Improved motion vectors in rainfall nowcasting using Burgers' equation

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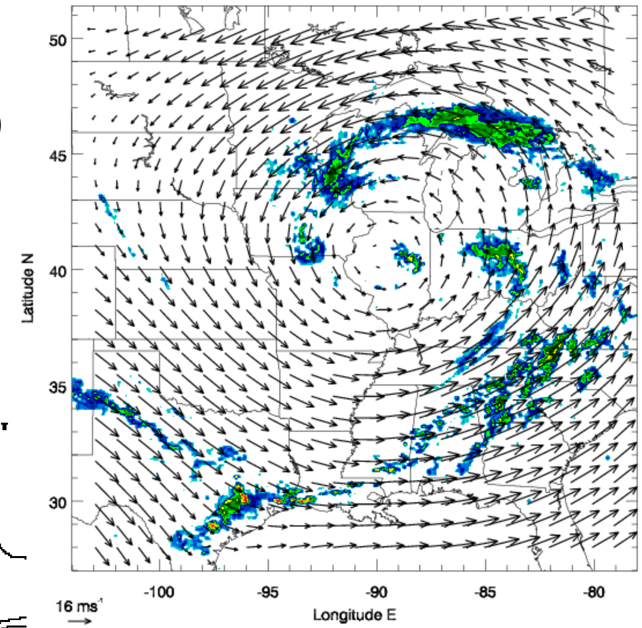
RADAR-BASED NOWCASTING

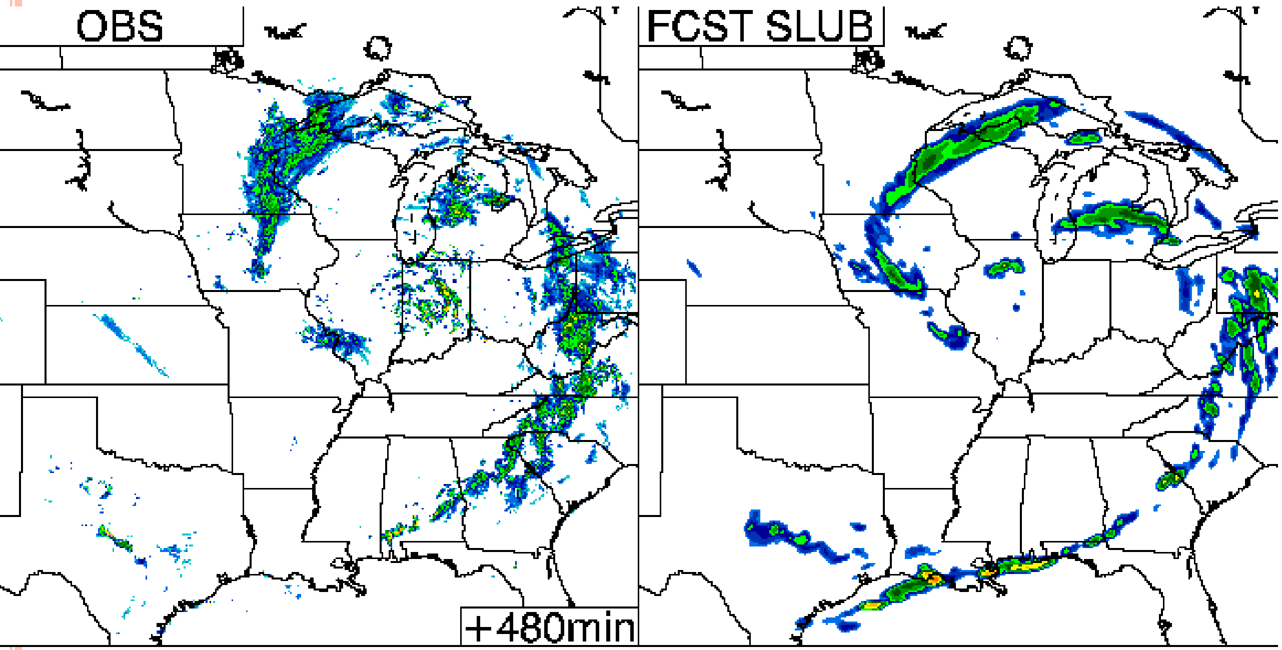
Ex) MAPLE

1. Motion fields of precip. (Variational Echo Tracking: VET)



3. Verification (compare fcst w/ obs)

2. Advect precip. fields: Semi-Lagrangian backward



$$\hat{R}(t_0+\tau,\vec{x}) = R(t_0,\vec{x}-\alpha)$$

Predicted field

Observed field

- Growth/decay (scale of predictability)
- <u>Non-stationary motion fields</u>

Germann and Zawadzki (2002)

Approach

Lagrangian extrapolation (MAPLE) $\hat{R}(t_0+\tau,\vec{x}) = R(t_0,\vec{x}-\alpha)$

OR

Conservation equation $$\frac{dR}{dt} = \frac{\partial R}{\partial t} + u\frac{\partial R}{\partial x} + v\frac{\partial R}{\partial y} = 0$$

Solve this simple advection(conservation) equation(AE) directly

$$\frac{\partial R}{\partial t} = -u\frac{\partial R}{\partial x} - v\frac{\partial R}{\partial y}$$

: Type 1

Add diffusion term for spatial filtering (smoothing):
advection diffusion equation (ADE)

$$\frac{\partial R}{\partial t} = -u\frac{\partial R}{\partial x} - v\frac{\partial R}{\partial y} + \nu\left(\frac{\partial^2 R}{\partial x^2} + \frac{\partial^2 R}{\partial y^2}\right)$$

: Type 2

Approach

However, above two equations assume that the motion vector field is stationary in time (constant motion vectors for entire forecast time)

Introduce Burgers' equation:

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} + s\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$\frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} + s\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

to allow non-stationarity of motion vectors. The *s* controls the degree of the smoothness.

Approach

Semi-Lagrangian extrapol.(MAPLE): $\hat{R}(t_0+\tau,\vec{x}) = R(t_0,\vec{x}-\alpha)$

Type 1: advection equation(AE)

$$\frac{\partial R}{\partial t} = -u\frac{\partial R}{\partial x} - v\frac{\partial R}{\partial y}$$

Type 2: advection diffusion equation(ADE)

$$\frac{\partial R}{\partial t} = -u\frac{\partial R}{\partial x} - v\frac{\partial R}{\partial y} + v\left(\frac{\partial^2 R}{\partial x^2} + \frac{\partial^2 R}{\partial y^2}\right)$$

Type 3: advection equation(AE) + Burgers' equation

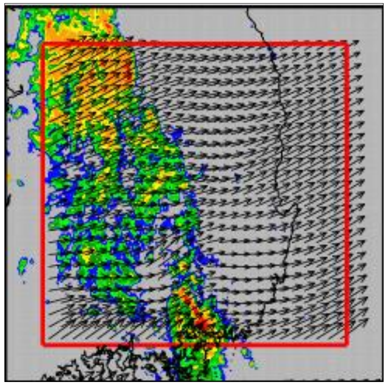
$$\frac{\partial R}{\partial t} = -u\frac{\partial R}{\partial x} - v\frac{\partial R}{\partial y} \quad + \quad \begin{aligned}\frac{\partial u}{\partial t} &= -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} + s\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)\\ \frac{\partial v}{\partial t} &= -u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} + s\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)\end{aligned}$$

Type 4: advection diffusion equation(ADE) + Burgers' equation

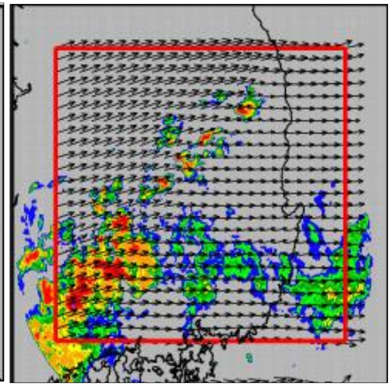
$$\frac{\partial R}{\partial t} = -u\frac{\partial R}{\partial x} - v\frac{\partial R}{\partial y} + v\left(\frac{\partial^2 R}{\partial x^2} + \frac{\partial^2 R}{\partial y^2}\right) + \begin{aligned}\frac{\partial u}{\partial t} &= -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} + s\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)\\ \frac{\partial v}{\partial t} &= -u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} + s\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)\end{aligned}$$

RAIN EVETNS

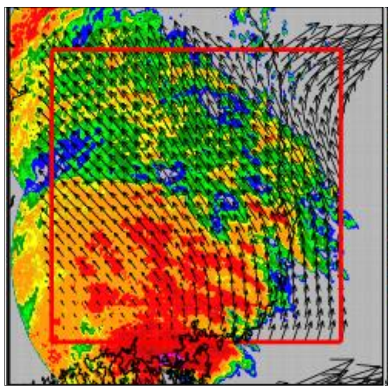
30 June 2012,0300



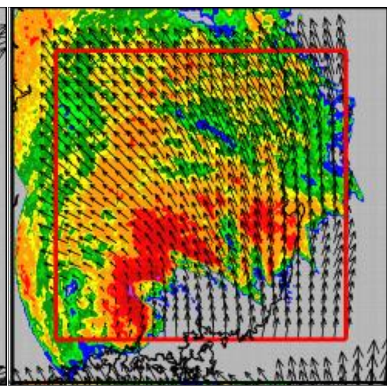
30 June 2012,1900



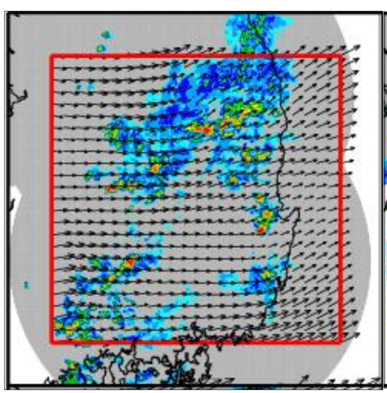
17 Sep 2012,1000



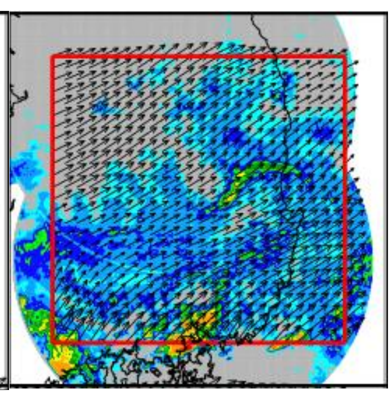
17 Sep 2012,1200



23 June 2014,1900



25 June 2014,1100



1. Cases

- 6 events, 2.5 min CAPPI composite from 3 radars
- 15 min nowcasting up to 3h

2. Computation domain

- Southeast area in South Korea
- 312 km x 312 km at 0.25 km resolution (1248 x 1248 pixels)
- Motion vectors : 10 km resolution
- Verification domain: 250 km x 250 km (red box)

Set-up

1. **Motion vectors (u, v):** derived from variational echo tracking (VET) with Δt = 2.5 min

2. **Advection (diffusion) equation:**
   - ODE for time: Explicit (forward) Runge-Kutta fourth order (RK4) with Δt = 0.1 min
   - Spatial derivative for R: Finite difference method with $\Delta x = \Delta y = 0.25$ km
   - $0.0 \le v \le 0.1$

3. **Burgers' equation:**
   - Spatial derivative for (u,v): Finite difference method with $\Delta x = \Delta y = 10$ km
   - s=0.2

SKILL SCORES, ERROR STATISTICS

❖ 2D Contingency table

	Forecast	
Obs	**$R>=R_{th}$**	**$R<R_{th}$**
$R>=R_{th}$	**Hit (a)**	**Miss (c)**
$R<R_{th}$	**False alarm (b)**	**Correct negative (d)**

❖ Categorical scores

Verification score	**Formula**
Probability of detection **(POD)**	a/(a+c)
False alarm ratio **(FAR)**	b/(a+b)
Critical success index **(CSI)**	a/(a+b+c)
Equitable threat score **(ETS)**	(a-w)/(a+b+c-w), w=(a+b)(a+c)/(a+b+c+d)

- **Correlation coefficient** :

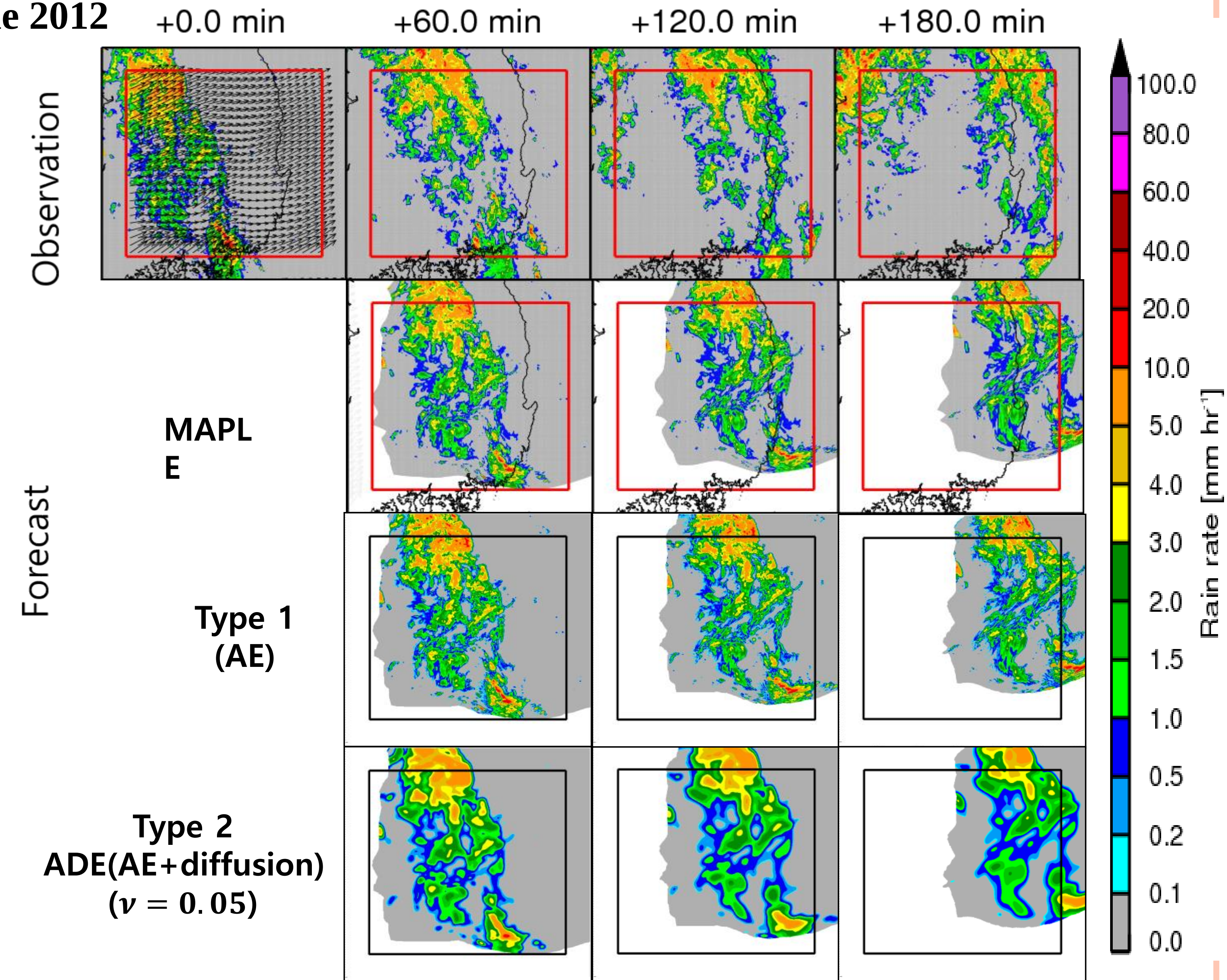
$$r(t) = \frac{\sum_{n(t)}(R_0(t)R_F(t))}{\left(\sum_{n(t)}(R_0)^2 \sum_{n(t)}(R_F)^2\right)^{1/2}},$$

- **Mean Absolute Error :** $\mathrm{MAE}(t) = \frac{1}{n(t)} \sum_{n(t)} |R_F(t) - R_O(t)|$

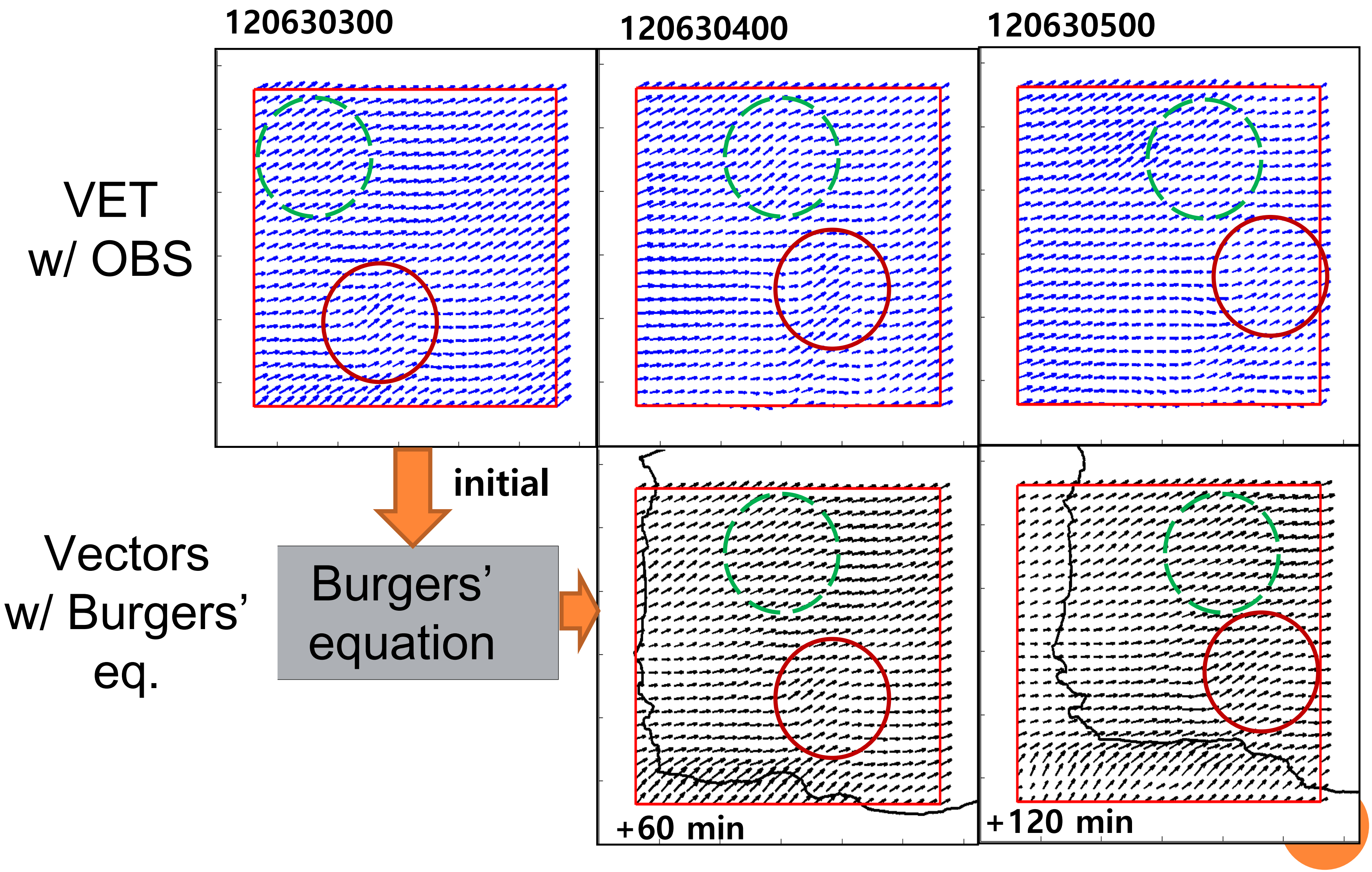
- **Conditional Mean Absolute Error** : $\mathrm{CMAE}(t) = \frac{1}{a(t)} \sum_{a(t)} |R_F(t) - R_O(t)|$

MAPLE vs. AE + DIFFUSION

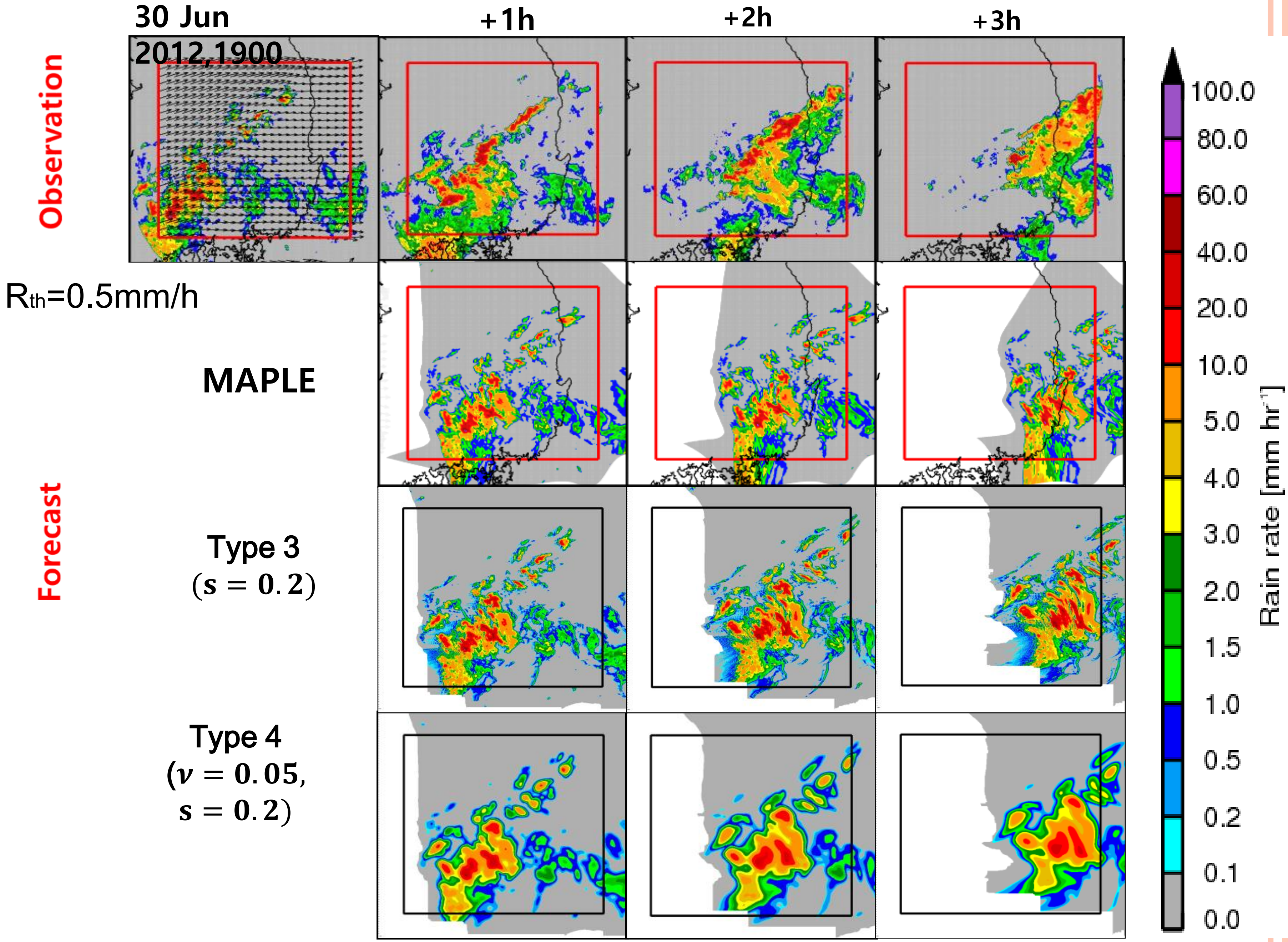
0300 LST
30 June 2012



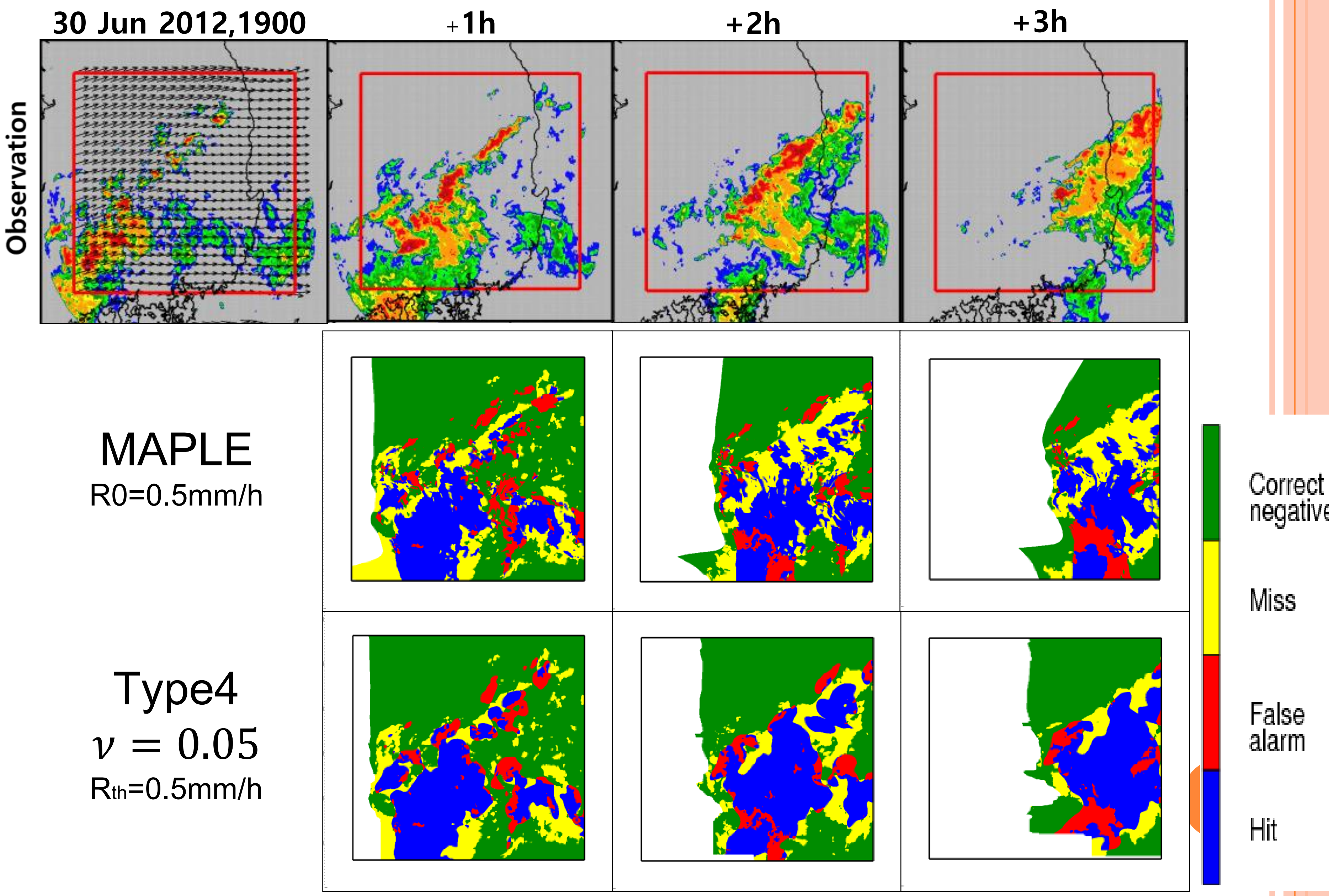
NON-STATIONARY MOTION VECTORS



MAPLE vs. AE + NON-STATIONARY + DIFFUSION

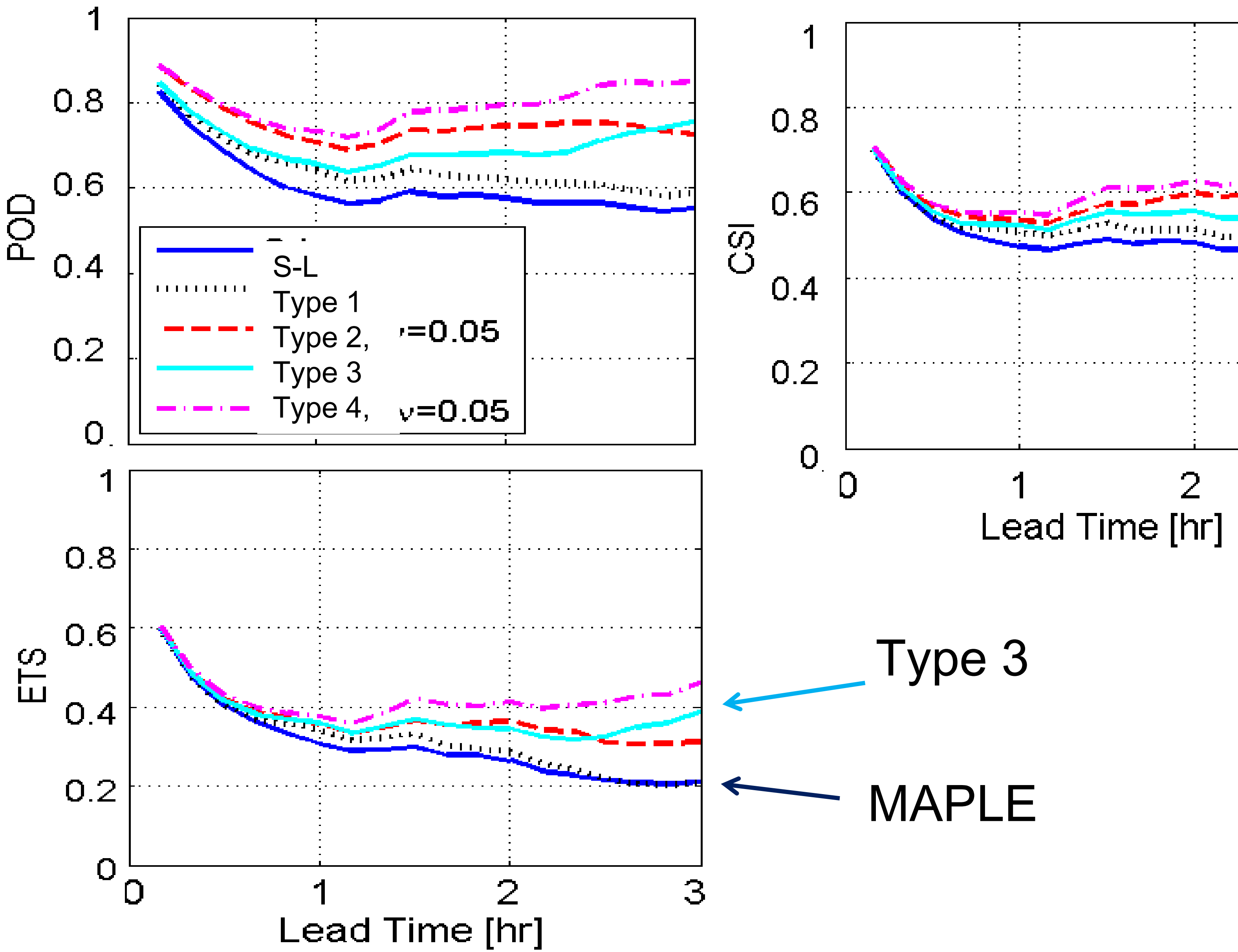


MAPLE vs. (AE + non-stationary + Diffusion)



MAPLE vs. (AE + non-stationary + Diffusion)

Skill scores R_{th} = 0.1mm/h (201206301900)

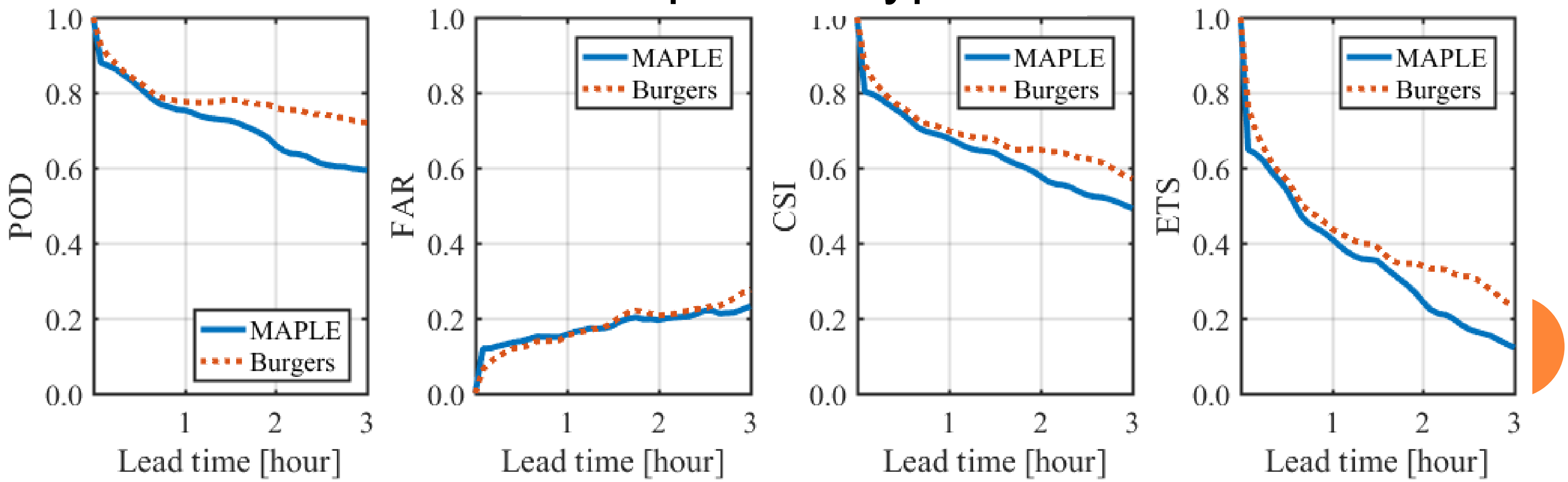


NON-STATIONARY MOTION VECTORS

$$\text{Mean}(|(u \cdot \nabla)\mathbf{u}|)$$

Events(LST)	+0 h	+1 h	+2 h	+3 h	Mean
30 Jun 2012, 0300	0.74	0.88	1.00	0.89	**0.91**
30 Jun 2012, 1900	0.81	2.10	0.79	0.59	**1.14**
17 Sep 2012, 0900	1.98	2.78	1.56	1.34	**1.90**
17 Sep 2012, 1200	1.62	1.34	1.10	0.72	**1.08**
23 Jun 2014, 1900	0.32	0.38	0.47	0.46	**0.43**
25 Aug 2014, 1100	0.60	0.65	0.83	1.00	**0.81**

Maple vs. Type 3



NON-STATIONARY MOTION VECTORS

$$\mathrm{Mean}(|(\boldsymbol{u} \cdot \nabla)\mathbf{u}|)$$

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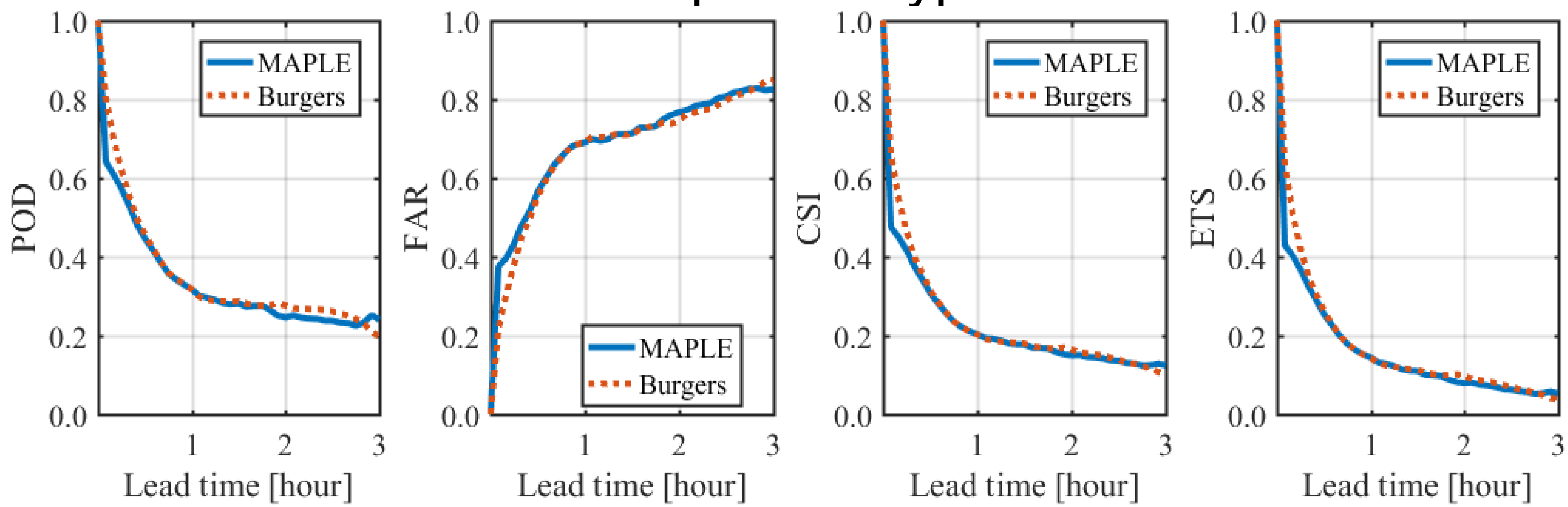
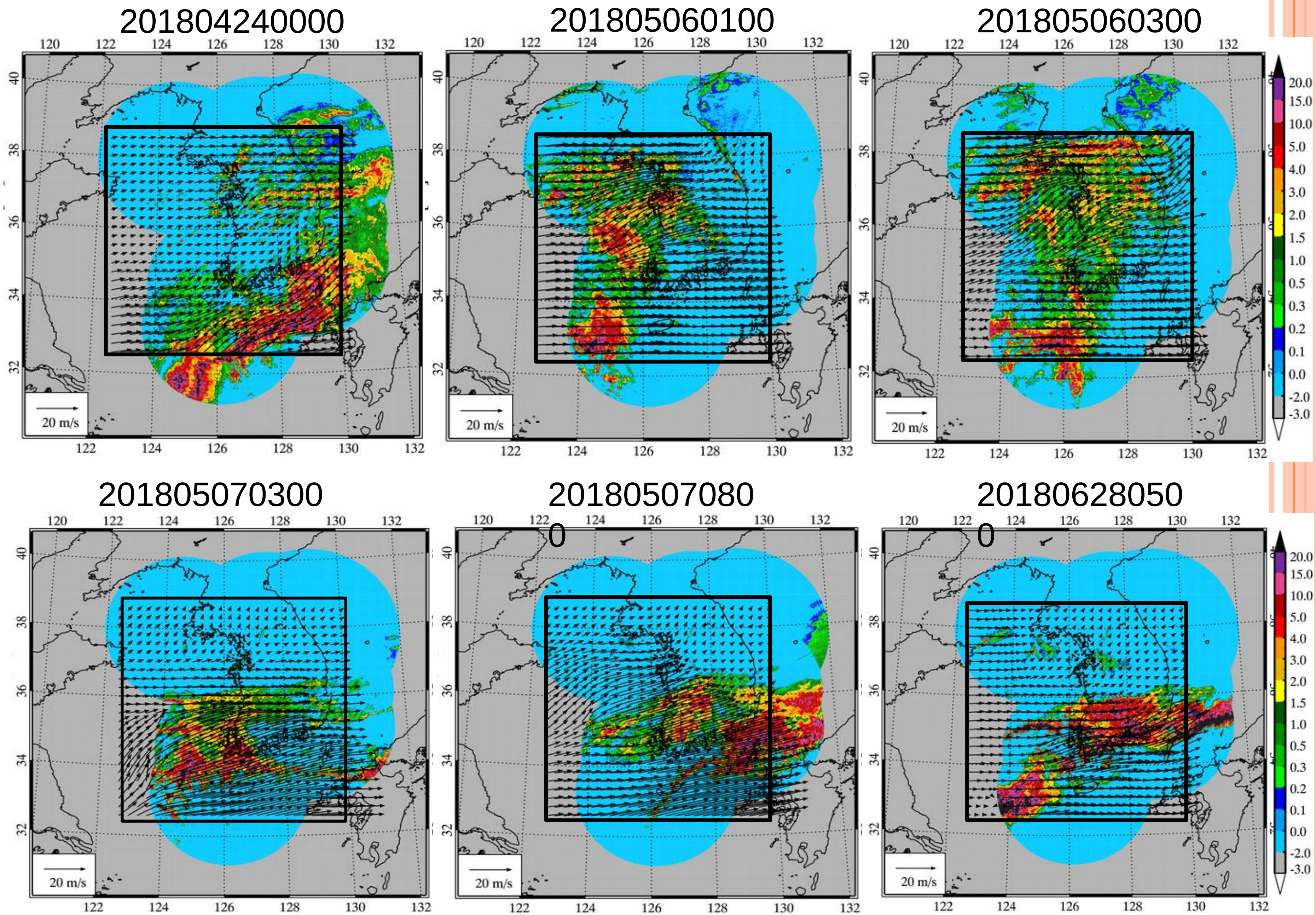
Maple vs. Type 3



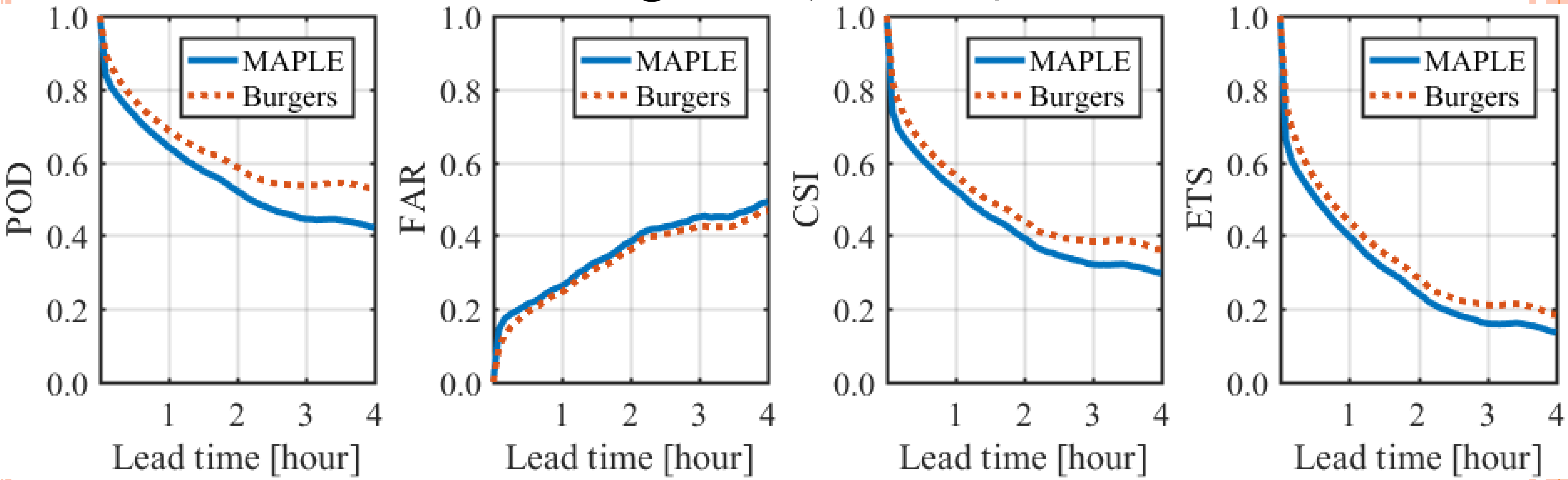
Test in larger domain

HSR composite, 1153 km x 1153 km at 1 km , Vector: 27 km, every 10 min)

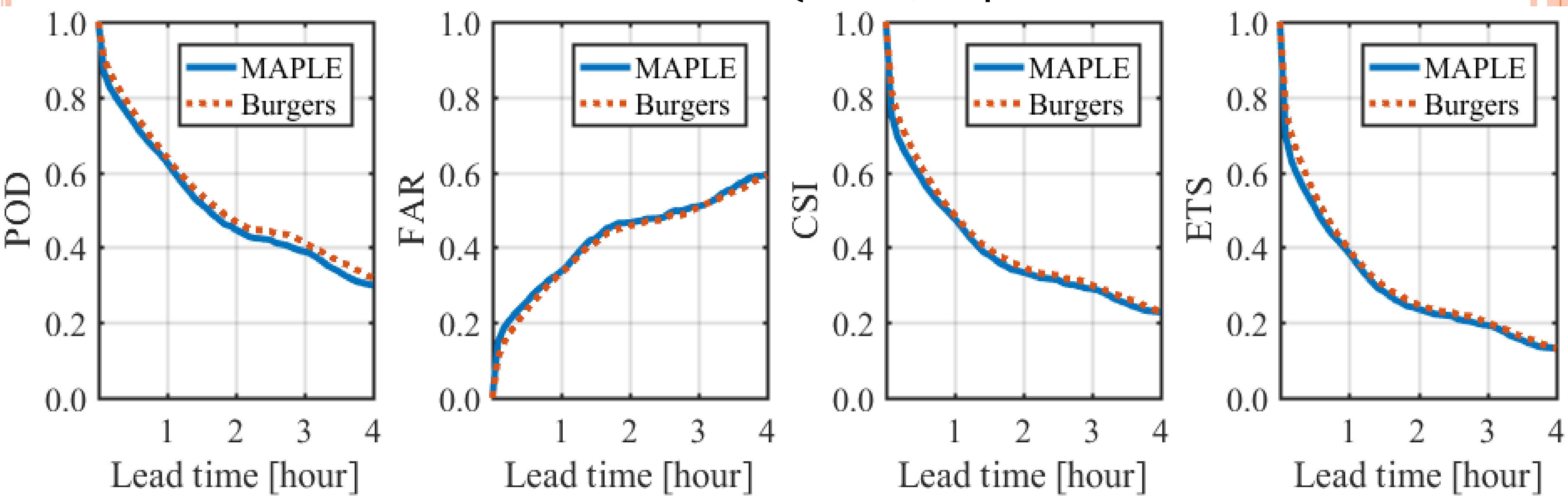


TEST IN LARGER DOMAIN: MAPLE VS. TYPE 3

Higher $|(\boldsymbol{u} \cdot \nabla)\mathbf{u}|$



Lower $|(\boldsymbol{u} \cdot \nabla)\mathbf{u}|$



SUMMARY

- Introduced nowcasting based on advection (diffusion) equation with Burgers' equation

- Performance:
  MAPLE ~ Advection eq. < Advection eq. + Burgers eq.
  (S-L ~ Type1 < Type2 < Type 3 < Type 4)

- Use of diffusion term and non-stationary motion vector improves forecasting skill scores

- When non-stationarity of motion fields is strong, the precipitation forecasts using Burgers' equation (Type3, Type 4) show significant improvement.