference will be heavily damped.

The reduced density matrix captures these behaviours and gives a great deal of useful information about entangled quantum systems. Its great virtue is the huge saving of computational power made by using this, rather than the – often very large – density matrix associated with the full set of entangled systems. It can be obtained from the total density matrix and is formally defined as: [3]

$$\rho_x = Tr_y(\rho_{xy})$$

There are, however, two issues which should be noted. First, the reduced density matrix relies on the assumption that operations are performed only on the subsystem to which it corresponds, and have no dependence on the other entangled system. If this is untrue, the reduced density matrix will not give reliable predictions. It is quite possible, for instance, that, although a system is perfectly entangled to another system, it will still display interference phenomena if an operator acts on both systems.

Second, although it is easy to see why the reduced density matrix proves practically useful, its connection to the linear quantum formalism is a subject of controversy (see Zeh 2003 pp. 35-37). Some work has been done on the subject of how the use of the reduced density matrix can be grounded in the formalism (e.g. d'Espagnat 1976, pp. 56-71), but these

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[3] The role of partial trace here, is essentially to average over Born rule weighted states of the environment to give a density matrix which gives the best measurement predictions possible in a description of just the local sub-system of interest. See Zeh pp. 35 for details.

accounts are sometimes hard to reconcile with specific interpretational frameworks.

One possible derivation of its use, mentioned by Zeh (2003), is to apply the Born rule to the total state vector, to obtain probabilities for possible eigenstates of the environmental system. As each eigenstate of the environment is associated with a well-defined state vector of the target system, this would seemingly provide a set of possible density matrices for the target system, and their associated probabilities. From this set it would then be easy to produce a density matrix for the target system as set out in section 2.2.3, using the Born rule probability of the environment to weight the component density matrices.

Zeh acknowledges, however, that this application of the Born rule is completely unjustified in the absence of some further interpretation. The Born rule in textbook quantum mechanics is only a reliable guide to the probabilities of particular measurement outcomes, and here we do not assume that any measurement is taking place. Whether and how the Born rule can be used to derive reduced density matrices from linear quantum mechanics will depend on how the Born rule is understood, and so will be interpretation-dependent.

Zeh makes no real attempt to derive the reduced density matrix, and confines himself to a discussion of the resulting theory if it is accepted.

Schlosshauer does not offer any kind of derivation either, but offers a proof that the diagonal elements of a reduced density matrix correspond to accurate probabilities of particular measurement results, assuming the Born rule (Schlosshauer 2007 pp. 45-46). This does not extend, however, to the off-diagonal elements which, as we have seen, are crucial to a theoretical understanding of decoherence. Schlosshauer also cites the envariance scheme proposed by Zurek 2005, which will be discussed in chapter 4. For now, though, we shall push on to examine some of the uses to which this framework is put by Schlosshauer amongst others. As I shall discuss, if this conception of decoherence is accepted, a large part of the quantum measurement problem, as set out, is apparently resolved.

2.2.5 Decoherence

We have seen, then, that the entanglement of a quantum system to external systems suppresses interference effects for the target system, and this is termed decoherence. Of course, for quantum systems to become entangled with their environment is very common, and essential in the measurement process. Decoherence is therefore extremely common, and appears to have a particular relevance in the case of measurement. Recent years have seen efforts to understand and test the effects of decoherence (e.g. Joos et al. 2003; Zeh 1970; Zurek 1991; 2005; 2014; Yoon-Ho et al. 2000), as well as several interpretations of quantum mechanics in which decoherence plays a very prominent role, as noted in the previous chapter.

The effect of decoherence, for a system interacting with an idealised, initially unentangled environment, is to cause the off-diagonal elements of the density matrix to tend to zero as $e^{-\Lambda t}$ where t is the time for which the system has been in contact with the environment, and Λ is a constant (Adler 2002 pp. 8). The rate and degree of entanglement is in most cases extremely fast, and Joos & Zeh (1985) argued that it would be orders of magnitude faster than the time taken for a measurement to be performed. This means that, after a significant period of contact with the environment, the density matrix describing the system in a quantum superposition state will closely approximate a density matrix that would correspond to a state of purely classical uncertainty as to the state of the system. In the limit in which off-diagonal elements reach zero, the system will develop under locally applied immediate operations completely without interference effects, and behave mathematically and empirically as if there were a purely classical uncertainty as to which of the possible states actually obtained.

It is important to note, however, that though the off-diagonal elements tend to zero, they never actually reach it. Concerns have been raised, therefore, that, as decoherence never actually recovers classicality, it cannot solve the measurement problem. Together with the difficulties of establishing the physical significance of reduced density matrices, it becomes clear that, while this reasoning accounts for the disappearance of distinctively quantum phenomena in practice, and to a large extent, in some sense it doesn't *really* recover the classical world of everyday experience. The *for all practical purposes* nature of this interference suppression will be discussed further in chapter 4.

For now, though, we shall look at the contribution that this theoretical notion of decoherence apparently makes to the resolution of the quantum measurement problem(s).

2.2.6 How Does Decoherence Contribute to Resolving Problems of Quantum Measurement?

Earlier in this chapter, I introduced 3 aspects of the quantum measurement problem distinguished by Schlosshauer 2007. Schlosshauer goes on to argue that the RDM conception of decoherence just developed is of direct relevance to resolving these problems. I will now discuss these problems and the interpretations to which they are relevant in more detail, and explain how Schlosshauer uses RDM decoherence to resolve (most of) the problems.

2.2.6.1 The Problem of the Non-Observability of Interference

Interference phenomena lie at the very heart of quantum mechanics. The behaviour of a particle passing through an interference experiment (e.g. a double slit apparatus) cannot be accurately predicted if it is assumed to always have a determinate position, without the addition of a Bohmian quantum potential or an equivalent. The occurrence of these phenomena is precisely what led to the development of quantum theory and the suspension of so many classical intuitions.

A key aspect of the measurement problem, however, is not the frequent occurrence of interference phenomena, but just how rarely they are actually observed. If quantum mechanics is supposed to be a fundamental theory, describing the behaviour of fundamental particles that make up our world, why are these quantum dynamics observed only by skilled scientists under very carefully prepared conditions? If Schrödinger's cat can really be described as a quantum mechanical object, then why isn't it possible to use cats in interference experiments rather than particles?

For realist interpretations of quantum mechanics that do not invoke wavefunction collapse, the answer to these questions (as set out by Schlosshauer) is decoherence. A quantum system, whose state becomes encoded within its environment by some variable, will after that point behave, under operations affecting only that system, in a way that is empirically and mathematically indistinguishable from a system in an unknown but determinate eigenstate of that variable (i.e. it will not display interference phenomena). Thus, one reason that it would be impossible to perform an interference experiment on Schrödinger's cat is because the live cat and dead cat states entail different positions for parts of the cat, and these positions will extremely rapidly become encoded within the cat's environment. Interference experiments performed on the cat’s mortality state will therefore fail to display any distinctively quantum interference phenomena. This should not be confused with wavefunction collapse. Decoherence of the cat’s mortality state does not mean that the cat is in a determinate state, and an interference experiment would still in principle be possible if it could be performed, not only on the cat, but on all features of the environment to which the cat has become entangled.

To take a more practical example, decoherence is probably the largest difficulty facing the developing field of quantum computation. The problem is that, in order to produce a quantum computer, you need to be able to produce particular quantum states that can be easily operated on, and that display interference phenomena. This requires that the quantum bits[4] encoding your state should not decohere with respect to the environment. It is extremely difficult to keep such states from decohering for any length of time.

If a quantum bit (e.g. the spin state of a spin half particle) becomes entangled to its environment by a particular degree of freedom (e.g. its z-spin state) then the RDM characteristics just shown ensure that the particle will not display interference phenomena in its Z spin state under operations performed on it. Or mathematically:

$$\langle \downarrow_z |\hat{\rho}| \uparrow_z \rangle = \langle \uparrow_z |\hat{\rho}| \downarrow_z \rangle = 0$$

This result is obtained by assuming linear unitary evolution of the Schrödinger equation without any form of collapse dynamics. The result therefore certainly holds for those interpretations which take a realist view of these dynamics. These include Everettian approaches, which take the wavefunction as the primary component in their ontology, and de Broglie-Bohm approaches, which include particles within their primary ontology but also reserve a fundamental role for the wavefunction, either as an object in its own right or as a law or disposition relating to particle dynamics.

This means that environment-induced decoherence (as captured by reduced density matrices) offers a clear solution to the problem of the non-observability of interference for these interpretations, seemingly without invoking any form of collapse postulate.

It is unclear whether the result is important to approaches which invoke collapse, such as GRW approaches. These approaches interrupt the linear Schrödinger dynamics with wavefunction collapses of one kind or another.

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[4] Quantum bits (known as Q-bits) are similar to bits as used in classical computing. However, whereas a classical bit occupies either state 1 or state 0, a Qbit may occupy a superposition state $\alpha|0\rangle + \beta|1\rangle$. This allows quantum computers to make use of interference phenomena in their processing. For more information see Rae 2008 chapter 12.

Whether or not the wavefunction would continue to develop linearly for periods longer than the timescales required for effective decoherence remains unclear and will depend on particular details of collapse dynamics within specific approaches (see Bub 1997 and Zurek 1993). GRW approaches consequently rely the least on decoherence and will not be discussed further in this chapter.

2.2.6.2 The Problem of the Preferred Basis

To explain the problem of the preferred basis and the role of decoherence in resolving it, I will present a variant on a Wigner's friend thought experiment, in which two non-commuting variables appear to be measured simultaneously. This closely resembles a similar thought experiment by (Vaidman, 1998).

Consider a physics lab in an impermeable box. Within the physics lab, Bob is performing an x-spin measurement on a spin half particle initially in a superposition state, using a Stern-Gerlach apparatus. The state of the lab immediately before measurement can be represented as:

$$(\alpha\big|0_x\rangle_{Spin\,state} + \beta\big|1_x\rangle_{spin\,state})\ |"ready"\rangle_{Bob}|t=0\rangle_{Environment}$$

Where the environment is taken to include all degrees of freedom within the lab, which are not degrees of freedom of the particle spin state or of Bob. (Hereafter, the ket descriptions will be abbreviated to a single letter.)

When the measurement takes place the state of the lab evolves as follows:

$$\begin{aligned}(\alpha|0_x\rangle_S + \beta|1_x\rangle_S)\ |"ready"\rangle_B|t=0\rangle_E \\ \rightarrow \alpha|0_x\rangle_S\,|0_x\rangle_B\big|t=1_{0_x}\big\rangle_E + \beta|1_x\rangle_S\,|1_x\rangle_B\big|t=1_{1_x}\big\rangle_E\end{aligned}$$

Thus the states of both Bob, and the environment become entangled with the x-spin state of the particle. From this point on, operations performed on the particle's x-spin state will not display interference phenomena. It will also be impossible for Bob to perform any operation on the particle or the environment within the lab capable of recovering the original particle state. If Bob were to make further measurements of the spin state of the particle, they would display the statistics expected if the particle were beginning in a determinate x-spin state.

Next, consider the lab supervisor Alice, who is standing outside the impermeable box containing the lab. Alice knows the procedure which Bob is following within the box, having agreed it with him in advance, and can easily establish the quantum state of the lab written above. Now though, she is able (in principle at least) to achieve a measurement of the z-spin state of the original particle spin state. As shown by Schlosshauer (2007, pp. 54), a simple basis transformation of the state of the lab provides a description in terms of the z-spin state of the original target particle.

To see this, we will combine the states of Bob and the rest of the lab environment. (Reasons for doing this will be discussed shortly.)

$$|0_x\rangle_B \left|t = 1_{0_x}\right\rangle_E = \left|t = 1_{0_x}\right\rangle_{Lab}$$

$$|1_x\rangle_B \left|t = 1_{1_x}\right\rangle_E = \left|t = 1_{1_x}\right\rangle_{Lab}$$

$$\alpha|0_x\rangle_S \left|t = 1_{0_x}\right\rangle_L + \beta|1_x\rangle_S \left|t = 1_{1_x}\right\rangle_L = \frac{1}{\sqrt{2}}(\alpha + \beta)|0_z\rangle_S \left|t = 1_{0_z}\right\rangle_L + \frac{1}{\sqrt{2}}(\alpha - \beta)|1_z\rangle_S \left|t = 1_{1_z}\right\rangle_L$$

(Eigenvector transformations from Rae 2008, pp. 126.)

Thus, by cleverly constructing an appropriate measurement, it is possible for Alice to effectively make a z-spin measurement on the original particle spin state.

It seems therefore that Alice and Bob have between them measured two non-commuting variables of the original state. This may appear to be a violation of the uncertainty principle. However, the viability of the uncertainty principle is maintained because, in the course of making a measurement, Alice has entangled the state of the contents of the lab to its environment by a set of degrees of freedom which do not commute with those of Bob's original measurement. Alice has not measured the state of Bob in a way that could possibly reveal the x-spin measurement result. Thus, Alice is not able to recover the results of both non-commuting measurements.

The issue raised by this result, however, is the following. Given that the state of the lab after Bob made his measurement encoded the state of the original particle in full without any loss of information, such that it is possible to recover a non-commuting measurement result from the state, in what sense was the original measurement a measurement of the x-spin state rather than the z-spin state?

Some interpretations may offer means to resolve this question without further reference to decoherence. For example, on a Bohmian account this process might be said to constitute an x-spin measurement, simply because for a period of time there is a counterfactual dependency between the positions of many particles in the lab, including those that make up Bob, and the position state of the measured particle passing through the Stern-Gerlach apparatus. The Bohmian presupposition of position, as a privileged variable, provides a possible rationale for characterising this measurement as a measurement of a particular spin basis.

Everettian interpretations, however, do not make any such presupposition of a privileged basis. The degrees of freedom of the lab state which encode the z-spin particle state are not position variables, or any classically recognisable variables. But this does not entitle us to disregard this encoding unless we presuppose the privileging of some quasi-classical basis. The Everettian, therefore, needs some other account of what it means to measure a particular variable. This is particularly important in establishing a conception of branching for Everettian many worlds approaches. The Everettian must give some account of whether branching occurs within the lab when Bob makes his spin measurement, and, if so, in which basis branching occurs. In this case, whether branching follows the particle’s x-spin, or z-spin state, and most importantly why it follows one of these states rather than the other.

The Everettian solution to this problem rests on an appeal to decoherence. There is a crucial difference between the Lab partitioned into histories based on the target particle’s x-spin state, and a partition based on the z-spin state. The difference is that while the Lab state in its entirety can be represented in a very similar way in both bases, the same is not true at all for Bob. (This is why it was necessary to combine Bob’s state with that of the wider lab when making this basis transformation.)

In the x-basis, the particle position state following the measurement differs in the two histories corresponding to the two measurement outcomes. As this state becomes entangled to its environment, many other positions come to differ between the two histories. Both of these histories contain a version of Bob with different position states. Bob is entangled to his environment by these positions, and so Bob considered as a subsystem is decohered, and will not display interference phenomena under operations affecting only Bob.

If instead we consider histories distinguished by the z-spin particle state, the set of degrees of freedom of the environment which encode this result will not be position but a set of radically nonclassical degrees of freedom. These degrees of freedom relate to the lab state as a whole, and cannot easily be divided up in such a way as to pick out a quasi-classical pattern identifiable as Bob. Each history taken individually, if considered in a position basis, would show Bob in a superposition of very different position states, and would very likely display interference phenomena between the two histories. The very great difference in the characteristics of these 2 pairs of histories, as they relate to Bob as a sub-system, stems from the original construction of the experiment. If the experiment had instead been constructed so as to entangle the particle’s z-spin state to its position state, and then to allow that position state to decohere, then the characteristics of the 2 pairs of histories would be reversed. Histories separated according to the particle's z-spin would be the ones to contain a well localised position state for Bob displaying little interference.

The response offered by Schlosshauer to this problem is simply to identify a preferred basis of branching histories with the diagonalization of the reduced density matrix of quasi classical objects such as Bob. For any quasi-classical object within the lab,[5] there is a very broad entanglement of position states, which will mean that, in the pair of histories originating from the particle x-spin state discussed, the object is decohered with respect to its environment, and does not display interference phenomena. Thus, a clear and reasonably mathematically rigorous criterion is given for identifying quasi classical histories from within a wavefunction, and establishing the point at which they come to branch off from one another. Namely, that branching in the history of a system occurs when the system's state becomes entangled to its environment in a basis in which the preceding history of the system would be described as a superposition.

Before moving on, I will identify two significant concerns relating to this solution of the problem of the preferred basis. The first, which will be discussed in more detail in chapters 4 and 5, is that this criterion is only able to pick out classical-like histories if the object taken as the subsystem of interest is quasi-classical to begin with. If, instead, some arbitrary selection of degrees of freedom of the lab were taken to be the subsystem of interest, there would still be a basis in which the reduced density matrix of the system was diagonalised, but the basis would be very unlikely to be classically recognisable. There has been a great deal of discussion within the Everettian literature of whether, and to what extent, this should be

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[5] This holds providing that the quasi-classical object considered has a small number of degrees of freedom compared to its environment (in this case the rest of the lab). If this condition fails to obtain, then effective decoherence becomes impossible, as there is insufficient environment to encode the state of the system in question. In realistic cases, this condition will always obtain for medium-sized dry goods, as the environment will not be limited to a single laboratory.

considered an objection to the application of decoherence to the problem of the preferred basis (e.g. Thébault & Dawid 2015, Kastner 2014, Zurek 2005). For now, though, it is simply important to note that decoherence is able to provide a rationale for identifying a preferred quasi-classical basis, only if we begin with a quasi-classical system of interest.

The second issue is how to make sense of the fact that after Bob makes a measurement of the particle's x-spin leading to decoherence of the particle's spin state in the x-spin basis, and producing two histories which can reasonably be identified as distinct, it is then possible for Alice to perform a measurement on the lab which reveals the particle's z-spin state, and in the process makes the x-spin state inaccessible. This issue is fundamentally conceptual. Firstly, because systems on the scale of laboratories cannot practically be maintained in isolation from the rest of the environment as has been supposed here. Even if they could, the operations that would have to be performed on the lab in order to recover the z-spin state would face extreme technical difficulties. And secondly, because if linear Schrödinger dynamics are assumed to apply throughout, there is no doubt as to characteristics of the physical evolution of the system as described by the wavefunction. The question is rather how we can make sense of a process of decoherence producing two distinct histories which are then recombined, thereby rendering records of the original measurement inaccessible.

This second problem will apply to any interpretation of quantum mechanics which does not endorse wavefunction collapse, and therefore allows for the possibility of histories recombining in this kind of process. Bohmian interpretations may be able to make sense of what it is to make a measurement without appealing to decoherence, but they still accept the same wavefunction dynamics, and consequently the possibility in principle that Alice could successfully make a z-spin measurement on a lab in which Bob has already made an x-spin measurement. This problem, and possible ways of responding to it, is a core theme in this thesis, and will be discussed in the next chapter.

2.2.6.3 The Problem of Outcomes

Finally, we come to the problem of how it is possible to reconcile the definite measurement outcomes obtained by observers within histories with the determinate superposition states of linear quantum mechanics.

Schlosshauer 2007 distinguishes two aspects of this problem: the specific and the generic. The generic problem of outcomes is the question of how it comes about that definite outcomes are (or seem to be) observed in cases of measurement. In terms of our Wigner's friend thought experiment, this is a question of how the absence of wavefunction collapse is to be reconciled with Bob's belief that he has successfully made a measurement of the particle's x-spin state and observed a definite outcome. Decoherence offers a solution to this problem by identifying two distinct fairly well-defined histories. In one of these histories, Bob has measured spin up, and in the other he has measured spin down. However, it is a determinate feature of both of these histories that Bob believes himself to have made a measurement and obtained an outcome. Thus, any recording operation of Bob concerning whether or not he had measured an outcome, which would yield a positive result in the event of the measured particle having determinately possessed either x-spin state, would also yield a positive result in the event of a superposition state. And precisely the same reasoning would apply to Alice's measurement in turn. It seems, therefore, that we can account for the experience of obtaining particular measurement outcomes, even if we do not actually obtain any.

The second problem identified by Schlosshauer is the specific problem of outcomes. This is the problem of how it comes about that we make the measurement that we do, as opposed to any other. In other words, if Bob finds himself having just made a spin up measurement, what determined that he should measure this result rather than spin down? Schlosshauer identifies this as the one feature of the measurement problem which he believes cannot be solved by appealing to decoherence. Instead, this problem is left to specific interpretations to offer a means of resolution.

2.3 Conclusion

In this chapter, I have introduced a useful way of subdividing the quantum measurement problem, as well as one clear and empirically well confirmed definition of decoherence. I have shown why decoherence seems to have clear relevance to the quantum measurement problem, for interpretations of quantum mechanics which reject the collapse postulate.

As noted, however, there are a range of concerns about this conception of decoherence. One concern which has been discussed in the literature relates to the difficulty of rigorously characterising decoherence without in some way presupposing the recovery of classicality which decoherence is being used to achieve. The most common presentation of this problem focuses on the difficulty of justifying the physical significance of the reduced density matrix without a seemingly illicit application of the Born rule (though the same problem can also be presented in other ways). Another is that, as the suppression of interference phenomena by decoherence is, in realistic cases, not total, decoherence in some sense is not truly able to recover the desired classicality, and so does not solve anything. These problems will be the subject of chapters 4 and 5. I will argue however that, at least when considered within the context of some interpretations, both of these problems meet with satisfactory responses.

Another problem I have noted here is that in many cases diagonalisation of a reduced density matrix in a particular basis will be a short-lived phenomenon. A single measurement of a quantum system clearly does not permanently disrupt its capacity for interference phenomena in the measured basis. If the preferred basis we are looking for needs to persist over an extended time period (as in Everettian interpretations), then a more complex conception of decoherence will need to be employed. Ways of formulating such a conception will be the subject of the next chapter.

A significant issue that such conceptions will need to consider can be seen in the Wigner’s friend example used above. When Bob makes his measurement, the x-spin particle state becomes entangled to the rest of the lab, and consequently the reduced density matrix of the particle is diagonalised in the x-spin basis. On the conception of decoherence presented here, this seems to indicate that x-spin is the preferred basis for this particle. After Alice's measurement, however, it is the z-spin particle state which ends up entangled to its environment, and so the z-spin basis which will be diagonalised. A conception of decoherence aiming to pick out a preferred basis over an extended time will need to say something about which of these contradictory bases we should take as preferred. I will argue that this challenge poses significant problems for interpretations which rely on such conceptions of decoherence.

3 History Spaces and the Decoherence Functional

In the discussion so far, extensive use has been made of the terms *decoherence* and also, less frequently, of *histories*. These concepts do not belong to any single interpretation of quantum mechanics, though some have been particularly influential in developing them. They refer to particular mathematically defined patterns within a system's wavefunction. These patterns have been shown to occur for many systems which physicists are interested in, as well as virtually all medium-sized dry goods such as we are generally familiar with. As such, they are in an important sense interpretation-neutral, between all those interpretations which retain the linear Schrödinger wavefunction dynamics, although their ontological significance is heavily interpretation-dependent.

This thesis will heavily rely on both of these concepts, and their application to particular interpretations. First, however, more rigorous mathematical definitions need to be given and considered for these concepts. In both cases, crucial developmental work has been done by advocates of consistent (or decoherent) histories approaches. This section will briefly outline the central distinguishing features of the consistent histories approach, formally introduce the concept of histories (which has already been used informally), and differentiate important distinctions in the meaning of the word decoherence.

In the philosophical literature, decoherence is often identified with the diagonalisation of a system’s reduced density matrix. In discussion, however, a number of subtly different definitions of decoherence are used, often without appropriate clarification. This section focusses mainly on two rigorously defined concepts, set out in Gell-Mann & Hartle 1990, which have come to be discussed within the wider literature (e.g. Wallace 2012). Both are sometimes referred to as decoherence.

The consistent histories interpretation of quantum mechanics was originally suggested by Griffiths 1984, and extensively developed in Omnès (1988 and 1999), and also developed into the very similar decoherent histories approach of Gell-Mann & Hartle 1990. There are some significant differences between these authors on the details of the approach, but all share a common core. Like Everettian and Bohmian interpretations, the consistent histories approach maintains the linear Schrödinger dynamics as universal, without wavefunction collapse.

Like many-worlds Everettians, the histories approach rejects the addition of hidden variables dynamics such as Bohmian particles[6]. Again like Everettians, they use decoherence, or the similar (but weaker) condition of consistency, to identify sequences of system states possessing quasi-classical dynamics from within the overall dynamics of the quantum state. They differ, however, in the metaphysical interpretation of these histories, and the assessment of their associated probabilities.

Everettians seek a single branching structure of histories, which they invest with strong metaphysical significance. It is in terms of this branching structure, and the probabilities obtained by Born rule projections of a system's state vector onto this basis, that all discussions of classical events and classical physics derive their meaning. There may be some degree of vagueness as to the precise character of this branching structure, but Everettian metaphysics relies on such a branching structure being robustly identifiable.

The consistent historian, by contrast, is not committed to there being a single such structure, or attributing any special metaphysical significance to

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[6] For the rest of this chapter I shall refer to Everettian many-worlds approaches simply as Everettian approaches, and distinguish them from decoherent and consistent histories approaches (which I will collectively refer to as many histories approaches). All of these approaches stem from work done by Everett 1957, but the term Everettian has come to refer most frequently to many-worlds interpretations.

a single structure in particular. The key interpretive principles are summarised by Griffiths 2017 along the following lines:

- Liberty: A physicist interested in a particular system may freely employ any number of different bases with associated sets of histories to analyse that system.
- Equality: No basis a physicist may choose to employ is more fundamental than any other; no single basis is singled out by nature.
- Incompatibility: Bases which use projectors which do not commute with one another, are *never* to be combined into a single quantum description. The (probabilistic) reasoning process starting from assumptions (or data) and leading to conclusions must be carried out using a *single* history basis.
- Utility: Some history bases are more useful than others for answering particular questions about a quantum system.

It is the principles of liberty and equality which clearly differentiate histories interpretations from Everettian interpretations. Whereas the Everettian seeks to identify a single basis to act as the foundation for a clear quasi-classical ontology, the histories approach allows for the use of multiple mutually incompatible bases, with no basis singled out as fundamental.

The significance of this position will be explained more clearly in due course. For now though, we will focus on understanding the concepts of history, consistency, and decoherence, as they are used in the histories literature.

3.1 Histories

So far, the histories we have discussed have been simple pairs corresponding to a single particle's spin eigenvectors. Intuitively, it is easy to imagine a pair of histories corresponding to each of the two possible outcomes of a particular measurement and evolving classically thereafter. Of course, a single pair of histories would be an exceptionally simple case. Realistically, the history in which Bob measured spin up would very quickly yield many additional histories as naturally occurring quantum superposition states decohere with respect to the rest of the lab. Precisely how many times these histories would subdivide will depend on the basis of the history space by which the physical system is analysed as well as the characteristics of that system.

The most general requirement for a history of a quantum system is that it should pick out a state of affairs for the system, with a corresponding projection operator, at a series of times in the system's evolution. For some such history α, we define a history operator:

$$\widehat{C_\alpha} = \prod\nolimits_m \widehat{P^m_{\alpha_m}}$$

Where m denotes a time index, and $\widehat{P^m_{\alpha_m}}$ is the projection operator, for the projection of a state $|\psi\rangle$, onto the state of affairs associated with history α at time m, denoted as α_m.

In other words, a history is defined as the product of a continuous series of projection operators over time. This means that a history will always specify a partial or complete state of the system at many (sometimes all) times within the systems evolution.

This is the most general form of a history, and will include many histories that have radically non-classical projectors $\widehat{P^m_{\alpha_m}}$, as well as many others for which there is no identifiable pattern to the projectors that make up the history at different times. It is hoped, however, that additional criteria can be added to pick out just quasi-classical histories which contain the types of useful regularities that we see in classical physics (and perhaps even justify a privileged metaphysical status for those histories).

For a set of histories, the set of projection operators for a time index do not necessarily form a complete basis. If the projection operators do form a complete basis at every time index, then the histories composed of sequences of these projections are said to be atomic. They pick out precisely one way that the world could be at every time. An atomic history of our laboratory might have projectors which specify a precise position for every particle, as well as x-spin states for every applicable particle.

A set of atomic history operators together form the basis vectors of a history space for the system. The set of histories that form the atomic basis of a history space must commute with one another.

The dimensionality of a history space will depend entirely on the basis of histories chosen. To take an extreme example of this, Bob's lab could be represented in a one-dimensional history space, if the projection operators of the history that make up the space's basis were perfectly aligned to the wavefunction of the lab at every time. This would be an adequate history space characterisation, but not a helpful one, as it would not give us any useful conceptual tools by which to understand that wavefunction.

Similarly, while many sets of histories that do not contain quasi-classical states, or feature quasi-classical regularities, could be found that would commute with one another, and form a mathematically adequate basis of the history space, we are generally only concerned with those histories that do offer these kinds of regularities when we talk about histories.

For any particular wavefunction there are any number of possible history spaces that could be applied depending on the choice of projectors at each time interval. This choice of complete basis for the history space will determine the number of atomic histories and their dynamics.

Non-atomic histories are formed of projectors which do not form a complete basis. This is because they only specify coarse-grained states at every time interval. They might, for example, specify particle positions as being within a particular range, rather than being a set of histories that distinguish particle positions with infinite precision. Consequently, they pick out a set of the atomic histories within the space rather than a single such history.

Gell-Mann and Hartle 1990 suggest 3 common kinds of history specification that would not correspond to a single perfectly fine-grained atomic history, but to some set of them. These forms of coarse-graining are: “(1) specifying observables not at all times, but only at some times (2) specifying at any one time not a complete set of observables, but only some of them (3) specifying for these observables not precise values, but only ranges of values.”

The references to histories made in the previous section were references to non-atomic histories. This is what *histories* is most often taken to mean. For many types of system, a decomposition into perfectly atomic histories will not yield any histories which show classically recognisable dynamics. For this reason, Gell-Mann and Hartle go so far as to argue that probabilities can only reasonably be assigned to non-atomic histories, and not to perfectly refined fine grainings of them.

So far, nothing I have said concerning histories specifies them as more than an arbitrarily chosen sequence of projectors. Before turning to introduce the concept of decoherence, and the requirements which are our main topic of interest, it is important to identify a more general requirement for the types of history in which we are interested. This requirement provides a dynamical connection between the sequence of projectors which make up a history. As Gell-Mann & Hartle put it:

> “…it is clear that the solution of the mathematical problem of enumerating the sets of decohering histories of a given Hilbert space has no physical content by itself. No description of the histories has been given. No reference has been made to a theory of the fundamental interactions. No distinction has been made

> between one vector in Hilbert space as a theory of the initial condition and any other.
>
> “We obtain a description of the sets of alternative histories of the universe when the operators corresponding to the fundamental fields are identified. We make contact with the theory of the fundamental interactions if the evolution of these fields is given by a fundamental Hamiltonian.” (Gell-Mann & Hartle 1990, pp. 14).

In other words, in order to be interesting histories of physical significance, the projectors of our histories must be in a basis in which some classically recognisable entity is identifiable, and more importantly one in which successive projectors follow patterns to be expected given the fundamental Hamiltonian for the system in question.

This is essentially a quasi-classicality criterion which is being indicated. Wallace 2012 also sees the need to invoke such a requirement on our history space if we are to obtain useful and informative histories.

Making this requirement precise presents some major difficulties, and neither Wallace nor Gell-Mann and Hartle attempt to do so. A first attempt would be to use the unitary transformations which in the Schrödinger picture connect states of the wavefunction at different times:

$$\rho_{t=k} = \widehat{U_{jk}}\, \rho_{t=j}\, \widehat{U_{jk}}^{-1}$$

Where ρ_t is the density matrix of a system at time t, and U_{jk} is the unitary transformation matrix for the change in the system state between time j, and time k. This transformation matrix is a direct function of the system’s Hamiltonian between these times.

Switching to the Heisenberg picture, this time dependence moves from the state to the projectors:

$$\widehat{P^{k}_{\alpha_k}} = \widehat{U_{jk}}^{-1}\, \widehat{P^{j}_{\alpha_j}}\, \widehat{U_{jk}}$$

It is tempting to think that precisely this relation might hold between history projectors at different times. Indeed, for quasi-classical histories undergoing entirely classical dynamics without any quantum probabilistic events, this relation would indeed hold for the types of history in which we are interested. The main problem is that this relation can offer no account of history branching, and a history will simply evolve linearly as a single history forever, and probably cease to be classical-like in the process. Nevertheless, some form of quasi-classicality criterion somewhat like this is essential to (and often tacitly assumed by) discussions of histories and history spaces. This criterion can be thought of in practice as ruling out bizarre and contrived examples. It means that the histories we consider are generally meant to be histories which roughly correspond to, or identifiably contain, classically recognisable entities, and in which the dynamics internal to histories generally follow intuitively natural classically plausible patterns.

Wallace 2012 goes so far as to make this relation between successive projectors (with appropriate allowances for history branching) an integral part of his definition of histories, and of a history space (pp. 91-92). The original architects of the histories approach do not go so far, treating histories as a more general term, but, as noted above, they clearly recognise histories of this type as being of particular interest and physical relevance.

We will now turn to examine two conceptions of decoherence defined as constraints on history spaces, and see how these add a vital criterion to the definition of those histories with which quantum interpretational projects are concerned.

3.2 The Decoherence Functional

The quasi-classicality criteria just discussed concern two aspects of our history space. The first is the choice of system whose history space we are considering. There are many systems we could choose which do not display any classically recognisable regularities. This is to be expected if we set out to consider the history space of a system composed of arbitrarily chosen and disparate degrees of freedom spread throughout the universe, rather than some more classically familiar system.

The second is the dynamics of those histories which we have chosen to form the basis of our history space. In the absence of branching, we require these to follow patterns of behaviour which are classically recognisable, and the behaviour to be predicted for the internal dynamics of a history given the Hamiltonian to which the system is subject.

Now we turn to criteria which have been developed to analyse the structure of such histories within a history space and their relations to one another. Gell-Mann and Hartle are seeking a criterion which, if fulfilled by a history space, would warrant applying probabilities to particular histories within the space. Wallace has the similar but distinct goal of finding a criterion which would warrant attributing metaphysical significance and an emergent reality to the structures contained within particular histories. In both cases they are looking for particular types of behaviour which indicate a particularly classical-like character in the histories which form a history space, and in the process refine our understanding of when history branching can usefully be considered to take place.

The central tool developed for this analysis is the decoherence functional:

$$D(\alpha,\beta) = \left\langle \psi \middle| \widehat{C_\alpha^\dagger}\widehat{C_\beta} \middle| \psi \right\rangle$$

For a pair of histories α and β, with history operators $\widehat{C_\alpha}$ and $\widehat{C_\beta}$, the decoherence functional D of that pair can be thought of as a measure of the overlap between the two histories over time. This overlap is then weighted according to the wavefunction amplitude in the region of Hilbert space at the time when the overlap occurs.

For an orthogonal pair of histories whose projectors do not overlap at any time, the decoherence functional will equal zero. The pair will also equal zero if their projectors do overlap, but the overlap is perfectly orthogonal to the system state vector.

It is worth noting that this functional can be applied just as easily to coarse-grained histories, as to atomic ones.

Two criteria based on this functional have been developed in the many histories literature. The first to be developed was *consistency*. Consistency is of very clear relevance to the problem of when probabilities can be attributed to particular histories, and is still advocated as the appropriate criterion by consistent historians such as Omnès.

For reasons that will be discussed later in this chapter, this criterion quickly lost popularity in the face of the second stronger criterion. This second criterion, *medium decoherence,* is less obviously relevant to issues of probability, but is in some ways conceptually and mathematically simpler than consistency. For this reason, we will examine the criterion of medium decoherence first before returning to consistency.

3.3 Medium decoherence

The medium decoherence requirement set out by Wallace 2012 can be expressed simply:

> A history space fulfils the medium decoherence criterion iff for any pair of distinct histories α and β which make up the space $D(\alpha,\beta) = 0$.[7]

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[7] There is some variation in precisely how this criterion is defined by different authors. This is Wallace's formulation. Gell-Mann and Hartle 1990 instead give the criterion as $D(\alpha,\beta) \approx 0$. I have followed Wallace here because I think it is useful to be able to clearly discuss the consequences of the decoherence functional equalling zero precisely (such as the branching decoherence theorem). All authors agree that realistic history spaces will never fulfil the precise form of this criterion, but only the approximate one. I will follow

In other words, it requires that distinct histories should never overlap with one another after becoming distinct.

This criterion is used by Wallace as a criterion for when histories count as robust metaphysically significant Everettian *worlds*.

Wallace 2012 pp. 93 makes a connection between history branching and the decoherence functional using the branching decoherence theorem, which he attributes to Griffiths 1993 (though much the same relationship is presented less formally in Gell-Mann and Hartle 1990)[8]:

> Branching-Decoherence Theorem: Suppose $\mathcal{P} = \left\{\widehat{P_J^{\iota}}\right\}$ is a history space and $|\psi\rangle$ is a quantum state. Then:
>
> i. If $\mathcal{P}$ has a branching structure (relative to $|\psi\rangle$) and α is a history then $\widehat{C_\alpha}|\psi\rangle \neq 0$ iff α is realised (with respect to $|\psi\rangle$).
> ii. If the set Hist of all histories α such that $\widehat{C_\alpha}|\psi\rangle \neq 0$ has a branching structure (that is, if no two histories in Hist agree on their ith but not on all previous projectors), then $\mathcal{P}$ also has a branching structure (relative to $|\psi\rangle$) and the realised histories in that branching structure are just the histories in Hist.
> iii. If $\mathcal{P}$ has a branching structure (relative to $|\psi\rangle$), $\mathcal{P}$ satisfies the [medium] decoherence condition.
> iv. If $\mathcal{P}$ satisfies the [medium] decoherence condition it is a coarse-graining of a ([medium] decoherent) history space which has a branching structure relative to $|\psi\rangle$.

The relationship expressed in this theorem is intuitively natural and easy to grasp.

(i) Offers a definition of what Wallace takes it to mean for a history to be *realised.*

Wallace in discussing this issue in terms of the strict medium decoherence criterion obtaining approximately.

[8] See Wallace 2012 appendix A for a proof of the branching-decoherence theorem.

(ii) Gives a natural definition of what it means for a set of histories to possess a branching structure (namely that once the projectors of two histories have come to disagree, they will not come to agree again at any future time).

(iii) Is a clear statement of the connection between a branching structure and the decoherence condition.

(iv) Says that if a history space satisfies the medium decoherence criterion then it is a coarse graining of another history space which satisfies branching. This is the most interesting of the points, and will be discussed further in a later section.

This serves to make it very clear what the relation is between the medium decoherence condition and a history space possessing a branching structure. It does not, however, make any obvious link to probability. The next section will look at the probability conditions that both Everettians and consistent historians wish their history spaces to display. It will introduce a criterion designed specifically to ensure that probabilities fulfil this condition. And it will show that this new criterion is entailed by the medium decoherence criterion.

3.4 Consistency

The probability condition that motivates the consistency criterion is directly linked to the absence of interference. In terms of probability calculus this can be expressed as:

$$\Pr(\alpha) = \sum_{\alpha_i \in Dec(\alpha)} \Pr(\alpha_i)$$

Where α is a non-atomic history, and α_i are the atomic histories which together form a complete decomposition of α.

To understand this, it is helpful to consider the double slit experiment. When considering the probability of a particle arriving at a particular point on the final screen, we usually have to consider the effects of interference between different particle trajectories. The condition for the absence of interference phenomena is that the probability of a particle ending up at a particular point on the screen is simply a sum of the probability of that particle following every trajectory that would lead to this final point. This is what we see when a detector is placed to record which slit the particle passed through. Without such a detector, however, interference between different particle trajectories is likely to change the probability of the particle reaching a particular point.

The consistency criterion is a direct attempt to capture this absence of interference as a constraint on histories within a history space.

In a history space formalism, this requirement can be written as (Wallace 2012, pp. 92-94):

$$\left\langle \psi \middle| \widehat{C_\alpha^\dagger} \widehat{C_\alpha} \middle| \psi \right\rangle = \sum_{\alpha_i \in Dec(\alpha)} \left\langle \psi \middle| \widehat{C_{\alpha_\iota}^\dagger} \widehat{C_{\alpha_\iota}} \middle| \psi \right\rangle$$

Where α denotes a particular history, $|\psi\rangle$ denotes the system state, and $Dec(\alpha)$ denotes a decomposition of α into atomic histories.

In terms of the decoherence functional, the condition that will ensure this for two distinct histories, α and β, is:

$$D(\alpha, \beta) \in \mathbb{I}$$

That is, the decoherence functional for any pair of distinct histories must be purely imaginary, with its real part equal to zero.

It is easy to see that this condition is entailed by the medium decoherence condition (which requires both real and imaginary parts to be equal to zero). Medium decoherence will therefore also ensure that probabilities sum in the desired fashion, and histories do not display interference phenomena.

Medium decoherence goes further than consistency, however, by requiring that the histories that form a history space should possess a branching structure. In order for interference between histories to be possible, they must come to agree on their projectors sometime after separating. Clearly, therefore, it is unsurprising that a condition which ensures a branching structure also ensures an absence of interference phenomena. There is a second situation, however, in which interference phenomena will not be displayed. This is if the phases of the histories in question agree at the time when they come to meet. In this case, the probabilities of the histories in question before and after they merge will obey the classical probability summation rule mentioned above.

Given that fulfilling the consistency criterion is enough to ensure that the history space does not display interference phenomena, and that interference phenomena are the key distinctively quantum phenomenon that separates quantum mechanics from the world of our everyday experience, it is easy to see why this criterion has been thought central to recovering classicality, and attributing probabilities. Griffiths 1984 and Omnès 1988 did indeed use consistency in this way, and Omnès continues to advocate an approach based on this criterion.

Griffiths, however, has since abandoned this criterion in favour of medium decoherence. Gell-Mann & Hartle also reject the consistency criterion in favour of medium decoherence, and medium decoherence is the criterion upon which modern Everettians such as David Wallace base their approach. Intriguingly, however, it is not particularly easy to understand what led all these authors to adopt the medium decoherence criterion in favour of consistency.

Gell-Mann & Hartle offer a rather cryptic explanation for rejecting the consistency condition in favour of medium decoherence:

> “[Consistency] is the condition used by Griffiths as the requirement for “consistent histories”. However, while, as we shall see, it is easy

> to identify physical situations in which the off-diagonal elements of D approximately vanish as the result of coarse graining, it is hard to think of a general mechanism that suppresses only their real parts. In the usual analysis of measurement, the off-diagonal parts of D approximately vanish. We shall, therefore, explore the stronger condition [medium decoherence] in what follows." (Gell-Mann & Hartle 1990, pp. 11).

Their reasoning seems to be that it is difficult to imagine the mechanism by which the weaker consistency requirement would be fulfilled without the stronger medium decoherence requirement being fulfilled, and so they mean to focus on the stronger rather than the weaker requirement. More consideration will be given to what might have been meant by this argument in the next section. On the face of it, however, this argument seems to do little to offset the very clear and direct relevance of the consistency criterion for recovering classical dynamics.

Though it may be unclear why Gell-Mann & Hartle originally introduced this strong criterion, there are some clear reasons for its popularity. Whereas the histories within a consistent history space may or may not form a branching structure of the type desired by the Everettian, the medium decoherence criterion implies that histories should form such a structure.

Though the support this gives for Everettian metaphysics may explain the popularity of this criterion, it is not used to justify it. The Everettian position presented by Wallace seeks to present decoherence as the basis of a branching structure of many worlds, not the other way around. Wallace however gives little real justification for choosing medium decoherence as his preferred criterion. The closest he comes to a direct justification of his choice when introducing the medium decoherence criterion is the following:

> “However, [consistency] does not seem to have any dynamical significance (in the way that decoherence proper [medium decoherence] has been shown to have), and composite systems satisfying weak but not full decoherence have been shown to have various unsatisfactory properties (Diósi 2004).” (Wallace 2012, pp. 98).

The reference to dynamical significance presumably relates to the Everettian need for a branching structure already discussed. More interesting is the reference to unsatisfactory properties. Wallace himself never explains what these unsatisfactory properties are, but he clearly considers Diósi 2004 to have found some.

The next section will look at Diósi 2004 and the features of consistent and medium decoherent history spaces which he discusses. Before moving on though, I wish to note that as these issues seem to first appear in the literature in a letter written in 2004, it remains unclear to me why medium decoherence was being endorsed in favour of consistency by authors such as Gell-Mann & Hartle 1990, long before this letter was published. This shift in the general consensus of the many histories literature has no clear explanation that I have been able to find, but it has had a profound effect on more recent literature, including modern Everettian interpretations.

3.5 History space composition

Surprisingly Diósi 2004, to which Wallace refers readers on this point, is not an extended paper considering the relative strengths of these criteria, but a two-page letter published in Physical Review Letters.

Much of this short letter is devoted to setting out the definitions of decoherence and consistency, in much the same way that Wallace does. He also introduces a proposal by Goldstein & Page for an alternative constraint, which will be ignored for the present discussion. Given the amount being presented in this short letter, his presentation of the problems he believes consistency to face is understandably brief. In this section I will do my best to expand on and clarify the issues with which he is concerned.

Diósi accepts that the consistency criterion yields histories which will obey the classical axioms of probability, and which can be assigned probabilities without contradiction. He believes, however, that there are two other desiderata which this criterion does not guarantee. These he names *the test of composition* and *the test of dynamical stability.*

3.5.1 The Test of Composition.

The test of composition looks at what happens if two distinct systems, each with their own history space, are considered as a single composite system with a composite history space.

Diósi considers two systems A and B, each of which has a history space of its own which fulfils the consistency criterion. He considers the system AB, which is a composite of A and B. Importantly, his analysis specifies that the history spaces of the two systems should be un-entangled, or in terms of system density matrices:

$$\rho^{AB} = \rho^{A} \otimes \rho^{B}$$

He then argues that there is no guarantee that this composite system will itself have a history space which fulfils the consistency criterion. His point becomes clear by looking at the relationship between the decoherence functionals of the relevant systems. Given the independence of the density matrices of the two systems mentioned above, this factorises to become:

$$D^{AB}(\alpha'\beta', \alpha\beta) = D^{A}(\alpha', \alpha) D^{B}(\beta', \beta)$$

Where α and α' are two distinct histories of the system A, and similarly for system B.

It is easy to see that, if the decoherence functionals of systems A and B are imaginary (as required by the consistency criterion), there is no guarantee that the decoherence functional of the composite system will also be imaginary. Just as easily, it can be seen that, if the decoherence functionals of A and B are both equal to zero (as required by the medium decoherence criterion), then the decoherence functional of the composite system will also equal zero.

Diósi takes it to be an appropriate expectation of any criterion seeking to pick out individual quasi-classical histories that, if individual histories are picked out for a particular system, then a composition of the history space of that system and the history space of a second which also fulfils the same criterion should itself form a history space which fulfils that criterion. Having succinctly shown that this is not the case for consistency, he moves on. I think the test of composition is an interesting and revealing one, and will now seek to further examine this counterintuitive result.

At first glance it is certainly easy to see why Diósi sees this as a problem with using the consistency criterion to decide when probabilities can be applied. It is counterintuitive, to say the least, to think that probabilities might be applicable to a description of the system A, but not be compatible with a description of the conjunction of that system and another. More than this, it seems bizarre to find that, while for a particular quantum system there might be no interference phenomena, for a conjunction of that system and another to which it is unentangled, interference phenomena might suddenly return.

The first thing to notice here is that a failure of consistency in the larger history space would not imply the presence of interference phenomena in the physical system that made up the smaller history spaces. That is, the histories that might possibly interfere with one another, are histories that are distinct in their descriptions of both system A and system B. If we instead look at pairs of histories that differ only in the dynamics of one system, we get:

$$D^{AB}(\alpha'\beta, \alpha\beta) = D^{A}(\alpha', \alpha)D^{B}(\beta, \beta) \in \mathbb{I}$$

$$D^{AB}(\alpha\beta', \alpha\beta) = D^{A}(\alpha, \alpha)D^{B}(\beta', \beta) \in \mathbb{I}$$

That is, as the decoherence functional of a history with itself is simply the Born rule probability of that history, and so always a real number, the decoherence functional of the composite system will still be imaginary. The effect of the second system will only change the magnitude of this term.

Consequently, the interference between histories that results in the composite history no longer being consistent is entirely down to interference between histories separated in ways that could not be represented in either of the single system history spaces. This goes some way to accounting for the seemingly bizarre possible appearance of interference terms in the composite history space.

Nevertheless, it does seem concerning to find that a criterion under consideration for use in deciding when probabilities can be assigned to states within a system should be so dependent on the choice of system. Diósi does, therefore, seem to have identified an apparently serious issue.

There is, however, something a little strange about the reasoning which underlies Diósi's analysis. Consistency is intended to be a criterion for when the physical phenomenon of decoherence has taken place. That is, when the state of the system has become entangled to its environment by particular degrees of freedom such that, under operations affecting those degrees of freedom of the system, no interference phenomena will be displayed. Nothing in Diósi's analysis represents or models the effect of system A or B being entangled by any degrees of freedom to their environment. In other words, nothing in the analysis specifies that decoherence (in the sense of the physical phenomenon) has taken place. The fact that the history spaces of A and B have been chosen such that they do not display interference phenomena (and so fulfil the consistency criterion) does not guarantee anything about the robustness of these dynamics. It is only when records are encoded outside of those systems that robust quasi-classical dynamics are to be expected.

This is certainly a disadvantage of the consistency criterion. It is overly generous. It applies not only in cases when encoding of the system state outside of the system leads to robust quasi-classicality, but also when a system's phases simply happen by chance to align in such a way that no interference phenomena are displayed. As a result, the criterion lets in systems whose consistent dynamics are very easily disrupted.

Indeed, this lack of stability is precisely the point which Diósi makes in his discussion of what he describes as the test of dynamic stability.

3.5.2 The Test of Dynamic Stability

This test involves examining the effect of introducing a briefly applied additional potential to the systems dynamics. Diósi shows that this brief perturbation will introduce a phase shift to the decoherence functional which will, quite possibly, lead to the resulting history space of the system failing the consistency criterion.

Clearly then, the consistency criterion admits history spaces of systems which are not correlated to their environments in the way we would expect for a system displaying the physical phenomenon of decoherence. As this entanglement to the environment is what gives rise to the stability of quasi-classical dynamics, and Diósi does nothing to represent this entanglement in his analysis, it is unsurprising to find that the history spaces he considers will not always display robustly classical dynamics despite fulfilling the consistency criterion.

What is more surprising, given that Diósi does nothing to represent this external entanglement, is that the medium decoherence criterion is sufficient to ensure stability in both cases. That is, a system which had no entanglement to its external environment, but which fulfilled the medium decoherence criterion, would display stable quasi-classical dynamics, both under composition with an external system, and an instantaneous perturbation to the Hamiltonian. It is reasonable to wonder, therefore, what it is about fulfilling the medium decoherence requirement which gives rise to this remarkable stability in the absence of any encoding of the system state onto the environment.

The answer, surprisingly, is that a system's fulfilling the medium decoherence criterion has absolutely nothing to do with that system being decohered with respect to its environment. The absence of interference phenomena within systems that fulfil the medium decoherence criterion is not due to the state of those systems being encoded outside of the system, but due to all past states of the system being encoded within the system itself.

To understand this, consider a particle passing through a double slit apparatus. If no record is made within the particles environment as to which slit it passes through, then interference will occur as histories in which the particle passed through the first slit and histories in which it passed through the second slit recombine, and phase relations between the different histories play out. In this case, the history space of the particle would fail to meet either the consistency or medium decoherence requirement.

If instead, a record of the slit the particle passed through became encoded somehow within its environment, then the phase relations of the histories in which the particle passed through each slit would be lost, and when the histories recombined their amplitudes would add linearly with no interference. This is a classic example of the physical phenomenon of decoherence, but the history space of the particle would not fulfil the medium decoherence criterion. The histories that make up the particle's history space do recombine after previously having separated. As such, while the history space of the particle fulfils the consistency criterion, it does not fulfil the medium decoherence criterion.

To see a system in which the medium decoherence criterion is fulfilled, you would have to consider one in which, having passed through the two slits, the particles continued on entirely different trajectories depending on which slit they went through. This would mean that the particle position projectors for the histories which went through the first slit would never agree with that of histories that went through the second slit. The history space for the particle passing through the apparatus would then have a branching structure and fulfil the medium decoherence criterion. As histories would never recombine, there would be no interference, whether or not the slot that the particle passed through was ever encoded outside of the system.

Of course, in the case where the slit which the particle passed through was recorded in the environment, there will generally be a system whose history space fulfils the medium decoherence criterion, and which includes quasi-classical particle trajectories for the particle with which we are concerned. The system will include both the particle’s positions, and at least 1 degree of freedom within the environment entangled to the slit through which the particle passed.

Medium decoherence, then, clearly does have a strong connection to the robust absence of interference phenomena. It is important to understand, however, that a local system displaying quasi-classical dynamics does not have any certainty of fulfilling this criterion. Rather, it fulfils the weaker consistency criterion. Medium decoherence can only be fulfilled by a system large enough that a present projector for a history of the system can encode a record of every quantum probabilistic event throughout the history of that system. Medium decoherence is therefore a very strong (arguably too strong) requirement, and chapter 6 will look in more detail at the question of whether it is really reasonable to suppose that this criterion will be fulfilled in the kinds of cases we meet in everyday life.

Consistency, on the other hand, is a significantly weaker criterion. It is weak enough to include histories which have nothing to do with the physical phenomenon of decoherence. In such cases, the property of the history space fulfilling the consistency criterion may well be very tenuous and easily disrupted. On the other hand, it can reasonably be applied to local systems that we can study rather than the extremely large systems needed to allow medium decoherence. Consequently, when we speak of a system as having decohered with respect to its environment, we typically mean that it fulfils the consistency criterion rather than the medium decoherence criterion.

The fact that the robustness of consistency, within the history space of a system which we might typically be interested in, depends on some far larger and more ill-defined system fulfilling the stronger medium decoherence criterion, poses an epistemic challenge which shall be a central topic of this thesis. That is, do we really have reason to suppose that systems with which we daily interact are part of a system which fulfils the medium decoherence requirement, and encodes records of everything that has happened over the histories of our system of interest? If so, what are these reasons? If not, what are the metaphysical consequences if the systems that make up the world around us cannot be relied upon to obey a probability sum rule?

Before moving on to look at these questions and their relevance to particular interpretations, there is one more connection which must be considered if we are to understand how different criteria of decoherence relate to one another. The next section will return to the diagonalisation of a system’s reduced density matrix (RDM), and examine how this connects to the two criteria on history spaces discussed in this section.

3.6 Connection to the Diagonalization of the RDM

The previous chapter used the RDM diagonalization conception of decoherence, which is used by authors such as Schlosshauer and Zurek. This conception connects very simply and clearly to the linear formalism of quantum mechanics, and has received a great deal of empirical support. This chapter has developed conceptions of decoherence as criteria applied to history spaces. I hope it has become clear that this approach is inspired by and heavily connected to the conception of environment induced decoherence presented at the start. Nonetheless, it is reasonable to wonder just how these conceptions of decoherence relate to the empirically and mathematically confirmed reduced density matrix formulation.

Unfortunately, the characteristics of these criteria, and the approaches which underlie them, are sufficiently different to one another that it is not particularly easy to relate them to one another. One reason for this is that RDM diagonalisation is an indicator of decoherence centred around a particular measurement and particular set of variables which become entangled to their environment. Consistency and medium decoherence, on the other hand, are constraints over a history space. Particular variables for particular measurements are an important characteristic of the histories which form the space, but connecting these variables through to a decoherence functional is a far from trivial exercise.

The second difficulty is that diagonalisation of a reduced density matrix is built on a fundamentally different picture of quantum mechanics to that which underlies histories approaches. In order for a reduced density matrix to become diagonalised as a consequence of interaction, the density matrix must be considered as dynamic. This clearly places RDM diagonalization approaches within the Schrödinger picture, in which density matrices are dynamic and measurement operators are static. Histories approaches on the other hand are built very much on the Heisenberg picture. The wavefunction (or density matrix) remains fixed, all dynamic evolution of the system is built into the projection operators. It is these dynamic sequences of operators which form the basis of a history space. Consequently, a formidable translation task is required in order to relate these conceptions of decoherence directly.

Perhaps for this reason, the decoherence literature does not seem to offer any detailed general explanation of how these conceptions of decoherence relate to one another[9]. I hope in the future there will be more work done to establish precise mathematical relations between these conceptions. For now though, I shall limit myself to a more qualitative comparison without attempting to rigorously explore their mathematical connections.

The first thing to note is that diagonalization of the reduced density matrix is clearly far closer to consistency than it is to medium decoherence. They both relate to interference loss in local systems due to environmental interaction rather than concerning the vast systems needed for medium decoherence.

They are both overgenerous in very similar ways. Consistency admits history spaces in which there is no environmental decoherence to remove interference phenomena, but instead the basis of projectors that form the histories simply happens to display phase alignments such that no interference is observed. In the same way, for the reduced density matrix (or pure density matrix) of a system, there is always a basis in which the matrix is diagonalized. This will be the case regardless of whether any encoding of a system's degrees of freedom onto its environment has taken place. Consequently, both RDM diagonalization and consistency will admit cases in which no environmentally induced decoherence has occurred.

The reason for the very close parallels is that both consistency and RDM diagonalisation are criteria built around the loss of interference phenomena. For RDM diagonalisation this means a particular set of degrees of freedom which cease to display interference phenomena, under operations affecting them. Consistency, on the other hand, concerns not only one set of degrees of freedom but a sequence of projectors over time which must not display interference phenomena. It seems, then, that consistency of the history space of the system will entail diagonalization of

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[9] Kiefer 2003 pp. 247-251 does offer some analysis of the connections between history space consistency and the diagonalisation of a system's reduced density matrix, but it is rather brief, and based on non-trivial assumptions about the nature of system-environment interaction.

the reduced density matrix of the system at a time k in cases where the basis of the reduced density matrix matches the projectors of the system’s histories at time k.

Diagonalization of the system’s reduced density matrix, however, does not entail the consistency of its history space in the same basis. The reason is that RDM diagonalization concerns a particular set of variables at a particular time, and is generally used in relation to a particular event which encoded degrees of freedom of the system into the environment. A history space fulfilling the consistency criterion, on the other hand, depends on the continual absence of interference phenomena over time and successive branching events. For consistency brought about by decoherence, environmental encoding is necessary after every quantum probabilistic event which occurs throughout the system’s history space.

RDM diagonalization is therefore closely tied to the consistency criterion. It is, however, a weaker criterion related to a particular set of degrees of freedom of a system, rather than a general constraint on a system’s entire history space. Given that it is diagonalisation of reduced density matrices which can be empirically tested in experimental tests of decoherence, it is interesting to find that this criterion is not only weaker than the medium decoherence criterion which has come to dominate in the literature, but is also weaker even than the consistency criterion. As already mentioned, chapter 6 will examine in far more detail the troubling gap between empirically verified forms of decoherence, and the far stronger forms being used to do explanatory and metaphysical work within modern interpretations of quantum mechanics.

For now though, the next section will summarise the terminology used in discussions of decoherence. I hope that by providing these (reasonably) clear definitions I can maintain (some) clarity between different conceptions of decoherence throughout this thesis.

3.7 Summary

- **History:** A history is an ordered series of projection operators of a systems state, which correspond to successive times. Generally, the sequence of projection operators are assumed to be related to one another by unitary time evolution operators which are a product of the system's Hamiltonian (at least in the absence of history branching). From this point on, I will be making this assumption about histories I refer to, unless I explicitly state otherwise.
- **Atomic histories:** These are any set of histories which form a complete basis of states in which to decompose the system’s wavefunction. For a quasi-classical history space, this will mean a basis which specifies a complete set of particle states (e.g. position) for the entire system at every time.
- **Non-atomic histories:** Most commonly discussed quasi-classical histories do not specify a system state of affairs with absolute precision at every time. These histories are coarse-grained, and usually only specify a macroscopic state of the system. Consequently, they describe a range of possible states of affairs and do not form a complete basis in terms of which a system's wavefunction can be decomposed. These histories are only partial decompositions of the system state, and are themselves composed of a number of atomic histories.
- **History space:** A history space is the space formed of the product of all the projection operators of a set of atomic histories, applied to the time dependent wavefunction of the system. These history state projectors form basis vectors of this space.
- **Medium decoherence criterion:** A history space fulfils the medium decoherence criterion if for any two histories at any time t: either the projectors of the two histories at every time prior to t agree; or, the projectors at time t are orthogonal to one another. In effect, this means that histories can branch, separating from one another

as time goes on, but cannot merge coming to agree on their states after previously disagreeing. This criterion ensures a branching structure of histories, unlike the consistency criterion. In terms of the decoherence functional this criterion is:

$$D(\alpha, \beta) = 0$$

- **Consistency criterion:** A history space will fulfil the consistency criterion if and only if there is no interference between the atomic histories which make up the space. Medium decoherence entails consistency, but unlike medium decoherence, consistency does not guarantee that a history space will have a branching structure. The absence of interference phenomena means that the Born rule probabilities of histories within the space will obey the usual probability sum rules seen in classical probability, without any influence from the relative phases of histories. This condition is generally brought about in a history space as a consequence of the system decohering with respect to its environment in an appropriate basis. The consistency criterion may also be fulfilled by history spaces for which this is not the case, however. In terms of the decoherence functional this criterion is:

$$D(\alpha, \beta) \in \mathbb{I}$$

- **Reduced Density Matrix diagonalization:** The reduced density matrix of a system entangled to its environment by some degrees of freedom will, in a basis of those degrees of freedom, be diagonalized (off-diagonal elements will go to zero). This means that, under operations affecting those degrees of freedom, the system will behave as if it is in a definitive but unknown state, rather than a superposition (i.e. there will be no interference phenomena).

  There are also cases in which the density matrix of the system that is not appropriately entangled to its environment still has a diagonal form. In other words, the density matrix is diagonalised without decoherence. In these cases of diagonalisation interference phenomena will still be absent, but the effect is likely to be very easily disrupted.

- **Decoherence:** Decoherence is the experimentally observed physical process whereby a quantum system, entangled by a particular variable to its environment, will behave under operations affecting only that system as if it were in a definite, but unknown, eigenstate of that variable. This is the phenomenon which underlies both reduced density matrix diagonalisation, and the consistency criterion. As noted however, both of these criteria are too generous, and can be fulfilled without the physical phenomenon of decoherence.

3.8 Conclusion

The purpose of this chapter has been to introduce two technical criteria of decoherence which appear within the philosophical literature, and examine the significance of these two criteria. I have argued that both are stronger than the reduced density matrix criterion presented in the previous chapter, and introduced the challenge of how we can know that a system fulfils these stronger criteria.

These criteria are based on the notions of history and history space. I have noted that the literature concerning histories is generally concerned with quasi-classical histories in which quasi-classical entities are easily identifiable and display quasi-classical dynamics. Concerns about this kind of presupposition of classicality will be discussed in the next chapter. Aside from this presupposition however, this conception of history spaces and the dynamics which they display stem directly from the linear Schrödinger dynamics. Consequently, like the conception of decoherence presented in the last chapter, it will be applicable in any interpretation which accepts these Schrödinger dynamics as representational and universal. There seems however to be a significant difference in how these histories are to be understood on different Everettian interpretations.

As I noted at the start of this chapter, many histories interpretations depart from many worlds interpretations in that they do not seek a single quasi-classical history space to invest with strong metaphysical significance. Instead of a single branching structure of Everettian worlds, they allow that a physicist may choose any history space which proves useful provided that the history space in question fulfils the relevant criterion (medium decoherence or consistency). They maintain that no history space which fulfils the relevant criterion is more fundamental than any other, and that different history spaces can be used whenever they are useful provided that incompatible histories are never combined into a single quantum description.

The primary focus of this thesis is a many worlds formulation of Everettian quantum mechanics. Before moving on however, I will briefly consider how the challenge of justifying the belief that we occupy a quasi-classical history within a history space which fulfils the medium decoherence criterion (i.e. an Everettian world) relates to these many histories interpretations.

It seems that decoherent histories approaches will be just as committed to showing that there is a basis of quasi-classical histories which fulfils the medium decoherence criterion as are many worlds Everettians. They seem to rely on the existence of such history spaces, though they are clearly interested in the possibility that there might be more than one such history space.

At first glance, it seems that consistent histories interpretations might be less committed to there being any history space which fulfils the medium decoherence criterion. To an extent this is indeed true. However, it is important to bear in mind that in the physical phenomenon of decoherence it is the encoding of a systems history within the environment which lead to the suppression of interference effects necessary for the history of the local system to fulfil the consistency criterion. Consequently, the most plausible justification for why history spaces of systems with which we might be concerned might fulfil the consistency criterion over long time periods would be because they are part of a larger history space which fulfils the medium decoherence criterion. This is not the only way in which local systems could fulfil this criterion, but alternatives rely on the phases of previously separated histories always agreeing when systems recombine without any explanation of why this should be the case.

It seems then, that though many histories interpretations may not all be directly committed to the existence of a single quasi-classical history space which fulfils the medium decoherence assumption, they still ultimately seem to be invested in there being at least one such history space.

The next two chapters will turn to examine specific technical issues which affect many histories interpretations which do not presuppose the Born rule for quantum probabilities. This is important to understanding the Everettian many worlds project, and the role of decoherence within it. Chapter 6 will return to history spaces, and examine possible means of justifying the assumption that we occupy a history within a history space which fulfils the medium decoherence criterion.

4 Circularity and the Born Rule

In the previous chapter, I introduced a number of ways in which the physical phenomenon of decoherence, and mathematical criteria which are to a greater or lesser extent based upon it, are used in particular interpretations to solve certain aspects of the quantum measurement problem. There are a number of standard objections to using decoherence in these ways. This chapter and the next chapter will introduce these objections. In this chapter I will focus on the objections presented by Ruth Kastner in Kastner 2014.

Some of these arguments are not relevant to the interpretations of decoherence with which I am primarily concerned in this thesis, but to Zurek’s quantum Darwinism project. Although, as I will argue, other interpretations seem able to avoid these problems, I believe it is important to understand Zurek's project, and the problems which it faces, in order to understand why other interpretations have the features which they do.

I will argue that for these problems as presented by Kastner, at least David Wallace's form of the Everett interpretation is able to offer satisfactory responses to these problems. However, I will also identify some places where, while the most natural interpretation of Kastner can be answered by Wallace, her underlying concern connects to deeper and more important problems for modern decoherence-based interpretations.

4.1 Circularity

In chapter 2, I presented Schlosshauer’s analysis of the role of decoherence in the quantum measurement problem. Schlosshauer shows how decoherence plays a major role in resolving a number of significant difficulties. He makes it clear, however, that decoherence alone is not sufficient to recover the classical dynamics with which we are familiar.

Crucially, as we saw in chapter 2, it is not able in itself to resolve the specific problem of outcomes. That is, while decoherence may be able to account for our belief that quantum measurements have determinate outcomes, it does not seem able to account for why we come to believe that a measurement had a particular determinate outcome, rather than an alternative. For this reason, Schlosshauer accepts that some further interpretation is needed, such as that offered in Bohmian or Everettian accounts.

Previously, however, there have been a number of attempts to suggest that decoherence might be able to allow a direct derivation of quasi-classical structures and their dynamics from the linear quantum formalism. If this were possible, it would seem to do away with the need for further interpretation, as the quantum to classical transition could be replaced with a view of classicality as a simple deductive consequence of fundamental quantum dynamics.

Zurek 1991 introduced a position he referred to as quantum Darwinism, which attempted to do exactly this. He sought to show that it was possible to derive the Born rule probabilities of quantum measurements without any presupposition of classical states, or other addition to the linear quantum dynamics. Though this position, as originally presented, has been substantially discredited (see Adler 2003), and now receives relatively little support, it is important to grasp it in order to understand the circularity objections still faced by decoherence based arguments. Furthermore, as we shall see later in this chapter, the Born rule derivation which Zurek provides remains popular with advocates of Everettian interpretations, and is used by Sebens and Carrol (2018) as a key part of their analysis of self-locating uncertainty.

4.2 Zurek's Born Rule Derivation

Premises

Zurek relies on seven premises. Four premises (0-3) are taken from the basic axioms of the standard quantum mechanical formalism, and three additional premises (i-iii) some of which are controversial.

Zurek's four premises taken from quantum mechanical axioms are (Zurek, 2014, pp. 1):

0) The "state of [a] composite system is a vector in the tensor product of constituent Hilbert spaces". That is $\mathcal{H}_{SA} = \mathcal{H}_S \otimes \mathcal{H}_A$
1) "States of quantum systems correspond to normalized vectors in a (complex) Hilbert space."
2) "Evolutions [of quantum systems] are unitary, $|S_t\rangle = U_t|S_0\rangle$" Zurek points out that this also implies the linearity of such evolutions.
3) "Immediate repetition of the measurement yields the same outcome" for both measurements.

None of these premises are particularly controversial.

Zurek adds three premises which he regards as facts (Zurek, 2014, pp. 3):

i) Locality: "A unitary [operator] must act on the system to change its state." And a system only changes its state when acted upon by such an operator.
ii) "[The] state of the system is all there is to predict measurement outcomes."
iii) "A composite state determines the state of subsystems (so [the] local state of [a subsystem] s is restored when the state of the whole $s\varepsilon$ is restored)."

None of these (except possibly for (ii)) is uncontroversial.

With these premises in hand, Zurek sets out to derive the Born rule by showing, first, that there is nothing other than coefficient amplitudes in the wavefunction which could be used to indicate probability, and then showing that it is possible to choose a decompositional basis in which different histories are broken down into sub histories of equal amplitude. He then uses the assumption of locality, and a state swap operator, to show that such sets must be equally probable, and thence derives the Born rule.

4.3 Entanglement-Assisted Invariance (Envariance)

In order to rule out other possible aspects of the wavefunction, which might influence or dictate a history's probability, Zurek begins by setting out to show that for a system entangled with its environment, the phases of coefficients must be irrelevant to the probabilities of measurement outcomes. This invariance to phase of entangled systems Zurek terms *Envariance*, and this is one of the major consequences of the physical process of decoherence.

The care taken in this step shows that Zurek is in many ways extremely rigorous in his methodology. The possibility of the direct physical influence of phase amplitudes on probability is neglected in many other attempts to derive the Born rule, such as that found in Wallace 2012.

Zurek considers a two-state spin particle s which has become entangled with its environment ε such that the total system is described by the state:

$$|\psi_{s\varepsilon}\rangle = \alpha e^{i\theta}|\uparrow\rangle|\varepsilon_{\uparrow}\rangle + \beta e^{i\phi}|\downarrow\rangle|\varepsilon_{\downarrow}\rangle$$

(Where α, and β are real numbers, and the phases have been taken into the complex exponential terms.)

Importantly, once these systems have become entangled, they are then separated and decoupled such that it is possible to perform operations on them individually.

Zurek then defines a phase shift operator:

$$u_s^{\varphi} = |\uparrow\rangle\langle\uparrow| + e^{i\phi}|\downarrow\rangle\langle\downarrow|$$

Which acts on s to shift the phase of $|\downarrow\rangle$ by $e^{i\varphi}$.

This operator will act on the total system $|\psi_{s\varepsilon}\rangle$ as follows:

$$u_s^{\varphi}|\psi_{s\varepsilon}\rangle = \alpha e^{i\theta}|\uparrow\rangle|\varepsilon_{\uparrow}\rangle + \beta e^{i(\phi+\varphi)}|\downarrow\rangle|\varepsilon_{\downarrow}\rangle$$

Now consider another similar operator which acts on the environment:

$$u_{\varepsilon}^{-\varphi} = |\varepsilon_{\uparrow}\rangle\langle\varepsilon_{\uparrow}| + e^{-i\varphi}|\varepsilon_{\downarrow}\rangle\langle\varepsilon_{\downarrow}|$$

When this operator is also applied to the total system, the system state is restored:

$$\begin{aligned} u_{\varepsilon}^{-\varphi}\left(u_s^{\varphi}|\psi_{s\varepsilon}\rangle\right) &= \alpha e^{i\theta}|\uparrow\rangle|\varepsilon_{\uparrow}\rangle + \beta e^{i(\phi+\varphi-\varphi)}|\downarrow\rangle|\varepsilon_{\downarrow}\rangle \\ &= \alpha e^{i\theta}|\uparrow\rangle|\varepsilon_{\uparrow}\rangle + \beta e^{i\phi}|\downarrow\rangle|\varepsilon_{\downarrow}\rangle \end{aligned}$$

As the systems were decoupled, and locality has been taken as a premise (i), it is impossible that $u_{\varepsilon}^{-\varphi}$, could in any way affect the state of s. Nonetheless, by premise (iii), as the state of the total system has been restored, the state of the subsystem is restored also. As the physical state of s cannot have changed with the action of operator $u_{\varepsilon}^{-\varphi}$, and must following its action be identical to the state prior to the action of either operator, phase shifts can have no physical significance for the states of perfectly entangled systems. And, as these states are by premise (ii) the only means by which we can predict measurement outcomes, phases should not affect the probabilities we assign to possible outcomes in quantum measurements.

Zurek therefore concludes that amplitudes must hold the key to obtaining quantum probabilities from wavefunctions. The wavefunction can be thought of as being made up of basis vectors, which correspond to physical outcome states, amplitudes, and phases. As Zurek has shown that phase amplitudes have no direct influence on probability, he now turns his attention to amplitudes.

The reader may be concerned that the wavefunction’s basis vectors seemed to go unconsidered here. In essence, Zurek relies on decoherence here too. This is just another instance of the problem of the preferred basis, and Zurek regards this as being solved by precisely the same decoherence based reasoning presented in chapter 2.

4.4 Born's Rule for Equiprobable States

Zurek first derives the Born rule for pairs of states with equal amplitudes, before going on to extend this to a more general derivation. To understand Zurek's proof of Born's rule for equally weighted pairs of states, consider the system $|\psi_{s\varepsilon}\rangle$ in the case where $\alpha = \beta = \frac{1}{\sqrt{2}}$:

$$|\psi_{s\varepsilon}\rangle = \frac{1}{\sqrt{2}} e^{i\theta} |\uparrow\rangle |\varepsilon_{\uparrow}\rangle + \frac{1}{\sqrt{2}} e^{i\phi} |\downarrow\rangle |\varepsilon_{\downarrow}\rangle$$

As in the previous section, Zurek focuses on the case in which the subsystems s and ε are decoupled and spatially separated. Zurek then considers the effect of operators which swap the states of the subsystems. The effect of the swap operator is to exchange system states; for example, an operator which, if applied to a spin up state, changed it to spin down, and if applied to spin down changed it to spin up, would be a simple swap operator. Locality once again ensures that operations performed on one subsystem do not influence the state of the other.

First, consider a swap operation applied to the state of s:

$$|\psi_{s\varepsilon}\rangle \Rightarrow \frac{1}{\sqrt{2}} e^{i\theta} |\downarrow\rangle |\varepsilon_{\uparrow}\rangle + \frac{1}{\sqrt{2}} e^{i\phi} |\uparrow\rangle |\varepsilon_{\downarrow}\rangle$$

Linearity demands that the probabilities of $|\uparrow\rangle$, and $|\downarrow\rangle$, are therefore exchanged, so that: $P_{\uparrow} = P_{\downarrow 0}$, and $P_{\downarrow} = P_{\uparrow 0}$.

Now consider applying a similar swap operation to the state of ε:

$$|\psi_{s\varepsilon}\rangle \Rightarrow \frac{1}{\sqrt{2}} e^{i\theta} |\downarrow\rangle |\varepsilon_{\downarrow}\rangle + \frac{1}{\sqrt{2}} e^{i\phi} |\uparrow\rangle |\varepsilon_{\uparrow}\rangle$$

The probabilities of $|\varepsilon_{\uparrow}\rangle$, and $|\varepsilon_{\downarrow}\rangle$, are therefore exchanged, so that: $P_{\varepsilon_{\uparrow}} = P_{\varepsilon_{\uparrow}0}$, and $P_{\varepsilon_{\downarrow}} = P_{\varepsilon_{\downarrow}0}$.

Now, however, in effect the only swap that has taken place is a swap between θ, and ϕ. As it was demonstrated in the previous section that the phases these terms determine can have no effect on the probabilities to be assigned to possible outcomes, we can therefore perform the phase shifts without influencing the probabilities of subsystems states. As such, we can perform the phase shift necessary to swap θ and ϕ, and restore $|\psi_{S\varepsilon}\rangle$ to its original state without influencing any probability.

The state of the total system having been restored by (ii) and (iii), we conclude that the probabilities appropriate to assign to possible outcomes are also restored, just as in the previous section.

We can therefore conclude:

$$P_{\uparrow} = P_{\downarrow 0} = P_{\downarrow} = P_{\uparrow 0} = \frac{1}{2}$$

$$P_{\varepsilon_{\uparrow}} = P_{\varepsilon_{\uparrow}0} = P_{\varepsilon_{\downarrow}} = P_{\varepsilon_{\downarrow}0} = \frac{1}{2}$$

4.5 Born's Rule for Uneven Superpositions

Zurek expands his derivation to the case of uneven superpositions by changing the basis states so as to produce many equiprobable states (Zurek 2014, pp. 10).

This can be done remarkably simply. Let us consider our two-state particle interacting with a measurement apparatus. The apparatus has two macroscopic states: $|A_{\uparrow}\rangle$, and $|A_{\downarrow}\rangle$. In order to fine-grain these states we define:

$$|A_{\uparrow}\rangle = \sum_{k=1}^{\mu} |a_k\rangle / \sqrt{\mu}$$

$$|A_\downarrow\rangle = \sum_{k=\mu+1}^{\mu+\nu} |a_k\rangle/\sqrt{\nu}$$

Where $\frac{\alpha^2}{\beta^2} = \frac{\mu}{\nu}$, where μ, and ν are natural numbers

The state of the system (neglecting normalisation) is therefore:

$$|\psi_{sA}\rangle \propto \sqrt{\mu} e^{i\theta} |\uparrow\rangle |A_\uparrow\rangle + \sqrt{\nu} e^{i\phi} |\downarrow\rangle |A_\downarrow\rangle = e^{i\theta} \sum_{k=1}^{\mu} |\uparrow\rangle |a_k\rangle + e^{i\phi} \sum_{k=\mu+1}^{\mu+\nu} |\downarrow\rangle |a_k\rangle$$

Which during the process of measurement will become entangled to many states of the environment:

$$|\psi_{sA\varepsilon}\rangle = e^{i\theta} \sum_{k=1}^{\mu} |\uparrow a_k\rangle |e_k\rangle + e^{i\phi} \sum_{k=\mu+1}^{\mu+\nu} |\downarrow a_k\rangle |e_k\rangle$$

By precisely the same reasoning as in the previous section these k states must be equiprobable. The probability of $|\uparrow a_k\rangle$ is therefore:[10]

$$P_\uparrow = \frac{\mu}{\mu+\nu} = \alpha^2$$

$$P_\downarrow = \frac{\nu}{\mu+\nu} = \beta^2$$

And this is precisely the Born rule for quantum probabilities.

4.6 Circularity in Quantum Darwinism

Zurek's argument has for a long time now faced accusations of circularity. A significant reason for this is that the original form of this argument, given by Zurek 1991, presented system states and the process of decoherence in terms of reduced density matrices. Using these matrices at all seems to implicitly presuppose the Born rule, essentially because they rely on the possibility of averaging over possible states of the environment, weighting

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them according to their Born rule amplitudes. Thus, Zurek’s putative derivation of the Born rule originally relied on its implicit presupposition.

Zurek 2005 himself identifies this problem and since then he has reformulated his derivation to the form shown here. This did not, however, put an end to accusations of circularity for his project. The principal reason for this is that Zurek still relies heavily on environment induced decoherence, and reduced density matrices remain the most common means of representing this process. Consequently it is reasonable to wonder if Zurek’s reliance on decoherence introduces a vicious circularity to his reasoning.

Ruth Kastner has been particularly vocal in arguing that his derivation still rests on implicit presuppositions of classicality in ways which undermine his analysis. Though she presents this primarily as a response to Zurek, the arguments appear to extend to the more general project of using decoherence to recover aspects of the classical macroscopic world.

Kastner 2014 makes a range of interrelated points which I think can be summarised under two headings:

1. Zurek’s use of decoherence relies on a particular choice of system environment split.
2. The theoretical models which underlie decoherence theory rely on the assumption of an environment made up of randomised phase correlations. This is not a plausible assumption in the absence of some form of collapse postulate.

I will examine these points in turn. Both of them are on a basic level accurate and important points. The first, I have already noted in chapter 2. Quantum Darwinism as presented by Zurek does not currently offer a means by which to respond to either point. Other interpretations however do seem to have readily available (albeit partial) answers. In a sense, it would be very easy to apply these answers to Zurek's quantum Darwinism.

It is tempting to offer these as friendly amendments in order to develop and defend Zurek’s position. I will not do so, however. The reason is that these solutions all involve adding either additional physical dynamics, or additional interpretational metaphysics, and doing either of these things would directly contradict the spirit of Zurek’s quantum Darwinism, as I understand it. Zurek’s aim was to show that the linear quantum formalism, and the decoherence theory that derives from it, were, in and of themselves, sufficient to recover all the essentials of the classical world with which we are generally familiar. This would effectively dissolve the quantum measurement problem without any need for any of the additions of the major quantum interpretational projects. For this reason, I will simply show why I believe Kastner's objections make Zurek’s project untenable, and then turn to look at the responses that can be offered within Wallace's Everettian quantum mechanics.

4.7 Objection 1: Decoherence Relies on a Particular Choice of System Environment Split

The primary purpose, for which Zurek relies on decoherence, is the selection of a set of states naturally preferred within the process of measurement, just as I set out in the previous chapter. At the same time, I also noted that using decoherence to obtain a preferred basis in this way relies on a particular choice of system and environment.

If we consider the degrees of freedom of a chair as a system, and form a reduced density matrix to represent that system, then the heavy entanglements of elements of the chair to its environment in a position basis, will ensure that the reduced density matrix is approximately diagonalised in that basis, so that the chair’s shape and position can act as an effectively preferred basis. However, if instead we consider a system made up of some other non-classical set of degrees of freedom, such as the spin states of a set of electrons in the chair and its environment, then the reduced density matrix formed will not be diagonalised in a classically recognisable basis such as position, but in some other profoundly nonclassical basis.

Kastner’s concern is that choosing such classical splits illicitly smuggles in what decoherence was meant to prove, by assuming a very particular division of system and environment, which she believes to be arbitrary. The natural response is that there is nothing arbitrary about seeking descriptions of those objects with which we are classically familiar rather than others. After all, all scientific theories have a particular subject matter which is chosen in accordance with our interests, and the things which we wish to model and understand. No one expects, for example, that our ideal gas laws, or the theory underlying them, should be able to derive theoretically the existence of gases, such as noble gases, to which they (approximately) apply. Rather, we are interested in these laws because we have found empirically that they are successful at modelling particular types of system in particular circumstances. It might be thought that, in precisely the same way, modelling a system such as a chair in decoherence theory is nothing more than investigating the dynamics of a system of particular interest to us. Kastner rejects this suggestion.

Her reasoning is that treating a system such as a chair as our modelled system can only be a non-arbitrary choice if there is a high degree of separation and independence between the chair and the rest of its environment, making the chair an approximately isolated system. But, given the very high degree of entanglement of all macroscopic objects, this is not ever going to be a realistic assumption. Given the incredible ease with which any two systems rapidly become entangled, the only system for which it is ever going to be a reasonable assumption is that of the universe as a whole. And as that system has no environment, environment induced decoherence theory could not be applied to it in any case.

Though it might be possible to argue that the observed dynamics of chairs mark them out as special in some way, she believes that even this could not serve to avoid the circularity of using such a division. “After all, the whole point of the ‘einselection’ program is to demonstrate that the *observed* divisions arise naturally from within the theory. To assume the divisions we *already* see in the world and then demonstrate that, based on those assumed divisions, the divisions arise ‘naturally,’ is clearly circular.” (Kastner 2014, pp. 10, original emphasis).

I believe that Kastner's argument is successful against very strong forms of decoherence-based reasoning. If decoherence is truly being used in an attempt to show that quasi-classical entities and their dynamics are directly and objectively derivable from the linear quantum dynamics, then Kastner is right to reject the program as viciously circular. As her criticism is primarily directed at Zurek, it may be an important and pertinent argument. It is difficult to judge what precisely Zurek regards his derivation as showing. The crucial question is whether his einselection program is indeed intended to demonstrate *that* the observed divisions of the classical world arise naturally from within quantum theory, or rather to demonstrate *how* the observed divisions of the classical world arise naturally from within quantum theory.

Which of these Zurek seeks to achieve remains rather unclear. It seems likely that he doesn't attribute any particular significance to the distinction and has no settled position on the question.

Seeking to demonstrate *that* the observed divisions of the classical world arise naturally from within quantum theory, merely by examination of the linear formalism and decoherence theory, seems clearly to fall prey to objections of circularity. It is not the case that decoherence gives chairs and their quasi-classical dynamics as a necessarily preferred structure derivable from the fundamental laws of physics. Indeed, it would seem all but miraculous if they were to do so, and this interpretation of Zurek's project is clearly rendered untenable.

On the other hand, if Zurek is understood as simply showing that, for quasi-classical structures, it is possible to show that quasi-classical dynamics will inevitably apply, then his program does not seem to face any difficulty arising from this argument of Kastner's. This is how I believe those interpretations of quantum mechanics which use decoherence theory (discussed in the previous chapter) generally understand it. Whether it is the particles following (relatively) quasi-classical trajectories found in Bohmian mechanics, the agent-like Information Gathering Utilising System (IGUS) discussed by Gell-Mann and Hartle[11], or the emergence-based understanding found in Wallace's Everettian quantum mechanics, all these interpretations have metaphysical and conceptual formulations designed to provide some elements of quasi-classicality which simply relies on decoherence to show how that continued classicality is consistent with, and ensured by, linear quantum dynamics. This is crucial not only to answering charges of circularity, but, as argued by Adler 2002, to showing how apparently stochastic dynamics are able to emerge from a fundamentally deterministic theory.

If Zurek’s analysis is indeed seeking to show how the observed divisions of the classical world are able to arise naturally from within quantum theory, then Kastner’s argument ceases to apply. However, if this is the case, Zurek’s project also seems to lose much of its unique interest. Zurek does not offer an interpretation to go with his formal analysis and derivation of the Born rule. He seems to aim his project at providing a means of recovering classicality without such an interpretation. And this sets Zurek's quantum Darwinism apart as a unique and ambitious project, fundamentally different from more recent interpretations which use the decoherence-based reasoning he pioneered. Unfortunately, as it stands, this project has no means of answering the charges of circularity levelled

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[11] Gell-Mann and Hartle 1990 pp. 24, introduce the notion of an IGUS as a generic form of complex adaptive system which resides within quantum mechanical histories, and changes its behaviour in response to past records of its environment. They take human beings to be examples of IGUSes. They argue that the patterns which IGUSes would come to exploit are those displayed by quasi-classical decoherent history spaces.

by Kastner. If on the other hand, Zurek is to be understood as simply providing a proof of consistency with classicality which is interpretation neutral (at least among those interpretations which endorse his additional assumptions), but not a solution of the quantum measurement problem without further interpretation, then his project becomes essential and pioneering ground work for those interpretations which rely on it, but not something to be considered as an interpretation in its own right.

4.8 Objection 2: Decoherence Relies on an Environment with Randomised Phase Correlations. This isn't a Plausible Assumption.

Kastner offers a second objection to Zurek's reasoning, which appears to have wider applicability, and which could potentially undermine all interpretations which rely on decoherence. Kastner believes, based on a derivation of time-dependent interference suppression given by Bub 1997, that decoherence relies on a very large environment of random unentangled states. Given however, the ease with which systems become entangled, she points out that it is extremely unlikely to have the kind of randomly varying environment needed for this derivation.

> “The crucial point that does not yet seem to have been fully appreciated is this: in the Everettian picture, everything is always coherently entangled, so pure states must be viewed as a fiction -- *but that means that it is also fiction that the putative 'environmental systems' are all randomly phased*… Everettian decoherentists have effectively assumed what they are trying to prove: macroscopic classicality only ‘emerges’ in this picture because a classical, non-quantum-correlated environment was illegitimately put in by hand from the beginning.” (Kastner 2014, pp. 4, original emphasis).

Kastner is certainly right that Bub's environment model is implausible in representing the environment as a large number of independent elements with no entanglements of phase correlations between either the many elements of the environment, or the system to be measured. She does not seem, however, to offer a clear and convincing argument for how this common inaccuracy in the theoretical models which underpinned decoherence, is supposed to lead those models into error. In this section, I will consider two problems which Kastner suggests this might pose. In both cases I will argue that as stated these are very unlikely to represent serious problems, as only very specific types of correlation between the system and its environment would lead to a failure of decoherence. Finally, I will argue that those possible phase correlations which do present a significant concern, are precisely those cases which might lead to history recombination.

4.8.1 Entanglement of Elements within the Environment

Kastner clearly feels that widespread entanglement within the environment is an important omission from the standard theoretical underpinnings of decoherence. She argues that neglecting this entanglement amounts to smuggling in classical dynamics without any means to justify them. It is unclear however precisely what she thinks is concerning about these correlations.

Standard decoherence theory relies on an environment with a large number of uncorrelated degrees of freedom becoming entangled with the system's measured property in order to effectively diagonalise the reduced density matrix of that system. A plausible understanding of Kastner's concern here seems to be that the environmental degrees of freedom might be sufficiently prone to entanglement with one another that they will not act as multiple independent measurements, each forming an additional entanglement with the measured property of the target system. If so, this would mean that, rather than being measured by a vast number of independent degrees of freedom, the system is measured only by a few.

This would seem to undermine environmental decoherence, which is typically presented as needing a very large number of measuring degrees of freedom in order to effectively suppress the off-diagonal elements of the systems reduced density matrix.

In fact however, this argument holds only in the idealized case in which the entanglement of the measured system to the first environmental element is perfect. In realistic cases, in which there will not be a one-to-one correlation between measured particle state and the state of the environmental element, other environmental elements will not be perfectly entangled to the state of the measured system, and so will be able to interact to further decohere the measured system. In the ideal case, where the entanglement produced by the first interaction is perfect, this alone will be sufficient to set the off-diagonal elements of the reduced density matrix of the measured system strictly equal to zero (see example given by Wallace 2012, pp. 77-81).

An alternative formulation of the same problem would be that, at any time after the environment has interacted with the measured two-state system, there are two states which the environment will occupy in different histories, one corresponding to each of the possible values. At any such time it is possible to select a basis set in which these branches differ by only two elements in the state vector of each environment (these two basis vectors describe the plane in Hilbert space upon which the state vectors of both branches will lie). Given that all the other elements that make up the environment's state vector are now entirely unaffected by the state of the measured system, the measured two-state system can only ever be entangled with these two environmental states. At first glance this formulation seems to present a problem, as these two environmental states are not specifically designed to produce ideal entanglement interactions, and typically single interactions do not produce the degree of entanglement necessary to substantially suppress interference effects. In fact, the degree of entanglement between these environmental degrees of freedom and the state of the measured system will increase with time, producing exactly the same interference suppression factor as a function of time as any other environmental basis selection.

This highlights an important point, that for a two-state observable it is always possible to select a basis according to which the observed system interacts with only 2 degrees of freedom of the environment, and that if environment induced decoherence could not be produced for a system interacting with a number of degrees of freedom equal to its own, decoherence would not be possible for any system.

In short, the concern that entanglements within a system's environment might render it effectively equivalent to a smaller environment, and so undermine the ability of that environment to decohere the system, relies on a common misunderstanding of what is required for the suppression of interference. In order for a reduced density matrix to become diagonalised, the system state must be encoded somewhere outside of itself in the basis of that matrix. This is usually regarded as depending on a very large number of degrees of freedom to robustly encode the system state. However, the same degree of interference suppression can be achieved by a single measurement if the single measurement leads to a sufficient degree of entanglement, and this entanglement remains robust. Thus, the entanglements between elements of the environment which are omitted from Bub's model do not seem to be problematic failings of that model.

4.8.2 Pre-Existing Correlations Between Measured Systems and Their Environment

The second possible understanding of Kastner's concern is that it relates to pre-existing entanglements between measured systems and their environments. Typically, quantum measurement processes are represented as pure quantum states becoming entangled to their environment and so decohering. One aspect of Kastner's concern seems to be the origin of these pure states. Without the collapse postulate, entanglements will become extremely widespread extremely quickly, and it might seem that the (clearly inaccurate) assumption of an environment that is initially independent of our measured system is crucial to decoherence theory.

At first glance, the near certainty of entanglement between the environment and the measured system which Kastner rightly identifies is certainly troubling. In fact, however, this concern is resolved by consideration of the process of preparing the target system for measurement. In particular, it must be remembered that quantum measurements of superposition states, of the type which decoherence is meant to facilitate, are performed on particles whose state vectors are known with a high degree of accuracy.

If the state vector were not known, it would be impossible to say with any certainty that the measured state of the particle after measurement was not the state of the particle all along. In order to obtain particles in known superposition states the particles must already have been measured. This certainly verifies Kastner’s claim that the target particle and environment must already be entangled. Importantly, however, decoherence would fail to diagonalise the reduced density matrix of the target particle only if the environment with which the particle becomes entangled during decoherence were already entangled to the property of the particle being measured. In any case in which the target particle is known to be in a superposition state for a particular measurement, this is because some non-commuting variable has already been measured, in order to identify the particle’s state in a non-commuting basis. Consequently, the environment will be heavily entangled to the particle observable first measured, and accordingly uncorrelated from the second measured observable.

In the more general case of naturally occurring quantum measurements which have not been carefully prepared in this way, much the same factors still apply. We may not usually possess the prior information about the quantum state which is measured to compare its outcome to any theoretical statistics, but its initial state will nonetheless have been encoded within the environment. This encoding will decohere the initial state of the system, producing a state which (for most purposes) can be thought of as a pure quantum state.

Consider the following example:

> A spin $\frac{1}{2}$ particle is obtained for the purposes of performing a measurement of a particle in a superposition state. The initial state of the particle is unknown. In order for a clearly probabilistic quantum measurement of a superposition state to be performed, a measurement must first be made in order to obtain a known state vector for the particle. Suppose that the spin of the particle is measured in the z basis. In a collapse-free framework this will involve the universal wavefunction entering a superposition state according to the possible outcomes of the measurement. The effect of this measurement is to produce histories in which the state vector of the particle is aligned to the spin z basis in Hilbert space. In each of these histories the environment will become heavily entangled with this spin z measurement. If a subsequent spin measurement is now made in the x basis, the value of this observable is initially entirely unentangled from the environment: thus there is no danger of existing entanglements between the environment and the target particle interfering with the ability of the environment to decohere the state of the measured observable, in this case x spin.

Of course, not all superposition measurements involve successive measurements of orthogonal variables. For a measurement made in some basis θ, at an angle between x and z, the environment will be partially entangled to the θ spin measurement. However, the effect of entangling the θ spin state to the environment will still be to suppress the off-diagonal elements in the reduced density matrix of the target particle described in the θ basis. The only case in which there will be no significant suppression of these elements will be in the case when θ is extremely closely aligned to z, as in this case these terms will already have been effectively suppressed.

It is interesting to note that, for those interpreters of quantum mechanics who do not advocate a collapse interpretation, local pure states are something which no one has ever measured. That is, truly pure states apply only to those entities which have never interacted, and which are completely unentangled from the rest of the universe. This of course is not the case for those particles used in physics experiments. Thus, when a physicist describes a particular particle as occupying a pure state, this claim cannot be interpreted literally on any view of quantum mechanics other than a collapse interpretation. Quantum Darwinism, as well as more common no-collapse interpretations, must make sense of this claim as meaning something like: the particle has been measured or operated on such that a large portion of the environment including the physicist are entangled to its state in a particular basis, and we can predict with very high accuracy what the result will be of a subsequent measurement in that basis if one is undertaken immediately.

4.8.3 History Recombination

In general, then, we do not need to be concerned about the implicit assumption of phase randomness found in decoherence theory. The vast majority of possible entanglement relations will still produce very similar results to those obtained by assuming phase randomness. However, there is a special subset of possible entanglement relations for which this is untrue. These are the entanglement relations which under subsequent operations to the system will in a short time period lead to history recombination and large-scale interference phenomena, as seen in the previous chapter.

It is unclear how far Kastner herself identifies this as a specific concern. The whole of her 2014 paper is built around a comparison to Boltzmann, and a putative proof of the second law of thermodynamics which proved to be circular. In the circumstances, this comparison seems rather apt for the subject of history recombination. Wallace 2012 does suggest that a branching structure of histories is ensured by some quantum equivalent of the assumed low entropy starting conditions, needed to safeguard the second law of thermodynamics. On the other hand, Kastner certainly never raises the issue of history recombination directly or engages with Wallace's suggestion here. As such, it seems most likely that Boltzmann's argument is serving as nothing more than an example of a viciously circular argument in physics.

It is unclear to what extent Kastner has considered the possibility of history recombination, and the threat it poses for decoherence based arguments. None the less, her very evident concerns about the possibility of long-standing phase correlations undermining decoherence theory point to just these possibilities. She correctly identifies that standard treatments of the subject invariably begin by assuming both the system and its environment to be in a pure uncorrelated state. This is an understandable decision, as the alternative would increase the mathematical complexity hugely, and in many cases wouldn't change the results at all. But it undoubtedly contributes to obscuring the issue of recombination and the origins of the branching history structure.

4.9 Conclusion

I began this chapter by introducing Zurek’s formal decoherence-based derivation of the Born rule. This derivation paved the way for more recent derivations, such as Wallace 2012 and Sebens & Carrol 2018, which play a major role in currently prominent attempts at interpreting quantum mechanics. These later works very closely follow the basic structure originally set out by Zurek. Decoherence-based Born rule derivations of this kind have particular significance for modern Everettian interpretations.

I then introduced accusations of circularity against this approach levelled by Kastner 2014 amongst others. The concerns presented by Kastner divide into two categories:

First, concern that presupposing a quasi-classical entity with a quasi-classical choice of basis states of interest implicitly presupposes the classicality which decoherence based arguments were meant to obtain. Kastner believes that this presupposition renders Zurek's derivation of the Born rule viciously circular. As Zurek himself notes in later papers, this is entirely true if his work is understood as a derivation showing *that* classical dynamics emerge from the linear quantum mechanics by decoherence. On the other hand, if the derivation is understood as simply showing *how* classical dynamics emerge from linear quantum mechanics, when it is applied to those types of systems which we already expect are going to display this kind of behaviour, then this circularity is not something which should concern us.

Second, Kastner is concerned about the standard assumption of random and uncorrelated phases within the environment in presentations of decoherence. I have argued that she is right to be concerned about these. She does not however distinguish those particular types of phase correlations which are going to present a serious concern. These, I believe, are precisely those correlations which are likely to lead to history recombination under subsequent evolution of the total system. This point will be discussed in far more detail later in this thesis. More generally, I have argued that phase correlations within the environment do not need to represent a serious concern.

The next chapter will look at the objections to decoherence based Born rule derivations raised by Thébault & Dawid 2015. These are in some ways similar to those raised by Kastner, but potentially far more challenging for the decoherence project.

5 Thébault and Dawid’s Inconsistency Objection(s)

In the last chapter, I looked at the Born rule derivation developed by Zurek, and objections of circularity raised by Kastner 2014. I argued that generally the objections of circularity rested on a mistaken understanding of the appropriate purpose of Zurek’s derivation.

In this chapter, I will turn to look at somewhat similar objections raised by Thébault & Dawid 2015. These objections are not directed against Zurek directly, but against Wallace 2012, who offers a derivation of his own which duplicates the bulk of Zurek's argument. Wallace departs from Zurek in two ways.

First, he offers a sophisticated decision theoretic characterisation of probability designed to make sense of how probabilities can be understood in the context of a fundamentally deterministic linear quantum mechanics without any additional dynamics. This careful and technical treatment of types of probability is largely beyond the scope of this thesis. It is also largely irrelevant to the arguments of Thébault & Dawid. The only point which will matter for the purposes of this chapter is that Wallace's derivation of the Born rule rests on a decision theoretic conception of probability. As will become clear, Thébault & Dawid have significant concerns about this conception.

The second difference between Wallace and Zurek is the emergence-based interpretational framework within which Wallace's view of the derivation is situated. I argued in the previous chapter that, within the context of this interpretational framework, Wallace does not need to show *that* classical dynamics and the Born rule are *derived* from the linear quantum mechanics, but rather to show *how* these things *emerge* from the linear quantum mechanics.

The first argument presented by Thébault & Dawid is a concern about circularity very similar to that seen in the previous chapter. This argument seems to be neatly answered by this distinction between attempting to recover classicality, as a derivative consequence of linear quantum mechanics, and simply showing how it emerges from the linear quantum mechanics. Thébault & Dawid seem to be aware of this response and to accept it. They present this form of circularity argument partly by way of context setting, and partly to emphasise the limitations on what can be achieved by decoherence, which they feel are often neglected. This chapter will review this argument because, although it is similar to the circularity arguments presented by Kastner, there are some important differences.

The two later arguments presented by Thébault & Dawid are more significant, and argue that this form of Born rule derivation is not merely circular but, in some sense, inconsistent. One argument claims that this form of derivation of the decision theoretic Born rule rests on the presupposition of a form of the Born rule which is far stronger, and (on Wallace's view) seems to be incompatible with the decision theoretic form. The other argument claims that the neglecting of off-diagonal elements of the reduced density matrix is unjustifiably different from the way in which other terms are treated, in a way which renders the Born rule derivation suspect. If these arguments proved successful it would pose a new and very significant challenge for decoherence-based Born rule derivations.

I will argue that, at least on the most natural understanding of the concerns raised by Thébault & Dawid, these concerns can reasonably be answered in ways suggested by Wallace in other contexts. Consequently, I believe that at least Wallace (who is the main target of Thébault & Dawid’s paper) should not be particularly concerned by the arguments they present. As I shall identify at the end of the chapter, however, there is another way of understanding these arguments which presents a far more significant difficulty.

5.1 Thébault & Dawid’s Circularity Objection

In the section of their paper focusing on circularity objections to decoherence-based Born rule derivations, Thébault & Dawid present a circularity objection produced by a number of authors including Kent 2010, primarily targeting the Born rule derivation provided in Zurek 2004. In fact, as Thébault & Dawid note, this objection was first presented by Zurek himself in Zurek 2005.

The difference between the derivation presented in the previous chapter of this thesis, which is taken from Zurek 2005, and the derivation which he offers in his paper a year earlier is the extent to which the reduced density matrix characterisation of decoherence plays a role. Presenting the 2005 version in the previous chapter, I noted at a certain point that Zurek presupposed a particular basis for measured states. I commented at the time that this preferred basis is precisely what decoherence typically provides. The standard means of presenting this, however, relies, as I showed in chapter 2, on the formalism of the reduced density matrix. In his presentation in earlier papers, Zurek does not confine the reduced density matrix to this minor implicit role. This formalism is prominent throughout his derivation.

The reason for this shift in Zurek's presentation is that he came to recognise that the reduced density matrix relies fundamentally on the Born rule. This is because the reduced density matrix of an entangled subsystem is formed by taking a partial trace over the density matrix of the entire closed system, which includes the environment of the subsystem. As I made clear in chapter 2, this process of taking a partial trace amounts to adding together a weighted mixture of density matrices for the subsystem, with the weightings of those different matrices based on the Born rule probabilities of different states of the environment. This means that any derivation of the Born rule which relies on reduced density matrices must clearly be circular.

Having recognised this problem, Zurek 2005 sets out to eliminate this reliance on reduced density matrices as far as possible. The derivation provided in the previous chapter does not directly rely on them at all – though, as Zurek acknowledges, it does rely on a preferred basis which would typically be obtained by consideration of a reduced density matrix as discussed in chapter 2. Zurek devotes a section in his paper to consideration of other means by which a preferred basis might be justified, though he does not seem to produce any very convincing suggestions.

Thébault & Dawid ignore the subtleties of Zurek's revised argument. As far as they are concerned, what has been established is the clear circularity of Zurek's Born rule derivation, as well as the related derivation offered by Wallace 2012.

It is interesting to note that Zurek's revised derivation clearly shows that there are alternatives to this circularity, albeit somewhat unappealing ones. What Zurek offers actually is a derivation which produces the Born rule without presupposing it. This derivation is valid and non-circular, as long as we accept just putting in the preferred basis states by hand with no further derivation. On some level, the explicit presupposition of a system of interest and the basis of measurement outcomes do not seem like particularly excessive premises from which to derive the Born rule in the way which Zurek does. This speaks to the clear and continued interest and importance of Zurek's result.

Nevertheless, explicitly presupposing a preferred basis in this way is clearly far from the derivation from first principles we might hope for, and would be unacceptable to many advocates of the decoherence program. Most relevantly to Thébault & Dawid’s argument, Everettians such as Wallace have set themselves the goal of developing an interpretation which does not presuppose a particular preferred basis, but allows the preferred basis to emerge from the underlying linear dynamics. As such, Everettians such as Wallace seem obliged to accept the circularity of Born rule derivations.

As Thébault & Dawid put it in their conclusion from this section:

> “Deutsch–Wallace type derivations of the Born rule as a subjective probability measure are inherently circular since they involve the prior assumption of the Born rule in order to establish the Born rule” (Thébault & Dawid 2015, pp. 1568).

Of course, this circularity is very similar to the circularity which arises from the presupposition of a particular choice of system environment split, which was discussed in the previous chapter. In the previous chapter I looked at how this kind of circularity could be made acceptable by shifting the purpose of the Born rule derivation from trying to show from first principles that the Born rule obtains without any form of presupposition (which I think is clearly impossible), to an attempt to show how the Born rule emerges from a particular approach to linear quantum mechanics. This appeal to emergence would undoubtedly be the response offered by Wallace, as Thébault & Dawid acknowledge. Thébault & Dawid suggest that the appropriate way to think of Zurek's Born rule derivation in this context is as a demonstration of consistency for the decoherence approach in interpretations such as Wallace’s.

Having introduced this circularity concern and the limitations on Zurek's derivation of the Born rule which it demonstrates, Thébault & Dawid now turn to examine in more detail this form of consistency. As the next section will discuss, however, they believe that the decision theoretic character of the Born rule, which is produced as the conclusion of this derivation, is in fact very different to the Born rule which must be assumed in order to make this derivation possible in the first place. They argue that this renders the kind of derivations of the Born rule presented by Zurek incoherent within Wallace’s Everettian quantum mechanics.

5.2 The Presupposition of the Born Rule

In order to understand the incoherence objection of Thébault & Dawid, it is important to note a surprising difference between the role which Zurek identifies the Born rule as playing in allowing his derivation, and the role Thébault & Dawid see it as playing.

Whereas Zurek regards reduced density matrices in general as implicitly resting upon the application of the Born rule to the wider environment, Thébault & Dawid seem more concerned with its role in warranting the neglecting of off-diagonal elements within the reduced density matrix. They write as follows:

> "…although such [environment induced decoherence] processes may act to re-scale the weightings of the off-diagonal elements of the density matrix to be very small, the interpretation of the smallness of those values as indicating either: (a) the neglectability of the corresponding component of the wavefunction; or (b) the robustness of branching structures within the wavefunction, will in the end rely on the prior assumption of the Born rule." (Thébault & Dawid 2015, pp. 1568)

The first thing to say here is that the neglectability of these off-diagonal elements does seem like an important factor in solving the problem of the preferred basis. It is the fact that these off-diagonal elements, and the interference phenomena to which they relate, will be negligible which, on the analysis of Schlosshauer 2007, warrants the treatment of these states as robust measurement outcomes states, and gives rise to the preferred basis of branching used in Wallace's interpretation. It is more difficult, however, to understand precisely how this neglectability connects to the Born rule.

A central conviction of Thébault & Dawid is that numbers cannot be neglected within a mathematical formalism without understanding what the physical significance of those numbers is and the purpose of the approximation. To make this point they point out that neglecting the Sahara Desert on account of its extremely low rainfall might be a reasonable approximation in a study of population distributions, but clearly not if the study directly concerns land area. They clearly believe that the Born rule is directly relevant to identifying the physical relevance of off-diagonal elements within reduced density matrices, and that this is what justifies the neglecting of those terms.

On some level, they are certainly correct that the Born rule is relevant to the physical significance of these elements. However, the link is not quite as direct as Thébault & Dawid seem to assume. Within a density matrix the diagonal elements are all real numbers corresponding to basis eigenstates of the subsystem. The Born rule very directly warrants regarding these elements within the matrix as probabilities. Off-diagonal elements, however, do not correspond to any eigenstates of the subsystem, and the values of these elements will typically be complex numbers.

The suggestion that the Born rule for quantum probabilities is responsible for giving physical significance to these complex numbers on the off-diagonal elements of the reduced density matrix clearly requires further explanation. No such explanation is given by Thébault & Dawid 2015, leaving this crucial connection in their argument rather mysterious. Later in this chapter, I will return to look at how I think the physical significance of off-diagonal elements within density matrices connects to the Born rule. For now though, I will go on to set out Thébault & Dawid’s inconsistency argument concerning the character of the Born rule which they believe is essential to making this form of approximation.

5.3 The Inconsistency Argument

Following on from the dependence of Zurek's derivation on the Born rule just discussed, Thébault & Dawid go on to present what they believe to be an inconsistency between the precise character of the Born rule which

Zurek's derivation relies on, and the character of the Born rule which it yields. Specifically, they claim that, while the Born rule which Wallace obtains from Zurek's derivation is a decision theoretic subjective Born rule, concerning the rational betting behaviour of an agent within an Everettian branch, the derivation itself requires a stronger form of the Born rule than this. Thébault & Dawid then point out that, as Wallace explicitly rules out the possibility of any more objective Born rule beyond the decision theoretic form which he derives, Wallace's scheme appears to be internally incoherent.

If true, this would represent a hugely damaging inconsistency within Wallace’s approach. Rather than being a complete and internally consistent emergent theory, the framework would depend on an objective probabilistic rule with no obvious place within Everettian quantum mechanics.

The reason that Thébault & Dawid believe that an objective form of the Born rule is needed here, and that the subjective form will not suffice, concerns the scope of applicability of the two rules. A purely decision theoretic Born rule is presumably only applicable in cases where a rational agent either does or could conceivably occupy the histories to which probabilities are being ascribed. In contrast, Thébault & Dawid believe that “Whilst we can reasonably consider generalising the argument to branches which potentially— rather than actually—contain agents, we cannot consistently apply the argument to ‘branches’ which are not approximately separated.” (Thébault & Dawid 2015, pp. 1568)

As discussed in chapter 3, the word branch, as used by Wallace, refers to a history in a history space which approximately fulfils the medium decoherence criterion. The scare quotes applied in this quote reflects the fact that histories which are not approximately separated would not, in Wallace's terms, be branches. It is a little difficult to follow precisely the connection Thébault & Dawid are making to off-diagonal elements within a reduced density matrix here. But it seems that, as far as Thébault & Dawid are concerned, off-diagonal elements correspond in some direct sense to histories in which this approximate separation does not obtain.

I suspect much of the reason for the lack of clarity here is that Thébault & Dawid are seeking to discuss separation of histories purely in terms of density matrices. As I made clear in chapter 3 the connection between these is not straight forward or easy to identify. In some cases, off-diagonal elements in reduced density matrices may well point to histories which are not Everettian branches, and could not contain anything recognisable as a rational agent. A little reflection, however, will show that this is very far from always being the case.

5.4 Interpreting the Off-Diagonal Elements

Consider a reduced density matrix describing the state of a spin-half particle in the z-spin basis:

$$\rho = \begin{pmatrix} a & ce^{-i\gamma} \\ ce^{i\gamma} & b \end{pmatrix}$$

Where a, b, and c are real positive numbers. γ determines the phase of the off-diagonal elements. a and b are the Born rule probabilities of spin up and spin down measurements respectively.

If the value of c is zero, then any subsequent operations will behave as if the particle is currently in a definite z-spin eigenstate; that is, no interference phenomena will be displayed. In terms of histories, such subsequent operations are going to yield two branches, one corresponding to each z-spin eigenstate. If other noncommuting spin measurements are made, the results will still behave as would be expected if the particle began in a definitive z-spin state.

Now, though, suppose that c is not zero. Thébault & Dawid clearly believe that this should correspond to the occurrence of histories which are not separated from one another, and display nonclassical phenomena. Is this the case?

The answer is that it depends on what subsequent operations the particle is subject to.

If the particle is simply going to be subject to z-spin measurement, or other entanglement of its z-spin state to the environment, then there will be no interference phenomena. This entanglement will come to suppress the interference terms in the off-diagonal elements, and two well separated histories will be produced.

Equally, if the particle is simply left in isolation without any form of perturbation, the state will remain unchanged, and can reasonably be decomposed as two histories, one corresponding to each z-spin eigenstate (or any other pair of spin eigenstates).

If however, the particle spin state is measured in some other basis, then the off-diagonal elements will significantly affect the measurement result. The result will no longer be as expected based purely on the probabilities of z-spin states given in this matrix. If the basis in which histories of this particle are being decomposed is the z-spin basis, then explaining the dynamics in this case cannot be done without accounting for interaction between the histories. This is the type of case in which separation between histories fails in the way that Thébault & Dawid are concerned with. The next question is whether, and if so why, they are right in their conviction that there could be no agents within histories undergoing this kind of process.

Approximate separation of histories is something which Wallace sees as an essential feature of a branch, and as far as Wallace is concerned only branches are amenable to a quasi-classical metaphysics which agents might occupy. It therefore seems prima facie plausible to believe that histories of the above kind could not contain agents. However, it is important to give some consideration to the meaning of the word *approximate* here. A quote from Gell-Mann and Hartle, which Wallace presents in discussing the character of this branching structure, is instructive as to his thinking on the subject of what constitutes a branching quasi-classical history space:

> “such that the individual histories obey, with high probability, effective classical equations of motion interrupted continually by small fluctuations and occasionally by large ones” (Wallace 2012, pp. 99)[12].

In other words, the intention is to weaken the absence of interference requirement in the identification of quasi-classical branches to allow frequent small-scale interference phenomena and occasional large-scale interference phenomena. This, of course, means that the history space containing these branches cannot strictly fulfil the medium decoherence criterion.

The motivation behind this weakening of the quasi-classicality requirement for branches is that, if strictly classical dynamics are required of a history space without any exceptions or flexibility, then very few history spaces indeed are selected as candidates, and these do not display the types of behaviour which we are used to. Nonclassical dynamics are essential to sustaining stable atoms, for example, and the only way to rule out these dynamics is to select histories which are very dissimilar to our day-to-day experiences, and in many of which states never change at all Kent 1996.

Wallace's interpretation therefore accepts some (rather unclear) degree of failure of separation between histories. Quite possibly, then, brief instances of failure of separation between histories, caused by off-diagonal reduced density matrix elements, would not disqualify histories from remaining as branches, so long as these failures of separation were sufficiently rare.

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[12] Wallace attributes this quote to Gell-Mann and Hartle, but does not give a reference to any specific text.

What, however, about those very low amplitude histories which display continual interference phenomena? There will be histories in which large-scale failures of interference suppression occur not just occasionally but continuously. Such histories clearly do not fit Wallace's conception of branches, and it is certainly arguable that they do not display the types of persistent pattern which would allow us to identify structures within them as being a rational agent. These, I take it, are the types of history with which Thébault & Dawid are concerned when they speak of off-diagonal elements within reduced density matrix relating to histories which are not approximately separated, and which could not even in principle contain rational agents.

The question which Wallace must answer is why we do not expect to find ourselves in, or even seem to be aware of, this type of history in our day-to-day lives. The obvious answer is that such histories are incredibly improbable. Given the high degree of suppression of interference terms for day-to-day macroscopic objects, the chances of such an object displaying interference phenomena are infinitesimal. Here we are considering histories in which such events occur on a very frequent basis. The result is that the Born rule probability of such a history will be so unimaginably small that it can be safely ignored.

The point, however, is that if all we can apply when considering the probability of such a history is a decision theoretic Born rule, and, as Thébault & Dawid suggest, this type of Born rule is not applicable to histories without agents, then we have no rationale for identifying branch amplitude with probability. Without this ability to relate branch amplitude to some physical quantity, it is nothing more than a very small number. And, as Thébault & Dawid make clear, the fact that a number is very small does not in itself warrant neglecting anything associated with that number. Without some means of assigning a credence to the possibility of ending up in these non-classical histories, the decision theoretic derivation of the Born rule offered by Zurek and Wallace has no way to justify neglecting such histories. And so no way to justify treating a basis which gives the amplitudes of such non-classical histories to be very small as a preferred basis.

This is how I understand Thébault & Dawid's argument in this section. There are some differences between my presentation and theirs, and I will consider an alternative way of understanding their argument in a later section. This, though, seems the most plausible understanding, and I think it can be clearly formalised as follows:

Premise 1: Deriving the Born rule depends on the ability to ascribe rational credences to any successor history of a history which an agent currently occupies.

Premise 2: Applying such rational credences depends on the possibility of a rational agent residing within each of those successor histories.

Premise 3: For some types of successor history which occur this is not actually possible on Wallace's account.

Premise 4: No other means of independently establishing the neglectability of such histories (such as a non-subjective Born rule) is available.

Conclusion: Wallace cannot offer a coherent account of the origin of the Born rule within his interpretation.

This is not an issue about which Wallace offers any direct comments. However, as I shall argue in the next section, I believe that his answer to a different problem gives a clear indication as to what his response in this case would be. I believe that he could (and would) successfully respond by rejecting premise 2.

5.5 Quantum Russian Roulette

Wallace 2012, pp. 369-372 seeks to answer the question of whether, on his account of quantum probability, it would be rational to play quantum Russian roulette.

Quantum Russian roulette is a game in which the player places a bet on the outcome of a quantum process, and arranges, in the event that they lose the bet, a swift and painless death for themself. It has been argued (e.g. Lewis 2004) that taking such a bet is a thoroughly rational thing for a convinced Everettian to do. This is because, although there are successor histories in which the player is dead, they do not contain any agent appropriately related to the player to be considered when the player is considering their possible future experiences, and deciding whether to take this bet.

Wallace however points out that, if all we should be concerned with is our possible future experiences, then this would have drastic implications in the classical case, not just the quantum. If the player were to bet on a similar classical game of Russian roulette, then here, too, the possible outcomes would be either the future experience of a material gain, or a future in which the player has no experiences. If our credence ascriptions concerning the future, or our decision-making practices, were really only concerned with our future experiences, then there would be no reason not to play classical Russian roulette. The possibility of death is something we are well used to ascribing credence as to, and adjusting our decisions on the basis of these credences is something we regularly do in day-to-day life. This is despite the fact that in the event of death (at least for some types of death) there would be no future experience connected to that event.

It therefore seems rather suspect to suggest that we should not ascribe credence as to the possibility of death in the quantum case. Wallace makes it clear that an agent considering whether to play quantum Russian roulette both can and should, as far as he is concerned, ascribe a credence to dying, and adjust their behaviour accordingly.

Thébault & Dawid's incoherence argument seems strikingly similar to the Russian roulette argument, in that it too relies on the claim that a rational agent is unable to ascribe credences or adjust their decision-making behaviour to respond to successor histories of the history which they currently occupy, in which that agent does not exist. Wallace clearly sees no reason why the fact that an agent does not exist in a successor history should prevent them from making decisions on the basis of that successor history's existence.

Thébault & Dawid do make a possibly relevant distinction between cases in which it is, and is not, possible to imagine an agent occupying a history. In the former case, they seem to believe that analogy to cases in which an agent does exist in the successor history may warrant the ascription of credences. But they do not believe such credence ascriptions can be extended to the latter case.

There is certainly a difference between the Russian roulette case and the case which Thébault & Dawid consider here. I cannot see any reason, however, why this difference would render an agent incapable of ascribing credences to successor histories in which they could not exist, just as easily as to successor histories in which they do not exist. Thébault & Dawid do not seem to offer any explanation of why credence ascriptions would be more difficult in this case.

It seems to me, therefore, that Wallace could respond to this argument from Thébault & Dawid by arguing that a rational agent could perfectly well ascribe credences to successor histories in which they could not exist, and adjust their behaviour on the basis of these credences. This would amount to rejecting premise 2, and so allow Wallace to escape Thébault & Dawid's conclusion. The decision theoretic Born rule derived by Wallace is able to offer physical significance for the relative amplitude of successor histories in an agent’s future (as probabilities), and so justify neglecting successor histories with extremely low amplitudes.

5.6 Distant Nonclassical Histories

So far, I have dealt with what seems to be the most natural reading of Thébault & Dawid’s argument. However, there is another way of interpreting this argument. I have focused on the case of a nonclassical history which is the successor of a history which an agent occupies. An alternative way of understanding Thébault & Dawid’s concern is as relating to histories which throughout their entire evolution are sufficiently nonclassical that they could not be considered as containing agents. In this case, an agent's consideration of their future histories in the way discussed in the previous section could not be relevant. Consequently, it is hard to see how probabilities could be made sense of for such histories.

Unlike the previous form of this argument, I have no idea how Wallace would respond to this concern. Indeed, it is an interesting question how you could make sense of the decision theoretic conception of probability in the context of such histories.

I do not feel, however, that this is an argument that would undermine Everettian quantum mechanics very profoundly. Ultimately, if probabilities can sensibly be ascribed to all histories which agents occupy and all successors of those histories, then it does not seem like a particular problem if this understanding of probability cannot be extended to histories which never have contained or could contain agents.

This of course brings to light a slightly disconcerting agent dependence in the notion of probability within Everettian quantum mechanics. This agent dependence is unsurprising given the way in which derivations of probability within modern Everettian interpretations proceed from considerations of what it is to be a rational agent. Wallace 2012 pp. 142-156 argues that the Born rule for probabilities which he is able to derive is objective in the sense that under certain assumptions about the nature of quantum mechanical probabilities, the Born rule is the only rule that could possibly be identified as yielding such probabilities. Nevertheless, this derivation still rests on the consideration of decisions made by a rational agent, and so will not necessarily extend to histories which could never contain an agent. It may be possible to resolve this problem, but even if it is not, this feature of Everettian quantum mechanics does not seem unacceptable, though some may find it unpalatable.

5.7 The Ontological Prejudice Objection

As well as the problem of incoherence just given, Thébault & Dawid also offer a second problem, which they summarise as follows:

> "In (effectively) eliminating off diagonal elements due to their low Born weight the Everettian must either also (effectively) eliminate similarly low weighted distinct states and thus subvert their own position or simply apply a principle of ontological prejudice, such that coherence effects are eliminated simply on the grounds of being coherence effects, irrespective of their Born weighting." (Thébault & Dawid 2015, pp. 1571).

One obvious response to this problem is to point out (as I already have done) that off-diagonal elements in a reduced density matrix are very clearly different to Born rule probabilities. They are complex numbers which do not correspond to any form of eigenstate, and although they may well affect the probabilities of future dynamics, this will depend on the Hamiltonian to which the system is subject. There are therefore very clear differences between off-diagonal elements within a reduced density matrix and the Born rule probabilities of histories, and so it seems rather difficult to motivate the idea that if one is neglected the other must also be.

On the other hand, it is true that Born rule probabilities still play an important, albeit indirect, role in justifying the neglecting of off-diagonal elements. They can be neglected because the Born rule probabilities associated with interference phenomena related to the off-diagonal elements within the reduced density matrix (if it is evolved under a Hamiltonian that will produce them) will be very low for systems where environmental decoherence has suppressed these elements. In some sense, therefore, it does seem that neglecting histories associated with off-diagonal elements is playing a role here, and it is reasonable to wonder why just these specific histories are being neglected. I will therefore spend a little time considering just why and in what sense off-diagonal elements and the interference phenomena to which they are associated are neglected as part of Zurek's derivation of the Born rule.

5.8 Neglecting Off-Diagonal Elements

In understanding the role of neglecting off-diagonal elements, it is significant to recall that this is not done at any point in the derivation taken from Zurek 2005, which was presented in the last chapter. The relative phases of terms within the wavefunction, which are what give rise to off-diagonal elements within a reduced density matrix, are not neglected by Zurek. Indeed, he devotes a section of argument to showing just why they must be irrelevant to the chances of different eigenstates in the selected basis.

Neglecting off-diagonal elements does not appear in the derivation of Zurek 2005 because it is consigned to the portion of reasoning which Zurek side-lines in this paper. That is, the neglecting of off-diagonal elements plays a role in the process of identifying a preferred basis, and not in the broader derivation of the Born rule.

The next question, then, is what role does the neglecting of off-diagonal elements have in providing a preferred basis?

As discussed in chapter 2, it is the suppression of a density matrix in one particular basis as a result of entanglement with the subsystem's environment which is the distinguishing feature of a preferred basis picked out by environmental decoherence. Indeed, this seems to be at the heart of what Thébault & Dawid regard as the reliance of Zurek's derivation on the neglecting of off-diagonal elements. They write:

> “The off diagonal elements of this matrix would correspond to coherence phenomena which render any separation of branches impossible. Compare this to a reduced density matrix without the off diagonal elements… [which] we *can* interpret as a proper mixture of pure states and so taking it ontologically seriously would seem to imply the discrete branching structure which the Everettian requires.” (Thébault & Dawid 2015, pp. 1565).

Thébault & Dawid clearly regard neglecting the existence of interference phenomena and histories which display them as a key step in obtaining an Everettian branching structure, and the Born rule to go with it. I think, though, that they misunderstand the intended character of this branching structure. It is not intended to be a branching structure of perfectly classical histories, from which all interference phenomena or small-scale interactions between histories have been ruled out by fiat, and whose similarity to the real dynamics is supported by the neglectability of off-diagonal elements in reduced density matrices.

Instead, the significance of the preferred basis given by decoherence, is as a basis in which the wavefunction can be decomposed, to produce a set of histories in which interference phenomena, though they do occur, will occur very infrequently on scales large enough to be noticed by inhabitants of those histories. This is not meant to give a set of histories which are perfectly separable in the way in which mixed states are. It is simply a means of warranting the treatment of a particular basis decomposition of the wavefunction as different from others. Different because under this decomposition interference phenomena are suppressed to a large extent, though not perfectly.

Clearly characterising this criterion does presuppose the Born rule, and is one aspect of the much-discussed circularity of Born rule derivations. I do not believe, however, that it relies on any form of ontological prejudice. Histories in which interference phenomena (even very substantial interference phenomena) occur are accepted features of the fundamental Everettian ontology. Some of these histories will not be sufficiently well separated to display quasi-classical dynamics, or contain quasi-classical entities, but this does not mean that they are neglected from the Everettian ontology. Such histories do have Born rule probabilities associated with them, and if they did not this might create significant problems for the Everettian account of probability.

Consequently, I think that Thébault & Dawid are mistaken in their charge of ontological prejudice. The importance of the neglectability of off-diagonal elements is to show that the basis in which a reduced density matrix is written will display quasi-classical dynamics with little interference phenomena. This has nothing to do with neglecting the existence of these phenomena within the Everettian ontology.

5.9 Conclusion

In this chapter I have responded to claims of incoherence within Wallace's Everettian quantum mechanics levelled by Thébault & Dawid 2015. I have argued that these objections rest on a misunderstanding of the character of the Born rule derivation offered by Zurek and Wallace, and the sense in which this derivation relies on the presupposition of the Born rule.

Thébault & Dawid began their paper by noting pre-existing arguments which establish the circularity of this form of Born rule derivation. This circularity is undeniable. I think it is a mistake however to view this as a fatal problem for all accounts which use this derivation. In particular, I believe that the role of this derivation within Wallace's Everettian quantum mechanics should not be seen as being to derive the Born rule purely from the theory of linear quantum mechanics without any presupposition of classical structures. I believe such a derivation to be impossible. Instead, Zurek’s derivation should be regarded as showing how the Born rule emerges for emergent quasi-classical structures which can be picked out of the wavefunction. The fact that these structures themselves depend for their physical significance upon probabilities, and so on the Born rule, does not undermine the existence of these structures as real patterns within the wavefunction.

Despite my belief that the specific concerns developed by Thébault & Dawid in this paper do not present a serious problem for Wallace, I think that they are right to ask for empirical evidence for the success of approximations and empirical structures within Wallace's interpretation. If such approximations and robust structures cannot be shown to be underpinned by empirical evidence of their reliability (using Wallace's interpretation of the physical significance of quantities) then this would seem to pose a significant problem for Wallace. This will be discussed in the next chapter.

In this chapter I have commented that Thébault & Dawid do not seem entirely clear as to the connection between reduced density matrix conceptions of decoherence, and the dynamics of histories. This is unsurprising given that, as I made clear in chapter 3, the connections between reduced density matrix conceptions of decoherence, and many histories conceptions of decoherence, are not at all easy to pin down.

In making sense of the connections between reduced density matrices and histories in this chapter, I have generally assumed a branching structure of histories which form a history space which approximately fulfils the medium decoherence criterion. I feel justified in doing this as Wallace assumes this to be the character of the history space in his interpretation, and Thébault & Dawid give no direct indication of wanting to question it. In a sense, however, this may have been uncharitable to Thébault & Dawid. Many of their concerns about the assumption of separability between histories could be interpreted as questioning the assumption of a history space which approximately fulfils the medium decoherence criterion. In this chapter I have taken these concerns as only regarding the types of small scale or rare interference phenomena which I believe can plausibly be accepted without undermining Wallace’s claim that medium decoherence approximately holds.

This of course leaves the obvious questions of whether the history space which we occupy really does approximately fulfil this criterion? And what the empirically grounded physical significance of this assumption is?

To answer these questions will require far more detailed consideration of history spaces, the medium decoherence criterion, and Wallace's assumed branching structure. This will be the subject of the next chapter.

6 Interpretation Neutral Justifications for Medium Decoherence

In chapter 2 we saw how the physical phenomenon of decoherence gives rise to effective suppression of interference phenomena, in a way which seems to hold exciting clues to resolving important aspects of the quantum measurement problem. Following on from this, we looked at theoretical criteria based on the phenomenon of decoherence.

The diagonalisation of the RDM of a system of interest is the weakest of these criteria. This criterion, however, is sufficient to ensure the short-term suppression of interference phenomena in local systems which are most commonly discussed in presentations of the physical phenomenon of decoherence. It is in terms of this criterion alone that Schlosshauer 2007 presents his solutions to aspects of the quantum measurement problem.

A stronger criterion is that of the consistency of a history space. This criterion amounts exactly to the absence of interference phenomena within that history space. It differs from RDM diagonalisation, in that it concerns a history extended in time, rather than an instantaneous state of the physical subsystem.

As discussed in chapter 3, however, both of these criteria lack robustness. To ensure the robust absence of major interference phenomena desired by many interpretations of quantum mechanics, not only in local systems but when considering the history space of the entire universe, the criterion of medium decoherence is needed. Medium decoherence entails both the weaker criteria, and also entails a branching structure of histories.

The result is that, while environment induced suppression of local interference phenomena, and the everyday appearance of classicality, only require the RDM diagonalisation conception of decoherence, the far stronger criterion of medium decoherence is needed to ensure that this absence of interference phenomena really applies more generally. This makes justifying the use of the stronger medium decoherence criterion very important for those realist interpretations of quantum mechanics which accept unitary linear Schrödinger dynamics without adding any form of collapse postulate, such as the Everett interpretation.

Later chapters will look at what these interpretations can offer in terms of interpretation specific justifications for the assumption of medium decoherence. In the present chapter, however, we will look at the possibility of justifying this assumption purely in terms of the linear quantum formalism and existing empirical evidence, without adding any interpretation specific assumptions or methodology. Although several of the justifications I will consider are suggested by Wallace in his works on Everettian QM, I believe these justifications (if they were successful) might also work just as well within other interpretational frameworks.

I will argue that in general these justifications are not sufficient to make the bold cosmological claim, that the quasi-classical history space we occupy possesses a branching structure, well supported. The most promising justifications are themselves bold and controversial cosmological claims, which, though they may be true, should not be smuggled in to interpretational frameworks as unconsidered implicit premises (as I believe they often are). The next chapter will then turn to focus purely on Wallace's Everettian QM, and examine the interpretation specific justifications it can offer for these cosmological claims.

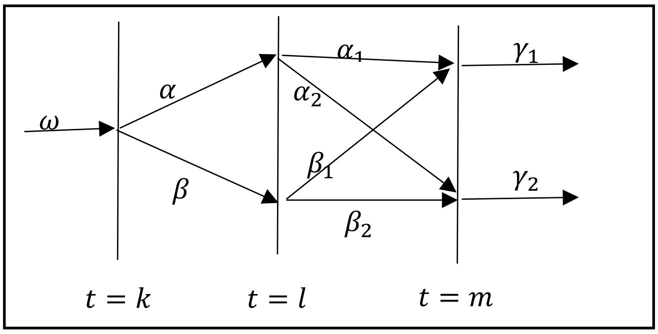
6.1 Empirical Evidence

The most obvious suggestion for a way to justify the medium decoherence assumption is by appeal to the apparently robustly classical dynamics of the world with which we are familiar. If the history space we inhabit does form part of a history space which fulfils the medium decoherence criterion, this will ensure the lasting and reliable consistency of local subsystems, and so ensure the robust approximately classical dynamics which the world around us seems to display. We have seen that for interpretations which do not use any form of collapse postulate, the medium decoherence criterion is required in order to reliably ensure robust and persistent quasi-classical dynamics. A very natural response is to consider the world with which we are familiar on a day-to-day basis and the classical dynamics it appears to robustly display and to conclude on this basis that the medium decoherence criterion must obtain.

I have already hinted at the reasons why I believe this argument to be flawed. In short, this is because, while medium decoherence is required to rule out the possibility of history recombination, and the failure of persistence of quasi-classical entities, it is only short-term local consistency (as captured by RDM diagonalisation) which is responsible for easily observable cases of interference loss. In other words, while medium decoherence is necessary for robust and persistent consistency and associated quasi-classicality, interference suppression experiments, in their general presentation, only rely on the short-term local consistency captured by the diagonalisation of a local system's RDM. I will argue that this far weaker conception of decoherence could perfectly well exist without a history space fulfilling the medium decoherence criterion. This will occur if initially separated histories later come to recombine with one another. This section will recap these points and consider the exceptional cases in which records of recombination could be found. The conclusion will be that, although records capable of establishing whether or not the wavefunction we inhabit is medium decoherent are in principle possible, they are in practice extremely difficult to obtain, and nothing about our experience of quasi-classicality in day-to-day life amounts to such a record.

To begin with, let us remind ourselves of the Wigner's friend thought experiment introduced in chapter 2. We considered a physics laboratory bounded by an impermeable box in which Bob is engaged in measuring the x-spin state of a spin half particle which has previously been determined to be in a superposition. Outside this lab is Alice who, after allowing time for Bob to complete his measurement, then undertakes a measurement of the entire laboratory designed to measure the z-spin state of the original particle. This second measurement, of course, does not commute with the original measurement which Bob made. This is not a breach of the uncertainty principle, however, as the process of the second measurement will necessarily render the result of Bob's original measurement inaccessible to both Alice and Bob.

To see why this overwriting of Bob's records is necessary, consider the following schematic representation of the history space of Bob's lab, in a quasi-classical basis (for medium-sized dry goods this will be the (approximate) position basis in which their reduced density matrices are diagonalised):



- At $t = k$, Bob makes his measurement.
- Bob's measurement yields two histories α, and β.
- At $t = l$ Alice begins applying a Hamiltonian to the lab which will over time render all records of the result of Bob’s measurement in the two histories to be only contained in the phase correlations of the lab, and not contained in any classically measurable property of it. This must necessarily involve some quantum process, which will overwrite the observables which previously encoded the measurement result (such a process is known as quantum erasure (Walborn, et al., 2002; Yoon-Ho, et al., 2000)).

- At $t = m$ this process is complete. The x-spin state record has moved from the measurable state of the lab, to phase correlations alone. The z-spin state can then pass from phase correlations back to the measurable state of the lab, allowing Alice to effectively measure this state by viewing the state of the lab.
- The histories after $t = m$ are distinguished only by the record they have of the z-spin state, and will have no record of results obtained from any x-spin measurement. As these histories are products of recombination, there is not really any fact of the matter about what x-spin records they once contained.

This example is simplified in several ways. In addition to the idealisations which go into this thought experiment, there is also the fact that for ease of discussion we have assumed that only two history branching events take place throughout the entire history of the lab. Nonetheless, although this process is bound to involve far more histories than are shown here, the general process would be much the same (just far more complicated to represent).

Let us now consider what empirical evidence within this process there could be that recombination had occurred. The most obvious evidence is that possessed by Alice. She obtains the final measurement result, which would have been impossible without the recombination of histories within the lab. If this experiment were run multiple times, she could on some runs open the door to the lab immediately after Bob's measurement, to confirm that the dynamics within the lab do indeed contain measurable properties which depend on x-spin measurements, and not on z-spin measurements prior to recombination. If, therefore, we had records of the dynamics of large-scale systems which we understood well enough to predict instances of recombination, and could remain unentangled from the system ourselves, then, like Alice, we could test the occurrence of such recombination against those records. Unfortunately we do not have such records. We rarely have sufficient understanding of macroscopic systems to predict their quantum dynamics, and are never realistically able to hold ourselves in isolation from such systems, let alone reproduce the same conditions for multiple runs. Such experiments are only achievable for very small systems which cannot really be said to offer a large enough environment to enable the physical process of decoherence.

When introducing decoherence, Wallace 2012 briefly uses a model based on two spin half particles. Entangling the state of the first particle with the state of the second by some degree of freedom has an effect on the results of measurements performed on the first particle very similar to decoherence. No one doubts that for such toy examples recombination of the type discussed in this thought experiment is possible, and such cases have been observed experimentally. However, the size of the environment used in such cases (the spin state of a single particle) is generally accepted to be far too small an environment to produce anything that could really be described as decoherence. Performing such experiments on very large scales will almost certainly always be beyond our capabilities.

The important question to ask, when considering whether the world we occupy is really medium decoherent, is not whether Alice can obtain empirical evidence of the history recombination which occurs in this thought experiment, but whether Bob can. We cannot generally hold ourselves in isolation from a large decohering environment, as is required for Alice’s measurement of recombination, and in those cases where we can (e.g. because the system is outside our past light-cone), sufficiently detailed knowledge and prediction of the system and environment to show history recombination within it will be beyond our technical capabilities. Our records of recombination will therefore resemble those of Bob (a part of the decohering, and recombining system), rather than those of Alice (watching from outside).

A significant complication for answering this question is that it is rather unclear whether a structure clearly identifiable as Bob would survive the processes performed on the lab by Alice in order to obtain her measurement. Any degree of freedom of the lab (or the subsection of it that is Bob) which has become entangled to the x-spin state of the particle will necessarily be overwritten during this process. Given the very large number of degrees of freedom this is likely to cover, the process may well be very disruptive to Bob as a pattern within the lab’s wavefunction. If Bob does not survive the recombination process, then clearly he is not in a position to establish that the recombination has taken place. For now, then, we will assume that the recombination disrupts aspects of Bob's state only where necessary, and that there is still an identifiable pattern corresponding to Bob throughout the recombination process. The question then is whether Bob could establish in such a case that recombination had taken place.

The answer, at least in this idealised thought experiment, is that he could. The easiest way in which he could do this, would be to make a record prior to the recombination process, not of the x-spin measurement which he makes, but of whether or not he has made a successful x-spin measurement. As this record would not be entangled to the x-spin state itself, it could survive the recombination process, unlike any record of a particular result. After recombination, Bob could effectively measure the z-spin state of the particle, either by direct measurement of the lab environment, or by simply asking Alice what her result was after she made her measurement and the lab was unsealed. If he was confident in both his successful measurement of the particle’s z-spin state, and the veracity of his record of having made an x-spin measurement, then, knowing these two measurements to be non-commuting, he would have clear evidence of history recombination.

A second way in which Bob could establish the recombination had taken place would be to know the starting conditions of the lab and its Hamiltonian throughout the period of the experiment very precisely. Armed with this information, he could use Schrödinger's equation to model the time evolution of the state of the lab and see precisely the behaviour of all histories throughout the period of the experiment including their recombination. Even in the context of a system the size of Bob's lab this is vastly beyond our technical capabilities to record or model with any accuracy[13]. Given that in realistic cases the systems involved in decoherence processes are unimaginably larger, there is no realistic prospect that we could establish that recombination did or did not take place in histories we occupy in this way.

In the case of this thought experiment, then, Bob is in a position to establish by empirical evidence that recombination has taken place. He could not establish that it was going to take place, except by knowing the Hamiltonian and state of the system very precisely. The question, then, is what records indicating that recombination of previously separated histories would look like in the less idealised conditions we are generally familiar with, and whether we have such records.

Unfortunately, such records will prove extremely difficult to find. The reason for this is that, unlike our thought experiment, the timescales of recoherence processes will typically be extremely long. Analysis by Zurek 1982 suggested that a typical recombination process may take place over a timescale greater than the lifespan of the universe. We will consider this fact further in the next section. For now, though, the important point is that, even for exceptionally short recoherence times, the period separating the original history branching and the subsequent recombination is likely to be far longer than the period of human history. There would certainly be no point in making a careful record of which property of a spin half particle was measured today, and waiting patiently in the hope that we could someday show that a non-commuting spin property was later measured. The chances of this happening in our future histories at all are low, and the

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[13] Bob must first have some means of encoding the complete state of the lab. Doing this inside the lab would produce a kind of fractal encoding problem as, unless the apparatus that Bob was using to model the lab was held in isolation from the rest of the lab, it would also be necessary to encode the state of the encoding of the lab. On the other hand, if this simulation were held in isolation from the rest of the lab, then it would leave Bob none the wiser about the character of the wave function he occupied.

chances of it happening within a time period for which we could compare the records are virtually non-existent.

In chapter 2 I argued that diagonalisation of a local system's RDM alone is sufficient to provide the standard responses to aspects of the quantum measurement problem, which decoherence is generally thought to provide, without any requirement for the system to be part of a history space fulfilling the medium decoherence criterion. This is unsurprising given that many less formal treatments of decoherence used in discussing these results simply identify decoherence with the diagonalisation of a system's reduced density matrix. Despite this, RDM diagonalisation does not guarantee robustly persistent quasi-classical dynamics. The question then arises whether we have empirical evidence that quasi-classical dynamics really are robust in the world around us. Or, in other words, whether recombination of previously separated histories occurs in the locally quasi-classical history space which we occupy.

In this section I have considered three possible situations in which an observer within a history might find themself. These are the situation of Alice outside of a history branching event, the situation of Bob after the initial measurement but prior to Alice's measurement, and the situation of Bob after Alice's measurement.

Alice is in the best position to produce records demonstrating the occurrence of recombination. Unfortunately, however, in realistic cases of large-scale decoherence phenomena we are never likely to be in Alice's position.

Prior to Alice's measurement, Bob cannot establish that the history he occupies is going to undergo recombination unless he knows with an extremely high level of accuracy not only the exact state of the history he occupies, but the state of the history with which recombination will occur, as well as the Hamiltonian governing the labs evolution. There is no experimental process he could perform on the history he occupies that would reveal that the history he occupies has not permanently branched.

On the other hand, if Bob makes appropriate records prior to Alice's measurement and then compares them to records he can obtain after the measurement, then he would in principle be able to establish that recombination had taken place. I have argued, however, that the ease with which this is possible is a product of the simplifications that have gone into this thought experiment. The timescales over which recombination typically occur are so long that obtaining effective measurements of quantum states prior to the original history separation, and finding records of what states were physically recorded at the time of separation, are both very unlikely to be possible.

I therefore conclude that, although empirical evidence of recombination events is in principle possible (at least after the fact), we currently do not possess any evidence capable of establishing whether such recombination events have ever occurred or not, and most likely we never shall do. I will now turn therefore to consider other reasons why we might believe that recombination events like these do not occur.

6.2 The Very Long Timescales of Recoherence

One popular suggestion is that the very long timescales over which recombination would happen are themselves clear reason to believe that the possibility can be ignored. That is, if recombination will take a time period longer than the lifespan of the universe, then we are free to neglect it as an aspect of our physical theories and assume medium decoherence. Zurek himself raised the suggestion as far back as 1982, correctly identifying that quantum recoherence times are of the Poincaré type, rising as the factorial of a system's degrees of freedom (or more precisely the degrees of freedom to which a quantum property becomes entangled). As the size of the closed system in which decoherence is taking place tends to infinity, “…the recurrence time becomes infinitely long, and in this sense the decay of the off-diagonal elements may be considered irreversible”. Unfortunately, this faces two major problems as a justification for assuming the medium decoherence criterion to hold for the systems which make up our environment.

The first problem is that Zurek's analysis is based on the time taken for the total amplitude of recombined histories to reach a certain (very low) level. In other words, Zurek regards recombination as having taken place for a system when the sum total of probability of histories of the system in which recombination has taken place reaches a certain level. This means, of course, that there will be some histories in which interference between previously separated histories takes place long before Zurek would regard the system as displaying recombination. Defining system recombination with reference to a particular threshold amplitude like this is necessary because extremely low amplitude histories in which recombination takes place will begin to appear from immediately after decoherence has taken place. These histories which display recombination will appear for any system which does not have an infinitely high energy potential preventing the erasure of records within the environment (i.e. any realistic system). There is therefore a quantum probabilistic aspect to the time taken for recombination phenomena to appear within any particular history. So, while typically recombination of a history may take an extremely long time, for any particular instance of decoherence there is no guarantee that we occupy a history in which recombination will take such a long time to occur.

The second problem is similar. It is that the extremely long timescale, over which significant levels of recombination is likely to take place, is calculated based on the assumption of a random, and initially uncorrelated, environment. As we saw in chapter 4, Kastner 2014 points out that such a model is clearly not an accurate representation of an environment, unless some form of collapse postulate is assumed to apply. Kastner's attempts to develop this problem are unclear, and generally not persuasive, but in the context of considering typical recombination times, and possible justifications for the medium decoherence criterion, her underlying point becomes important. Particular arrangements of elements within the environment may in some cases make the overwriting of records to produce a substantial total amplitude of recombination a far faster process. Records contained within the physical state of the environment are overwritten all the time, shifting to become purely encoded within phase correlations, and so inaccessible. Which elements will undergo this process is very difficult to predict, and will depend on particular details in the state of the environment, and the phase correlations between elements of that environment. Modelling the environment as an uncorrelated set of random states may be adequate to give a typical time period for substantial amplitude recombination, but there will be a broad probabilistic distribution of time periods. This probabilistic distribution does not represent any form of quantum Born rule probability, but simply our ignorance of the precise state of the environment and the way in which that state will evolve.

Having shown that for any particular set of histories there will be a probabilistic distribution as to the time period taken for recombination between those histories, for both quantum and epistemic reasons, I must now explain why I regard this as undermining any appeal to long recoherence times as a justification for the medium decoherence assumption.

Nothing I have argued so far suggests that the typical recombination time following a particular quantum event for the set of histories produced will be anything other than extremely long (possibly even longer than the lifetime of the universe). Given the staggering size of this estimate, it might be thought that it doesn't matter whether the time taken for recombination for a particular history should be represented as a probability distribution, because the portion of that probability distribution that represents recombination of the history within a time period of interest to us is still incredibly small.

The reason why I believe these probability distributions to be important is that, when considering whether the history we occupy will undergo recombination events during the course of our lifetime, it is not just the set of histories produced by a single quantum event which is of concern to us. There will be a probability of recombination associated with every quantum history branching event which has taken place at any time during the evolution of the universe prior to this point. The probability of recombination associated with any individual event may be incredibly small, but the total number of history branching events prior to this time is astronomically large.

I do not claim to know what the resulting probability of the history we occupy undergoing a recombination event during the course of our lifetimes is. Nor do I believe it could easily be established. The reasoning offered by Zurek 1982 is, as I have already indicated, only a very rough approximation and based on very sweeping assumptions. The total number of history branching events over the history of our universe cannot be easily established either, in fact it doesn’t seem to have a well-defined answer. It only makes sense to speak of histories in the context of a particular basis. I have been tacitly assuming a quasi-classical basis in which the RDM’s of quasi-classical entities are diagonalised. During the early stages of the universe, however, it does not seem that there are entities we could easily identify as quasi-classical, and so we seemingly could not define a basis of quasi-classical histories, or say with any confidence how many quantum events would have occurred which led to history branching.

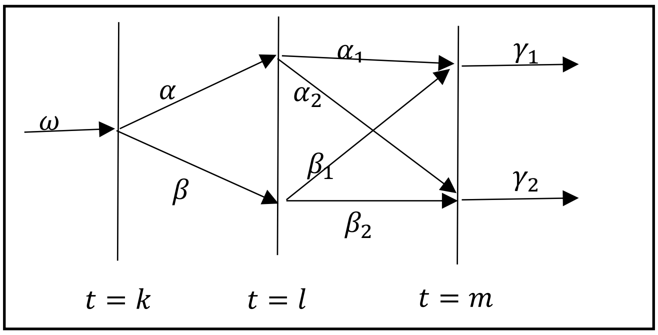
It is possible that recombination events typically occur in the histories we occupy multiple times every day. Alternatively, it is possible that no recombination events of previously decohered systems have taken place at any time over the history of our world so far. However, even if recombination events between histories are extremely rare over the course of the universe to date, it is worth noting that this won't remain the case forever. The probability of recombination events between the histories produced from a particular quantum event will drop very sharply during the initial decoherence process (assuming sufficiently many randomly uncorrelated degrees of freedom in the environment), but over time it may well begin to rise again as some of the records left behind in the environment begin to be overwritten. On the other hand, the number of history sets produced by quantum events between which recombination events are possible will only rise with time (assuming that no recombination events take place). The result is, that while it cannot be established whether recombination is a frequent event in the histories we occupy right now, the frequency is only going to rise with time as the number of past branching events increases. This trend will continue so long as recombination events remain rare.

The next section will look at an example designed to make sense of what a real-world instance of recombination might look like for the histories involved, and individuals who resided within them. This is intended to make clear why such large-scale recombination events would most likely not be noticeable to people residing within the histories, and why they nonetheless represent an important departure from classical dynamics for the histories concerned.

First, though, I wish to point out that the discussion in this section has still made an important, and potentially suspect, tacit assumption which encourages the belief that medium decoherence is likely to hold. Following the existing literature, discussion so far has been phrased in terms of recombination of previously separated histories, which arise from some prior history branching event. Though I have fallen in with this for the purposes of discussing recombination in this section (and will continue to do so in the next section), it is worth noting that history merger events of the type we are concerned with in discussions of recombination do not necessarily follow from history branching at an earlier time. To suppose that they do, is to implicitly suppose that there was some time prior to the present at which the wavefunction only projected onto a single history basis vector in the space of quasi-classical histories. In other words, by assuming that all histories with which the history we occupy could merge must have branched from our own history at some previous point, we are making a very sweeping assumption about early states of the universal wavefunction. If this assumption does not hold, this might make recombination events extremely common over the course of our history. This assumption will be discussed further in a later section.

6.3 A Simple Example

To try to make clear what a practical instance of recombination might look like in physical terms, and the issues it would present, consider another example with the same schematic form as the Alice and Bob case considered earlier:



- $t = k$ is a time very far in the past. At this time some physical event causes the original history to branch into two histories which become macroscopically distinct in some way. The example I shall consider here is interactions between atmospheric particles leading to a small but macroscopic difference in the spatial distribution of falling raindrops. This example, and the single pair of histories it is assumed to produce, is chosen for its relative simplicity, but still seems perfectly plausible as an example of recombination. In order that recombination may have a significant effect on future dynamics, we also need to assume that the histories produced both have amplitudes of the same order of magnitude. Within minutes the macroscopic traces of the original event are lost. Decoherence has however encoded the event in a very large number of disparate degrees of freedom and the two histories remain distinct.

- Billions of years later, at $t = l$, a physicist performs a measurement on a spin half particle. The two histories α, and β, have remained macroscopically indistinguishable, and this event occurs simultaneously in both. Both histories then split yielding histories α_1, α_2, and β_1, β_2. The microscopic encoding of the raindrop positions still persists, but is being slowly overwritten with time.
- At $t = m$, some time after the measurement performed at $t = l$, the last microscopic degrees of freedom encoding the original raindrop distribution pattern are finally overwritten. The branches α_1, β_1, and α_2, β_2, now coincide within the history space and will recombine, most probably producing interference phenomena.
- The Born rule probability of a spin up measurement at $l < t < m$, will be $|\alpha_1|^2 + |\beta_1|^2$. At $t > m$, it will be $|\gamma_1|^2$. But due to interference between the two histories, unless the phases of the two history state vectors α and β agree perfectly, then $|\alpha_1|^2 + |\beta_1|^2 \neq |\gamma_1|^2$. That is, the total Born rule probability of histories in which a spin up measurement took place at $t = l$, may change at $t = m$.

First of all, let us consider that final result. A physicist makes a quantum measurement. We assume that probability within a particular history can be understood as an amplitude measure over successor histories divided by the amplitude of the immediately preceding history, as indicated by the Born rule. Thus, the probability of our scientist making a spin up measurement in history α is $\frac{|\alpha_1|^2}{|\alpha|^2} = P_1$ and, given that the experiments performed on the two histories are assumed to be effectively identical in history β, It will be $\frac{|\beta_1|^2}{|\beta|^2} = P_1$ also. These are the probabilities of spin up measurements having been made immediately following the measurement. After the recombination event however the probability amplitude associated with histories in which spin up was measured will be $|\gamma_1|^2 = P_1{}'$, and depending on the phase amplitudes of the combining histories it is very likely that $P_1 \neq P_1{}'$.[14]

On an Everettian account this seems to mean that the amplitude associated with histories in which spin up was measured will change at a time considerably after the measurement itself. If probability is to be identified with branch amplitude, this would seem to mean that the sum probability of histories whose internal dynamics originate from a spin up measurement will change considerably after that measurement is made.

Similarly, on the Bohmian account, there will be a set of initial trajectories which, under the influence of this form of wavefunction, will correspond to the results of a particular spin measurement between $t = l$ and $t = m$, and then rearrange themselves so as to follow the trajectory that would be expected if the other measurement has been made at $t = l$, after $t = m$. In this way, a very similar change of probability after the event will be produced.

Other ways of interpreting probability in these cases may be available within each of these interpretations, which would produce less counterintuitive accounts. Probability and the metaphysics of histories are not simple in either of these interpretations. Unfortunately, specific Bohmian responses are beyond the scope of this thesis, but would be a fruitful area for future research. Later chapters of this thesis will look in more detail at the interpretation specific responses allowed by Everettian quantum mechanics. For now, though, I wish to highlight that this disconcerting behaviour is at the heart of why recombination is thought to

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[14] For the sake of simplicity, I have assumed the total amplitude of all histories represented here to be normalised $|\omega|^2 = 1$.

undermine the robust quasi-classicality of consistent histories approaches, which do not rely on the further claim of medium decoherence.

The need for medium decoherence is often discussed in terms of a need to preserve the applicability of classical axioms of probability. In fact, though, I believe the issue goes beyond this. There is no axiom of probability dictating that, if Bob tosses a coin at time $t1$, the chance that he gets heads at that time will still be the same as the chance that at a later time $t2$ he will occupy a future in which the coin came down heads at time $t1$. This principle is so basic to the assumed dynamics of our world that as far as I am aware no one has ever formulated it as an axiom of probability[15]. Yet for a history space which does not fulfil the medium decoherence criterion, the possibility of interference between recombining histories seems to directly call this principle into question.

It is also worth noting in this case, as previously emphasised, the great difficulty of obtaining records of the profound effects of recombination. In both histories γ_1 and γ_2, records of one measurement result will exist, with no indication that that measurement result might ever have changed. It will be virtually impossible for people occupying either of these histories to establish that histories in which records have seemingly always indicated a particular result now exist with higher amplitude than they did immediately following the original measurement.

I have argued in this and the previous section that the very long typical time periods taken for recombination do not give us a reason to believe that recombination events are particularly uncommon at the present time. Even if, as is customary, we implicitly assume that at some previous time the universal state vector projected onto a single consistent history vector (or at least very few vectors), the extremely large number of history branching events that will have taken place since that time still means that

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[15] Though this is certainly not a standard axiom of probability, a principle somewhat along these lines is discussed by Belnap & Green 1994.

the probability of recombination events at this time cannot be shown to be negligible. If this assumption is not made, then there is no reason to suppose that the frequency of such events should not be as high as, or even higher than, the frequency of history branching events at the present time. If such recombination events do take place on a regular basis, this will substantially disrupt the quasi-classical character of the histories we occupy, but will not generally provide us with records capable of showing that such events are taking place.

The next three sections will look at responses to this issue suggested (though not developed in much detail) by Wallace 2012. I will argue that the first of these suggestions is of no real value as a justification, and can be disregarded. The second (or at least one possible reading of it) is more persuasive, but relies on another cosmological claim about our universe, which is just as controversial as assuming medium decoherence itself. The third suggestion is the most interesting, and potentially fruitful. Its viability however, will depend on the interpretational and conceptual framework within which it is applied.

6.4 The Intuitive Locality of Dynamics

I stated in chapter 3 that Wallace 2012 introduces medium decoherence with very little discussion of its significance or relation to the alternative criteria of consistency and RDM diagonalisation. This is true, and when he does this in chapter 3 there is no critical discussion of these criteria or the possibility of recombination. In chapter 9, however, Wallace turns to look at the time asymmetry of classical dynamics, and the question of how this emerges from the fundamentally time symmetric dynamics of the unitary quantum theory. In this chapter Wallace never discusses recombination directly or any connection to the medium decoherence criterion essential for formal aspects of the branching structure developed in chapter 3. Nevertheless, Wallace is clearly aware that the time asymmetry of this structure is vital to his interpretation and that such an asymmetry is a surprising feature of an interpretation which claims to originate from simply taking time symmetric linear quantum mechanics at its face value.

In chapter 9 Wallace therefore makes some attempt at sketching a justification for the introduction of this time asymmetry. He offers sketches rather than fully developed arguments. He writes: “In any case, my purpose here is not to provide detailed dynamical hypotheses but to identify those hypotheses that we need.” (Wallace 2012, pp. 347). In other words, Wallace's aim is to provide some indication of the hypotheses that would ground belief in the kind of time asymmetric branching structure which would provide histories containing real and robust quasi-classical dynamics, and is not to present or support a particular detailed physical hypothesis which would account for how such a time asymmetry came about.

The next three sections will look at and try to develop arguments that Wallace identifies, and review their significance.

The first of these arguments is based on the intuitive locality of macroscopic dynamics. Wallace writes:

> “Since macroscopic properties are typically local, and correlative information tends to be highly delocalised, heuristically one would expect that generally the details of the correlations are mostly irrelevant to the macroscopic properties – only in very special cases will they be arranged in just such a way as to lead to longer-term effects on the macroproperties.” (Wallace 2012, pp. 346).

The argument here seems to rely heavily on our intuitive belief in local entities whose dynamics are determined locally. This generally seems to be an effective principle when it comes to our day-to-day experience of the world around us, at least as far as macroscopic objects are concerned.

The suggestion is that recombination events and macroscopic interference phenomena would involve the properties and dynamics of macroscopic entities being affected by nonlocal factors and nonlocal changes. Given that this is in sharp contrast with our intuitions about the behaviour of quasi-classical macroscopic entities, it seems that we have prima facie reason to believe that such events do not take place, or at least do not take place often.

It is certainly true that recombination events will involve changes being brought about as a result of nonlocal phase correlations. We saw in the previous example how the final records of an event being overwritten (wherever in the universe those records might be) affected the probabilities of the measurement performed by a physicist, and the results of that measurement. These records are likely to be in the form of local entities with local characteristics, and in the course of this recombination event those characteristics are changed as a result of a profoundly nonlocal influence.

On the other hand, it seems reasonable to doubt whether our intuitions of locality can be trusted in the context of interpretations of quantum mechanics. For those length scales over which quantum mechanics is generally seen to apply, it is an intuition that we have already been forced to abandon. The properties and dynamics of electrons are very often affected to a substantial extent by correlations with other particles at a distance from the electron in question.

Given that the approach of realist interpretations of quantum mechanics, such as the Bohmian and Everettian, is to extend quantum mechanics from being a purely microscopic theory to include the macroscopic world as well, it seems unreasonable to assume that the intuitions of locality we have already had to give up in the microscopic domain will remain tenable in the macroscopic context.

Moreover, as technology improves, it is increasingly unclear that such nonlocal phenomena can really be consigned to the purely microscopic. Tests of Bell's inequalities such as Aspect, Grangier, and Roger 1982 have led to demonstrations of entanglement and interference phenomena on ever larger scales such as Ockeloen-Korppi et al 2018. These show that, at least in those cases where decoherence with respect to the wider environment in general is prevented, size seems to be no barrier to nonlocal entanglements.

As I have said, Wallace certainly does not regard this argument as offering any very strong evidence for time asymmetry. Given the arguments I have already provided concerning the extreme difficulty of obtaining records capable of providing evidence for or against the occurrence of history recombination, I would go so far as to suggest that our intuitions on the subject are of little or no value as evidence of any kind.

6.5 Cosmological Arguments

Another argument which Wallace indicates but never fully develops relates to the extremely large and expanding environment within which records of branching events can be encoded.

> "These traces generally become extremely delocalised, and are therefore not erasable by local physical processes. In principle one can imagine that eventually they relocalize and become erased – indeed, this will certainly happen (on absurdly long timescales) for spatially finite systems – but it seems heuristically reasonable to expect that on any realistic timescale (and for spatially infinite systems, perhaps on any timescale at all) the traces persist." (Wallace 2012, pp. 346).

One aspect of the argument presented here is simply the argument from the long typical recombination times discussed in the previous section. I believe, though, that there may be a second argument which Wallace has in mind here[16].

The suggestion is that as a matter of fact the universe which we occupy is not a finite closed system (or at least not a finite system of constant size). This is true both in the sense that the volume occupied by astronomical entities within the universe has been shown to be expanding, and in that cosmic microwave background radiation emanating from macroscopic entities continues to travel out into empty space with no reason (as far as we know) why it should ever stop.

If the number of ways in which a past event is recorded continues to increase indefinitely over time, then this will progressively reduce the probability of recombination. Thus, if the universe were infinite, or expanding in such a way that records of past events could continue to multiply indefinitely, then it might be that the probability of recombination of a particular pair of histories continues to diminish over time at a sufficient rate that the total probability of a recombination event involving the history we occupy will remain negligible.

This is the strongest argument offered by Wallace 2012 for believing the consistent quasi-classical history space we occupy to have a truly branching structure. However, it is still very far from definitive.

Firstly, the claim that the universe is expanding in such a way as to allow for the multiplication of records in this way is a bold and potentially controversial cosmological claim. It seems likely that, in order to keep pace with the ever-growing number of past branching events, the growth in the number of records would have to happen very generally and very rapidly. Moreover, in order to prevent recombination there must not only be persistent or multiplying records of a fixed finite number of past events, but records of every further branching event as it takes place. This seems

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[16] I am grateful to Simon Saunders for bringing the second aspect of this argument to my attention.

to call for a remarkably persistent and rapid expansion in the degrees of freedom capable of recording past events, and it is far from clear that the expansion of our universe which we observe is sufficient for the purpose.

Secondly, if decoherence-based approaches to quantum mechanics really rely on this continuous expansion of recording degrees of freedom, then interpretations which rely on decoherence are relying on a bold and controversial cosmological claim. If this is really the case, then it is a fact about decoherence-based interpretations of quantum mechanics which needs to receive far more attention and discussion. This reliance on controversial cosmological facts certainly seems to be at odds with the description Wallace gives of his interpretation as being purely a matter of understanding the consequences of taking the linear quantum formalism seriously.

Thirdly, the relevance of this argument still seems to rest on the assumption that at some past time the total number of histories within the consistent quasi-classical history space was small. Or at least, some constraint preventing it from including a portion of high amplitude histories which already included this expanded number of degrees of freedom. The point is that, while the steady expansion of our past light cone offers a steadily growing number of degrees of freedom with which it is possible for past events to be entangled, it also offers a steadily increasing number of degrees of freedom which may have been in superposition states. If so, becoming entangled to these states would provide yet another source of histories involving ourselves, which may subsequently recombine. In short, expanding the number of degrees of freedom available to decohered systems improves the situation only if those degrees of freedom begin as, to some degree, simple unentangled eigenstates with no entanglement to each other, or the rest of our universe.

6.6 Past Hypothesis

The majority of Wallace's argument in chapter 9 is not directed at the general question of whether recombination events take place and if so with what frequency, but more specifically at what could give rise to time asymmetric branching dynamics. The answer Wallace gives is that, just like the time asymmetric macro dynamics which emerge from time symmetric statistical mechanics, the time asymmetry is imposed as a result of some form of boundary conditions.

Wallace considers at some length various boundary condition assumptions and the conditions needed to maintain them. He considers various formalisations of this requirement, but seems never to reach a formulation which he is entirely happy with. In general, he seems to believe that time asymmetry in both classical and quantum mechanics will depend on some assumed past boundary condition and the existence of dynamics which will maintain some important aspects of that initial condition.

Already in this chapter, I have raised the issue that the plausibility of the medium decoherence criterion will depend on the character of the universal wavefunction at an earlier time. This assumption, as well as the assumption that some (rather ill-defined) factor in the character of this earlier wavefunction will be maintained, are precisely what Wallace is attempting to introduce here[17].

The formulation of these assumptions which he seems most happy with are as follows:

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[17] Chen (Forthcoming) offers an ‘Initial Projection Hypothesis’ which is also intended to play the role of a past hypothesis within quantum mechanics. Chen’s proposal however does not seem like it will be of any help in securing the medium decoherence assumption. His proposal is to treat the initial state of the universe as a density matrix for an equal mixture of all quantum states compatible with a classical past hypothesis. Chen gives no consideration to histories or the possibility of history recombination following this starting point. It seems likely that this mix of states would make such recombination far more probable than if the initial state were taken to be any single component wave function of this initial density matrix.

> "**Simple Dynamical Conjecture** (for a given system with coarse-graining C). Any distribution whose structure is at all simple is forward predictable by C; any distribution not so predictable is highly complicated and as such is not specifiable in any simple way except by stipulating that it is generated via evolving some other distribution in time (e.g. by starting with a simple distribution, evolving it forwards in time, and then time reversing it)." (Wallace 2012, pp. 348).

Making sense of how this criterion applies to quantum mechanics is made more complicated by the fact that Wallace is attempting to formulate this criterion in a way which is applicable to those time irreversible coarse-grainings which make up the microscopic world in both classical and quantum mechanics. The issue is compounded by the fact that (as Wallace accepts) the notion of simplicity being used here is extremely difficult to pin down.

A first step in understanding Wallace's intention is to understand that, in the quantum context, coarse-graining here means coarse-graining of the wavefunction into (generally not unitary) histories. I think, therefore, a good summary of the quantum form of Wallace's conjecture here would be something like the following:

Quantum Simple Dynamical Conjecture [rephrasing]: Consider a wavefunction which can be coarse-grained into a particular structure of histories. Any wavefunction whose structure is at all simple is forward predictable by examining the internal dynamics of those histories; any wavefunction that is not so predictable is highly complicated and as such is not specifiable in any simple way except by stipulating that it is generated via evolving some other wavefunction in time.

The arguments offered by Wallace for this dynamical conjecture are those discussed in the previous two sections. As I have already argued that neither of these is particularly convincing, I think the justification of this dynamical conjecture remains in need of serious critical consideration.

The additional claim which Wallace introduces, is a direct assumption about the state of the wavefunction at an earlier time, very much along the same lines as the past hypothesis coined by David Albert for statistical mechanics:

> "**Simple Past Hypothesis (quantum version).** The initial quantum state of the universe is simple." (Wallace 2012, pp. 354)

Clearly, when this hypothesis is added to the Simple Dynamical Conjecture previously introduced, the result is that the history of the wavefunction of the universe is forward predictable in terms of a particular set of histories, and will not involve recombination events, as these could not be predicted in terms of the internal dynamics of those histories.

Of course, as I have already argued that we do not have a convincing reason to endorse the simple dynamical conjecture, I am unsurprisingly dubious about the simple past hypothesis as a means of guaranteeing the branching structure which Wallace needs. More generally, however, it does seem that Wallace is correct in maintaining that some asymmetric boundary condition must be employed if there is to be any hope of obtaining a time asymmetric branching structure from time symmetric linear dynamics. For now then, I offer the following as suggested forms of these criteria:

Revised dynamical conjecture: Consider a wavefunction of the universe, and a corresponding history space which contains quasi-classical structures with diagonalized reduced density matrices. If at a time the number of history states onto which the universal wavefunction projects within the space with significant amplitude is very small, and consequently the amplitude associated with recombination events between histories is also extremely small, then this state of affairs will persist to a reasonable degree over any reasonable time-scale that may be considered. This means that it is possible to forward predict the evolution of individual histories and their amplitudes in terms of the internal dynamics of those histories without any need to consider the wider dynamics of the history space.

Revised quantum past hypothesis: The initial quantum state of the universe projects onto a very small number of histories within the consistent quasi-classical history space.

I have already discussed reasons for believing some form of the dynamical conjecture presented here. Wallace seeks a precedent for his appeal to a past hypothesis, in the classical past hypothesis of Albert 2000. This may provide a precedent, but it is not in itself any form of argument for endorsing this major assumption about the structure of our universe. Justifying such an assumption, and understanding its implications, is not something that I believe can be done in an interpretation neutral way. For this reason, I will not deal with this question in this chapter apart from commenting that there does not seem to be any such interpretation neutral justification.

The next two chapters will abandon interpretation neutrality. I will focus on Wallace's Everettian quantum mechanics and investigate the question of how it is possible to justify the assumption that the history space we occupy fulfils the medium decoherence criterion within Wallace's interpretation. As part of this, I will give careful consideration to past hypotheses of the type used by Albert and Wallace and the question of how they are to be justified.

6.7 Conclusion

In this chapter I have looked at interpretation neutral reasons to suppose that the universal wavefunction might fulfil the medium decoherence criterion in a quasi-classical basis. Medium decoherence goes far beyond the simple diagonalisation of reduced density matrices of local systems for which we have direct empirical evidence. I have argued that it is very difficult to find any empirically based reason to believe that this strong criterion obtains. The most convincing justifications, discussed here, themselves rely on highly controversial cosmological claims about our universe, which seem themselves to stand in need of justification.

If the medium decoherence criterion cannot be assumed to obtain then, on any interpretation which accepts the reality of the universal wavefunction without collapse, there is the real possibility that the histories we occupy are interrupted on a regular basis by recombination events. Precisely what these events entail depends somewhat on the interpretation in question, but in any case they will represent a serious departure from the dynamics of classical physics which we generally assume apply (to a good approximation) to the world around us.

Though the drastically nonclassical dynamics involved in these events, and the overwriting of past records they entail, may at first sight seem implausible, these are just the consequences to be expected from standard linear quantum mechanics if taken at face value and applied universally. The reason we have for believing that such events take place is that it seems to be directly indicated by one of our best scientific theories. This is precisely the same reason that Everettians have long given for believing in the existence of many worlds whose presence we cannot usually detect.

There are special states that the wavefunction could occupy which would prevent these interference events. We have discussed several assumptions which may go some way to ensuring that our history space fulfils the medium decoherence criterion and does not display recombination. To do the job, these assumptions will need to ensure some form of past hypothesis concerning an earlier state of the wavefunction, and some assumption about the dynamics of the universal wavefunction which will ensure that some elements of the assumed past conditions still persists. Such assumptions include assuming the visible universe with which states can become entangled to be expanding in such a way as to provide many new uncorrelated degrees of freedom. And, as Wallace does, directly assuming the past state of the wavefunction to have been simple (in a quasi-classical basis).

Up until this point, I have done my best to make this thesis a general discussion of the issues associated with using decoherence in interpretations of quantum mechanics without a collapse postulate. While much of my discussion has focused on Everettian approaches to decoherence, because of the degree to which this predominates in the literature, I have done my best to make my conclusions more generally applicable. From this point on, however, I will turn to consider interpretation specific factors of relevance to the characterisation and applicability of decoherence. Unfortunately, I will not be able to deal in detail with all the interpretations of quantum mechanics which are featured in this thesis up to this point. I have decided to focus on Wallace's Everettian quantum mechanics as this is a well-developed and well-known interpretation, which deals extensively (and generally clearly) with issues related to decoherence. In the conclusion, however, I will make a few preliminary remarks about how Bohmian interpretations might deal with similar issues, which I hope might be an area for future research.

The next two chapters will look carefully at interpretation specific ways in which Wallace might seek to justify the medium decoherence assumption, and the implications for his interpretation if this assumption is dropped.

7 Everettian Justifications for Medium Decoherence

This thesis began by looking at (and seeking to clarify) various conceptions of decoherence. In general terms, I have explained why particular interpretations have come to rely on the strong, medium decoherence conception of decoherence, and explained why justifying the use of this criterion poses substantial difficulties. The previous chapter examined a variety of interpretation neutral reasons for believing that the medium decoherence criterion might obtain for systems with which we are generally interested. The conclusion was that, while the possibilities for interpretation neutral justifications of medium decoherence are certainly not hopeless, they are unpersuasive as it stands. This chapter will turn from interpretation neutral considerations to look in more detail at Wallace's formulation of Everettian quantum mechanics, to see whether this can offer any interpretation specific reasons to believe that the medium decoherence criterion should obtain. In particular, I will examine in more detail Wallace's suggestion that an appeal to the past simplicity of the universe's wavefunction state could be justified by making this condition a Humean law of nature.

I will argue that a Humean account of laws of nature is difficult to reconcile with Everettian metaphysics (at least as set out by Wallace). This is because the Humean account of laws describes them as regularities over a mosaic of locally instantiated particular properties. Everettian quantum mechanics, however, does not offer any such mosaic. While it is reasonable to suppose that some flexibility in this conception of a mosaic may be possible, I will argue that Wallace’s presentation of Everettian quantum mechanics still seems to rule out all plausible candidates.

Before getting to discussion of such justifications, however, I will begin by recapitulating the reasons why Everettian interpretations in general rely on decoherence, and then look in more detail at precisely the role it is playing within Wallace's formulation of this approach.

7.1 Two Problems of the Preferred Basis

In chapter 2 we looked at three problems which Schlosshauer sees decoherence as solving. By decoherence Schlosshauer seems to mean the diagonalisation of a local system's reduced density matrix as a result of interaction with its environment. This section will make clear precisely where the problems which Wallace is seeking to solve depart from those discussed by Schlosshauer, in a way that requires him to abandon Schlosshauer's criterion in favour of the medium decoherence criterion.

Using just this simple characterisation of decoherence Schlosshauer claims to solve three major aspects of the quantum measurement problem. As I commented in chapter 2, one of these, namely the generic problem of outcomes, does not really have very much to do with decoherence except in so far as it relates to the dynamics of a local system which is part of a nonlocal linearly evolving wavefunction which does not collapse.

I wish now to focus more carefully on the problem of the preferred basis, particularly with regard to its role in Everettian quantum mechanics. The problem, as set out by Schlosshauer, is primarily a question of identifying what constitutes a measurement of a particular property. The example he uses (Schlosshauer 2007, pp. 54) is that of a spin half particle coming into contact with an apparatus intended to measure its z-spin state.

The apparatus is defined such that

$$|0_z\rangle_{System}|"ready"\rangle_{Apparatus} \rightarrow |0_z\rangle_{System}|0_z\rangle_{Apparatus}$$

$$|1_z\rangle_{System}|"ready"\rangle_{Apparatus} \rightarrow |1_z\rangle_{System}|1_z\rangle_{Apparatus}$$

For some general particle state the interaction will then be:

$$(\alpha|0_z\rangle_S + \beta|1_z\rangle_S)\ |"ready"\rangle_A \rightarrow \alpha|0_z\rangle_S\,|0_z\rangle_A + \beta|1_z\rangle_S\,|1_z\rangle_A$$

As Schlosshauer points out, a simple basis transformation applied to the final state seems to change the quantity being measured in this process.

$$\alpha|0_z\rangle_S\,|0_z\rangle_B + \beta|1_z\rangle_S\,|1_z\rangle_B$$
$$= \frac{1}{\sqrt{2}}(\alpha+\beta)|0_x\rangle_S\,|0_x\rangle_A + \frac{1}{\sqrt{2}}(\alpha-\beta)|1_x\rangle_S\,|1_x\rangle_A$$

After this transformation, the form in which the state is written appears to show that the apparatus has become entangled to the particle by its x-spin state. The problem of the preferred basis as Schlosshauer understands it is as follows:

> "if we interpret in the spirit of the von Neumann scheme…, this formation of system-apparatus correlations as a complete measurement, this state of affairs seems to imply the following. Once A has measured the spin of S along the z axis, A may be considered as having measured also the spin of S along the x axis… Thus our device A would appear to have simultaneously measured two *noncommuting* observables of the system… In apparent contradiction with the laws of quantum mechanics." (Schlosshauer 2007, pp. 54, original emphasis).

In other words, by means of a basis transformation it appears that we can trivially transform what was intended to be a measurement of z-spin to a measurement of x-spin. The problem is finding some reasoned basis for claiming that we made one of these measurements and not the other.

The solution, as already presented in chapter 2, rests on consideration of the interaction between the measurement apparatus and its environment. As the measuring apparatus in this case is designed to measure z-spin, it will presumably be designed to have some macroscopic distinction, such as the position of a pointer, become entangled to the z-spin state of the target particle. This pointer will in turn very quickly become extensively entangled to its environment. As a result, in this basis, the off-diagonal elements in the reduced density matrix of the measurement apparatus and target particle will become very close to zero. Of course, there are elements of the measuring apparatus which become entangled not to the z-spin state, but to the x-spin state, but these degrees of freedom are not of the type to become entangled to the apparatus's environment in the same way. Consequently, because of the design of the apparatus, environment-induced diagonalisation of the system's reduced density matrix will occur for only a single observable (z-spin in this case). Schlosshauer takes this to be a reasoned justification for identifying z-spin as the measured property rather than any other.

When introducing this problem and Schlosshauer's solution in chapter 2, I said that it held important clues to providing a branching structure for Everettian many worlds interpretations of the type presented by David Wallace. This is certainly true; however, a solution to the form of this problem presented by Schlosshauer does not, in and of itself, provide a means to identify a branching structure of robustly distinct worlds of the type needed for an Everettian multi-verse. Identifying this structure is a different and more challenging form of the problem of the preferred basis, and it is central to understanding why Wallace employs the medium decoherence criterion in his analysis of the branching structure. The problem of the preferred basis for Everettian quantum mechanics is the problem of identifying when portions of the wavefunction are and are not distinct Everettian worlds.

Decoherence, as Schlosshauer characterises it (i.e. diagonalisation of a system's RDM) is not a condition applicable to very long timescales. In due course the system and apparatus for which a reduced density matrix was produced will be tidied away, or rearranged for the next experiment, or permanently dismantled and recycled. Working out whether the reduced density matrix of the system remains diagonalised at a later time might well prove extremely difficult.

Moreover, it is clearly not a viable criterion for world separation. We should not in general expect that a measurement apparatus in which decoherence has once occurred will never again display any form of interference phenomena. An interferometer apparatus might be used many times in a day. This shows that whichever degrees of freedom decohere when a measurement is made, clearly do not remain decohered indefinitely. Otherwise interference phenomena within these elements would be suppressed, on subsequent uses of the apparatus, and no interference phenomena could be observed. If the criterion for Everettian world separation were simply the diagonalization of the RDM of a localised system, then world separation would often be very short lived indeed.

This loss of diagonalisation does not mean that the original measured state would no longer be entangled to some elements of the wider environment (a lab book being a likely example). It almost certainly would be. But these entangled elements will be widely spread, and may well not include the localised system of degrees of freedom whose reduced density matrix was originally diagonalized in the environmental decoherence process.

This short-term approach to identifying the phenomenon of decoherence is eminently practical when it comes to deciding whether or not an interaction which has just taken place should count as a measurement of a particular observable. Consequently, it clearly offers significant help to the Everettian when it comes to establishing which types of interaction would result in world branching, and according to which basis that branching would take place.

It fails, however, when it comes to answering the question of whether or not two histories which have previously branched remain distinct at a later time. As I have argued extensively in previous chapters, establishing that histories remain distinct at a later time relies ultimately on the assumption of medium decoherence. The key difference in the problem of the preferred basis as set out by Wallace, as opposed to the problem as set out by Schlosshauer, is that Wallace is concerned not only with establishing the basis by which histories branch, but with establishing the basis by which you can decompose the universal wavefunction into individuated worlds.

The second way in which the consequences of medium decoherence depart from what is achieved by Schlosshauer's conception, concerns the problem of the non-observability of interference. As characterised by Schlosshauer, this is the problem of why we do not generally observe interference phenomena on scales other than the extremely small. If the universal wavefunction fulfils the medium decoherence criterion, then the answer is simple. Any entity whose state is continuously entangled with elements within its environment will not display interference phenomena in the basis by which the environmental entanglement takes place. These states will have become entangled to the environment and remain there for ever more without ever being lost, thus permanently preventing the separated histories from recombining. Without this criterion, however, the door is open to large-scale interference phenomena. Schlosshauer argues that continuous entanglement to the environment will suppress interference phenomena displayed by medium-sized dry goods on a day-to-day basis. I argued extensively in the previous chapter, firstly that this does not guarantee that interference phenomena will not occur, and secondly that it does ensure that the scale on which they occur is sufficiently large (both in terms of time and the size of system involved) as to be effectively unobservable.

A second distinction between the dynamics ensured by Schlosshauer's criterion and Wallace's, is that whereas RDM diagonalisation makes large-scale interference phenomena extremely difficult to observe, for the medium decoherence criterion to (approximately) obtain, as Wallace believes, would mean that large-scale interference phenomena (approximately) never occur at all. As we shall see in the next chapter this is crucial to securing the Everettian conception of probability.

Here then are the two key ways in which the problems which Wallace seeks to solve by invoking environmental decoherence depart from those discussed by Schlosshauer. The next section will look in more detail at how

Wallace characterises his branching structure and the role it plays in securing quasi-classical structures for Wallace’s metaphysics.

7.2 The Role of the Preferred Basis in Everettian Metaphysics

In chapters 4 and 5, I looked at objections to decoherence-based reasoning which stemmed from suspicions of the *for all practical purposes* nature of the reasoning involved. I noted then that much of the response to these issues offered by David Wallace stems from his use of Dennettian concepts of emergence. In essence, much of the metaphysical foundations of Wallace's approach rest on his endorsement of what Wallace coins as Dennett's criterion:

> **Dennett’s Criterion:** a macro object is a pattern and the existence of a pattern as a real thing depends on the usefulness -- in particular the explanatory power and predictive reliability -- of theories which admit that pattern in their ontology. (Wallace 2012, pp. 50).

This criterion is clearly intended to accept vaguely defined and approximate patterns as real, provided that they are sufficiently robust to be useful for explanatory and predictive purposes. To demonstrate this point, here is one of Wallace’s often repeated comments in response to the question of how many Everettian worlds exist in his view:

> “Decoherence causes the universe to develop an emergent branching structure. The existence of this branching structure is a robust (albeit emergent) feature of reality; so is the mod-squared amplitude for any *macroscopically described* history. But there is *no* non-arbitrary decomposition of macroscopically-described histories into ‘finest-grained’ histories, and *no* non-arbitrary way of counting those histories.” (Wallace 2012, pp. 101. Original emphasis).

This is representative of Wallace's overarching characterisation of the metaphysics of his Everettian multi-verse. The precise characterisation of entities, described within his interpretation, will inevitably become vague in some cases and on some scales. As far as Wallace is concerned though, this is not a problem for his view so long as the general patterns which, following from Dennett's criterion, are what constitute emergent macroscopic entities, really are robust enough to provide ongoing predictive and explanatory power.

This need for robustness is linked to the difference between the problem of the preferred basis, as considered by Schlosshauer, and that considered by Wallace. In a sense, the problem of the preferred basis considered by Schlosshauer, and his solution to it, are both clearly concerned with patterns within localised subsections of the wavefunction, just as Wallace is. The patterns picked out by RDM diagonalisation, however, have nothing to guarantee their ongoing robustness, and consequently nothing to guarantee that ongoing predictive reliability. If these patterns are not in fact robust then it seems that they would fail to fulfil Dennett's criterion, and so not describe anything which Wallace would consider a macroscopic object.

If on the other hand a quantum system which decoheres, becoming entangled to its wider environment in some basis, in fact forms part of a wider history space which (approximately) fulfils the medium decoherence criterion, then the entanglement of the system state will persist (approximately) forever, and the associated interference phenomena will remain (approximately) suppressed (approximately) for ever.

Given this robustness, it is easy to see why Wallace believes that the patterns contained within a history space which approximately fulfils the medium decoherence criterion would be robust enough to fulfil Dennett's criterion for inclusion in our ontology. If we accept that criterion, and that the universal wavefunction (or the portion of it we occupy) almost perfectly fulfils the medium decoherence criterion, then it is difficult to see how we could fail to see Wallace's scheme for Everettian quantum mechanics as recovering the manifest quasi-classical image of the world around us in day-to-day life (ignoring the question of probability for the time being).

Clearly then, securing the claim that the portion of the wavefunction we occupy approximately fulfils the medium decoherence criterion should be a major priority for all advocates of Wallace’s project. In the previous chapter I considered a wide range of interpretation neutral reasons for believing that the medium decoherence criterion might obtain. I argued, however, that none of these reasons was particularly convincing. I will now turn to consider the suggestion made by Wallace that a simple initial state of the universal wavefunction might constitute a Humean law of nature. The suggestion was mentioned in the previous chapter, but as I will make clear it is not a suggestion that can be evaluated independently of the metaphysical apparatus of particular interpretations. As this suggestion was made by Wallace, it will be particularly interesting to examine how it fares in the context of his own form of the Everett interpretation. Examining this, and the issues which arise from it, will be the main topic of this chapter.

7.3 Humean Laws and Medium Decoherence

Humean laws of nature depart from other conceptions of laws in that they do not govern the behaviour of entities in the world, but rather supervene on that behaviour. Lewis 1986, who was one of the major modern champions of this view, took Humean supervenience to be “the doctrine that all there is in the world is a vast mosaic of local matters of particular fact, just one little thing and then another” (Lewis 1986, ix), with the laws of nature being regularities over the particulars which make up this vast mosaic. Specifically, laws of nature are seen as being those regularities which give the best balance of theoretical simplicity and predictive strength within some theoretical system. There are many concerns relating both to how precisely this simplicity and strength should be characterised, and whether this is an adequate characterisation of laws at all. For a discussion of many of these issues see Ned Hall 2015 and Carrol 2016. For present purposes though I will ignore most of these questions and assume that, at least for non-quantum cases where a mosaic of the type Lewis has in mind can be supplied, then best regularities over that mosaic can be identified as laws of nature in much the way that Lewis suggests.

In this chapter, I will focus on two questions regarding Wallace's appeal to Humean laws. Firstly, I will consider what could count as a mosaic over which these laws might supervene in the context of Everettian quantum mechanics. Secondly, I will look directly at Wallace's suggestion that an (in some sense simple) initial state of the universal wavefunction might possibly fit the criteria to be a law of nature in such a context.

Before going on to these points, however, I will very briefly recap why, and in what sense, these simple initial conditions for the starting state of the universal wavefunction would provide support for Wallace's belief that the medium decoherence criterion approximately obtains for the universal wavefunction.

In chapter 9 of his book, Wallace introduces two principles, neither of which he rigorously defines. These are his *quantum past hypothesis,* which states that the universal wavefunction begins in a “simple” state, and *simple dynamical conjecture,* which seems to amount to the conjecture that a once “simple” state will remain that way on reasonable timescales. Simplicity here refers to some vaguely identified condition on the state of the universal wavefunction (which is certainly not simple to define). The central purpose of requiring simplicity is clear however, and this is to rule out history recombination.

Just how Wallace’s concept of simplicity is to be characterised is something which he leaves extremely unclear, and it seems likely that specifying it precisely would prove extremely difficult. I made some attempts to find a more precise formulation in the previous chapter, though it is hard to be sure whether they were really in the spirit of what Wallace had in mind. For the present chapter, I will use a perhaps simplistic conception of these principles which, though possibly suspect in some ways, certainly captures the spirit of Wallace's proposal.

Consider the initial segment of a single history within the history space of a closed system and the successor histories produced as it (and its successors) undergo history branching events. As discussed previously, if particular subsystems of interest within the total system are assumed, then the bases in which these subsystems interact with the rest of the system will give rise to a basis in which the reduced density matrices of these subsystems are diagonalised. Following standard practice in histories approaches, we will take our histories to be picked out by the bases in which these reduced density matrices are diagonalised. Now consider the question of whether after some time interval history recombination has occurred within the history space.

History recombination will occur if and only if two histories come to agree on their projectors at a time after previously disagreeing. Whether or not this will happen depends on the character of other histories which form the history space of our system, and the Hilbert space which they occupy. If at a starting time there are histories in our space corresponding to a large proportion of accessible points within the Hilbert space of our system, then it is highly probable that our considered history and its successors will quickly come to recombine with other histories. I take the quantum past hypothesis to amount to the assumption that this is not the case for the starting conditions of the system which is that portion of the universe which we occupy.

Assuming that there are not histories which correspond to a large proportion of points in the Hilbert space of a system at our initially considered time, the progressive branching of the successor histories we are considering will after some period of time inevitably make this the case. I take the simple dynamical conjecture to be the assumption that the time for this process to happen is long compared to any timescales we are likely to be interested in.

The second of these points I have already discussed in the previous chapter and I have nothing to add in the specific context of Everettian quantum mechanics. My position remains that it is extremely difficult to say one way or another whether this conjecture is plausible or not.

In this chapter I will follow up on this brief suggestion of Wallace, that assuming medium decoherence might be justified by some form of quantum past hypothesis. It should be remembered in what follows that this is a speculative suggestion by Wallace, and not a position which he develops or defends in any clear fashion. What follows is intended as an exploration of this possibility, not a critique or counter argument to Wallace.

It should also be remembered, though, that the claim that our universe fulfils the medium decoherence criterion is something which Wallace relies on heavily. Consequently, while Wallace is not committed to the viability of a Humean quantum past hypothesis, it would be a great support to a claim which he does rely on.

7.4 Albert's Classical Past Hypothesis

The past hypothesis in thermodynamics (a term coined by Albert (2000)) is (roughly) the hypothesis that at the start of the (classical) history of the universe it occupied a particular low entropy state. The precise characterisation of this hypothesis is controversial – not least because the characterisations that Albert gives do not always seem consistent with one another. Brown 2017 identifies three different characterisations of this hypothesis within Albert 2000, and argues that these characterisations are not all equivalent. The intended gist is clear enough, however.

This hypothesis, and its intended consequence, that entropy begins in a low state and increases progressively over the course of the universe, is essential to our ability to make many reasonable judgements both when retrodicting past states and the history of our universe, and predicting future ones. Both the precise character of this classical past hypothesis, and the degree of justification we have for believing it, are complex issues which I will largely ignore. For an overview of this literature see Callender 2016. For discussion more directly relevant to Wallace's understanding of this hypothesis see Wallace's own discussion (2012, pp. 324-358), much of which is concerned with the classical form of the past hypothesis, and the extensive discussion offered by way of response by Brown 2017.

The other thing to be noted about the classical past hypothesis is that it is often seen as likely to constitute a law of nature on a Humean understanding of laws. In the words of Callender 2016, “It is likely that the specification of a special initial condition would emerge as an axiom in such a system, for such a constraint may well make the laws much more powerful than they otherwise would be.”

Wallace's quantum past hypothesis, then, is a close analogy of Albert's classical past hypothesis, and Wallace hopes that it too might be considered a Humean law of nature. I will return to the question of just how close the analogy between the quantum and classical past hypotheses are, but for now the next section will focus on the questions of what it would mean for something to be a Humean law within an Everettian interpretation, and what would constitute the mosaic onto which Humean laws are supposed to supervene.

7.5 In Search of an Everettian Mosaic

As previously mentioned, this Humean mosaic in classical cases was generally seen as being locally instantiated properties in four-dimensional space-time. Central to this conception was the idea that there were no necessary connections between the distinct local particulars which make up the Humean mosaic and that these particulars could be freely recombined into other possible configurations. In this section I will take the essential elements of a Humean mosaic to be freely recombinable particulars with no necessary connections between them.[18] Searching for a candidate mosaic which fulfils these requirements is the aim of this section.

Quantum mechanics poses a very clear prima facie challenge for the traditional mosaic. Central to the theory is the possibility of quantum entanglement between spatially distinct particulars. These entanglements seem at first glance to represent a clear example of necessary connections between spatially separated particulars.[19]

Within the context of the Bohmian interpretation there is something very reminiscent of the traditional Humean mosaic, namely the space-time positions and trajectories of Bohmian particles. The ontological status of the wavefunction, and the connections between particular particles that it seems to contain, still remain challenges for such interpretations, which have received a wide variety of responses see for example Miller 2013, Bhogal & Perry 2015, Dewar 2016. There is however a clear consensus as to the subvenient basis onto which Humean laws are meant to supervene in the context of this interpretation, and this is the positions of Bohmian particles.

In Everettian quantum mechanics there is no such straightforward answer. Quasi-classical Everettian worlds contain approximately classical patterns

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[18] This recombinability of particulars is crucial within the modern Humean project as it is the basis of the Lewisean conception of possibility and necessity in terms of *possible worlds* (very different to the Everettian conception of worlds). These *possible worlds* are related to our own by difference in some of the particulars which make up the Humean mosaic. For more on this see Hall 2015 or Lewis 1986.

[19] Darby 2015 argues persuasively that entanglement does not have to be understood in terms of necessary connections, and so does not necessarily undermine Humean metaphysics. His analysis is strictly interpretation neutral, however, and so does not rule out the possibility that particular interpretations of QM may understand entanglement relations in a way incompatible with Humean metaphysics.

which behave like simple local structures under most interactions, and so may seem like a promising choice for the subvenient basis of an Everettian Humeanism. Unfortunately, on Wallace's view these structures are merely emergent patterns within a profoundly nonlocal wavefunction. Their apparently local dynamics rely fundamentally on the profoundly nonlocal fact of their entanglement to many other degrees of freedom within their extended environment. Consequently, they do not seem able to offer either the locality which is traditionally expected in a Humean mosaic, or the freedom of recombination among elements of the mosaic on which the Lewisian account of possibility (which is at the heart of modern Humeanism about laws) relies.

Similarly, the wavefunction itself, if considered purely in three-space, does not seem like a promising candidate for the subvenient basis of Humean laws. Like the emergent quasi-classical structures which it instantiates, it is fundamentally profoundly nonlocal, and it is even less clear what a recombination of the mosaic would mean where the mosaic is something as complex as a wavefunction in three-space than it is in the context of these emergent structures.

As far as I can see, then, it does not seem at all promising to seek a candidate for a Humean mosaic in any three-space account of Everettian quantum mechanics. None of the structures found by examining Everettian quantum mechanics as described in three-space seem to display the necessary capacity for recombination independent of one another. As such, a Humean account of laws which sought to use any of these structures as its subvenient basis would be forced to make a radical departure from the way that Humean laws of nature have been conceived of in the literature since the work of Lewis 1986.

The alternative would be to identify the mosaic, not with any structures in three-space, but with the wavefunction in some higher dimensional space such as 3N configuration space. 3N configurations space means the higher dimensional space whose dimensionality is three times the number of particles present in three space. In this space, any arrangement of particles in three space is represented as a single vector. Strictly speaking, of course, this still doesn't capture everything we want to know in the context of quantum mechanics, as properties such as particles’ spin are not included. Nonetheless, adding extra dimensions to capture particles’ spin is a (reasonably) simple addition to make, and Ney 2016 pp. 14 makes clear that it is this expanded space rather than simple configuration space which she considers the wavefunction to occupy.

Advocates of what has become known as wavefunction realism have long argued that the wavefunction is best conceived of in the context of this higher dimensional space. This view, originating with Albert 1996 and more recently defended by Ney 2013 and North 2013, takes the wavefunction as an entity which should be considered as fundamentally occupying this higher dimensional space. The major advantage of this view is that viewed in the context of this higher dimensional space, quantum mechanics no longer describes nonlocal properties or interactions. Consequently, the wavefunction can be represented by simply assigning a complex number to every point in this space.

This corresponds far more closely to the traditional notions of a Humean mosaic. It is a space composed of local properties instantiated at points, just as traditionally expected. Moreover, these locally applied coefficients could easily be rearranged independently of one another (excepting possible issues of renormalisation), meaning that recombining the mosaic into other possible configurations corresponding to other possible worlds (in the Lewisian sense) could be easily considered. In the context of wavefunction realism, therefore, it seems as though there really is a viable candidate for the Humean mosaic. Laws of nature could potentially be conceived as regularities over this mosaic, and assuming that Wallace's quantum past hypothesis actually obtains, and offers a good enough balance of simplicity and strength to those systems of laws which include it, it could potentially be a Humean law of nature on this view.

There are, however, many people who believe that the wavefunction realists are fundamentally mistaken in their commitment to conceiving of the wavefunction as occupying a higher dimensional space in this way. One particularly persuasive opponent of this view is David Wallace himself.

Wallace (forthcoming) makes a wide range of generally persuasive arguments against conceiving of the wavefunction in this way. These begin with the point already noted, that configuration space (the usually discussed candidate for this higher dimensional space) caters only to position, and not to other quantum properties such as spin. Less easily corrected problems arise when consideration is given to relativistic extensions of quantum mechanics. Wallace points out that many of our leading candidates for theories of relativistic quantum mechanics do not appear to reside in a space where particle position is primary or even, in some cases, a clearly defined property. He also points out that locality, as it is obtained by representing the wavefunction in a higher dimensional space, does not have anything to do with locality as it is generally conceived of in day-to-day life (that is, as locality in three-space) and as such locality in the sense achieved is a rather dubious virtue.

Wallace’s general theme in this paper, with which he continues, is to attack the basic rationale for believing that our mathematical descriptions of the wavefunction suggest it to be fundamentally an entity in configuration space. Essentially, he believes this to be an error made by people who have taken the ways in which the wavefunction is often written in non-relativistic cases far too seriously.

It seems, then, that at least as far as the position advocated by Wallace is concerned, wavefunction realism in configuration space is not to be endorsed as an Everettian mosaic.

Part of the reason for Wallace's opposition no doubt stems from the fact that configurations space as customarily presented could be seen as giving a privileged status to the position basis. This is at odds with Wallace's project of presenting an interpretation which does not rely on such a preferred basis. This is not the only reason for Wallace's objection to this position, however.

7.6 What is Wallace Talking About?

Throughout his development of Everettian quantum mechanics, there is a fundamental question which Wallace explicitly does not answer. This question is what the fundamental ontology of quantum mechanics is.

Wallace says a great deal about the ontology of Everettian quantum mechanics, but all of this ontology is composed of emergent structures – patterns within the wavefunction whose fundamental nature is never explained.

Both in his concluding remarks in Wallace (forthcoming), and in chapter 8 of his book on Everettian quantum mechanics (2012), Wallace explicitly states that he does not know what the correct fundamental ontology of quantum mechanics is. Moreover, he seems far from optimistic about the possibilities for answering this question. He writes:

> “I suspect that looking for `the' ontology of a framework theory is a category error and that we would do better to reformulate the question in terms of the ontology of specific quantum theories, such as the standard model of particle physics (and also to recognise that these are unlikely to be fundamental theories, so that hopes to learn about fundamental ontology from those theories are probably vain).” (Wallace Forthcoming, pp. 11).

The recognition of ignorance about the fundamental ontology of quantum mechanics may be entirely appropriate here. Certainly, it seems very plausible that committing to a particular ontology on the basis of our present knowledge might be premature. However, it does present a striking problem for anyone seeking a subvenient basis of Humean laws within Everettian quantum mechanics. As I have already argued, the emergent structures which feature in Wallace's form of Everettian quantum mechanics do not seem to have the necessary character to function as such a subvenient basis. Given that Wallace is also unwilling to commit to any particular fundamental ontology, it seems to be left open to speculation whether or not any viable subvenient basis for Humean laws within Everettian quantum mechanics exists.

Wave function realism in some updated form capable of allowing for relativistic cases remains a possible fundamental ontology, and if the fundamental ontology of Everettian quantum mechanics is really of this form then it might be possible to apply a Humean account of laws as regularities over this mosaic in a higher dimensional space. Alternatively, if the fundamental ontology was in line with the priority monism advocated by Schaffer 2016, then recombination of particulars within a mosaic independent of one another would not be possible, and a Humean account of laws (at least one of traditional form) would be ruled out too.

It seems, therefore, that treating Wallace's quantum past hypothesis as a Humean law of nature within Everettian quantum mechanics is on some level a potentially workable possibility, but given the present state of Everettian ontology it is a distant possibility rather than a presently viable argument. Moreover, it is a possibility which rests on progress in establishing fundamental quantum ontology in a way which Wallace himself seems very doubtful about.

7.7 Comparing the Classical and Quantum Past Hypotheses

Before leaving the topic of the quantum past hypothesis there is one more point that I wish to make. It concerns a subtle but significant dis-analogy between the past hypothesis concerning entropy of the early universe presented by Albert 2000, and that suggested by Wallace.

Consider the following quote:

> “What will follow (more particularly) from the world’s present macrocondition + the uniform microdistribution over that macrocondition + the laws of motion is… that any book describing the Roman Empire is *far* more likely to have fluctuated out of molecular chaos than to have arisen as some sort of distant causal consequence of the *existence* of that empire; and no amount of *redundancy* among various such books, or among such books and archaeological artifacts and whatever else you may be able to come up with, will change that one iota. Period.” (Albert 2000, pp. 115. Original emphasis).

The point Albert is making is that everyday inferences about past states of the world, which we make based on the present state of the world, cannot be supported simply by our knowledge of the present, and the laws of motion. These inferences rely on a crucial additional premise. They rely on assuming that the universe previously occupied a lower entropy state than it does at present, and that over the time period considered total entropy has been increasing. That is, they rely on some form of classical past hypothesis.

If this hypothesis is not assumed, then our epistemic access to the past is almost unimaginably weaker than we generally take it to be. This provides a very clear incentive for endorsing the classical past hypothesis. If, as most people do, we believe that the existence of books about the Roman Empire should give us a high degree of confidence that the Empire itself once existed, then we seem to be obliged to endorse some form of this hypothesis.

On the other hand, the same does not seem to be true in the case of the quantum past hypothesis. If we assume that the total entropy within histories will generally increase over time, in accordance with the classical past hypothesis, then the existence of a large number of books about the Roman Empire make it highly probable that the Roman Empire existed in a large proportion of the predecessor histories which led to the one which we currently occupy. There will also be predecessor histories of this one in which those books came about with no causal connection to any such Empire, but these will almost certainly make up an extremely small proportion of the total amplitude of predecessor histories at any past time.[20]

The point is that, as I argued extensively in the previous chapter, branch recombination events will only affect the distribution of history amplitudes over the history space. Records we have of past events within the history we occupy will be accurate records of events (neglecting the usual accidents[21]) regardless of whether the history recombination has taken place or not. The thing that may well have been disrupted if history recombination has taken place is the amplitude of the history we occupy. As this (except in the highly exceptional cases discussed in the previous chapter) is not something that we have records for or direct empirical access to, it would not obviously make us wrong about any of our natural beliefs concerning the world we occupy.

It seems to me, therefore, that our epistemic access to our past survives the rejection of the quantum past hypothesis far better than it survives the rejection of the classical past hypothesis. One of the strongest motivations for endorsing the classical past hypothesis, therefore, seems to be irrelevant in the quantum case. Consequently, I do not feel that the quantum past hypothesis shares the same intuitive appeal as the classical past hypothesis.

The only issue that this would present relates to the Everettian treatment of probability. This will be discussed in the next chapter. For now though it

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[20] It should be noted of course, that when the assumption of medium decoherence is dropped the total amplitude of predecessor histories is very likely to change at different times. In particular, it is entirely possible that the total amplitude of our present history is greater than the total amplitude of predecessor histories which lead to it at some particular time. As discussed in the previous chapter, this is because of the possibility of constructive interference between recombining histories.

[21] Unlike in the classical case, of course, such accidents will certainly occur in a small proportion of the predecessor histories which lead to our present one, but these will be a very small proportion of the total.

is simply worth noting that this seems to represent a clear dis-analogy between the classical and quantum cases and, I would suggest, undermines the intuitive appeal of accepting the quantum past hypothesis.

7.8 Conclusion

In this chapter I have sought to look in more detail at the specifically Everettian uses to which the phenomenon of decoherence is put as set out by David Wallace. I have then focused on Wallace's quantum past hypothesis, and the suggestion he makes that this could be understood as a Humean law of nature. I have argued that, while this hypothesis may quite possibly be true, the case for viewing it as a law of nature is significantly undermined by the present state of Everettian ontology.

I have also set out an apparent dis-analogy between the quantum past hypothesis and the classical past hypothesis of David Albert, which I believe undermines the intuitive appeal of the quantum past hypothesis.

Introducing Wallace’s quantum past hypothesis, and defending it as a Humean law of nature, was the last of Wallace’s suggested justifications for assuming medium decoherence to be considered. It may be possible to develop such an argument, but it would require adding metaphysical commitments beyond those of Wallace’s Everettian interpretation, and these do not seem like safe commitments given the present state of our theories.

In this chapter and the previous chapter then, all the suggestions made by Wallace as justifications of the medium decoherence assumption (and some others besides) have been considered. None of them seem to offer persuasive reason to endorse this assumption. This assumption, however, is crucial to Wallace’s interpretation, as a source of robust patterns through which quasi-classical entities can be identified, allowing the recovery of our manifest image of the world.

There remains one possibility which I still wish to examine by way of a possible interpretation specific defence of the medium decoherence criterion. This would be a defence by way of indispensability. The next chapter will examine in more detail the impact on the Everettian derivation of the Born rule, and the broader Everettian metaphysics which depend on it, of dropping the medium decoherence assumption. I will consider firstly the question of whether the medium decoherence criterion is indispensable to the effective functioning of this metaphysics and its ability to recover our experiences of quasi-classical reality. I will then turn to consider the question of whether, if it is indispensable to the Everettian framework, this could in itself be the basis of a direct argument for believing in it.

8 Everett Without Medium Decoherence

The last two chapters have considered many reasons for endorsing the assumption that the medium decoherence criterion obtains for the universe we occupy. I have argued that all of these arguments are unconvincing. This is not to say that the universe could not fulfil this criterion (for now at least), but the few arguments found in the literature for this position as well as the other plausible suggestions I have considered do not seem like convincing justifications for believing it.

This chapter is intended to serve two purposes. Firstly, it will look in more detail at just what the problems are which are produced for Wallace's Everettian quantum mechanics if this assumption is abandoned. Secondly, it will identify what I believe to be the most hopeful prospect for a justification of the medium decoherence assumption.

The justification I have in mind is to appeal to the assumption’s explanatory indispensability in a way rather similar to that sometimes used to argue for mathematical Platonism. The form of this argument will closely follow the indispensability argument for mathematical Platonism as given by Baker 2009 pp. 613:

> P1) We ought rationally to believe in the existence of any entity that plays an indispensable explanatory role in our best scientific theories.
>
> P2) Mathematical objects play an indispensable explanatory role in science.
>
> C) Hence, we ought rationally to believe in the existence of mathematical objects.

The argument I propose is as follows:

> P1) We ought rationally to commit to all and only those physical claims about our universe that play an indispensable explanatory role in our best scientific theories.
>
> P2) The claim that our universe fulfils the medium decoherence criterion plays an explanatorily indispensable role within our theory of quantum physics, which is one of our best scientific theories.
>
> C) Hence, we ought to commit to the claim that our universe fulfils the medium decoherence criterion.

Premise 1 is closely connected to arguments about conformational holism within the existing literature on mathematical Platonism. I will touch on these arguments towards the end of this chapter.

There is one clear dis-analogy between the indispensability argument presented here and the argument as used for mathematical Platonism which may concern the reader. This is that in the case of mathematical Platonism this form of argument is used to argue for an ontological commitment (to Platonic numbers), rather than a general claim about the structure of the universe. While I acknowledge this clear dis-analogy, I don’t believe that it undermines the use of this form of argument in support of the claim of medium decoherence. The reason for this is that the indispensability argument in mathematics itself gains much of its persuasive power by being analogous to general scientific reasoning about the commitments entailed by our theories. I believe that it is general scientific practice to believe those claims about our universe which are indispensable to our best scientific theories, just as much as it is scientific practice to believe in the existence of entities which are indispensable.

For the purposes of this chapter I will treat *our theory of quantum physics* as meaning Wallace’s Everettian theory of quantum physics. This is of course an approach which advocates of other interpretations would certainly reject. The concluding chapter will suggest that the medium decoherence assumption appears to be indispensable to at least some other interpretations of quantum physics, but even so, there will certainly remain other interpretations of quantum mechanics for which the assumption is not indispensable. The argument will therefore also need to rely on Everettian quantum mechanics being a better theory than quantum mechanics interpreted in some other way.

I also take the meaning of this dispensability to be the same as that found in indispensability arguments in the philosophy of maths. Colyvan 2015 writes “What we require for an entity to be ‘dispensable’ is for it to be eliminable and that the theory resulting from the entity's elimination be an attractive theory.” The first task of this chapter will therefore be to argue that if the medium decoherence assumption is dispensed with, Everettian quantum mechanics ceases to be an attractive theory.

From what was said concerning the implications of the medium decoherence assumption in the previous chapter, the reader would be forgiven for wondering if it is really so necessary as an assumption for Wallace's Everettian quantum mechanics. In the last chapter I focused on the need to clearly delineate robustly distinct Everettian worlds. This is certainly something that Wallace is committed to, but given that (as I have already argued) the failure of this separation is extremely unlikely to have easily observable physical consequences, it is unclear just why Wallace is so committed.

Now I will turn to look at Everettian accounts of probability and the drastic impact which abandoning the medium decoherence assumption has for them. I will argue that this impact is at the heart of why medium decoherence is indispensable to Everettian quantum mechanics.

8.1 Deriving the Born Rule

In chapter 4 I looked at a scheme for deriving the Born rule presented by Zurek. Since Zurek first presented the scheme, many authors have presented their own schemes for deriving the Born rule for linear quantum mechanics without a collapse postulate. Though many of these schemes have been conceptually more sophisticated than that offered by Zurek, they share the same basic approach of showing that the modulus squared of a state’s coefficient should rationally be treated as probability by agents described within a linearly evolving wavefunction.

These approaches can sensibly be divided into two broad camps: the decision theoretic approaches of Deutsch 1999, Saunders 2004, and Wallace 2010, and the epistemic approaches of Sebens & Carroll 2018, and Zurek 2005.

The decision theoretic approach begins from the assumption of a branching structure of worlds in accordance with Wallace's general metaphysical project. It then proceeds to show that for agents within these worlds the modulus squared of the coefficient of a successor history state vector fulfils the functional role of the probability of a future state. A range of arguments concerning the nature of probability are then used to argue for accepting the modulus squared of this coefficient as quantum probability, just as set out in the Born rule.

The epistemic approaches on the other hand do not begin from such general Everettian metaphysical assumptions. They attempt to show that, for an agent uncertain about their future location within the wavefunction, the modulus squared of the coefficient of future histories gives the epistemically justified credence ascriptions for their future state. These approaches are less specific about the metaphysical structure into which these probabilities fit. They are certainly compatible with a variety of Everettian interpretations including Wallace’s.

All of these approaches rest on reasoning very much like that originally set out by Zurek which was discussed in chapter 4. Zurek's argument is given in a form very similar to that found in Sebens & Carroll 2018, pp. 17-20 and Wallace 2012 ch3. This reasoning is used to show that, for a branching structure of histories of a system which decoheres with respect to its environment, Born rule amplitudes behave as probabilities are generally expected to. And that no other easily identifiable quantity within the description of this process fulfils the general expectations on the behaviour of probability (Zurek, 2005, 2014). I will not present the reasoning for these results again in this chapter. For a structure of branching histories, I will simply take these results as proven.

In this chapter I will focus on the changes brought about if the medium decoherence assumption fails, and consequently, the histories of the system cannot be assumed to possess a branching structure. I will focus primarily on Wallace's work, but I will also refer to Sebens & Carroll as an example of a Born rule derivation which seems compatible with Wallace’s project, and which, at least prima facie, does not seem to rely on the assumption of medium decoherence in the same way.

8.2 Macro Interference and Wallace’s Justification of the Born Rule

The first task then is to review the effects of the failure of medium decoherence discussed in chapter 6. I will briefly re-present this result in order to make clear precisely how it leads to a failure of wavefunction amplitudes to fill the functional role of probability within the context of Wallace style Everettian metaphysics.

In the fifth chapter of his book Wallace 2012 gives an extended and very careful discussion of decision theoretic notions of probability. His intention in this chapter is to argue that, because history amplitudes are able to play the functional role of probability for an agent within those histories, they can be regarded as actually being in some sense an objective form of probability for such an agent. This is a subtle and interesting treatment to which Wallace devotes the bulk of his discussion of deriving the Born rule for Everettian quantum mechanics. For my purposes in this chapter, however, I believe it can be set aside. The reason is that this chapter is concerned really with showing that the concept of probability is one that can be given a rigorous analysis within the deterministic theory of Everettian quantum mechanics. The basic derivation of the Born rule on which Wallace relies actually happens in the previous chapter. He makes this clear in chapter 5.

> “At its mathematical core, the argument I will present is not really decision theoretic at all: it rests on the same symmetry considerations as the proof of the Born rule which I presented in section 4.13. The reason for appealing to decision theory, as I have been at pains to stress, is simply that it provides us with a sharp unambiguous notion of probability applicable to a context – that of a branching universe – in which it has been questioned whether probability makes sense at all.” (Wallace 2012, pp. 159).

I am prepared to grant Wallace's claim that decision theory provides a viable account of probability within Everettian quantum mechanics suitable to all those purposes, provided that the symmetry considerations to which Wallace alludes really can provide a means to derive the Born rule and that this rule is able to fill the functional role of classical probability. I will therefore focus my attention on his argument in section 4.13, which in fact is very largely the same argument originally presented by Zurek, which I presented in chapter 2.

I will briefly set out the type of branching structure to which Wallace applies this reasoning, and then turn to consider cases of history recombination, and how these undermine that reasoning.

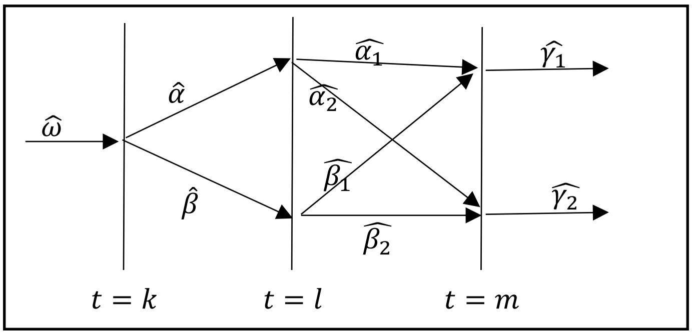
Wallace takes the general form of a quantum probabilistic branching process to be:

$$\widehat{\chi_{t=0}} \rightarrow \sum_{i} \widehat{\phi}_{\iota}$$

Here an initial history state yields some number of successor histories which differ on their projectors from one another by degrees of freedom which become encoded within the wider environment. Recombination of histories is something which Wallace assumes not to take place in accordance with the general metaphysics of his project. With this characterisation in hand, he follows precisely the same symmetries-based reasoning as Zurek. He uses this to initially show that for a pair of successor histories of equal amplitude, the probability ascribed should be equal, just as given by the Born rule. He then considers decompositions of more complex sets of histories into a larger set of histories of equal amplitude and argues that these should have equal probability as one another for just the same reason. Finally, he shows very simply that in consequence, the probability for any general set of history states ϕ_i and associated coefficients α_i, the appropriate probabilities to ascribe are just as given by the Born rule:

$$P(\phi_i) = \left\langle \psi \middle| \widehat{\phi_\iota}^{\dagger} \widehat{\phi_\iota} \middle| \psi \right\rangle$$

Now I will turn to look at what happens if the medium decoherence criterion does not obtain, and consequently, history recombination and interference do occur. To do this, I will return to the schematic representation of the process presented in chapter 6.



As previously, the branching event that takes place at $t = k$ will be some event in the distant past which yields two macroscopically similar histories within which records of the event are stored, but gradually erased with time, until they are lost entirely at $t = m$. I will focus on how probabilities would be described by agents within history $\hat{\alpha}$, and its successors.

At $t = l$ a quantum probabilistic event occurs in both $\hat{\alpha}$ and $\hat{\beta}$, which is observed by an agent within each of those histories. For the present discussion we will take this to be the measurement of a spin-1/2 particle initially in a superposition. The first question to be considered is what the appropriate credence ascription is for an agent in history $\hat{\alpha}$ for the outcome of the measurement.

If we concern ourselves only with probability ascriptions over the successor histories $\widehat{\alpha_1}$ and $\widehat{\alpha_2}$, then the symmetries-based analysis suggested by Wallace seems perfectly appropriate, and capable of giving clear probabilities for both possible outcomes (outcome 1 being obtained in history $\widehat{\alpha_1}$, and outcome 2 being obtained in $\widehat{\alpha_2}$). The trouble is, that as I argued in chapter 6, the histories which recombine at $t = m$, will very generally interfere with one another. This occurrence may be seen as in itself enough to invalidate Wallace's analysis, but if it can be applied to the probability ascriptions of our agent in $\hat{\alpha}$, then they are still perfectly able to find sensible credence ascriptions for the histories $\widehat{\gamma_1}$ and $\widehat{\gamma_2}$, but these credences will generally not match those appropriate to give to the histories $\widehat{\alpha_1}$ and $\widehat{\alpha_2}$. In consequence, it seems that our agent would be compelled to ascribe one probability to outcome one having been obtained at $t = l$ between $t = l$ and $t = m$, and a different probability to the same outcome of the measurement after $t = m$.

To be clear here, this process does not involve any second measurement which has different probabilities for its outcomes. The only measurement being considered is the measurement made at $t = l$. That measurement, however, seems to change the probabilities of its possible outcomes significantly after the measurement itself takes place. This is clearly a striking departure from our traditional notions of probability.

Even if we are happy to accept that our agent in $\hat{\alpha}$, should ascribe two contradictory probabilities to a single event in this way, we have a still more challenging problem. This is the question of what probability an agent in $\widehat{\alpha_1}$ should ascribe to the successor history $\widehat{\gamma_1}$. Given that the only distinguishing feature of $\widehat{\gamma_1}$ is a result 1 being obtained for a measurement at $t = l$, which in history $\widehat{\alpha_1}$ has already been performed and recorded as producing result 1, the classically expected answer is certainty. But it is very unclear how this can be reconciled with the difference in Born rule probabilities between $\widehat{\alpha_1}$ and $\widehat{\gamma}_1$.

In the circumstances, it seems clear to me that arguing for the identification of Born rule amplitudes as probabilities on the grounds that they play the functional role of classical probability is not a tenable strategy in the context of history recombination. Wallace's symmetries argument relies on the assumption of a branching structure of histories as produced if the medium decoherence criterion obtains. Dropping this assumption therefore has fatal consequences for Wallace's Born rule derivation.

8.3 The Epistemic Separability Principle

Perhaps it is unsurprising to find that an interpretation with medium decoherence at its heart does not provide a derivation of the Born rule which survives dropping this assumption. Before moving on however, I will briefly consider the derivation of the Born rule given by Sebens and Carroll 2018. The reason is that this derivation does not make any explicit mention of medium decoherence. If this really were a Born rule derivation which could be incorporated into Wallace's project, but which did not rely on the presupposition of medium decoherence, then this might be an important step in saving Wallace's project from the effects of history recombination.

Unfortunately, however, while medium decoherence is not employed directly by Sebens and Carroll, they rely on what they term an epistemic separability principle. The quantum form of this principle is given as follows (Sebens & Carroll 2018, pp. 17):

> “**ESP-QM** Suppose that an experiment has just measured observable $\widehat{\mathrm{O}}$ of system S and registered some eigenvalue $\mathrm{O_i}$ on each branch of the wavefunction. The probability that agent A ought to assign to the detector D having registered $\mathrm{O_i}$ when the

universal wavefunction is Ψ, $\mathrm{P}(\mathrm{O_i}|\Psi)$, only depends on the reduced density matrix of A and D, $\widehat{\rho_{\mathrm{AD}}}$:

$$\mathrm{P}(\mathrm{O_i}|\Psi) = \mathrm{P}(\mathrm{O_i}|\widehat{\rho_{\mathrm{AD}}})$$

Consequently, the impact of any changes in records outside of the system which directly undergoes the quantum probabilistic process is, by virtue of this principle, irrelevant to the appropriate credence ascriptions of an agent concerning the results of that process. Sebens and Carroll justify this principle by comparison to the classical cases to which it is very generally appropriate.

This principle is used as grounds to neglect the wider environment for the purposes of Sebens & Carroll's analysis. Given that history recombination of the type I am discussing relies on records within the wider environment being erased, this amounts to the assumption that such records and potential erasure events are irrelevant to the probabilities that an agent should ascribe to results of an experiment within their local environment.

There seem to be two possible responses to this state of affairs. The first is to say that this principle implicitly rests on the assumption of medium decoherence, and the Born rule developed by Sebens and Carroll is simply not applicable to cases where medium decoherence does not obtain. This seems the more natural understanding of Sebens & Carroll’s intention. A second possibility, however, is to treat this principle as the basis for a new quantum probability rule which differs from the traditional Born rule.

This would be the suggestion that if this principle is applied when analysing the recombination case discussed above, then the probability that an agent within $\hat{\alpha}$ should ascribe to obtaining result 1 is simply the probability that their immediate successor history is $\widehat{\alpha_1}$. The Born rule amplitude of $\widehat{\gamma_1}$ would be irrelevant.

I suspect that if it was generally applied this new probability rule could be shown to produce inconsistent probability ascriptions in some cases. Even if it did not, it would certainly be a striking departure from the Born rule, and require a significant revision of quantum mechanics. Such a revisionist project does not look likely to prove fruitful.

It seems, therefore, that Sebens & Carroll do not provide a Born rule derivation that is capable of surviving the failure of the medium decoherence assumption. I will now turn to consider why this failure matters to Wallace's broader project.

8.4 The Problem of Low Amplitude Histories

In this chapter I have argued that there is a significant problem in the application of the Born rule to Everettian quantum mechanics in the way that Wallace suggests. However the reader may wonder, given my discussion in chapter 6, whether this is really an important problem. The cases which I have used to develop a problem for Born rule probabilities in Everettian quantum mechanics are cases in which, as I have argued previously, we are very unlikely to have any evidence of anything untoward having taken place. The fact that it is difficult to make sense of probability in such cases might therefore be thought a rather abstract theoretical problem, rather than an insurmountable difficulty for a putative interpretation of quantum mechanics. And given that my purpose in this chapter is to outline an indispensability argument for believing the medium decoherence criterion to obtain, I do need to show that there is, or at least seems to be, a really serious and insurmountable problem. To explain why I believe medium decoherence really is indispensable to Wallace's project, let us return to a problem already discussed in chapter 3.

The problem is that, while the branching multi-verse of Everettian quantum mechanics definitely includes worlds which correspond to the types of experience and apparent dynamics with which we are familiar, it also includes many histories[22] which do not display anything like these classical dynamics. This is because for a quasi-classical entity within a history, such as a table, there is a persistent low Born rule probability of it spontaneously relocating to any other position in the universe. This is just one example of the very many types of radically nonclassical behaviour that some low amplitude histories within the wavefunction will display. The problem this poses for Everettian quantum mechanics is to explain why we can reasonably expect the quasi-classical dynamics we observe on a day-to-day basis and not these outliers.

Barrett, writing before the development of modern Born rule derivations, expresses this problem well:

> “Consider trying to use splitting worlds theory to make bets about where the Eiffel Tower is *right now.* Suppose that the universal wavefunction assigns a small but nonzero amplitude to me having all the beliefs that I have as I write this and the Eiffel Tower being in Pittsburgh (as it presumably would given the usual dynamics). So I bet a friend $10 that the Eiffel Tower is in Pittsburgh right now. This is presumably a bad bet, *but why?...* There is nothing in the splitting worlds theory as it stands that tells me which sort of world I inhabit right now or even which sort I should *expect* to inhabit right now.
> “As it stands then, the splitting worlds theory does not even explain why one should expect to record the usual quantum statistics *right now*.” (Barrett 1999, pp. 166. Original emphasis).

With the Born rule in hand this problem is very simple to answer. We can neglect these extremely low amplitude histories because they are very improbable. This is precisely the answer that Wallace points to when responding to this question (Wallace 2012, pp. 248). But if, as I have argued, dropping the medium decoherence assumption fatally undermines

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[22] As previously noted, technical issues relating to Wallace's characterisation of a world mean that many of these histories will not fulfil the criteria to count as a world.

Everettian Born rule derivations, then it also returns the Everettian to the position of being unable to answer this basic question.

The problem posed is explanatory rather than empirical. The histories which display the quasi-classical dynamics with which we are generally familiar do exist, and will continue to exist. Our observation of such dynamics is an inevitability within linearly evolving deterministic quantum theory. The trouble is that histories in which radically nonclassical behaviour is displayed are also inevitabilities. With medium decoherence assumed, Wallace's project is able to explain why it comes about that our observed experience is of histories of the former type and not of the latter.

More generally, if medium decoherence is granted, Wallace’s project is able to *explain* quantum probability ascriptions, and our existing statistical records. These records cannot be used as the basis of a direct empirical argument for medium decoherence, but the ability of Wallace's project to explain them clearly represents a very great theoretical strength.

The reason these records cannot be used for a direct empirical argument is that we are not comparing two theories which make competing probabilistic claims. We are comparing a theory which makes probabilistic claims to one in which probability is simply not an applicable concept. Both are capable of predicting the occurrence of records of precisely the types we possess. The grounds for deciding between them must rest on their theoretical virtues rather than direct empirical comparison.

I believe however that the level of difference between the explanatory virtues of these theories is sufficient that, so long as we are only considering Everettian quantum mechanics with and without the assumption of medium decoherence, medium decoherence can be taken as clearly indispensable.

8.5 Indispensability

This state of affairs in which a theory relies on a physical claim about the universe which is indispensable to the theory, but which the theory does not empirically confirm may seem a little puzzling. The key thing to understand is that this claim about the universe is not needed for any empirical prediction, but is indispensable to explaining why observations of these types are not simply inevitable features of one determinately predicted history amongst many, but features of the type of history in which we should expect to find ourselves. This explanatory power is of such great importance to the attractiveness of the interpretation that I believe Everettian quantum mechanics without this power ceases to be an attractive theory (as Barrett 1999 also suggests). Consequently, it seems to me that this explanatory indispensability provides a promising argument in support of the medium decoherence assumption, as set out at the start of this chapter.

This form of argument faces objections. Maddy 1992 argues that we should not think of empirical confirmation for a theory as representing empirical confirmation for every ontological claim which is indispensable to that theory. She bases this claim on the observed practice of scientists, who frequently seem to have varying levels of credence for different parts of a theory. Consequently, simply showing that a physical claim about the universe is indispensable to a theory does not necessarily mean that we have significant justification for accepting that claim.

Another possible concern is that though the assumption of medium decoherence is explanatorily indispensable to the theory of quantum mechanics, it might still not be sufficient to commit us to the assumption. Saatsi 2016 argues that for mathematics to be in some sense indispensable to scientific theories is not necessarily enough to commit us to accepting mathematical entities into our ontology. He argues that whether or not we should be committed to accepting such entities will depend on the nature of the explanatory role they are playing within the theory, as well as the theory of explanation we have in mind. In particular, he suggests (Saatsi 2016 pp. 1060) that the explanatory role of mathematical entities (indispensable though it may be) is *thin* meaning that it acts as a representation of something else which is what really plays the *thick* explanatory role. In the case of the medium decoherence assumption, it seems to me that if the assumption is, in any sense a representation of something explanatorily indispensable, which must be accepted as real, then all that Wallace needs of the assumption will be established, and the essence of this indispensability argument will have succeeded. Establishing this however will require a level of engagement with the philosophy of explanation literature which is beyond the scope of this thesis.

These arguments originate in the context of indispensability arguments for the existence of mathematical entities, but they may also apply in the context of medium decoherence. Various questions therefore still remain concerning the justification of medium decoherence in this way. Nevertheless, an indispensability argument of the type I have suggested in this chapter does at least seem like a promising basis on which a justification of medium decoherence might be developed.

Of course, throughout this chapter I have ignored other interpretations of quantum mechanics. In order for the indispensability argument I have described to succeed, it would be necessary to show either that medium decoherence was indispensable to other interpretations of quantum mechanics as well, or that Everettian quantum mechanics was in some sense theoretically superior to interpretations which did not need this assumption.

This seems to me to be an entirely appropriate demand. Medium decoherence is a very broad claim about the physical characteristics of our universe, which is essential to the account of Everettian quantum mechanics developed by Wallace. I have not been able to find any convincing justification for this claim about our universe either in the existing literature or my own investigation. While this assumption is not untenable, it does represent a real and significant disadvantage of the Everettian interpretation. Noticing this point seems particularly important given that Wallace often presents his interpretation as simply a matter of taking the linear quantum mechanics seriously, without any additions or further assumptions.

8.6 Conclusion

The main purpose of this chapter has been to outline a possible justification of the medium decoherence assumption. This justification is to argue that medium decoherence is indispensable to what seems to be one of our best scientific theories. This, I suggest, can be used as the basis for an indispensability argument for accepting this assumption.

I have argued that medium decoherence is indispensable to the Everettian Born rule derivations available in the literature, and that these derivations are indispensable to Wallace's Everettian project generally. This first step in the indispensability argument seems to me indisputable as this interpretation stands, and likely to remain that way under foreseeable developments.

Beyond Everettian quantum mechanics, however, the argument depends on complex and disputed issues relating to theory confirmation. It also depends on a favourable comparison being made between Everettian quantum mechanics and other possible interpretations. The second of these issues will be dealt with at slightly greater length in the final chapter. The first is a question which is beyond the scope of this thesis.

9 Conclusion

In the last chapter I argued that it might be possible to justify assuming the universe we occupy satisfies the medium decoherence criterion, in order to support Wallace's Everettian quantum mechanics, on the grounds of the indispensability of this assumption. For such an argument to be successful, there must be no equivalently good (or better) theory which does not rely on this assumption. That is, in order to justify the medium decoherence assumption within Everettian quantum mechanics, it is necessary that all other interpretations of quantum mechanics are either less good theories, in some sense, or also rely on the assumption of medium decoherence.

In this chapter, I will review the results of this thesis to remind the reader of the significance of medium decoherence, and the difficulties involved in justifying the assumption that this condition really applies to the history space of our universe in a quasi-classical basis. I will then close this thesis with a preliminary examination of other interpretations of quantum mechanics to see if there is an interpretation other than Everettian quantum mechanics which does an equivalently good, or better, job, without relying on this assumption.

I will argue that the only viable candidate for such an interpretation is GRW, and that it is far from clear that this interpretation could at present be considered to be as appealing as Everettian quantum mechanics.

Finally, I will argue that Bohmian mechanics, though still reliant on the same medium decoherence assumption, may be in a better position to justify this assumption within its interpretational framework. I will not investigate these possibilities in any detail here, but will indicate why I believe them to be hopeful avenues for future research, which I hope to see developed further.

9.1 A Review of this Thesis

In the first chapter of this thesis, I introduced the basic problems which arise when we try to reconcile linear and deterministic quantum mechanics, as captured by the Schrödinger equation, with the world of our everyday experience. I outlined four different types of interpretation which seek to solve this problem.

I also gave a broad characterisation of decoherence – the process by which a quantum system, when entangled to its environment, will come to behave under subsequent operations, performed in the basis by which it is entangled, without displaying distinctively quantum interference phenomena. This phenomenon presents the exciting prospect of a means to distinguish those areas of the world which we can expect to display distinctively quantum interference phenomena, from those which will not. As a result, three of the four families of interpretation set out in this chapter use it to help distinguish when linear quantum mechanics should or should not be taken as a useful and appropriate guide to physical behaviour.

Importantly, the phenomenon of decoherence in the sense set out here is a direct consequence of linear Schrödinger dynamics rather than being a change or addition to the theory. Consequently, it will feature (in a similar form) in any interpretation which incorporates the linear Schrödinger dynamics as a real and universal feature of reality (e.g. Everettian and Bohmian interpretations). The rest of this thesis has focused on examining different ways of understanding decoherence within a universally applicable linear and deterministically evolving wave function.

Chapter 2 began by returning to the quantum measurement problem, and introducing a subdivision of the problem set out by Schlosshauer 2007. Schlosshauer divides the problem into the *problem of the non-observability of interference*, the *problem of the preferred basis,* and two distinct *problems of outcomes.* Of these, the most important over the course of this thesis has been the problem of the preferred basis. That is, the problem of explaining how within linear quantum mechanics it is possible for a measurement to be a measurement of one property of a system rather than another noncommuting property.

I then set out the first rigorously defined conception of decoherence given in this thesis. This is the conception of decoherence as the diagonalisation of a local system's reduced density matrix in a particular basis, as a result of interaction with its environment. As I emphasised, this conception of decoherence is empirically well confirmed, and, aside from relying on a rather suspect application of the Born rule and a particular choice of system-environment split, as discussed in chapter 4, it has a clear and direct connection to the formalism of linear quantum mechanics. This relatively clear and direct connection to the standard formalism and empirical evidence is a distinctive feature of this weakest conception of decoherence.

Following Schlosshauer, I went on to set out how this conception of decoherence seems to offer a means of resolving three of these four aspects of the quantum measurement problem. In the context of the problem of the preferred basis, this is done by identifying the measured basis with the basis in which the reduced density matrix of the system is diagonalised over the course of the measurement process. Consequently, a natural choice for a criterion, to determine when quasi-classical histories of a local system branch off from one another, is precisely this process of diagonalisation. This will ensure that under operations performed only on the local system there will be no interference phenomena between distinct histories. I noted however, that there is nothing about fulfilling this criterion that will ensure these interference-free dynamics persist in the long-term, and that if such interference effects returned this would seem to present both Bohmian and Everettian interpretations with puzzling distinctly nonclassical dynamics.

Chapter 3 set out two different criteria of decoherence, both of which aim to ensure robustly interference free quasi-classical dynamics within histories. To explain these, I set out a more detailed conception of a history, and a history space, as well as outlining the many histories interpretations which developed these concepts. I also introduced the decoherence functional in terms of which these two criteria, *consistency* and *medium decoherence,* are defined. Unlike the diagonalisation of a local system's reduced density matrix, which is a short-term criterion for when decoherence takes place within a single history and its immediate successors, these new criteria are conditions over a system's history space which ensure particular dynamics over the entire duration of the system considered.

Consistency, the weaker of these criteria, ensures that there is no interference between the different histories which make up the history space of a system. This is of very obvious relevance to our everyday notions of the classical world and what it would mean for a history space to display quasi-classical dynamics. However, within the Everettian literature, consistency has declined significantly in popularity as a criterion of decoherence. Instead, Wallace, in his presentation of the many worlds interpretation, as well as authors such as Gell-Mann, Hartle and Griffiths within the many histories literature, have come to advocate the stronger criterion of medium decoherence. Medium decoherence entails consistency, and also requires that histories, having once separated from one another, should never at any later time recombine.

I argued in chapter 3 that consistency is too weak as a conception of decoherence as it is fulfilled in cases where a system's history space does not display interference between histories, but where this absence of interference is not guaranteed by entanglement to the wider environment. As such, this criterion can be fulfilled in cases where the physical phenomenon of decoherence has not taken place.

Medium decoherence, on the other hand, seems to be too strong a criterion. For a system’s history space to fulfil this criterion, each history within the space must always retain records of every past branching of that history. Whereas the physical phenomenon of decoherence concerns records being encoded outside of a system, the medium decoherence criterion requires that the results of all probability events over the system’s history should remain always encoded within the system itself. A consequence of this is that any history space which fulfils the medium decoherence criterion will have a branching structure. That is, histories will branch off from one another but never recombine, just as desired for Wallace's many worlds interpretation.

Clearly, the medium decoherence criterion is a far stronger criterion than the diagonalisation of a system’s reduced density matrix, which raises the question of why we should believe that the history space of the universe we occupy really fulfils this criterion and has such a branching structure.

Chapter 4 left the subject of medium decoherence in order to further examine how reasoning based on environment induced decoherence is used to develop an interpretation of quantum mechanics which seeks its origin purely in terms of linear Schrödinger dynamics and a universally applying wave function. The biggest challenge for such approaches is how to reconcile a linear and deterministic theory with the probabilistic nature of the observations which underpin quantum mechanics. In chapter 4 I presented a derivation of the Born rule developed by Zurek, and showed how, based on certain assumptions, it is possible to derive the Born rule as the rational credence ascription of an agent who is part of a history which undergoes a branching process.

Zurek's derivation is compelling, but faces criticism from authors such as Kent 2010 and Kastner 2014 who claim that it is viciously circular. The remainder of chapter 4 focused on examining and responding to two criticisms raised by Kastner 2014. In both cases, I argued that the circular nature of Zurek's derivation, identified by Kastner, is a legitimate criticism of Zurek's attempt to show that Born rule probabilities for quasi-classical states follow as a necessary consequence of the linear Schrödinger formalism. However, I also argued that Kastner is mistaken in extending these objections from the use of this reasoning within Zurek’s quantum Darwinist project (which attempts this direct derivation), to its use within broader Everettian projects such as Wallace’s, which simply use this reasoning to show how such probabilistic quasi-classical states robustly emerge from the linear formalism.

A second point raised by Kastner is that typical models of decoherence used for calculating decoherence times rely on the assumption of randomised phase correlations within the environment to which the system becomes entangled. She points out that this assumption does not seem plausible in reality. Kastner does not give any clear example of when such phase correlations could present a significant problem for decoherence or Zurek's Born rule derivation. I reviewed several concerns about this approximation which she might have had in mind, and argued that the only one which need concern us is the possibility that phase correlations within the environment will give rise to subsequent history recombination.

In chapter 5 I consider two concerns presented by Thébault & Dawid 2015 which are related to those of Kastner, and which are presented primarily as a challenge to Wallace. The first of these concerns is that Thébault & Dawid believe that the derivation of the Born rule, as given by authors such as Zurek and Wallace, presupposes not simply the subjective Born rule which the derivation yields, but a stronger objective Born rule. They believe that such an objective Born rule is inconsistent with the nature of the Everettian project, thus leading to an inconsistency within the theoretical framework. The second concern is that the neglecting of off diagonal elements within the reduced density matrix is ad hoc and an example of ontological prejudice given that diagonal elements within the matrix are not disregarded in the same way.

I argued in chapter 5 that neither of these concerns, as presented by Thébault & Dawid, need trouble Everettians such as Wallace. I argued that a derivation of the Born rule in the style given by Zurek and Wallace, while it must presuppose a subjective form of the Born rule, does not need a stronger objective form of the Born rule. I also argued that the neglecting of off diagonal elements within reduced density matrices, as discussed by Thébault & Dawid, is not something that need concern us. I noted, however, that this supposed neglectability of off diagonal elements is only benign within the context of a branching structure of histories. I assumed such a branching structure in the discussion in chapter 5, as Wallace assumes such a structure, and Thébault & Dawid never indicate that they wish to question this assumption.

In chapter 6, I examined the dynamics of individual histories within a history space which does not possess a branching structure, and looked at a variety of arguments for why we might believe that the history space of our universe possesses such a branching structure. I began that chapter by setting out cases in which environment-induced diagonalisation of a local system’s reduced density matrix takes place without the permanent history separation needed for such a history to occupy a history space which fulfils the medium decoherence criterion. I looked at two such examples: a Wigner's friend style thought experiment originally discussed in chapter 2, and a more realistic, though still very simplified, case of history recombination. I argued that, although it is in principle possible to have empirical evidence of this kind of history recombination, and the interference phenomena which are likely to result, the very long typical recoherence timescales make such records extremely difficult to obtain in practice. As such, it seems that we do not have direct empirical evidence for the medium decoherence assumption.

On the other hand, though these recombination events are very difficult to prove or disprove empirically, they do seem to have a profound effect on the dynamics of the recombining histories. This is because destructive or constructive interference between the recombining histories leads to changes in history amplitude, independent of the internal dynamics of the individual recombining histories. This seems to mean that the probabilities associated with different histories change as a consequence of a profoundly nonlocal property of the history space in a way that is very difficult to reconcile with the quasi-classical internal dynamics of the histories in question.

Having shown that history recombination leads to profoundly nonclassical dynamics, I then turned to consider a variety of reasons why it might be reasonable to assume that the history space we occupy does possess a branching structure, and so does not display these dynamics. I argued, however, that these arguments were all either unconvincing, or rested on bold cosmological claims which themselves stand in need of careful justification.

One such argument, suggested by Wallace, is that there should be a *quantum past hypothesis* postulating an initial state of the universal wave function, which when coupled with a *simple dynamical conjecture* will ensure a time asymmetric branching structure for the history space of our universe. Wallace goes on to suggest that the quantum past hypothesis might be defensible as a Humean law of nature rather than simply as a cosmological claim on which his interpretation depends.

In chapter 7 I set out to examine this suggestion of using the Humean conception of laws to offer a means of justifying the quantum past hypothesis and the medium decoherence assumption, rather than relying directly on a bold cosmological claim about the universe we live in. Examining this claim will depend on the metaphysics available which varies between particular interpretations. For this reason, from this point on, the thesis focuses purely on Wallace's many worlds Everettian interpretation.

I argued that it is difficult to reconcile a Humean conception of laws of nature with Wallace's Everettian quantum mechanics as it stands. The problem is that such a conception requires a subvenient basis of recombinable particular facts. Laws of nature, on a Humean view, are nothing more than regularities in the arrangement of particulars in this subvenient basis. As it stands, it is very unclear on Wallace's interpretation what could fill the role of this subvenient basis.

Much of the reason why this problem is so difficult to resolve is that Wallace himself is explicitly noncommittal as to what the fundamental ontology of his interpretation is. A suggestion advocated by Ney 2013 and North 2013, of taking the wave function in configuration space as the fundamental ontology, might offer a potential subvenient basis for Humean laws. Such a proposal, however, would be at odds with the intended basis neutrality of Everettian quantum mechanics, and has been strongly rejected by Wallace himself as an unnecessary change to the theory which makes it significantly harder to reconcile with the theory of relativity.

I therefore conclude that as it stands it would be very difficult to incorporate the quantum past hypothesis into Wallace's Everettian quantum mechanics as a Humean law of nature.

In chapter 8 I continued to focus purely on Wallace's interpretation, and looked in more detail at what the consequences would be for it if the medium decoherence assumption was abandoned. I argued that this assumption is of pivotal importance to the conception of probability within Wallace's interpretation, and that it could not be dropped without fundamentally undermining Wallace's derivation of the Born rule.

This poses a serious problem for Wallace's interpretation as, although without the Born rule his interpretation is still capable of predicting and describing histories very much like the one we occupy, without an account of probability there is no means of explaining why we should expect to find ourselves in a history which displays approximately classical dynamics. As Barrett 1999 puts it, without a justified means of identifying amplitudes with probabilities, we cannot explain why it is more reasonable to expect the Eiffel tower to currently reside in Paris than in Pittsburgh.

I argued that the extreme loss of explanatory power, which results within Wallace's interpretation if the medium decoherence assumption is dropped, could itself be seen as the basis for an indispensability argument to support this assumption. I believe that this is probably the best justification that can be offered for the medium decoherence assumption within Everettian quantum mechanics, as it does not rely on the direct assumption of other controversial cosmological claims, unlike several of the other arguments considered in this thesis. However, this indispensability argument relies on their being no other equally virtuous similar theory which is able to dispense with the medium decoherence assumption.

9.2 The Indispensability of Medium Decoherence

A thorough comparison of the full range of interpretations of quantum mechanics and their relative virtues is beyond the scope of this thesis. However, this section will look briefly at which interpretations of quantum mechanics rely on the medium decoherence assumption, and why Wallace’s Everettian quantum mechanics might be thought superior to those interpretations which do not rely on it. If Wallace's interpretation is indeed superior to all such interpretations, then this goes a long way towards justifying the use of the indispensability argument outlined in the previous chapter.

It was mentioned in the first chapter that Bohmian mechanics uses a local *effective wave function*, rather than the complete universal wave function, to predict the dynamics of particles within local systems. I noted at the time that Goldstein 2013 regards this use of an effective wave function as being the same phenomenon which has come to be known as decoherence. I also pointed out in chapter 6 that the possibility of interference phenomena between previously separated histories, when they come to recombine, is a direct product of the linearly evolving universal wave function, and as such is a result that applies to Bohmian mechanics just as much as it does to Everettian quantum mechanics.

What such interference phenomena mean for the Bohmian interpretation is a question that I will turn to in the next section. It seems clear, however, that any Bohmian interpretation which is committed to the viability of effective wave functions, must endorse the medium decoherence assumption. This is because effective wave functions rely on the assumption that only the local states within a particular history will affect a system's future evolution. The nonlocal entanglement phenomena which give rise to history recombination directly undermine this assumption. Consequently Bohmian mechanics, like Everettian quantum mechanics, seems to rely on a commitment to the medium decoherence assumption.

In chapter 1 I also noted that pragmatist interpretations of the type presented by Healey 2012 and Friederich 2015 make use of decoherence to demarcate non-representational purely quantum claims from non-quantum measurement claims which are representational. It remains unclear to me just what conception of decoherence is really needed here. Consequently, it is also unclear to me whether or not these pragmatist interpretations need to be committed in any sense to the medium decoherence assumption.

Even if these interpretations are not committed to this assumption however, I do not think that pragmatist interpretations offer an alternative theory to Wallace's Everettian quantum mechanics. This is because the pragmatist accounts are not realist interpretations seeking to describe the state of reality which underlies the applicability of quantum mechanics. As a result, the project in which they are engaged is not sufficiently similar to that of Wallace's Everettian quantum mechanics that comparison could usefully be made as to their theoretical virtues.

The final interpretation presented in chapter 1 was the GRW collapse interpretation. This interpretation, unlike the others presented in chapter 1, does not rely on decoherence. Environment induced decoherence may be displayed in the system’s wave function prior to the spontaneous localisation of the system’s wave function, but this is not of particular importance to the interpretation. Moreover, frequent spontaneous localisations will prevent the development of any long-lasting large-scale entangled states. Consequently, GRW has no need to add the medium decoherence assumption, as history recombination of the type discussed in this thesis is already ruled out by the assumption of spontaneous position localisations.

Here then, is an interpretation which does not rely on the medium decoherence assumption. Therefore, if the GRW interpretation is equally as attractive in other respects as the Everett interpretation, it would indicate that the medium decoherence assumption is in fact dispensable, and remove the possibility of defending it by means of an indispensability argument.

I will not attempt a detailed comparison of the theoretical virtues of Wallace's Everett interpretation with the various forms of GRW interpretation. However, I will briefly identify two reasons why the Everett interpretation might reasonably be thought the more attractive theory. The first is the difficulty of reconciling wave function collapse with relativity. The spontaneous wave function localisations described in the GRW theory are difficult to reconcile with relativity theory as, on the standard formulation of GRW, these localisations seem to involve a superluminal change in the states of wave functions for entangled systems. This problem is the subject of ongoing research and may well not be insurmountable. For a detailed presentation of the problem, as well as an account of ongoing efforts to resolve it, see Maudlin 2002 and Tumulka 2006. For the time being, however, this tension between GRW and relativity remains a significant disadvantage to the interpretation.

The second reason why Wallace’s Everettian interpretation might be thought more attractive than the GRW interpretation is that, whereas Wallace's interpretation relies on the medium decoherence assumption to rule out the possibility of problematic history recombination, within the GRW interpretation this is achieved by spontaneous wave function localisations. Both of these are additions to the standard linear quantum formalism. For the time being, neither of these additions to the standard formalism is supported by any empirical evidence[23]. I would suggest, however, that the addition of collapse dynamics to the standard formalism is a far more radical assumption than the assumption of a particular type of structure for the history space of our universe. The collapse dynamics of GRW rely on the identification of position as a preferred basis and involve the introduction of discontinuous and fundamentally stochastic dynamics unlike anything seen in our other fundamental physical theories. Given the emphasis which Wallace places on conformity to the standard formalism without unnecessary alterations or additions, it seems likely to me that he and other Everettians would still regard Everettian quantum mechanics as considerably more attractive even if it does require one very general assumption about our history space.

9.3 A Bohmian Defence of Medium Decoherence

In the previous section, I said that, as Bohmian mechanics seems to rely on the medium decoherence assumption just as Everettian quantum mechanics does, the relative appeal of these two interpretations could not undermine the indispensability of the medium decoherence assumption. However, although both of these interpretations rely on the medium decoherence assumption, this does not mean that they rely on it for the same purpose, or that it has the same status within both interpretations.

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[23] Though, as noted in the first chapter, GRW is empirically distinct from other interpretations of quantum mechanics, and may someday receive independent empirical support. This is presently beyond our technical capabilities, however.

This section will offer some preliminary comments on the significance of the medium decoherence assumption within Bohmian mechanics, and possible ways of justifying the assumption within that interpretation. I will suggest that, based on this preliminary examination, it seems possible that the medium decoherence assumption may be easier to justify within Bohmian quantum mechanics than it is for Everettian quantum mechanics.

The first difference from Everettian quantum mechanics lies in the reason for the interpretation to commit to this assumption. Within Everettian quantum mechanics, the most important role which medium decoherence plays is to provide a robust basis of branching histories which can be identified as worlds, which emerge from the linearly evolving quantum state. And, following from this, to ensure that history amplitudes persist in playing the functional role necessary to be identified as probabilities.

Within Bohmian mechanics on the other hand, both the problem of the preferred basis and the problem of the origins of probability find solutions in the dynamics of Bohmian particles. Quantum probability is accounted for as a consequence of our ignorance of the initial positions of Bohmian particles, and the proportions of possible trajectories which would lead them to particular points. Particle position is given as a preferred basis, and the process of measurement can be analysed in terms of a counterfactual dependency between the final positions of the particles of the measuring apparatus and the trajectories of the measured particles.

Instead, the medium decoherence assumption seems to be needed just in order to justify the use of effective wave functions, and the reliability of the dynamics which they predict. Consequently, it seems that this assumption may be somewhat less pivotal within Bohmian mechanics than it is within Everettian quantum mechanics. Nevertheless, it still seems to play a very important role. The universal wave function is epistemically very hard to access, unlike locally determined effective wave functions. Thus, if the medium decoherence assumption is abandoned, then the Bohmian may find it very difficult to make useful predictions.

Another striking difference between the status of the medium decoherence assumption in Bohmian and Everettian quantum mechanics, is the ease with which it could be defended as a Humean law of nature. In chapter 7, I argued that within Everettian quantum mechanics it is very difficult to find a subvenient basis onto which Humean laws could supervene. Within Bohmian mechanics, on the other hand, the arrangement of particle positions is a readily available candidate for a mosaic of freely re-combinable particulars onto which Humean laws of nature could supervene. Thus, it seems likely that identifying the medium decoherence assumption as a Humean law of nature would be a perfectly reasonable strategy for supporting the assumption in a Bohmian context. Indeed, authors such as Dewar 2016 already argue for regarding the wave function itself as a Humean law of nature which supervenes on the positions of Bohmian particles in a similar way. At least for those Bohmians who already think of the universal wave function in this way, it seems that incorporating the medium decoherence assumption as a law of nature might be a very natural and very minimal addition to the interpretation.

These comments are only intended to identify a fruitful seeming area for future research, and do not amount to a well-developed argument. More work needs to be done to properly understand the full significance of medium decoherence within Bohmian mechanics and its relation to the use of effective wave functions. More work is also needed in examining the many variations of Bohmian mechanics, and how these different approaches would regard the incorporation of medium decoherence as a Humean law of nature. Superficially, however, it does seem that the relative ease with which some Bohmian interpretations could incorporate the medium decoherence assumption might represent a minor incentive in favour of a Bohmian rather than an Everettian interpretation[24].

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[24] Of course, this advantage may possibly be offset by other disadvantages of the Bohmian interpretation. One example being that, like the GRW interpretation, it is more difficult than Everettian quantum mechanics to reconcile with relativity.

9.4 Final Remarks

This thesis has investigated the nature and role of decoherence as the concept is used within interpretations of quantum mechanics. I have set out three different conceptions of decoherence and done my best to make clear the relations between them. There is further work to be done in mathematically pinning down just how a definition of decoherence in terms of a reduced density matrix relates to definitions of decoherence given in terms of history spaces, but the general nature of their relation to one another is already clear. The weakest of these criteria, diagonalisation of a reduced density matrix, is empirically well confirmed and, I have argued, conceptually well founded within a variety of interpretational frameworks. The strongest of these criteria, on the other hand, is medium decoherence, which is conceptually well defined, but troublingly disconnected from both empirical evidence and the linear quantum formalism.

The conception of decoherence which plays a role in modern Everettian and Bohmian interpretations of quantum mechanics is this strongest form, raising the questions of why this strong conception of decoherence is needed, and how such a strong conception can be justified. In this thesis I have focused primarily on Wallace's Everettian quantum mechanics, and I have argued that this strong conception of decoherence is essential to providing a branching structure of approximately isolated worlds, and allowing the functional identification of Born rule probabilities.

I have considered a variety of concerns relating to the use of decoherence in Everettian interpretations of quantum mechanics. I have argued that, as long as medium decoherence, and the branching structure it provides, are assumed, these concerns can be answered within Wallace's interpretational framework. I have subsequently raised my own concern, however, about how the assumption of medium decoherence can be justified. I have considered a wide variety of candidate justifications and argued that most are unpersuasive, and others amount to bold cosmological claims which themselves stand in need of justification. Finally, I have proposed the indispensability of the medium decoherence assumption to an otherwise powerful and appealing interpretation as the best justification for accepting medium decoherence. Depending on this assumption is a significant cost for Wallace's interpretation, which aims to be nothing more than an interpretation of robust patterns which arise within the linear quantum formalism without any additional formalism or assumption. Overall however, I believe that Wallace's interpretation is sufficiently appealing to make the addition of this assumption worthwhile.

In this chapter I have turned back to look at the role of decoherence in other interpretations. I have examined other interpretations to see if any of them are able to offer a similarly appealing interpretation which does not rely on the medium decoherence assumption. If such an interpretation could be found, it would undermine the claim that the assumption of medium decoherence is really indispensable. I have argued that the only candidate for such an interpretation is some form of GRW collapse interpretation, and that it is far from clear that such interpretations can really match the appeal of Wallace's Everettian interpretation.

Finally, I have offered some preliminary observations regarding the role that medium decoherence appears to play within Bohmian quantum mechanics, and possible means of justifying it. I have suggested that these may be significantly different from the situation of medium decoherence within Everettian quantum mechanics.

I hope that future work may be done to examine medium decoherence within Bohmian mechanics, and that in the future more consideration may be given to the conception of decoherence at play in arguments which rely upon it.

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