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Using Schematic-Based and Cognitive Strategy Instruction to Improve Math Word Problem Solving for Students with Math Difficulties

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A Dissertation
Submitted to the Faculty of Old Dominion University
in Partial Fulfillment of the Requirements
for the Degree of
Doctor of Philosophy
Special Education Concentration

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Dedication

This dissertation is dedicated to all of my former students who have shared their hearts and minds throughout many adventures in learning with me. I hope you learned a great deal from me; but I learned more from you. Through you, I came to understand the meaning of Ralph Waldo Emerson's words, "What lies behind us and what lies before us are tiny matters compared to what lies within us."

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Abstract

Using Schematic-Based and Cognitive Strategy Instruction to Improve Math Word Problem Solving for Students with Math Difficulties

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For students with math difficulties (MD), math word problem solving is especially challenging. The purpose of this study was to examine a math word problem solving strategy, bar model drawing, to support students with MD. The study extended previous research that suggested that schematic-based instruction (SBI) training delivered within an explicit instruction framework can be effective in teaching various math skills related to word problem solving. As a more generic schema approach, bar model drawing may serve as an effective form of SBI that can be developed across word problems. Moreover, the bar model approach has the potential to enhance students' awareness of cognitive strategies through paraphrasing, visualizing, hypothesizing about problem solutions, and checking work, all of which are explicitly taught through the use of the bar-model drawing protocol.

A multiple-baseline design replicated across groups was used to evaluate the effects of the intervention of bar model drawing on student performance on math world problem solving. Student performance was investigated in terms of increased accurate use of cognitive strategies and overall accuracy of math word problem solving. Both of these

dependent variables increased and remained stable throughout intervention, and remained high during the maintenance phase of the research. Pre and posttesting results were also favorable. Participants reported high social validity for the intervention. However, the results of the research also yielded some surprises and raised some questions. Conclusions drawn from the data include a discussion of the implications for action and recommendations for further research. Limitations of the study are also discussed.

CHAPTER ONE

INTRODUCTION

Chapter one describes concerns related to student performance in mathematics, specifically in the area of math word problem solving. Students who have mathematics difficulties (MD) especially struggle in math word problem solving. These difficulties may be attributed to cognitive and metacognitive deficits (Watson & Gable, 2013). Students with these deficits can be supported through the use of schematic-based instruction (SBI) and cognitive strategy instruction (CSI; Jitendra et al., 1998; Xin, 2008; Montague & Applegate, 1993; Rosenzweig, Krawec, & Montague, 2011). However, there are gaps in the current literature supporting the use of SBI and CSI. The stated purpose and subsequent research questions address these gaps in the research. This chapter will provide an overview of the problem, along with gaps in the current research, a rationale for the study and statement of the problem, research questions, and will include a glossary of key terms that are integral to the research.

Problem Context

Student performance in mathematics. America continues to lag behind many of its peers in mathematics and mathematics instruction. In 2008, the National Mathematics Advisory Panel (NMAP) admonished that without improvement in this area, the United States' leadership role is in jeopardy. The Panel cited statistics demonstrating the gravity of the issue of mathematics literacy, pointing out that 27% of eighth graders fail to accurately shade 1/3 of a rectangle. Furthermore, the problem appears to extend into adulthood; seventy-one percent of all adults in the United States cannot calculate a 10% tip. In 2011, the most recent administration of the Trends in International Mathematics and Science Study (TIMSS) reported that average mathematics scores for fourth-grade students in eight countries and other education systems outranked the United States. Eleven education systems outranked eighth-grade students from the United States. Countries considered economic rivals to the U.S., including Singapore, Korea, China, and Japan, outranked U.S. fourth- and eighth-grade students. Fourth-grade students in the Russian Federation rivaled U.S. fourth graders and the Russian Federation's eighth-grade students scored higher than their eighth-grade peers in the U.S. (Provasnik et al., 2012).

Math word problem solving. The National Council of Teachers of Mathematics (NCTM; Cai & Lester, 2010) asserts that math word problem solving must be a fundamental part of mathematics, pointing out the interdependence between problem solving and successful conceptualization of mathematics across content and grade levels. However, math word problem solving continues to be a source of difficulty for many students in the United States. The above mentioned report issued by the NMAP (2008) cited an example in which 45% of eighth-grade students were not able to solve a word problem that involved dividing fractions. In response to the importance of math word problem solving and the continued difficulty students display in this area, the NCTM has given problem solving priority by listing it first in its process standards since first highlighting it as a critical standard in 2000.

Characteristics of students with MD. Math word problem solving is especially difficult for students with MD. Estimates for the prevalence of MD vary, from 3-9% of the entire school-age population (Fuchs et al., 2010; Swanson, 2012). This large variance reflects a lack of clarity and uniformity as yet in the identification and classification of MD, or a "lack of consensus" among researchers in this relatively nascent field of research, especially when compared to the field of reading disabilities (Mazzocco, Devlin, & McKenney, 2008, p. 319; Watson & Gable, 2013).

Researchers have identified cognitive characteristics of students with MD that negatively impact their ability to solve math word problems. Working memory (WM) deficits have been linked to mathematics disabilities (Fuchs et al., 2014; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Swanson, 2012; Swanson, Jerman, & Zheng, 2009). WM refers to the concurrent storage and processing of information (Baddeley, 1992; Watson & Gable, 2010). WM deficits in students can be evidenced by sluggish, often inaccurate processing of classroom instruction. In addition, students with WM deficits often have difficulty planning tasks, filtering relevant and irrelevant information, and regulating attention (Swanson, Orosco, & Lussier, 2014; Watson & Gable, 2010; Li & Geary, 2013). Math word problem solving requires all of these skills in addition to a host of others, such as reading ability, including decoding and comprehension, and procedural and conceptual knowledge of mathematics. Demonstration of these skills can further tax WM, further exacerbating the problem of poor academic performance.

Metacognitive deficits have also been linked to MD (Desoete, 2009). Metacognition refers to knowledge and regulation of cognitive activity and processes (Krawec & Montague, 2012; Palinscar & Brown, 1987). Brown (1978) asserted this knowledge about one's own cognition is perhaps more vital than cognition itself. Metacognition includes attention and self-regulation, or cognitive monitoring. Students with metacognitive abilities can recognize deficiencies and lapses in their thinking, and recheck and revise their activities (Akbari, Khayer, & Abedi, 2014). On the other hand, students with metacognitive deficits often have difficulty distinguishing between reality verses what is not realistic (Yong & Kiong, 2005). For example, a student with metacognitive deficits may be certain she can pass a high-stakes mathematics test, even though she has passed no benchmark tests throughout the school year.

Math Word Problem Solving Support Strategies for Students with MD

Researchers have established CSI as an empirically-supported strategy for assisting students with MD in math word problem solving. CSI typically involves a representational aspect. Research also supports SBI as an effective intervention in supporting students in word problem solving. SBI typically integrates cognitive strategies within the explicit instruction of the strategy. Thus, each strategy complements the other, and could easily be bundled for greater possible effect. A brief overview of SBI, CSI, and the gaps that exist in the research for both strategies will be discussed in this section.

SBI. To address the cognitive deficits of students with MD, researchers have investigated the effectiveness of schematic-based instruction (SBI). SBI is based on the schema theory, which emphasizes the need for students to conceptualize the problem schema, the underlying structure of the problem, in order to successfully solve math word problems (Jitendra et al., 2013). Swanson, Lussier, and Orosco (2013) asserted that visual-schematic strategies supported the visual-spatial WM of students with MD. SBI has produced favorable results for supporting students with MD in math word problem solving across problem types and student age groups (Jitendra et al., 1998; Xin, 2008; Xin, Jitendra, & Deatline-Buchman, 2005)

CSI. Researchers have found that students with deficits in metacognition can be supported in math word problem solving by building awareness of task demand and providing direct instruction of appropriate word problem solving strategies (Krawec & Montague, 2012; Montague, 2007). Cognitive strategy instruction (CSI) addresses these cognitive and metacognitive deficits. CSI combines and inserts metacognitive strategies into structured cognitive sequences (Krawec & Montague, 2012). CSI has consistently yielded positive effects for students of varying age and ability groups (Fuchs et al. 2005; Garrett, Mazzocco, & Baker, 2006; Montague & Applegate, 1993; Rosenzweig, Krawec, & Montague, 2011).

Research gaps. Although SBI has yielded some positive results, it is not supported by the research that supports CSI either in quantity or span of years (see literature review charts, Appendices A and B). In addition, much of the research on the topic of SBI has been conducted by only a handful of researchers. Since evidence-based practices identified in research necessitate that the effect of an intervention be replicated across a range of researchers (Horner et al., 2005), more research is needed. Also, there is a dearth of research that systematically combines CSI and SBI. There is a need to investigate the effectiveness of the SBI strategy on students with learning disabilities (LD) and students who are at risk of mathematics failure. Finally, studies focusing on SBI have utilized graphic organizers with limited application, rather than as a method that can be used more broadly and generically across word problem types. That is, one type of schematic diagram, the bar model, which can be used across word problem types may be an approach that offers promise to the field of special education. (Ginsburg, Leinwand, Anstrom, & Pollock, 2005). The need is increased by the reality that this research on the effectiveness of strategies to solve math word problems are minimal.

Rationale for this Study

The current research will investigate a problem-solving intervention for students with MD. It is important and timely because it will attempt to advance research in SBI, combine CSI and SBI, and investigate the effectiveness of bar model drawing as a form of SBI that can support students with MD in math word problem solving. Findings may assist practitioners to better address the challenges of students with MD across grade levels. The research will extend previous studies in the field of mathematics, specifically in math word problem solving for students with MD.

Statement of Purpose

The purpose of the research is to extend previous research that suggests that SBI with explicit instruction and embedded CSI can be effective in teaching various mathematics skills related to word problem solving. The strategy for the current study uses a more generic schema, the bar model, as a form of SBI that can be developed across word problems. It is hypothesized that the bar model will support students while they solve math word problems and enhance students' awareness of cognitive strategies through paraphrasing, visualizing, hypothesizing about problem solutions, and checking work (i.e., CSI strategies), all of which are explicitly taught through the use of a bar model drawing protocol. Since much of the previous research in SBI did not disaggregate the data, this study investigated the effectiveness of the bar model as a specific schematic math word problem solving strategy with students with MD. Consequently, this study had two hypotheses:

1. Explicit instruction of SBI with bar model drawing and cognitive strategies as an intervention protocol will improve cognitive strategy awareness of students with MD.

2. Explicit instruction of SBI with bar model drawing and cognitive strategies will increase word problem solving accuracy of students with MD.

Research Questions

This empirical research study will have two research questions:

1. *To what extent will explicit instruction of the bar model drawing strategy improve the use of cognitive strategies of urban students labeled either with MD when solving math word problems?*

2. *To what extent will explicit instruction of the bar model drawing strategy and the use of CSI strategies increase the ability of urban students with MD to accurately solve math word problems?*

Glossary of Terms

This study used the following definitions to establish operational definitions. These operational definitions defined the concepts and contributed to consistency throughout the research.

Bar model drawing. A representational structure in which to build word problem solving schema, also referred to as a *strip diagram.* The instructional sequence of bar model drawing follows this order for each lesson: 1. Read the entire problem. 2. Rewrite the question being asked in sentence form, leaving a space for the answer. 3. Determine *who* or *what* is involved with the problem. 4. Draw the unit bar(s). 5. Chunk the problem and identify the missing variable. 6. Correctly adjust the unit bar(s) and compute (for which students may use calculators) to solve the problem. 7. Write the answer in the previously written sentence, making sure the answer makes sense.

Cognitive strategy instruction. For this study, CSI was defined as the accurate use of paraphrasing (i.e., rewriting the question as an answer statement), visualizing (i.e., constructing a bar model), hypothesizing about problem solutions (i.e., manipulating the bar model), and checking work (i.e., writing the answer in the previously written answer statement and ensuring it makes sense), all of which will be explicitly taught through the use of the bar model drawing strategy protocol.

Explicit instruction. This involved a step-by-step presentation of a strategy, along with teacher modeling incorporating think-aloud procedures, providing specific examples, and including opportunities for guided and independent practice with feedback.

Summary

This study is organized into five chapters. Chapter one provided an overview of the problem, the gaps in research, research questions, and a list of definitions used in the study. Chapter Two presents a review of literature related to SBI and CSI, math word problem solving and students with MD. Chapter Three describes the research design and methodology of the study, including the participants, instruments used to gather data, and the procedures followed are also described. An analysis of the data and a discussion of the findings are presented in Chapter Four. Chapter Five includes a summary of the results, conclusions, recommendations, and implications of the study. Finally, a list of references and appendices of materials used in the implementation of the study are provided.

CHAPTER 2

REVIEW OF THE LITERATURE

The National Center for Education Statistics reported that 13.1% of the students enrolled in public schools in the United States during the 2009-2010 school year were identified as having a disability and served under the Individuals with Disabilities Education Act (IDEA; U. S. Department of Education, 2012). Within this overall percentage, the largest individual disability population served is students with learning disabilities (LD). This group makes up 4.9% of the entire student population, or 37.5% of the population of students with disabilities. Within this nearly 5% of the total school population identified as having LD, it is unclear how many students have a math learning disability (MD).

In comparison with the extensive research that has been conducted in the field of reading disabilities since the term *learning disability* was first coined in 1963, math disability (MD) is a relatively nascent field of research (National Mathematics Advisory Panel, 2008; Watson & Gable, 2013). Despite the large population of students affected, MD is not as well researched or understood as reading disabilities (Garrett, Mazzocco, & Baker, 2006; Mazzocco, 2005). Furthermore, a lack of clarity and uniformity as yet in the identification and classification of MD, or a "lack of consensus" among researchers (Mazzocco, Devlin, & McKenney, 2008, p. 319) has resulted in nebulous definitions of MD (Watson & Gable, 2013). In fact, many researchers use the acronym *MD* to mean *mathematics difficulty* (e.g., Powell, Fuchs, Fuchs, Cirino, & Fletcher, 2009; Vuvokic & Siegal, 2010). For example, Mazzocco and colleagues (2008) highlighted the interchangeable use of *mathematics difficulty* and *mathematics disability* by using the term MLD in their research to refer to students who had mathematics difficulties and disabilities. Seethaler and Fuchs (2010) reiterated the synonomous use of the two terms in their comment, "In the research literature and this article, *mathematics disability* is operationalized as low mathematics performance and referred to as *mathematics difficulty*" (MD; p. 38).

This chapter will explain the characteristics of MD, various foundational perspectives for solving math word problems, and the research evaluating cognitive strategy instruction, followed by empirical evidence to support the use of of schematic based model with explicit instruction to assist students with MD to solve math word problems.

Characteristics of MD

Cognitive characteristics. Working memory (WM) deficits may hinder students with MD in successfully completing various math tasks, particularly math word problem solving (Andersson & Lyxell, 2007). WM has been defined as cognitive activity in which information is both preserved and processed simultaneously (Swanson, Jerman, & Zheng, 2008). The most prominent model of WM is the one proposed by Baddeley (Baddeley, 2000, 2002; Baddeley & Hitch, 1974). In this WM model, two store systems, the phonological loop and the visuo-spatial sketchpad, deal with verbal information and visual-spatial information, respectively. In math, the phonological loop is necessary for encoding math operations and storing information in complex math problems, while the visuo-spatial sketchpad is implicated in solving multi-digit operations and problem solving (Meyer, Salimpoor, Wu, Geary, & Menon, 2010). An episodic buffer provides an interface between the store systems, allowing multiple sources and modes of information to be considered and manipulated simultaneously to complete a cognitive task and the central executive functions as a gateway and controls the limited attentional capacity of WM (Baddeley, 2000, 2002). Researchers have applied this model in an attempt to investigate the cognitive weaknesses that may be associated with MD.

When comparing students with MD to typically-achieving (TA) students, researchers (e.g., Meyer et al., 2010; Swanson, 2012; Vukovic & Siegal, 2010) have found that age and grade level, particularly in elementary school, impact cognitive performance of students diagnosed with MD. Meyer and colleagues (2010) investigated the components of WM most accessed by students in second and third grade when exercising numerical operations (i.e., arithmetic) and math reasoning (i.e., problem solving) competencies. They found that second graders relied on the central executive and phonological loop, and strengths in these WM components predicted performance on math reasoning. However, third-grade students relied more heavily on the visuo-spatial sketchpad which predicted performance on numerical operations and math reasoning. Vukovic and Siegel (2010) found similar results; many students identified as having MD across at least two points in time did not show typical cognitive deficits that impact calculation until third grade. Swanson's (2012) study also confirmed that impairments in the visuo-spatial sketchpad of WM stabilized after third grade and correlated with ongoing MD in higher grades and into adulthood.

Characteristics in math skill sets. In investigating weaknesses in math skill sets implicated in MD, researchers often divide these skills into procedural and conceptual skills. Procedural skills involve computational fluency or fact retrieval; conceptual skills involve number sense or problem solving skills (Seethaler & Fuchs, 2010). Seethaler and Fuchs (2010) found that conceptual skills, or number sense, in kindergarten was a better predictor of MD than were procedural skills. The research of Jordan, Glutting, and Ramineni (2010) concurred with this finding. Through their longitudinal work with students from first through third grades, the authors indicated that number sense is a strong predictor of later mathematics achievement. Jordan et al. asserted that while number sense was correlated with strengths and weaknesses in later calculation skills, it was even more strongly correlated with later applied problem solving ability.

Implications for best practices in support of students with MD. Math word problem solving is a multifaceted task that requires simultaneously decoding information presented linguisticly and applying math concepts, creating representations, identifying and carrying out appropriate procedural operations, and accurately executing calculations, which requires math fact retrieval (Garrett et al., 2006; Jordan & Montani, 1997; Palinscar & Brown, 1987; Zheng, Flynn, & Swanson, 2013). These skills and tasks become more challenging when the students performing them have a learning disability (LD) and accompanying deficits in WM which may hinder the students' ability to successfully solve math word problems (Andersson & Lyxell, 2007). Students with MD are often poor problem solvers

(Garrett, et al., 2006; Rosenzweig, Krawec, & Montague, 2011). One reason for this could be the focus of word problems on conceptual understanding, rather than rule-driven procedural computation (Maccini & Gagnon, 2002). For many students, including students with LD, the ability to accurately solve math word problems continues to elude and frustrate them beyond their school years (Montague, 2008; Montague & Bos, 1990).

Vukovic and Siegel (2010) highlighted the importance of teaching math in a manner that fosters mathematical thinking. They further pointed out that a math education that focuses on procedural skills and fluency, instead of conceptual understanding, does not facilitate mathematics literacy. An integrated understanding of both conceptual and procedural knowledge leads to math proficiency (Rittle-Johnson, Siegler, & Alibali, 2001).

Foundational Perspectives

One way to understand the development of math word problem solving is from the perspective of Piaget's theory of constructivism, through which he asserted that children are not blank slates or sponges, absorbing knowledge delivered by a teacher, but rather, constructivists creating their own understanding based on acquired tools and prior knowledge (Van de Walle, 2004). As Piaget (2006) developed his theory of cognitive development, he employed the example of a mathematician solving a problem while formulating his definition of *intelligence* and *schema*. Piaget defined *schema* as

> an incorporation of new situations into the previous schemata, a sort of continuous assimilation of new objects or new situations to the actions already schematized . . . which function as practical concepts. Here is the structuring of intelligence. Most important in this structuring is the base, the point of departure of all subsequent operational constructions. (p. 100)

The schema theory eventually emerged from the early cognitive approach in order to address and explain the acquisition of complex cognitive activities, such as strategic learning, in which a student can use prior knowledge of a concept (i.e., schema) to analyze newly received information in order to form new understanding (Reynolds, Sinatra, & Jetton, 1996). Schemata are triggered when a student attempts to comprehend and organize a new concept, such as a math word problem; schemata are assembled by continuously adding new layers of knowledge to form deeper and broader understanding of concepts (Steele & Johanning, 2004). Steele and Johanning (2004) call schema building "the wider applicability of the schema" (p. 67), or generalizability. Thus, schema building and the application of familiar schemata to new situations indicate acquisition of knowledge and cognitive development. Van Garderen, Scheureman, and Jackson (2013) linked effective use of schemata to cognitive development and more effective performance in solving mathematics word problems. Krawec (2014) found that schema building was even more critical for students with LD than TA students in supporting problem solving accuracy.

Cognitive development related to mathematics skills can be defined as understanding and using declarative knowledge (e.g., math facts), procedural knowledge (e.g., steps for solving word problems), and conceptual knowledge (e.g., understanding relationships between part and whole) (Montague & Jitendra, 2006). In problem solving, students must have not only those types of knowledge, but also awareness of their own cognition or metacognition. The term *metacognition* can be defined as "knowledge or beliefs about . . . ways to affect the course and outcome of cognitive enterprises" (Flavell, 1979, p. 907), or "the relation between task and strategy" (Reid & Lienemann, 2006. p. 27). Students who have mastered word problem solving may engage in many cognitive and metacognitive strategies, perhaps intuitively with no direct instruction. For example, successful students may reread the problem or parts of the problem, identify and highlight important information, visualize the problem, make a plan for solving the problem, estimate the answer, and work both forward and backward, and detect and correct errors (Montague, 2007; Montague & Jitendra, 2006). In other words, they think and make a plan (metacognition) to apply the knowledge they already possess to a new problem in order to successfully solve it (cognition). Students with LD, however, are typically noted for deficits in both metacognitive and cognitive performance, particularly in the area of math problem solving, lacking mastery of effective strategies, struggling to choose appropriate strategies for a given task, and displaying difficulty in differentiating between effective and ineffective strategies (e.g., When I use the blue pencil, I will get the problems right.) (Montague & Bos, 1990; Montague & Jitendra, 2006; Reid & Lienemann, 2006).

Polya (1957) is credited with first developing a math word problem solving strategy referred to as the Four-Stage Model to provide sequential support for struggling students (Powell, 2011; Pressley & Woloshyn, 1999). Polya's four step model, which includes understanding the problem, devising a plan, carrying out the plan, and checking the result, forms the foundational framework for many math problem solving approaches still used today. This sequence corresponds well to what Flavell (1979) later termed as *metacognitive action or strategy*. Palincsar and Brown (1987) compared the metacognitive differences between students identified as having LD and their non-disabled peers, noting the potential benefits of direct instruction in explicit cognitive and metacognitive strategies for the former population, asserting that this instructional approach should include increasing students' awareness of task requirements, effective strategies to support task completion, and self-monitoring strategies. In support of this, Montague and Bos (1990) interviewed eighth-grade students who had been diagnosed as having MD, along with students who demonstrated through testing weak, average, or excellent math problem solving performance. The researchers were able to establish a relationship between proficient problem solving and application of cognitive and metacognitive strategies similar to or reflective of Polya's approach. Montague (2003) later established a math problem-solving sequence, and eventually developed the math word problem solving program, *Solve It!*, which featured the sequence as its foundation. Research conducted to evaluate the effectiveness of metacognitive and cognitive strategy instruction (CSI) on mathematics word-problem solving skills of students with LD is examined next.

Cognitive Strategy Instruction

Several ex post facto studies have demonstrated the cognitive and metacognitive deficits of students with LD when solving math word problems. These studies have underscored the critical need to support students with LD by addressing cognitive weaknesses, since metacognitive and cognitive deficits not only impede mathematics problem solving, but also contribute to students' developing serious doubts in their abilities. As a result, cognitive and metacognitive strategies to minimize the effects of cognitive deficits have been researched. CSI has been defined as a structured approach that teaches students cognitive strategies to support their learning within an explicit instruction framework. CSI embeds metacognitive strategies within this approach, including self-regulation strategies (Krawec & Montague, 2012).

Metacognitive and cognitive strategy performance in ex post facto research. In 2001, Hanich, Jordan, Kaplan, and Dick investigated the relationship between mathematical cognition and problem solving in second-grade students who had MD, math and reading difficulties, reading difficulties, or were considered typically-achieving (TA) in both math and reading. Students in Hanich et al.'s (2001) research were given a comprehensive set of author-created story problems—change, combine, equalize, and compare. As expected, participants with MD performed worse on word problems than students with reading difficulties or TA students; the former also used automatic retrieval less than the latter. This emphasizes the WM deficit in many students with MD.

In a longitudinal study that began with third-grade participants and ended when the participants were in fifth grade, Compton, Fuchs, Fuchs, Lambert, and Hamlett (2012) attempted to determine if the cognitive strengths and weaknesses of students with LD matched the academic areas affected by their LD. The authors divided LD into narrowly defined categories: reading comprehension LD, word reading LD, applied problems LD, and calculations LD. The researchers found that the cognitive strengths and weaknesses of the students were correlated with the area of LD in students. Students who had been found having LD in the area of applied problems exhibited low performance on concept formation. This supports the notion that students who struggle to solve math word problems might benefit from direct instruction in CSI. Moreover, the study confirmed that deficits in defined categories do not disappear or diminish over time without the aid of intervention.

In 1993, Montague and Applegate worked with 90 students from sixth, seventh, and eighth grades (30 participants each in LD, average achieving, and gifted categories) randomly selected from a larger pool. Montague administered her own Mathematical Problem Solving Assessment (MPSA) (Montague & Bos, 1990), among other mathematical achievement tests. The MPSA, which would later be determined a valid assessment (Krawec, Huang, Montague, Kressler, & de Alba, 2013), measured mastery and use of metacognitive and cognitive strategies and word problem solving performance, along with attitude, and perception of performance. As expected, the average-achieving and gifted students outperformed students with LD in their ability to represent a math word problem and used other strategies to successfully solve a problem. Montague and Applegate (1993) asserted that the inability of students with LD to represent word problems may directly correlate with the failure to select the appropriate operation required to solve a word problem. In line with this, students with LD conveyed serious doubts about their ability to solve math word problems.

Garrett and colleagues (2006) compared metacognitive performance in students with MD and their TA peers. Specifically, the researchers looked at "offline" metacognitive processes—that is, metacognition that occurs before (i.e., predictive skills) and following (i.e., evaluation of task) the actual task of word problem solving. This study, like the one previously discussed, was a longitudinal ex post facto design, following students from second through fourth grades. The researchers discovered that across grades over time, students determined to have MD were consistently less accurate than their TA peers in their ability to predict or evaluate their successful completion of the task. In light of the findings, the authors asserted that practitioners should not assume that students with MD will naturally develop metacognitive skills over time, but rather, should be explicitly taught metacognitive strategies to support academic tasks.

In 2011, Rosenzweig and colleagues (including Montague) again investigated metacognitive and cognitive strategy use. The researchers worked with 73 eighth graders, comparing students with LD to low- and average-achieving students, separated into these categories according to scores on the Florida high-stakes test. The eighth graders were audio taped and instructed to think out loud as they solved a one-, two-, and three-step math word problem--a total of three word problems. These questions had been used in previous research and had been determined to have discriminant validity, or in other words discriminate between math word problem solving mastery and mastery of separate, related skills. From the audiotapes, tallies were compiled for cognitive, productive metacognitive, and nonproductive metacognitive verbalizations. The authors found that all students behaved more metacognitively as the difficulty of the word problems increased; however, as problems increased in difficulty students with LD increased in nonproductive metacognitive verbalizations, while average-achieving students increased in productive metacognitive verbalizations. The researchers speculated that the students with LD may have "exhausted their metacognitive resources" (p. 515) when problems became more difficult. Thus, they pointed out a valuable finding from their research:

> More metacognitive activity does not necessarily mean better metacognitive activity or better problem solving. For metacognitive strategies to have a positive impact on problem solving, they need to be anchored in developmentally appropriate cognitive skills. (p. 516)

The researchers asserted that a think-aloud could help a teacher or practitioner differentiate between the types of supports a student may need, either cognitive (i.e., concept or skill development) or metacognitive (e.g., direct instruction in self-monitoring).

Metacognitive and cognitive strategy performance in intervention research. Other empirical studies explored the effectiveness of CSI, which includes not just cognitive, but metacognitive strategies, as well. Montague, Applegate, and Marquard (1993) used a pretest/posttest control group design to compare the effectiveness of cognitive strategy instruction, metacognitive strategy instruction, and a combination of both cognitive and metacognitive strategy instruction (i.e., CSI). The authors compared 72 middle school students placed in a school district's LD program in Florida, dividing these students into the conditions described previously. The students in the three groups receiving intervention were compared with 24 "normally-achieving" peers for pretest/posttest comparison. Participants in the cognitive condition received direct instruction in a prescribed sequence that supports problem solving tasks: (1) Read for understanding; (2) Put the problem in your own words; (3) Visualize or construct a diagram; (4) Hypothesize a plan to solve the problem; (5) Estimate; (6) Compute; and (7) Check. Students in the metacognitive condition were taught the strategy *Say, Ask, Check*, which includes paraphrasing, self-questioning, and checking the problem. Interestingly, this research found similar gains were made across conditions from pretest to posttest. Combined pretest scores were 3.76, 4.35, 4.04, and 7.83 (out of a possible ten) for the cognitive, metacognitive, combined cognitive/metacognitive, and normally-achieving peers, respectively; posttest scores rose to 6.80, 6.43, 6.79 (out of 10) for the treatment groups, with the control group's score staying the same. These results could possibly reflect the overlapping of cognitive and metacognitive strategies between the conditions (e.g., paraphrasing, checking). Despite the gains made over this study's four-month duration, the students with LD still did not meet the achievement of their

TA peers. The authors (1993) asserted that there is variability in the time that individual students require before a new strategy becomes "part of the cognitive response pattern" (p. 229). The authors pointed out that CSI could support a student with LD so that he or she has the ability and confidence to participate in the general education math program.

Hutchinson (1993) employed a single-subject design, working with twenty students with LD who received math assistance in a resource setting in two middle schools. Of these twenty students, eight were randomly assigned to a comparison group, while the remaining twelve received a form of CSI intervention. Prior to intervention, students were tape recorded as they thought aloud while problem solving. The intervention was then provided in the form of direct instruction in self-questioning and a cognitive problem solving sequence that included drawing a representation of the problem, identifying the necessary operation to solve the problem, and checking the answer to the problem after solving. Hutchinson focused on more complex relational algebra problems. While pretest/posttest scores remained low and constant for the control group as a whole (.06% to .08%), the intervention group made great gains, from .03% at pretest to 93.17% collectively. In addition, the recorded think-alouds of the students who received intervention improved dramatically, particularly in the area of representation and metacognitive awareness. In light of her findings, Hutchinson advised direct instruction in CSI even for algebra for students with LD.

Fuchs and colleagues (2005) worked with struggling first-grade students considered at-risk of LD, providing a metacognitive and cognitive math intervention based on the CRA sequence. Researchers compared a group of students at risk of LD who received the intervention ($n = 70$), a group of students at risk of LD who did not receive the intervention ($n = 69$), and a group of TA peers ($n = 437$). They also compared student task performance in various areas (including word problem solving) to assessed cognitive abilities. Following intervention, students at-risk of LD exceeded the performance of at-risk students who did not receive the intervention. The intervention group, however, still did not meet the performance ability of the typically-achieving peers. The authors also noted that math word problem solving performance correlated with WM function, confirming what has previously been discussed. In light of their findings, the authors recommended early tutoring as a preventive effort to minimize the effects of LD in students' academic careers in the future.

Montague, Enders, and Dietz (2011) compared students with LD, low-achieving, and average-achieving students in an intervention group ($n = 319$) to those in a control group ($n = 460$). Forty middle schools in a large district were matched on high-stakes performance levels and then randomly assigned to conditions to determine which students were assigned to which condition. By the time this study was conducted, the combined metacognitive and cognitive strategies described above in Montague et al. (1993) had become a well-respected, research-based program titled *Solve It!* (Montague, 2003). Cognitive instruction consisted of direct instruction of a problem solving sequence: Read, paraphrase, visualize, hypothesize (a plan), estimate, compute, and check; metacognitive instruction, *Say, Ask, Check*, was intertwined within each of the cognitive steps. In their 2011 study, Montague et al. sought to compare the effects of CSI in the form of *Solve It!* (Montague, 2003) to regular class instruction, as well as across ability levels. Results showed that from pretest to posttest, students across ability levels made uniform gains, while the control group remained the same in achievement level. This finding indicates that CSI, often considered appropriate as a tier two or three intervention, may be effective as a classroom tier one intervention.

Recently, Krawec and colleagues (2013) conducted a study to determine the effects of CSI in the form of *Solve It!* across the middle school students determined to be LD (n = 77) or TA (n = 77). While their research validated earlier findings that CSI produced effective results regardless of ability level, students with LD were raised to abilities commensurate with the TA control group, emphasizing the value of CSI in math word problem solving. Montague, Krawec, Enders, and Dietz (2014) examined the effectiveness of CSI in the form of *Solve It!* with 1,059 7th grade students. The results confirmed earlier findings, and the authors stressed the value of using CSI in inclusive classroom settings.

Summary. Studies reviewed that focused on metacognitive and cognitive performance furnished evidence that students with MD consistently display deficits in those areas (which includes WM), when compared to their TA peers. In 2012, Compton et al.'s longitudinal research helped to establish that over time cognitive deficits in students with MD do not diminish without targeted intervention efforts. While elementary school studies did not address student perceptions, Montague and Applegate (1993), working with middle school students, noted that the poor performance of students with MD had self-doubt of their math ability. This could become a factor in the "negative shift" (Wigfield, Eccles, Mac Iver, Reuman, & Midgley, 1991, p. 564) that can occur in middle school that is associated with school failure.

Studies that incorporated interventions uniformly demonstrated the value of direct instruction in CSI in assisting students at-risk for or with LD in the area of math word problem solving; Hutchinson (1993) extended this to algebra. Fuchs and colleagues (2005), working with first graders, recommended early tutoring of students at-risk of failure in math, based on their research findings. Montague, Applegate, and Marquard (1993) pointed out that the duration of intervention training will vary from student to student, specifically associating intervention support for students with MD in middle school with confidence building. Finally, Montague, Enders, and Dietz (2011) suggested that CSI may be appropriate across ability levels, thus making it suitable as a Tier One classroom intervention.

While CSI typically involves a representational aspect (e.g. concrete-representational-abstract sequence in Fuchs et al. [2005], and the representational component included in some studies using Montague's *Solve It* [2003]), some research has indicated that greater focus should be placed on teaching students to create schematic representations for word problem solving (Jitendra & Hoff, 1996;

Jitendra & Star, 2011). Schematic-based instruction (SBI) could correlate well with CSI. Future research could combine these two major conceptual supports into one intervention.

Schematic-Based Instruction

Schemata are triggered when a student attempts to comprehend and organize a new concept, such as a math word problem; schemata are assembled by continuously adding new layers of knowledge to form deeper and broader understanding of concepts (Steele & Johanning, 2004). Hegarty and Kozhevnikov (1999) defined the use of schematic representation in math instruction as "representing the spatial relationships between objects and imagining spatial transformations," while a pictorial representation is defined as a "vivid and detailed visual image" (p. 685). Van Garderen and Montague (2003) compared students' use of pictorial and schematic diagrams during problem solving and found that students who used pictorial representations solved math word problems incorrectly about 70% of the time, while students who employed schematic representations solved the same word problems correctly about 76% of the time. More recently, the research of van Garderen and colleagues (2013) supported the earlier findings that schematic diagrams better equipped students in math problem solving accuracy, emphasizing the conceptual correlation between schemas and problem solving.

Schematic-based instruction (SBI) integrates the use of systematic explicit instruction found effective in math instruction (Montague, 2008) with the use of visual representations and incorporates cognitive processes involved in problem execution, such as paraphrasing, visualizing, hypothesizing about problem solutions, and checking work (The National Mathematics Advisory Panel, 2008; Palinscar & Brown, 1987; Rosenzweig, Krawec, & Montague, 2011). Jitendra and Star (2011) summarized the main instructional steps of SBI as problem comprehension, problem representation, planning, and problem solution. It is possible that the use of SBI, which incorporates visual representations and explicit instruction of cognitive strategies, may be one way to improve percentage calculations in middle-school students.

Researchers and evidence-based practices. Jitendra authored or co-authored the majority of articles on the topic of SBI under consideration in this literature review. Jitendra, considered the author of seminal works on the topic of math word problem solving and SBI, published her first article on math word problem solving in a peer-reviewed journal in 1993 (Jitendra & Kameenui); her first article describing a study on SBI was published in 1996 (Jitendra & Hoff). Since then, she has continued to refine and strengthen the research on the topic of SBI involving visual representations, authoring or coauthoring some 20 articles involving math word problem solving, including multiple articles with Xin (2008). Van Garderen (2007), a close associate of Montague who is responsible for landmark studies involving math word problem solving and cognitive strategy instruction, is closely associated with SBI (Montague & Bos, 1990; van Garderen & Montague, 2003). Despite past research, standards established for identification of evidence-based practices suggest that experimental effects must be replicated across different researchers (Horner et al., 2005); specifically, Kratochwill et al. (2013) recommend "at least three research teams with no overlapping authorship" (p. 33). This standard indicates the need for further research to establish SBI as an evidence-based practice.

Studies that incorporated cognitive strategies into SBI instruction. The schematic-based instructional procedure varied somewhat from study to study, although all studies incorporated direct or explicit instruction as a means to improve math performance. More recent studies also incorporated cognitive components with the SBI intervention. Jitendra and Hoff (1996), Jitendra and colleagues (1998), Jitendra, Hoff, and Beck (1999) and Jitendra, DiPipi, and Perron-Jones (2002) trained students to identify problem schemata, select, and use the appropriate diagram. In these interventions, students were trained to pinpoint the missing element in the problem with a question mark. When Jitendra, Griffin, Deatline-Buchman, and Sczesniak (2007), and Jitendra and colleagues (2009, 2013) conducted their SBI research, they added a four-step mnemonic component, FOPS (1. Find the problem type. 2. Organize the information using a diagram. 3. Plan to solve the problem. 4. Solve the problem). While these studies did not compare the implementation of a more structured mnemonic as a cognitive strategy to their previous less structured problem solving protocol, the research yielded positive outcomes, and the researchers continued to employ the more structured cognitive strategy routine.

In 2005, Xin and colleagues taught students a five step strategy that applied specifically to the word problem types under consideration (i.e., multiplicative compare and proportion problems). Later, Xin (2008) developed a four-step checklist to support students as they solved word problems. While the checklist was still applied to the specific word problem types under consideration (two forms of multiplication problems), it was developed with the specific intent to ground students' learning of SBI. These steps, similar to FOPS (i.e, 1. Find the problem type. 2. Represent the information using a diagram. 3. Plan for a solution. 4. Solve and Check.) better integrated cognitive strategies into the SBI research. The authors reported that students were more successful when they consistently applied the learned strategy during maintenance and generalization testing.

In 2007, van Garderen, for the first time, combined major components of Montague's work on cognitive strategy instruction (CSI) for math word problem solving, providing students with a previously researched, more formal cognitive structure (e.g., Montague, 2003; Montague & Bos, 1990; Montague, Enders, & Dietz, 2011) as the basis for SBI. This involved explicit instruction in reading the problem for understanding, visualizing the problem, planning how to solve the problem, and checking the answer. As part of the cognitive strategies that were incorporated into each of these studies, however, none sought to ensure students' comprehension of the problem at the outset of problem solving in any way, such as requiring students to create an answer sentence leaving a blank for the answer. This is an oversight that needs to be corrected in future research.

Disaggregation of data in studies. A common limitation found across the studies reviewed was that the effectiveness of SBI instruction in supporting students with MD was not disaggregated from the effectiveness of SBI instruction in supporting other participating students considered at-risk of failure in math, but not MD. Four of the studies that included participants with MD and students at risk of failure included their performance data with students with other disabilities or English Language Learners (ELL) and did not disaggregate the data (i.e., Griffin & Jitendra, 2009; Jitendra et al., 1998; Jitendra et al., 2009; Xin et al., 2005). In 2007, Jitendra and colleagues compared the performance of students with MD only to the few participants in the study who were receiving ESL or Title 1 services. Consequently, it is difficult to compare results, since the populations varied among studies. There is a need for new SBI math research that quantifies the responses of students with MD. This study attempted to address this need.

Math problems researched and generalizable outcomes in SBI. Explicit instruction of a number of different math word problem types were involved in the eleven studies in this review of the empirical literature. All five studies involving elementary school students in grades two through four (Jitendra et al., 1998; Jitendra et al., 2013; Jitendra, Griffin, Haria, et al., 2007; Griffin & Jitendra, 2009; Jitendra & Hoff, 1996) made use of *change*, *group*, and *compare* word problems. Xin's (2008) study with grade five participants included instruction of group and multiplicative comparison word problems. Xin and colleagues (2005) focused on instruction involving multiplicative comparison and proportion word problems for students in grades six through eight. Jitendra et al. (1999) worked on one- and two-step change, group, and comparison word problems with students in grades six and seven. Jitendra and colleagues (2009) worked with seventh-grade students on ratio and proportion word problems. Jitendra et al. (2002) taught vary and multiplicative comparison word problems, while van Garderen (2007) used one- and two-step addition and subtraction word problems in their respective interventions with students in grade eight. Each study involved a limited number of problem types taught--never more than three in any one study. The research involved explicit instruction in identifying and differentiating between a limited range of word problem types, followed by the application of schematic diagrams in the form of graphic organizers specifically designed for each word problem type. On the other hand, other visual representations may be more effective due to their more universal usability across word problem types, avoiding the need for graphic organizers that apply to only one word problem type. Further investigation in this area is necessary.

In Singapore, one of the leading nations in mathematics proficiency, students from a very early age begin math training by using a schematic representation known as bar-model drawing, a strategy which appears to support students' math word problem solving (Ginsburg et al., 2005). Although there is a dearth of research on bar-model drawing (Ng & Lee, 2009), these visual representations align well with the research-driven concrete-representational-abstract (CRA) sequence which transitions students through mathematical conceptual understanding through the use of manipulatives, then schematic diagrams, and finally, through abstract mathematical symbols (Flores, 2009; Maccini & Ruhl, 2000). This approach may be equal to or more effective due to their generalizability than other well-researched schematic diagrams that come in the form of graphic organizers, in which students are trained to use a specific organizer for a specific word problem type. Although the proposed research will follow previous SBI research in focusing on a limited problem type, its use of bar-model drawing may have the potential to be a generalizable strategy across multiple grade levels.

Empirical Gaps in the Literature

This review of the literature revealed that there are gaps in the available research that need to be examined. First, many existing studies have been conducted by similar or overlapping research teams. There is a need for other researchers to investigate SBI in order for it to be considered an accepted evidence-based practice. In addition, few studies formally considered the cognitive link to SBI and none considered how cognitive strategies could be incorporated into SBI at the outset of problem solving. Also, there is a need to investigate the effectiveness of the SBI strategy with students with MD. Finally, previous studies have focused on SBI using a focused graphic organizer with limited application, rather than a method that can be used more broadly and generically across word problem types.

To summarize, the purpose of this study was to extend previous research that suggests that SBI with explicit instruction can be effective in teaching various math skills related to word problem solving. The strategy proposed for the current study uses a more generic schema approach--a Bar Model--as a form of SBI that can be developed across word problems. It is hypothesized that the bar model strategy will support students while they solve math word problems. Moreover, the bar model approach has the potential to enhance students' awareness of cognitive strategies through paraphrasing, visualizing, hypothesizing about problem solutions, and checking work, all of which are explicitly taught through the use of the bar-model drawing protocol. Since previous research did not disaggregate the data, this study investigated the effectiveness of a specific schematic math word problem solving strategy with students with MD.

CHAPTER 3

METHODOLOGY

Introduction

The research methodology described in this chapter was preceded by a pilot study conducted for five weeks during the Summer 2012 semester. This study will be briefly summarized prior to describing the methodology for the dissertation research.

The purpose of the pilot study was to investigate the feasibility of a model of direct math instruction that utilized bar model drawing as a strategy to support the ability of students to solve math word problems. Secondarily, the pilot study provided the investigator with a way to evaluate and refine the procedures (e.g., teaching protocols) and measures (e.g., student assessment measures, data collection tools, reliability measures, treatment fidelity, social validity measures) for the dissertation research. Moreover, the results of the pilot provided preliminary data to access the effectiveness of the proposed model in increasing the students' accuracy in solving word problems.

The independent variable for the descriptive pilot study was direct instruction in the use of bar model drawing as a representational strategy for math word problem solving. The dependent variable was increased accuracy in math word problem solving, as determined by posttest performance compared to pretest performance.

The four participants chosen for the pilot study were rising fifth graders in an urban Virginia public school. One of the participants was identified as having a math learning disability; another was identified as having a learning disability in the area of reading; and the other two participants did not receive any special education services. All four participants struggled in the area of math, evidenced by their failure to receive a passing score on the most recent state high-stakes math test.

Pretests for the pilot study included the *Woodcock-Johnson III (WJ-III) Applied Problems* (Woodcock, McGrew, Mather, & Schrank, 2001) subtest, as well as math word problems taken from released Virginia Standards of Learning (SOL) tests for grades three, four, and five. When pretest/posttest data were compared, gains were made by participants in solving the math word problems that were taken from released Virginia SOL tests. Pre/post assessments of the 3rd grade problems indicated participants had performed better on the 3rd grade pretest questions (m = 68%, SD = 33.68), allowing only modest gains on the posttest (m = 93.25%, SD = 4.5). In comparison, participants struggled with the 4th (m = 41.75%, SD = 32.12) and 5th (m = 54.25%, SD = 28.25) grade pretests, subsequently showing marked improvement in 4th (m = 83.5%, SD = 19.05) and 5th (m = 95%, SD = 10) grade posttesting. (Refer to Figure 3.1) Though the results suggested the effectiveness of the bar model drawing strategy, the small pilot study indicated the need for further research on the effectiveness of this intervention.

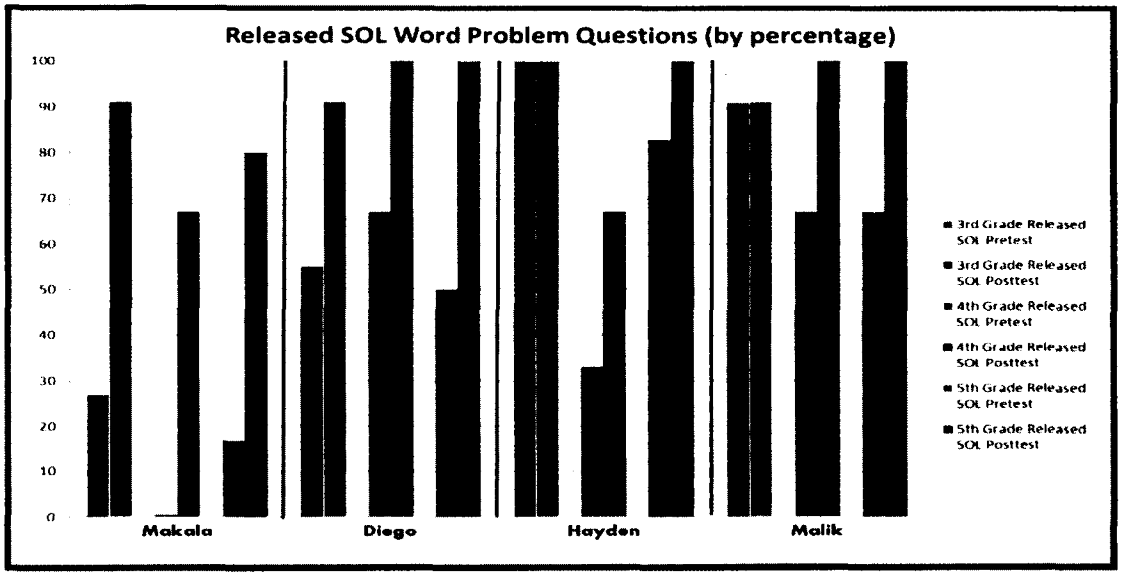


Figure 3.1. Descriptive Pilot Study Results

The remainder of this chapter will describe the methodology for the dissertation research to examine the effectiveness of bar model drawing as a form of schematic-based instruction (SBI) that incorporates cognitive strategy instruction (CSI). It includes the research questions, a discussion of the research design, a description of participants and the materials used, and the procedures for the study. Also, inter-observer agreement, procedural fidelity, and social validity procedures and measures will be detailed, as well as data analysis methodology.

Research Questions

The purpose of this research was to extend previous research that suggests that SBI can be an effective support for students with MD as they learn to solve math word problems. Previous research in SBI used specific schemas to teach specific word problems (e.g., one schema for addition word problems involving grouping and another schema for subtraction word problems involving comparisons); however, bar model drawing incorporates direct instruction of one schema (i.e., the bar model) to teach students how to solve different types of word problems. Bar model drawing also seamlessly incorporates CSI within the protocol.

The research addressed two questions:

1. *To what extent will explicit instruction of the bar model drawing strategy improve the use of cognitive strategies of urban students labeled either with MD when solving math word problems?*

2. *To what extent will explicit instruction of the bar model drawing strategy and the use of CSI strategies increase the ability of urban students with MD to accurately solve math word problems?*

Research Design

A multiple-baseline design replicated across groups was used to evaluate the effects of the intervention of bar model drawing on student performance on math world problem solving. This design has been used by researchers for over forty years to effectively demonstrate functional relationships between educational interventions, which cannot be "taken away" (i.e., withdrawal or reversal designs) once taught, and mastery of skills has been achieved (Gast, 2010; Hersen & Barlow, 1976; Kennedy, 2005). The design model used was a quasi-replication of Flores (2009), in which she conducted a math intervention involving concrete-representational-abstract (CRA) math instruction.

According to a report prepared for What Works Clearinghouse by a panel of researchers (Kratochwill et al., 2010), single subject designs may only achieve evidence standards by meeting four criteria:

1. The independent variable, or intervention, must be methodically, intentionally manipulated by the researcher.
2. The study must include interrater reliability on each condition, meeting at least minimal standards of agreement.
3. The study must demonstrate the effect of the intervention over three points in time or over three phase repetitions.
4. Each phase must have at least three data points.

This research met these standards for single subject design. Manipulation of the independent variable, bar model drawing instruction, was carefully planned in advance and was carried out accordingly to study the intervention's effects on students with MD. Inter-observer agreement was assessed for 35% of data points in each phase resulting in 91% agreement. The research was replicated across three dyads of participants and each phase (baseline and intervention) included at least five data points for each dyad. Since this research meets the criteria for single subject design standards, it may be analyzed to determine if there is evidence of an effect (Kratochwill et al. 2010).

The independent variable for each research question was explicit instruction of word problem solving using bar model drawing combined with cognitive strategy instruction. The dependent variable in question one was the frequency of accurate use of cognitive strategies while solving word problems. The dependent variable in question two was accuracy in word problem solving. Both dependent variables were measured through criterion checks that followed each lesson, mastery checks that occurred midway and following the intervention, and pre and posttests that consisted of word problems compiled from released Virginia Standards of Learning (SOL) tests.

Participants

A university-based Institutional Review Board approved the research (see Appendix C). Participants were six 3rd-grade students from a small urban public school district that serves about 1,300 students from grades K-12. The school district granted permission for the researcher, a former teacher from that school district, to work with students within that system. This school district is located in a city ranked as having one of the top ten highest child poverty rates in Virginia (Voices for Virginia's Children, 2012). Moreover, the elementary school has been *Accredited with Warning* by the Virginia Board of Education for two consecutive years, and has experienced federal sanctions due to failure to meeting federal annual measurable objectives (Virginia Department of Education, 2013).

Initially, a pool of possible participants were identified as having an identified learning disability by the school district or at risk of failure in math based on benchmark testing and current grades. Students diagnosed with disabilities other than learning disabilities (e.g., autism, emotional disabilities) or who had comorbid disabilities, including attention deficit hyperactivity disorder (ADHD), and students for whom English is a second language were not eligible to participate in the study. Students at risk of failure based on attendance issues were not eligible. Letters were sent to parents of third-grade students meeting one or both of the inclusionary criteria requesting permission to test their students in mathematical skills. A brief overview of the purpose of the testing was included in the letter (see Appendix D). Parents of nine students agreed to the testing. Seven of these students were then identified by the researcher as having a math difficulty, defined as scoring at or below the 16th percentile (i.e., one standard deviation below the mean) on the *KeyMath--3* (*KM—3;* Connolly, 2007) assessment and scoring below 80% on 15 word problems taken from released SOL tests. Two students did not meet the qualifying score on the *KM--3* (Connolly, 2007) assessment, falling within one standard deviation from the mean instead of below one standard deviation. Finally, parental consent and student assent were sought for the seven participants found eligible based on testing. Permission was granted and informed consent forms were signed by parents of six of the eligible participants. The parent of the seventh participant declined his participation in the study.

Characteristics of the six participants are presented in Table 3.1.

Table 3.1

Participant Demographic Characteristics

	Participants					
Characteristics	**Child One**	**Child Two**	**Child Three**	**Child Four**	**Child Five**	**Child Six**
Age	10.3	9.5	9.11	8.9	9.10	9.7
Grade	3	3	3	3	3	3
Gender	F	M	F	F	F	F
Ethnicity	Black	Black	Black	Black	Black	Black
Identified Disability	None	None	None	None	LD	None
Reading Level Grade Equivalent	3.7	3.0	1.5	2.7	1.0	1.7
***KeyMath–3* Ranking**	2%	5%	2%	4%	7%	3%
Released SOL percentage	**27%**	**6%**	**27%**	**13%**	**20%**	**33%**

Note. SOL = Standards of Learning

Setting

All research, including testing, baseline, intervention, and posttesting, was conducted at the participants' elementary school. A conference room off of the library was provided for conducting the research. The room contained a large rectangular table with the capacity to seat six students, typical of a classroom table used for group work. It also contained three "beanbag" chairs. Participants participated in all interventions with the researcher at the table, but had the option of completing criterion probes and mastery checks on the beanbag chairs. The room was devoid of decoration of any type. Much of the room served as a storage area for defunct equipment, such as overhead projectors, televisions, video players, and even one film strip projector from the 1970s. The school's laminating machine was also located in the room. Testing took place on a one-to-one basis, while baseline and intervention sessions took place in dyads. Pretesting took place during school hours. Baseline, intervention, and posttesting took place daily after school in the room provided.

Many students at this school frequently received after school tutoring and participants viewed working with the researcher as a special form of after school tutoring and willingly stayed daily. Participants received a small snack daily, which is the expectation of all students participating in normal after school tutoring activities. Participants also received a small prize (worth $1.50 or under) for every lesson they completed successfully. All participants established a good rapport with the researcher and seemed eager to elicit the researcher's attention. Participants from the first two dyads continued to come to the conference room to visit briefly at the end of each day during bus call after they had completed their roles in the study.

Materials

Participants were assessed through the administration of the *KeyMath Diagnostic Assessment, Third Edition, Form A* (Connolly, 2007) assessment that served as a screening instrument. The *KM--3* is a comprehensive assessment of mathematical skills. The assessment is organized into three main areas of mathematical skills that are comprised of more specific subtests. The three main components are Basic Concepts, Operations, and Applications. The subtests included in the area of Basic Concepts are: Numeration, Algebra, Geometry, Measurement, and Data Analysis and Probability. The subtests included in the area of Operations are Mental Computation and Estimation, Addition and Subtraction, and Multiplication and Division. The last area, Applications, includes two subtests, Foundations of Problem Solving and Applied Problem Solving. Students were administered all *WJ—3* subtests.

SOL test questions taken from third and fourth grade released tests from 2007-2010 were also used as a screening instrument. Chosen word problems involved computation of basic mathematical operations (i.e., addition, subtraction, and multiplication). There were no SOL word problems involving division. Word problems that involved probability, rounding, and estimation were excluded. Word problems that involved fractional concepts were also excluded (see Appendix E).

In addition to the screening instruments, participants were interviewed prior to baseline and intervention using twelve questions taken from the *Mathematical Problem-Solving Assessment—Short Form* (*MPSA—SF*; Montague, 2003). These interview questions investigated participants' self-perception and attitudes toward math, as well as cognitive and strategic knowledge. All three of these instruments were readministered following the completion of the intervention to serve as pretest/posttest measures.

A bar model drawing protocol adapted from Forsten, 2010 (see Appendix F), and mathematical word-problem questions were developed by the researcher for baseline, intervention, and criterion probes and mastery checks. (These mathematical word problem questions are discussed in detail in the following section.) A social validity survey was given to participants upon the completion of the intervention (See Appendix G). All of the participants had access to standard classroom-issue calculators.

Procedures

Baseline. For each baseline session, participants were given eight math word problems that represented the eight levels of instructional concepts that would be taught in the intervention condition (see Appendix H). Each baseline test differed only in terms of story context and numerical values, and continued to represent the eight levels of instructional concepts. Participants were assigned to dyads via a random drawing. After the first dyad, Child One and Child Two, completed five baseline sessions with a non ascending trend, they were introduced to the intervention condition. Participants randomly chosen to be the second dyad, Child Three and Child Four, and the third dyad, Child Five and Child Six, were also probed at five different points as they remained in baseline. When a stable baseline was achieved for each dyad, reflecting the percentage of word problems solved accurately and the numbers of cognitive strategies used in problem solving, intervention was begun for that dyad. Correct solutions of problems included both the accurate answer and use of cognitive strategies, since not just numerical accuracy, but learning a process that enhances number sense and equips students with a conceptual tool is necessary in order to assist students in achieving mastery of math word problem solving.

Throughout the study, participants were allowed to use a calculator as needed, and received any requested help with reading the word problems.

Intervention. Intervention procedures will be described in three sections: 1. General procedures, 2. Criterion probes, and 3. Instructional sequence.

General procedures. Intervention involved the introduction of eight math word problem solving lessons that increased in difficulty and were based upon mastery of previous lessons. Each instructional session ranged from 25 to 40 minutes; sessions were conducted at the end of each school day. Due to unforeseen circumstances, such as parents picking a child up early or teachers entering the room to use the laminating machine, an intervention session did not always necessarily align with one day, but could overlap to two days. During each session of the intervention, the researcher first administered a criterion probe to check for understanding of the previous lesson taught (for example, see Appendix I). Criterion mastery was set at 100% accuracy. If the participant demonstrated mastery of both the intervention strategy and the word problem type taught in the lesson, the participant continued on to the next lesson or the researcher remediated the previous lesson. After ascertaining what the participant knew about the current lesson, the teacher modeled the bar model drawing strategy, specifically as it applied to the particular word problems in a given lesson and provided examples. The content of lessons varied according to skills taught. During all intervention sessions, the researcher used explicit instruction to teach the lessons. Explicit instruction is defined as a step-by-step presentation of a strategy, along with teacher modeling incorporating think-aloud procedures, providing specific examples, and including opportunities for guided and independent practice with praise and corrective feedback. Corrective feedback included explicit correction, clearly indicating participants' incorrect use of steps while problem solving and teacher modeling of correct use and application of the protocol, and elicitation, drawing participants to the correct use of the strategy of bar model drawing by asking questions and asking participants to reformulate their work (Tedick & Gortari, 1998). Although lessons were scripted (see Appendix J), the researcher used the script as a guide, rather than reading from it verbatim. Lessons adhered to the strategy sequence protocol based on Forsten (2010). The instructional sequence followed the same order for each lesson: 1. Read the entire problem. 2. Rewrite the question being asked in sentence form, leaving a space for the answer. 3. Determine *who* or *what* is involved with the problem. 4. Draw the unit bar(s). 5. Chunk the problem and identify the missing variable. 6. Correctly adjust the unit bar(s) and compute (for which participants could use calculators) to solve the problem. 7. Write the answer in the previously written sentence, making sure the answer makes sense. Participants were provided with a copy of the protocol and allowed to use it as long as they needed. By the time participants reached the concluding mastery checks following the last intervention, all participants phased out the protocol sheet on their own, having memorized the steps through extensive practice throughout the intervention phase. The researcher used only the strategies directly associated with explicit instruction and those stated in the strategy sequence protocol.

Cognitive strategies were implicitly included in the strategy sequence protocol, outlined above. These included accurate use of paraphrasing (i.e., rewriting the question as an answer statement), visualizing (i.e., constructing a bar model), hypothesizing about problem solutions (i.e., manipulating the bar model), and checking work (i.e., writing the answer in the previously written answer statement and ensuring it makes sense), all of which were explicitly taught through the use of the bar model drawing strategy protocol.

Criterion probes. Each lesson began with a four-question criterion probe of the previous session's material formatted to the same specifications described in the baseline probe condition. If a participant was not able to correctly solve problems from material covered in the previous lesson, that lesson was reviewed and remediated. In addition, a cumulative four-question mastery check of Lessons One through Four was given following individual mastery of those four lessons. The participants were required to demonstrate 100% on the mastery check before continuing on to Lesson Five. Following demonstration of mastery of Lessons Five through Eight as evidenced by individual lesson criterion probes, another four-question mastery check of those cumulative four lessons were given following the same protocol as the first mastery check (Lessons One through Four). Participants were also required to demonstrate 100% mastery on the second mastery check. Then a cumulative eight-question mastery check of all lessons was given (see Appendix K). The participants were also required to demonstrate 100% mastery on the final cumulative mastery check.

In the event that one participant in a dyad required remediation of a previous lesson while one demonstrated mastery, the participant demonstrating mastery was given an independent math activity that did not directly relate to math word problem solving (e.g., Hot Dot flashcards for practicing telling time and counting money; Geoboard activities, or VersaTiles geometry practice). Participants working independently were given a choice of activities.

Once the first dyad of participants demonstrated 100% mastery on the mastery check for Lessons Four through Eight and the final cumulative mastery check, the second dyad, Child Three and Child Four, simultaneously began intervention, following the same intervention protocol outlined for Child One and Child Two. Once Child Three and Child Four reached mastery on the final mastery checks, the third dyad, Child Five and Child Six began intervention.

Instructional sequence. Word problem instruction was delivered sequentially beginning with word problems that involved addition with one variable which could be solved using a discrete bar model (see Appendix L). For example, in the word problem, *Olivia ate 3 cookies after lunch and 2 more cookies after dinner. How many did she eat all together?*, there is only one variable, cookies, and it can be solved by drawing three discrete bars, and then two more, for a total of five bars. Lesson Two involved subtraction with one variable that could be solved using a discrete model (e.g., *Five birds were sitting in a tree. Three flew away. How many birds are still sitting in the tree?*). Lesson Three taught participants to solve addition problems that have more than one variable, but could still be solved using a discrete model. For example, in *Jeannette saw 4 snakes and 2 frogs while she was hiking. How many amphibians did she see all together?*, there are two variables, snakes and frogs. The problem can still be solved with each bar model representing one-to-one correspondence. Lesson Four involved subtraction with more than one variable using a discrete model (e.g., *Seven cats and five dogs live on Virginia Avenue. How many more cats are there than dogs?*).

The continuous model was introduced next to support participants in solving word problems in which bar model drawing can no longer be used with one-to-one correspondence. Lesson Five taught addition word problems with one or more variables using the continuous bar model. For example, in *Sarah owned 53 fiction and 31 nonfiction books. How many books does Sarah have in all?*, the participant can no longer draw a bar representing one-to-one correspondence. Instead, she will draw "continuous" bars. Lesson Six followed with subtraction involving one or more variables and the continuous bar model.

In Lesson Seven, participants were introduced to multiplication with one or more variables that could be solved using the continuous or discrete model. Lesson Eight involved addition and subtraction with one or more variables that could be solved using the part-whole bar model. For example, the problem, *There were 321 baseball fans in the stadium. 203 were Phillies fans. The rest were Mets fans. How many Mets fans were there?* is solved by specifically manipulating the continuous bar model to represent the whole, the part, and the other missing part. These word problems correspond to five different mathematical word problem types in other SBI literature that uses a specific schema for a specific type of mathematical word problem. These schemata are known as Change Schema, Group Schema, Compare Schema, Vary Schema, Equal-Group Schema, and Part-Whole Schema (Jitendra, DiPipi, & Perron-Jones, 2002; Jitendra, Griffin, Deatline-Buchman, & Sczesniak, 2007; Jitendra & Kameenui, 1996; for examples, see Appendix M).

Generalization. The fifteen word problems taken from released SOL tests and administered as a posttest served as a generalization measure for both research questions. These were analyzed for accurate use of cognitive strategies and overall accuracy in math word problem solving.

Maintenance. One week after each participant achieved mastery, he/she was probed using the same eight-question mastery check that was representative of each of the target lessons.

Treatment Fidelity, Inter-Observer Agreement, and Social Validity

All intervention sessions were videotaped. Treatment fidelity (both content and process) was assessed by a doctoral student in the same cohort as the researcher. She viewed 35% of the taped sessions for each dyad (randomly selected by using the Integer Generator on Random.org) to ensure that the researcher adhered to the content and intervention procedures. Refer to the Content and Procedural Fidelity Checklist in Appendix N.

All baseline probes, 35% of all intervention probes for each dyad (randomly selected by using the Integer Generator on Random.org), and all mastery checks were graded by a doctoral student to ensure that these had reached 100% accuracy and

100% strategy use. Since inaccurate computation was taken into consideration, meaning a computational-type error did not automatically produce an incorrect response when determining correct use of cognitive strategies if all other components of the word problem solution were correct, mastery checks were graded separately by the primary researcher and another researcher, who compared the assessment scores to produce a reliability measure. A criterion level of 85% and above inter-observer agreement was established to ensure accuracy of data collected. Inter observer agreement was calculated by reporting agreements on occurrences or accuracy divided by agreements plus disagreement (A/[A+D]) met 85% or greater for each dyad.

A social validity survey was administered to participants upon completion of the study. This survey was comprised of five questions employing a five-point Likert scale to measure attitude toward and usefulness of bar model drawing for math word problem solving. Social validity measured the participants' attitudes and perceptions of bar model drawing, including its perceived effectiveness, feasibility of use, and potential of future use by the participants. These factors are related to socially important outcomes, a quality indicator for single subject research (Horner et al., 2005). Pretest and posttest questions from the interviews based on questions from the *Mathematical Problem Solving Assessment—Short Form* (*MPSA-SF*; Montague, 2003) were examined qualitatively for themes and patterns that emerged across participants. These interviews also provided information on social validity, as well as knowledge and mastery of effective strategies for math word problem solving.

Data Analyses

Data from baseline, intervention, and maintenance for each participant were collected, graphed, and assessed daily for purposes of formative evaluation of intervention effects. Summatively, visual analyses were conducted on graphs to determine level change, trend, variability, and points of non overlapping data, in order to ultimately determine if a functional relation existed between the independent and dependent variables and effect sizes. The split-middle method of trend estimation which can quantify graphed data, was used. In a comparison of overlap methods for quantitatively analyzing single-subject data, the split-middle method was found to have the lowest error percentage (Wolery, Busick, Reichow, & Barton, 2010).

Pre- and post-test comparisons were also used to examine summative growth in mastery across time. In addition, the number of occurrences of the effective application of each of the four cognitive strategies used on the posttest comprised of 15 word problems taken from released Standards of Learning (SOL) tests were tallied and analyzed (See Appendix O), serving as a generalization measure. The social validity survey and *MPSA-SF* were examined qualitatively.

Summary

This chapter outlined the methodology for the dissertation research examining the effects explicit instruction of word problem solving using bar model drawing as a form of SBI combined with cognitive strategy instruction. It included the research questions, and a discussion of the research design. It also provided detailed information about the participants, the materials used, and the procedures. Inter-observer agreement, procedural fidelity, and social validity were also detailed. Finally, it outlined the formative and summative assessments and data analyses used to evaluate intervention effects.

CHAPTER 4

RESULTS

This study examined the effectiveness of the bar model drawing as a strategy to support students with math difficulties (MD) to improve their use of cognitive strategies and accurately solve math word problems. This chapter is organized in terms of the two specific research questions posed in Chapter 1. First, it examines whether participants' use of cognitive strategies improved after learning the bar model strategy to solve math word problems. Secondly, it reports on the effectiveness of the bar model drawing strategy in increasing the accuracy of the math word problems solved by the participants with MD. Results are provided to answer both research questions and are discussed separately.

A multiple-baseline design replicated across groups was used to evaluate the effectiveness of the bar model drawing strategy on student performance on math world problem solving. Baseline data were collected on each participant until baseline data were stable. Intervention was then implemented and continued until each participant reached criterion. Systematic visual analyses were conducted to examine the stability, level change, and trend direction of participants' performance within and between phases. Specifically, when at least 80% of data points fell within 20% of the median and trend lines, the data were considered stable. Relative and absolute level changes between phases are reported. Trend direction was identified by examining whether the direction of the data path was zero celerating (flat), accelerating, or decelerating. Split-middle analysis was used to construct a trend line. Points of non-overlapping data (PND) are reported to determine effect size.

The research took place over a 16-week period, including the time the researcher began screening participants to the time she collected the maintenance data from the last dyad of students. Six third-grade students with MD participated in the study. They were randomly assigned in dyads to three tiers of intervention. Each tier of instruction consisted of baseline, intervention, generalization, and maintenance phases. As participants demonstrated mastery in each tier, based on the cumulative mastery check, he or she progressed to the next tier. In addition, data were collected using participant interviews based on questions taken from the *Mathematical Problem-Solving Assessment—Short Form* (*MPSA—SF*; Montague, 2003) and a social validity survey was distributed. Pre and posttesting results based on the *KM—3* assessment and released Virginia SOL word problem questions, which served as a generalization measure, are also discussed. Each research question is answered individually.

Research Question 1

To what extent will explicit instruction of the bar model drawing strategy improve the use of cognitive strategies of urban students labeled with MD when solving math word problems?

Visual Analyses of Data

Baseline and intervention for each participant is discussed. Refer to *Figure 4.1* for a graph of the results and to Table 4.1 for the means across phases.

Baseline. Systematic visual analyses of within-condition phases indicated that none of the six participants accurately used cognitive strategies during the five baseline sessions. This resulted in a median, mean, and range of 0% and a trend line of zero.

Intervention. Intervention results are reported by participant.

Child One. Child One received a total of ten intervention sessions. The use of cognitive strategies immediately increased to 100% when the intervention was implemented and maintained at that level for eight of the ten intervention sessions (mean = 87.5% correct; range = 25% - 100%). Overall, the trend line for Child One was stable with 80% of data points falling on the trend line.

Child Two. Across nine sessions, the median for Child Two immediately increased to 100% when the intervention phase was introduced. Child Two had a mean of 88.9% (range of 0%-100%) for the percent of accurately used cognitive strategies. The level was stable, with eight out of nine intervention points falling on the median, and a relative and absolute level change of zero.

Child Three. Like Child One and Child Two, the median increased to 100% during the intervention phase for Child Three as she demonstrated mastery of cognitive strategy use across the nine sessions. The mean was 88.9%, with a range of 0%-100%. The level was stable with eight out of nine points falling on the median. The relative and absolute level changes were both zero. The trend during intervention was stable with eight out of nine points falling on the trend line.

Child Four. The median increased to 100% during the ten sessions of the intervention phase for Child Four, reflecting effective cognitive strategy use. The mean was 90% and the range was 25%-100%. The level was stable with eight out of ten intervention points falling on the median.

Child Five. Since all eight intervention points were 100%, reflecting perfect mastery of each lesson on cognitive strategy use across different math word problem types, the median, mean, and range for Child Five were all 100%, reflecting stability. The relative and absolute level changes were zero. The trend, too, was perfectly stable with eight out of eight points falling on the trend line.

Child Six. Child Six, showed the same results for the intervention phase as Child Five. She demonstrated 100% mastery across all eight intervention sessions (mean = 100%, median = 100%, range = 100%), thus achieving stability across the intervention phase. With all eight points falling on the trend line it was stable, with relative and absolute level changes of zero.

Summary of analyses between conditions. Since baseline and intervention phases were stable within conditions for all participants, the relative and absolute changes in level for all participants between conditions increased from 0% to 100%, demonstrating a positive effect. The PND were 100% for Child One, Child Four, Child Five, and Child Six. The PND for Child Two and Child Three were 88.9%, reflecting a large effect size between all participants' baseline and intervention conditions.

Summary of analyses across conditions. As noted above, all participant baseline conditions were similar, showing no use of cognitive strategies to support math word problem solving during baseline. When comparing intervention conditions for all participants, the median level rose to 100% and all levels were stable with a zero celerating trend and stable direction. Means indicated growth from a range of 0% to 0% in baseline to a range of 85% to 100% during intervention.

Summary of visual analyses of data. Within conditions, between conditions, and across conditions analyses reveal the presence of a functional relation between the intervention and accurate use of cognitive strategies through the bar model drawing model intervention. When analyzing the data within conditions, a median and mean of 0% for all baseline points rose to a median of 100% and a mean ranging from 85%-100% for the intervention phases. Between conditions analyses showed positive changes in relative and absolute levels from baseline to intervention, rising from 0% to 100% for all participants. PND (ranging from 88.9% to 100%) demonstrated that the large majority of data points during interventions did not overlap with baseline data points.

Generalization. Fifteen word problems taken from released SOL math tests were analyzed for accurate use of cognitive strategies. Accurate use of cognitive strategies divided by opportunities to use cognitive strategies (i.e., four cognitive strategies for fifteen questions) yielded scores ranging from 10.71% to 92.50%, with a mean score across students of 54.33%. This measure will be discussed in greater detail in the pre/post measure.

Maintenance. A maintenance probe was administered at least one week following the completion of intervention for each participant. Students demonstrated that they were able to maintain their accurate use of strategies with maintenance scores ranging from 75% to 100%, with a mean score of 91.8%.

Frequency of Bar Model Drawing and the Use of Accurate Cognitive Strategies to Solve Word Problems

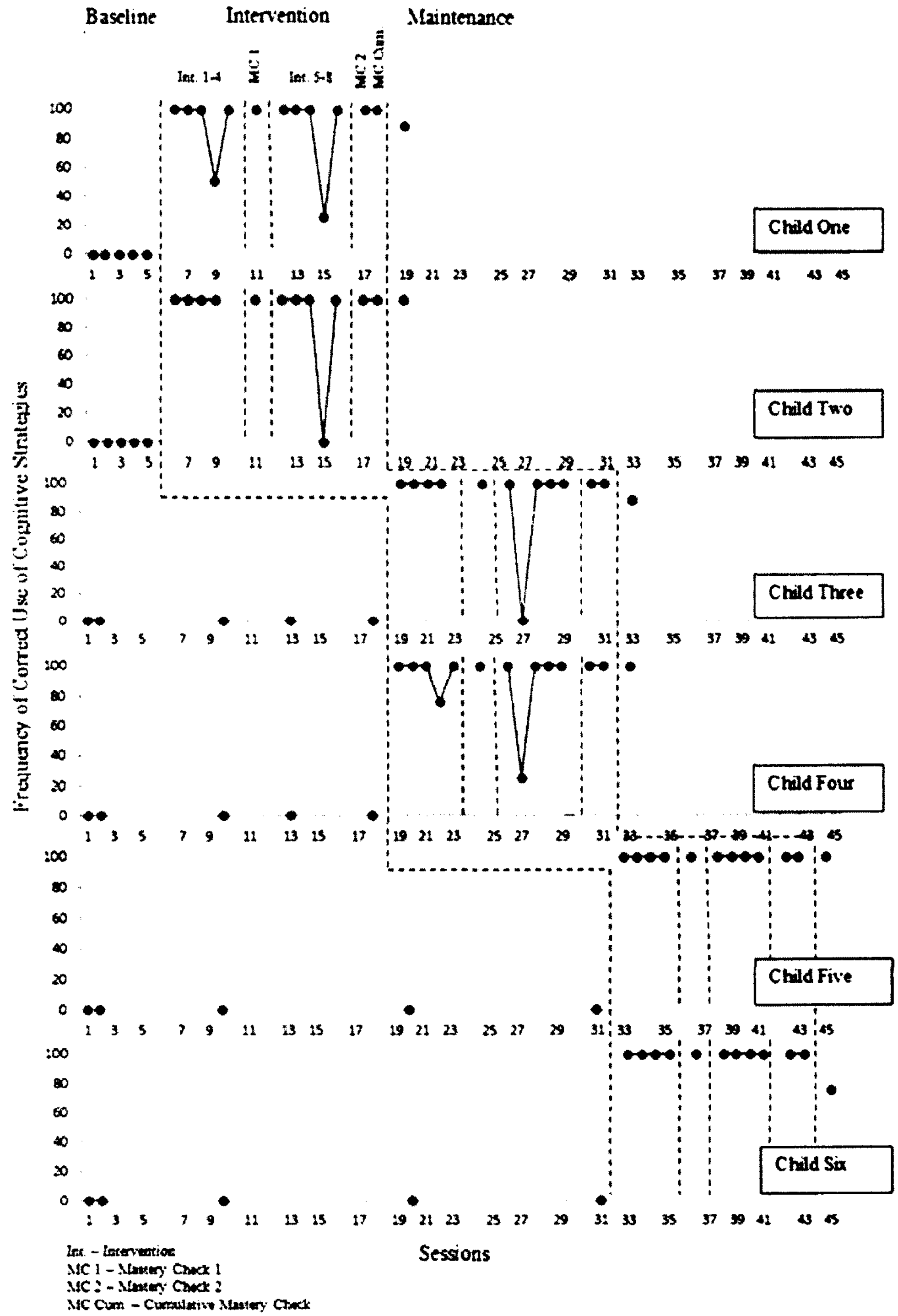


Figure 4.1.

Table 4.1. Phase Means for Accurate Use of Cognitive Strategies

	Accurate Use of Cognitive Strategies Phase Means			
	Baseline	Intervention	Generalization	Maintenance
Child One	0	87.50	92.50	88
Child Two	0	88.89	50.00	100
Child Three	0	88.89	28.57	88
Child Four	0	90.00	82.14	100
Child Five	0	100.00	60.71	100
Child Six	0	100.00	10.71	75
Average	0	92.55	54.33	91.83

Research Question 2

To what extent will explicit instruction of the bar model drawing strategy increase the ability of urban students with MD to accurately solve math word problems?

Visual Analyses of Data

Baseline, intervention, generalization, and maintenance for each participant is discussed. Refer to *Figure 4.2* for a graph of the results and Table 4.2 for the means across phases.

Child One. Visual analyses of within-condition phases indicated that Child One's accuracy in word problem solving during baseline showed a median of 50%, with a mean of 47.6%, and a range of 38%-50% correct responses to word problems.

The baseline level was stable, with four out of five points falling within 20% of the median range. The relative and absolute level changes were zero. At baseline, the trend was stable, with four out of five points falling within a 20% range of the trend line.

When Child One reached the intervention phase, the median rose to 100%, with a mean of 85%, and a range of 25% – 100%. The level was stable with eight out of ten points falling at the median. The relative and absolute level changes were zero. The intervention trend, like the baseline trend, was stable. Child One demonstrated 100% accuracy on Mastery Checks 1 and 2 and the final cumulative mastery check. She scored 67% on a generalization measure. Her accuracy remained relatively high on the final maintenance check, at 75%.

Between condition analyses indicated that the trend for Child One's baseline and intervention phases was stable with no change in direction. The relative and absolute changes in level showed a 50 point gain. The PND for Child One in accuracy of math word problem solving using bar model drawing was 80%, showing a large intervention effect.

Child Two. Visual analyses of within-condition phases indicated that Child Two's accuracy in word problem solving across five baseline probes yielded a median of 20%, with a mean of 24.2%, and a range of 13% - 50%. The stability at baseline was variable, with only one of five baseline points falling within 20% of the median level. The relative level change reflected a deterioration of 12.5%, and the absolute change showed 37% deterioration. Trend stability was variable with only three out of five, or 60%, of baseline points falling within 20% of the trend line.

The median rose to 100% during the intervention phase with a mean of 88.9% and a range of 0% - 100%. The intervention phase showed level stability, with eight out of nine intervention points falling on the median line. The trend was stable, with eight out of nine points falling on the trend line. Child Two demonstrated 100% accuracy on Mastery Checks 1 and 2 and the final cumulative mastery check. He scored 67% on a generalization measure. He scored 88% for accuracy on the final maintenance check.

Visual analyses of between conditions phases indicated that Child Two's low accuracy in math word problem solving developed from decelerating at baseline to zero celerating at intervention, and from variable to stable. The relative change in level rose from 19% to 100%, an 81 point increase; the absolute change in level increased from 13% to 100%, an 87 point improvement. The PND was 89%, indicating that the improvement in accuracy in math word problem solving through the support of bar model drawing is strong.

Child Three. Visual analyses of within-condition phases indicated that Child Three's median accuracy in math word problem solving across five points of baseline was 13%, with a mean of 17.8% and a range of 0% - 38%. The stability level at baseline was variable, with only two of the five baseline points falling within 20% of the median level. The relative change level was zero and the absolute change level was 13%. The trend was variable, with two of the five baseline points falling within 20% of the trend line.

During intervention, Child Three's median for word problem solving accuracy across nine intervention points rose to 100% with a mean of 88.9% and a range of 0% to 100%. The level was stable with eight of nine intervention points falling on the median at 100%. The relative and absolute level changes were both zero. The trend at intervention was stable. Child Three demonstrated 100% accuracy on Mastery Checks 1 and 2 and the final cumulative mastery check. Her ability to generalize was high, reflected in a score of 87%. Her accuracy remained high on the final maintenance check, at 88%.

Between condition analyses indicated that the trend for Child Three's baseline and intervention phases was stable with no change in direction. The relative change in level rose from 19% to 100%, an 81 point increase; the absolute change in level increased from 13% to 100%, an 87 point improvement. The PND for Child Three in accuracy of math word problem solving using bar model drawing was 89%, a large effect size.

Child Four. A within condition visual analysis of five points of baseline for Child Four reveals a median of 25% accuracy in word problem solving, with a mean of 25.2% and a range of 13% to 38%. The stability of the level is variable with only three of the five points falling within 20% of the median. The relative level change is -.5, but the absolute level change is 19. The baseline trend reflects variable stability, with only three out of five points falling within 20% of the trend line.

Across ten intervention points, Child Four's median rose to 100%, with a mean of 90% and a range from 25% to 100%. The level stability at intervention was stable, with eight intervention points lying on the median line. The relative and absolute level changes were zero. The trend line was stable. Child Four demonstrated 100% accuracy on Mastery Checks 1 and 2 and the final cumulative mastery check. Her ability to generalize was relatively high, as shown by a generalization score of 73%. Her score of 100% on the final maintenance check reflects very good ability to maintain the strategies taught during intervention.

Between condition analyses for Child Four reveals a trend direction change from decelerating to zero celerating from baseline to intervention. The trend stability changed from variable to stable. Both the relative and absolute level changes rose from 25 to 100. The PND effect size was large at 90%, with only one intervention point overlapping the baseline points.

Child Five. Within condition analyses revealed a 38% median for accuracy in word problem solving for Child Five, with a 35.2% mean and a range of 25% to 50%. The stability of the five points of baseline is variable, with only two points falling within 20% of the median. The relative level change is -12.5% and the absolute level change is -22%. Thus, the baseline phase for Child Five demonstrated a variable trend.

Child Five demonstrated 100% mastery across eight intervention points. No intervention lesson required remediation. Thus, her median and mean both stood at 100% with a stable median line. The trend line was stable. Child Five demonstrated

100% accuracy on Mastery Checks 1 and 2 and the final cumulative mastery check. Her generalization score was 60%. Her accuracy remained relatively high on the final maintenance check, at 88%.

Between condition analyses for Child Five showed a directional change in trend from decelerating to zero celerating across baseline and intervention with trend stability changing from decelerating to stable. Stability change in trend went from variable in baseline to stable during intervention. The relative change in level rose from 31.5% to 100%, and the absolute change went from 38% to 100%. The effect size as shown by PND was large at 100% with no data point in intervention overlapping a baseline point.

Child Six. Visual analyses of within-condition phases indicated that Child Six's accuracy in word problem solving during baseline showed a median of 0%, with a mean of 12.6%, and a range of 0% to 50%. The baseline level was variable, with only three out of five points falling within 20% of the median range. The relative level change was -6.5% and the absolute level change was -13%. At baseline, the trend for Child Six was variable, with two out of five points falling within a 20% range of the trend line.

Like Child Five, Child Six attained perfect mastery of all interventions, or lessons, requiring no remediation of any lesson across eight intervention points. Her median and mean were 100%. The median and trend lines were one and the same, showing stability. Like all participants before her, Child Six demonstrated 100% accuracy on Mastery Checks 1 and 2 and the final cumulative mastery check.

Generalization was relatively high, with a score of 73%. Her accuracy remained relatively high on the final maintenance check, at 75%.

Between condition analyses indicated that the trend for Child Six between baseline and intervention phases went from decelerating to stable, and from variable to stable. Both the relative and absolute changes in level showed a 100% point gain. The PND for Child Six in accuracy of math word problem solving using bar model drawing was 100%, showing large effects of the intervention.

Across similar conditions for all participants. When comparing all participants' baseline conditions for initial accuracy of solving math word problem solving, median levels ranged from 0% to 50%, with a mean range of 12.6% to 47.6%. While Child One showed level and trend stability at baseline, the rest of the participants showed variable stability in trend and level. When comparing intervention conditions for all participants, the median level rose to 100% and all levels were stable with a zero celerating trend and stable direction. The means showed growth for all participants during intervention, ranging from 85%-100%.

Summary of visual analyses. Analyses within conditions, between conditions, and across similar conditions analyses revealed the presence of a functional relation between the intervention and accuracy of word problem solving through the bar model drawing model. When analyzing the data within conditions, a median ranging from 0% to 50% for baseline points rose to a median of 100% and to a mean ranging from 85%-100% for the intervention phases. Between conditions analyses showed positive changes in relative levels from baseline to intervention, rising from a range of 19% - 50% at baseline to 100% at intervention for all participants. PND (ranging from 80% to 100%) demonstrate that most data points of probes during interventions did not overlap with baseline data.

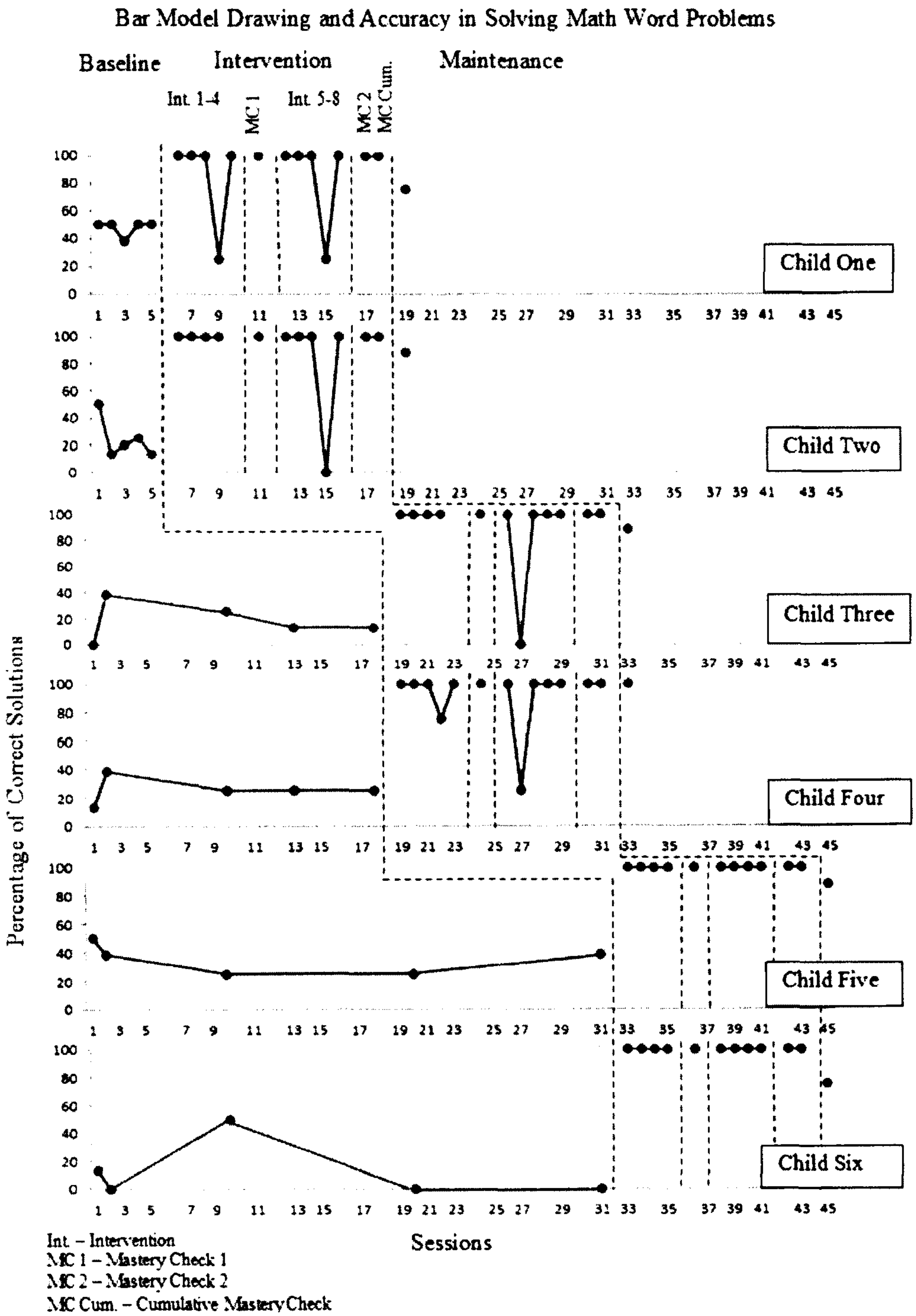


Figure 4.2. Bar model drawing and accuracy in solving math word problems.

Table 4.2. Phase Means for Accuracy in Solving Math Word Problems

	Accuracy in Solving Math Word Problems Using Bar Model Drawing — Phase Means			
	Baseline	Intervention	Generalization	Maintenance
Child One	47.60	85.00	66.67	75
Child Two	24.20	88.89	66.67	88
Child Three	17.80	88.89	86.67	88
Child Four	25.20	90.00	73.33	100
Child Five	35.20	100.00	60.00	88
Child Six	12.60	100.00	73.33	75
Average	27.10	92.13	71.11	85.67

Pre/Post Assessment Results

Data were collected using participant interviews based on questions taken from the *Mathematical Problem-Solving Assessment—Short Form* (*MPSA—SF*; Montague, 2003). Pre and posttesting results based on released Virginia SOL word problem questions and the *KM—3* assessment are also presented. Finally, a social validity survey is discussed.

Pre and Post Interviews. Pre and post interviews were conducted with participants to determine their perceptions of their math and word problem skills, their attitudes toward math and word problem solving, and their knowledge of the application of cognitive strategies. Twelve selected questions for these structured interviews were taken from the *Mathematical Problem Solving Assessment—Short*

Form (MPSA-SF, Montague, 2003), a 40-question assessment typically used with middle school students (Krawec et al., 2013; Montague, 1997; Montague & Applegate, 1993). It was hypothesized that changes in response to the same questions following intervention may reflect changes in attitudes toward math and demonstrate word problem solving and transformations in understanding and appreciation for cognitive strategies for solving math word problems (see Table 4.3).

Questions 1-3 gauged participants' perception of their overall math and word problem solving skills, participants collectively reported a total 49 points out of a possible 54 points on the pretest ($M = 8.17$ out of a possible 9 points for each participant). Following intervention, participants collectively reported a total 50 points out of a possible 54 points ($M = 8.33$ out of a possible 9 points for each participant). Collectively, participants reported that they had positive attitudes towards math before intervention possibly as a result of an overall lack of realistic views of abilities, as described in Chapter Two. Child One and Child Two reported an increased ability to solve math word problems from pre to post interview (Child One, score 2 to 3, pre to post; Child Two, score 1 to 3, pre to post), while Child Three reported a one-point drop in ability to solve word problems, perhaps reflecting a more thorough knowledge of the steps involved in accurately solving a math word problem.

Questions 4 and 5 in the interview were written to reflect attitudes toward math and word problem solving. Participants collectively reported 30 out of 36 points ($M = 5$ out of a possible 6 points for each participant) reflecting a positive attitude during pretesting. During pretesting, only one participant, Child Six, reported that she only liked math sometimes and did not usually like solving math word problems. Following intervention, participants again collectively reported 30 out of 36 points ($M = 5$), indicating a positive attitude toward math and word problem solving. While Child One reported a one point gain for each of the two questions regarding attitudes toward math and word problem solving, Child Six reported that she did not like math at all during the post interview; however, she verbally stated that her disregard for math was due to struggles she was experiencing within the classroom.

Questions 6 through 12 of the interview related to knowledge and accurate use of cognitive strategies. Although the publisher of the *MPSA-SF* (Montague, 2003) requested that the questions taken from the assessment not be replicated, these questions asked participants about cognitive strategy use, such as their strategies for understanding and planning to solve the problem, making a representation of the problem, and checking the problem. During the pretest interviews, participants collectively reported 32 out of a possible 126 points ($M = 5.33$ out of a possible 21 points for each participant). When participants were asked what he/she did when he/she did not understand a word problem, all replied that they elicited help from teachers. When asked how they remembered the important details in a word problem, four participants reported that they "reread the problem," while two failed to identify a strategy. Four participants could not think of a strategy to use when asked how they planned or made a representation to help them solve word problems. The other two participants came up with one general strategy for each question, such as deciding on the math operation. Following intervention, participants collectively scored 78 out of a possible 126 points ($M = 13$ out of a possible 21 points for each participant), more than doubling their reported use of cognitive strategies from the pre intervention interview. Participants verbally reported using the cognitive strategies they had learned through the direct instruction used during intervention.

Pre and post interviews demonstrated participants' stability in their perceptions of skill in math and math word problem solving and their attitudes toward math and math word problem solving. However, participants demonstrated gains in their knowledge and reported use of cognitive strategies while solving math word problems.

Table 4.3

Pre and Post Interview Results Adapted from the *MPSA--SF*

	Questions 1-3 Math and Word Problem Solving Skills Perceptions			Questions 4-5 Attitudes Toward Math and Word Problem Solving			Questions 6-12 Knowledge of Cognitive Strategy Use			Overall Total		
	Pre	Post	Point Gain	Pre	Post	Point Gain	Pre	Post	Point Gain	Pre	Post	Point Gain
Child One	7	9	2	4	6	2	6	16	10	17	31	[illegible]
Child Two	7	8	1	5	5	0	8	14	6	20	27	[illegible]
Child Three	9	8	-1	6	6	0	2	14	[illegible]	17	28	[illegible]
Child Four	9	9	0	6	6	0	3	8	[illegible]	18	23	[illegible]
Child Five	9	9	0	6	6	0	8	12	4	23	27	[illegible]
Child Six	8	7	-1	3	1	-2	5	14	[illegible]	16	22	[illegible]
Mean Point Gain			.17			0			[illegible]			[illegible]

Pre and Post SOL Questions. Students were given word problems taken from released Virginia SOL tests prior to and following intervention. On pre and posttests, question one was the same, and questions two through fifteen varied only by noun and proper noun changes. Problem order remained the same. Posttest released SOL questions were assessed for accuracy and correct use of cognitive strategies and served as generalization measures. Each question, except for Question 1, in which participants had to choose the correct representation for a problem rather than drawing their own representation, was analyzed for use of paraphrasing, visualizing, hypothesizing, and checking. In terms of accuracy, participant scores on the pretest ranged from 6% to 33% correct (M = 21%).

All participants' posttest scores rose, ranging from 60% to 87% correct (M = 71.17%; see *Figure 4.3*). On the posttest, Child One refused to answer four questions (Numbers 2, 13, 14, 15) and Child Two left one question blank (Number 13). Excluding these five questions, participants answered 77 of a total 85 questions accurately (i.e., six participants multiplied by 15 questions each with five unanswered questions subtracted). Two questions, numbers 5 and 10, represented Lesson Eight (i.e., part/whole). Three participants did not answer question 5 correctly, and two participants did not answer question 10 correctly, representing the largest fraction, 5 of 13, incorrect answers. The remaining 8 questions answered incorrectly were scattered and did not correspond to any one lesson taught or problem number. Patterns in accurate use of cognitive strategies did not seem to be present.

Participants who used a higher number of cognitive strategies on the posttest did not necessarily achieve higher accuracy scores.

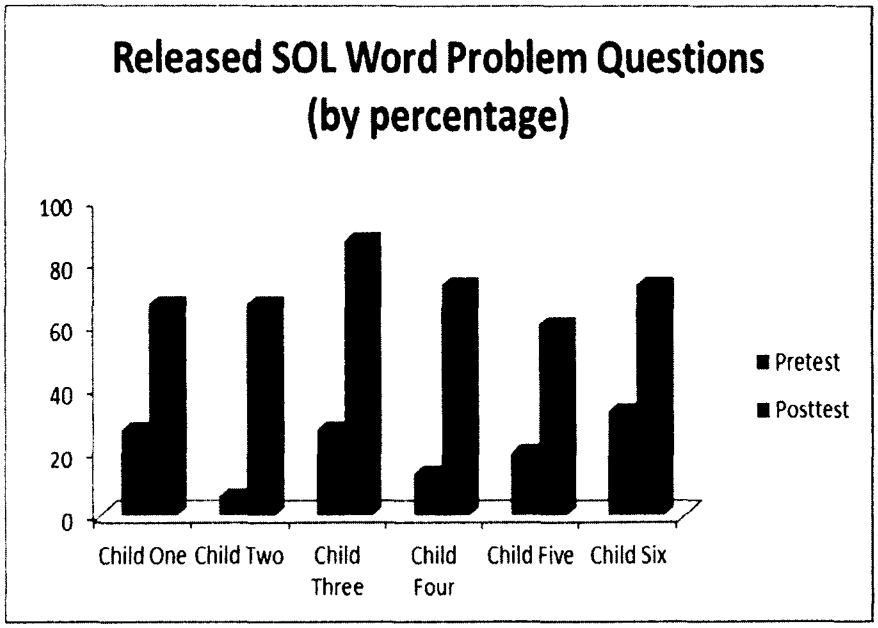


Figure 4.3

Pre and Post *KeyMath--3* Assessments. The *KeyMath Diagnostic Assessment, Third Edition, Form A* (Connolly, 2007) is a comprehensive assessment of mathematical skills. The assessment is organized into three main areas of mathematical skills that are comprised of more specific subtests. The three main components are Basic Concepts, Operations, and Applications. The subtests included in the area of Basic Concepts are: Numeration, Algebra, Geometry, Measurement, and Data Analysis and Probability. The subtests included in the area of Operations are Mental Computation and Estimation, Addition and Subtraction, and Multiplication and Division. The last area, Applications, includes two subtests, Foundations of Problem Solving and Applied Problem Solving. The *KM—3* was administered to participants prior to and following intervention. Their performance on each administration is discussed.

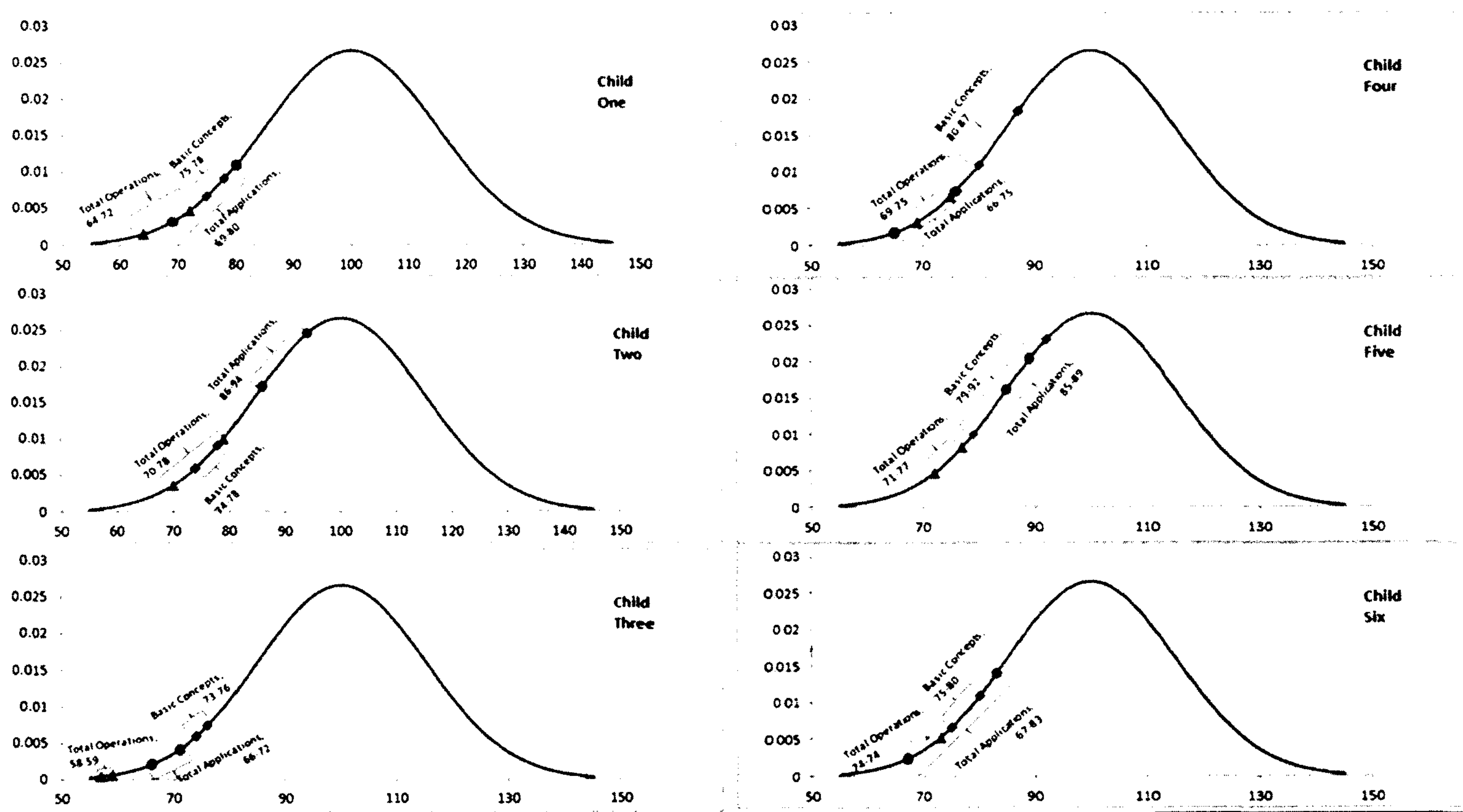
Form A of *KM—3* (Connolly, 2007) was administered as both the pre and post assessment due to a lack of availability of *Form B.* The *KM—3* manual reports high test-retest reliability (.97 for Total Test) and a small practice effect, about 1/5 of a standard deviation (i.e., $SD = 3$ on subtests; $SD = 15$ on three main areas and total test). Participant results are found below in *Table 4.4.*

On the pretest, all participants fell within the first through the seventh percentiles, in the below average and well-below average ranges. During posttesting, all participants made point gains (range = 3 - 9, $M = 6.17$) on the Total Test, with scores between the third and seventeenth percentiles, in the below average to just within the average range. The participant (Child Three) with the lowest pretest score made the lowest overall point gain (i.e., 3 points), while the participant with the highest pretest score (Child Five) made the greatest overall point gain (i.e., 9 points). Participant gains in relation to the normal distribution for the *KM—3* are illustrated in *Figure 4.4.* Furthermore, while participants made gains in all three areas, they made the largest gains ($M = 9$ points) in the Applications cluster, which focused on problem solving.

Table 4.4

Pre and Post *KeyMath--3* Results

	KM—3 Basic Concepts Cluster			*KM—3* Total Operations Cluster			*KM—3* Total Applications Cluster			Total *KeyMath--3*		
	Pre SS	Post SS	Point Gain	Pre SS	Post SS	Point Gain	Pre SS	Post SS	Point Gain	Pre SS	Post SS	Point Gain
Child One	75	78	[illegible]	64	72	[illegible]	69	80	[illegible]	70	76	[illegible]
Child Two	74	78	4	70	78	[illegible]	86	94	[illegible]	74	80	[illegible]
Child Three	73	76	3	58	59	[illegible]	66	72	[illegible]	68	71	[illegible]
Child Four	80	87	7	69	75	6	66	75	[illegible]	73	80	[illegible]
Child Five	79	92	13	71	77	6	85	89	4	77	86	[illegible]
Child Six	75	80	5	74	74	0	67	83	16	72	78	[illegible]
Mean Point Gain			5.83			4.83			9.0			[illegible]



Subtests

Basic Concepts
- Numeration
- Algebra
- Geometry
- Measurement
- Data Analysis and Probability

Operations
- Mental Computation and Estimation
- Addition and Subtraction
- Multiplication and Division

Applied Problem Solving
- Foundations of Problem Solving
- Applied Problem Solving

Figure 4.4. Pre and Post growth on *KeyMath--3* subtests in relation to the normal distribution.

Social Validity Survey Results

Participants were given a social validity survey to determine their perceptions regarding ease of learning and use of bar model drawing and their perceptions of its practical application in word problem solving. Participants completed the five-question Likert-style survey anonymously. The surveys were collected by the school secretary who passed all the surveys to the researcher. All surveys from six participants were returned. *Table 4.5* shows the results of the survey.

Results revealed that the participants had positive perceptions of the use of model drawing as a tool to help them solve word problems. For statements 1 and 2, five participants strongly agreed (score of 5) that they liked learning how to use bar model drawing and that it was a helpful strategy, while one participant was noncommittal for each of these questions (marking a 3). Four participants reported that they felt that it was not difficult to learn how to draw models for word problems (marking 1), while one participant circled both 1 and 2, and the remaining participant strongly agreed (marking 5) that it was difficult to learn the strategy. Only one participant expressed doubt that he or she would use bar model drawing in the classroom, scoring a 2 out of 5, while the rest of the participants strongly agreed that they would use the strategy in their classroom. All participants indicated that they strongly believed that bar model drawing should be taught to other children their age. These results demonstrate a strong social validity of the bar model drawing strategy to the participants in this research.

Table 4.5

Social validity statements and scores

Social Validity Statements	Average Score (out of possible 5)
I liked learning how to draw models and solving word problems using model drawing.	4.67
Drawing models for word problems helps me solve the problems.	4.67
It was difficult for me to learn how to draw models for word problems.	1.75
I will draw models when I have to solve math problems in my classroom.	4.5
I think other kids my age should be taught how to draw models for word problems.	5

Procedural Fidelity and Inter-Observer Agreement

All intervention sessions were videotaped. Treatment fidelity (both content and process) was assessed by a doctoral student using a checklist created by the researcher to ensure that the researcher adhered to the content and intervention procedures. The doctoral student viewed at least 35% of taped sessions for each dyad. Intervention sessions were randomly selected by using the Integer Generator on Random.org. The doctoral student determined that the researcher followed the intervention checklist with 100% procedural fidelity.

All baseline probes, at least 35% of all intervention probes, for each dyad (randomly selected by using the Integer Generator on Random.org), and all mastery checks were graded by a doctoral student to ensure that these had reached 100% in strategy and accuracy use. A criterion level of 85% or above was established to ensure accuracy of data collected. Inter-observer agreement was calculated by reporting agreements of occurrences or accuracy divided by agreements plus disagreements (A/[A+D]). Inter-observer agreement for the research was 91%.

Conclusion

In this chapter, the results of the dependent measures of increased accurate use of cognitive strategies and overall accuracy of math word problem solving were summarized and reported. It was found that the independent measure of bar model drawing had a strong, positive effect on both dependent measures. Both dependent variables increased and remained stable throughout intervention, and remained high during the maintenance phase of the research. For each research question, the results were presented for individual participants and the overall summary of results for all participants was provided. For research question one, it was found that a median and mean of 0% for all baseline points rose to a median of 100% and a mean ranging from 85%-100% for the intervention phases. Results on research question one, examining whether participants' use of cognitive strategies improved after learning the bar model strategy to solve math word problems, indicated that there may be a functional relation between bar model drawing training and accurate use of cognitive strategies. For research question two, it was found that a median ranging from 0% to 50% for baseline points rose to a median of 100% and to a mean ranging from 85%-100% for the intervention phases. Results on research question two regarding the effectiveness of the bar model drawing strategy in increasing the accuracy of the math word problems demonstrated the possibility that bar model drawing training increases overall accuracy when students with MD solve math word problems.

In addition, qualitative data were gathered. The results of pre and posttest and interviews showed that participants increased in their knowledge of cognitive strategies. Participants reported high social validity for the intervention. Pre and posttesting results were also favorable. Participants were able to more accurately solve questions taken from released SOL tests, and demonstrated growth in overall math skills as shown on the KM—3. Chapter 5 will discuss implications of these results along with recommendations for further research and concluding remarks.

CHAPTER 5

DISCUSSION

This chapter presents a summary of the study and a discussion of results and implications of the research. Additionally, suggestions are offered regarding the potential impact of the study on practice and recommendations for further research. Finally, limitations of the study are also discussed.

Summary of the Study

The purpose of this study was to determine if a schematic-based instructional (SBI) strategy, bar model drawing, would increase third-grade students' with MD accurate use of cognitive strategies and overall accuracy in solving five types of math word problems. Using a multiple-baseline replicated across groups design, the researcher provided explicit instruction in bar model drawing across five types of mathematical word problems to six third-grade participants with MD. Cognitive strategy instruction (CSI) was embedded in the bar model drawing strategy sequence protocol. The following research questions guided this study:

1. To what extent will explicit instruction of the bar model drawing strategy improve the use of cognitive strategies of urban students labeled with MD when solving math word problems?
2. To what extent will explicit instruction of the bar model drawing strategy increase the ability of urban students with MD to accurately solve math word problems?

The hypothesis that the bar model drawing strategy would support students in increasing their accurate use of cognitive strategies while solving math word problems was confirmed in the study. Visual analyses of the single subject data of this study indicated that there is a functional relationship between bar model drawing and increased accurate use of cognitive strategies and overall accuracy in solving math word problems. A large effect size between all participants' baseline and intervention conditions demonstrated promising results. Furthermore, post intervention interviews and a social validity survey revealed that participants valued the instruction and felt they would be able to make practical application of it. Explicit teaching of the bar model drawing protocol enhanced students' awareness of cognitive strategies through paraphrasing, visualizing, hypothesizing about problem solutions, and checking work, all of which are important steps in solving word problems.

The hypothesis that the bar model drawing strategy would support students in increasing their overall accuracy in math word problem solving was confirmed in this study. Visual analyses of the single subject data of this study indicated that this study appears to have resulted in demonstrating a functional relation between bar model drawing and increased accuracy in word problem solving. Large effect sizes between all participants' baseline and intervention conditions validated the hypothesis. Furthermore, pre and posttesting using word problems from Virginia released SOL tests and the *KM—3* (Connolly, 2007) reinforced the results.

This study is consistent with previous research that has demonstrated the value of direct instruction in CSI in assisting students with MD to correctly solve math word problems (Krawec et al., 2013; Montague et al., 1993, 2011). However, this investigation extended previous findings suggesting that SBI with explicit instruction can be effective in teaching students to answer different types of math word problems (Jitendra et al., 1996, 2002, 2009, 2013). The literature available on SBI describes the use of several schemas to solve different types of word problems (e.g., Change, Group, and Compare schemas for addition and subtraction and Vary schema for multiplication word problems); however, in the current study, a more generic schema approach, bar model drawing, was used as a form of SBI, showing that it can be applied across different math word problems. The bar model drawing uses only one schema (i.e., the bar model) for many types of math word problems (e.g., change, group, compare, vary, equal-group, and part-whole) involving different math operations (e.g., addition, subtraction, and multiplication).

Discussion of Results

Research Question One. To what extent will explicit instruction of the bar model drawing strategy improve the use of cognitive strategies of urban students with MD when solving math word problems?

Visual analyses of individual participant's performances during intervention, on pre and posttests, and the outcomes of interviews that included structured questions regarding use of cognitive strategies suggest that the use of bar model drawing to solve math word problems is an effective forum for improving participants' use of cognitive strategies. Prior to intervention, participants' baseline performances showed no use of cognitive strategies. During intervention, participants were able to successfully implement the use of cognitive strategies. Although some remediation was necessary, participants' median level of cognitive strategy use rose to 100% during intervention. The levels remained high during the maintenance phase, which occurred at least one week after intervention using novel word problems.

Instruction in the use of four cognitive strategies was included within the direct instruction of bar model drawing. These four cognitive strategies were paraphrasing (i.e., rewriting the question as an answer statement), visualizing (i.e., constructing a bar model), hypothesizing about problem solutions (i.e., manipulating the bar model), and checking work (i.e., writing the answer in the previously written answer statement and ensuring it makes sense), all of which were explicitly taught through the use of the bar-model drawing strategy protocol.

In an attempt to ascertain what cognitive strategy or strategies proved most useful to participants, the number of occurrences of the effective application of each of the four cognitive strategies used on the posttest comprised of 15 word problems taken from released SOL tests were tallied and analyzed. Accurate strategy use was compared with overall success in solving each of the word problems correctly. As noted in Chapter 4, participants demonstrated growth in their ability to solve the SOL word problems. However, participants varied widely in their use of the strategies on this measure. Child One and Child Four used all four strategies consistently. Child

Two used visualizing only once. Child Three failed to use the paraphrasing or checking work strategies. Child Five used the strategies inconsistently across word problems, using each strategy correctly between 8-10 times. Finally, Child Six used very few cognitive strategies, using visualizing twice and hypothesizing four times across the entire test. Child One and Child Four were no more successful in posttesting than participants who did not consistently use the cognitive strategies; participants who favored one or two strategies and declined the use of the others were no more successful or unsuccessful than Child Six, who declined using almost all strategies.

Participants found one cognitive strategy, paraphrasing, extremely difficult and distasteful. In a pilot study that was conducted with 2012, four rising fifth graders did not seem to experience difficulty rewriting the question as an answer statement as the third-grade participants in this study. However, at times, the fifth grade participants in the pilot study did vocalize their perceptions that the rewriting of the question was tedious. In this study, all six of the third-grade participants struggled with rewriting the question as an answer statement.

Each lesson taught during the intervention phase of this study included a discussion of the process of turning questions into answer statements. The third-grade participants experienced minimal difficulty with paraphrasing on Lessons One and Two, which involved simple addition and subtraction problems with one variable. For example, when the word problem posed *a young man who ate 3 candy bars after lunch, and 2 more candy bars after dinner*, participants had little or no difficulty constructing an accurate answer statement, such as *Joshua ate ____ candy bars*. Only one question in Lesson Two posed a little difficulty for participants: *Melanie had $12, but she spent $6. How much does she have left?* Only two participants' answer statements were completely accurate: *Melanie had $____* and *Melanie had ____ dollers* [sic]. Of the remaining four answer statements, two stated *Melanie had ____ money*, while another read, *Melanie had ____ but* [sic]. The last statement read, *She have ____ left*. Both the researcher and Ph.D. student conducting fidelity checks concluded that the answer statements were close approximations that were age and grade appropriate. On the other hand, when Lesson Three introduced problems with more than one variable, participants were not able to initially verbalize correct answer statements. After much discussion in the intervention sessions, participants were able to write accurate answer statements only if they could categorize the two variables. For example, *roses and violets* became *flowers,* and *bracelets and rings* became *jewelry.* Participants who were not able to construct a general term demonstrated great difficulty trying to include both variables in the answer statement, tending to include only one. For example, participants who failed to categorize and use the term *flowers* constructed an inaccurate statement, such as *They picked ____ roses for her mom.* Much more work and time went into teaching participants to construct accurate answer statements than teaching participants the actual drawing of representational bar models.

Lesson Four involved subtraction with more than one variable using a discrete model. This lesson was very difficult for participants attempting to construct answer statements. For example, answer statements for one question, *There are 4 children and 3 adults buying tickets to a movie. How many more children's tickets were bought than adult tickets?,* included (1) *There are ____ adult then* [sic] *children,* (2) *There were ____ more children tickets,* (3) *There are ____ children ticket,* (4) *There are ____ child then* [sic] *adult,* (5) *There were ____ more c,* and (6) *There are ____ children.* Participants' work following lessons using continuous bar models had similar patterns in difficulty forming answer statements: In general, when problems included one variable, such as 60 stamps in a collection, or when several variables could be conceptualized as one variable (e.g., third graders in Virginia must know that *carnivores, herbivores*, and *omnivores* are all *consumers*), participants could formulate an answer statement with little difficulty. The larger numbers included in later lessons/intervention sessions (e.g., 29 *carnivores,* 56 *herbivores*, and 24 *omnivores* in a forest) did not appear to be an issue; however, in Lesson Six, which compared two variables and involved subtraction using a continuous bar model to discover *How much more money, How many feet farther,* participants struggled again to construct answer statements, just as they had in Lesson 4. Lessons 7 (multiplication) and 8 (part-whole) followed the same pattern. Participants loved drawing representational bar models, but regularly pleaded not to have to write answer statements which was required in the protocol.

It is possible that this problem with writing answer statements may have been associated with the participants' reading levels. Only two participants, Child One and Two, read on the third-grade level. Three participants read on first grade level, and one on a second grade level. Since the rising fifth grade participants in the pilot study conducted prior to this research did not experience the same difficulty with writing answer statements, perhaps this cognitive strategy would be more appropriate for participants reading on at least a fourth grade level. The level of literacy of participants seems to be a crucial factor in participants' ability to write answer statements.

The struggle with forming answer statements raises some questions: Is this weakness or lack of development in the area of language associated with, or separate from, the math weaknesses participants demonstrated during pretesting? Would bar model drawing have been more or less effective if the step of constructing an answer statement had been taken out of the bar model drawing protocol? The questions need to be addressed in future research.

Research Question Two. To what extent will explicit instruction of the bar model drawing strategy increase the ability of urban students with MD to accurately solve math word problems?

Visual analyses of the single subject data, along with pre and posttesting in the form of grade-appropriate word problems from released SOL tests and the *KM—3* suggested that the use of bar model drawing is an effective strategy for improving students' accuracy in solving math word problems. During the baseline phase prior to intervention, participants' accuracy in sample math word problems was low, with a mean range of accuracy of 12.6 to 47.6%. Perhaps due to lack of interest or being ill-equipped in even the most basic math concepts, including number sense, four out of six participants (Child Two, Child Four, Child Five, Child Six) displayed decelerating trend lines, meaning that their accuracy decreased over time. The other two participants (Child One, Child Three) maintained stable baselines that demonstrated consistently low accuracy. During intervention, participants were able to successfully and accurately solve five different types of word problems across eight to ten sessions of intervention. Although some remediation was necessary, participants' median level, ranging from 0% to 50%, rose to 100% during intervention. Levels remained high during the maintenance phase.

When solving the posttest word problems of the sample released SOL, participants showed gains in accuracy, with a mean accuracy gain across participants of 51.17%. Participants also demonstrated gains between pre and posttesting on the *KM—3,* with a mean gain of 6.17 points across participants. Despite these results, several questions regarding participants' ability to accurately solve word problems were raised.

Child Three achieved the smallest point, three points, which is the typical gain for practice effect between the *KM—3* pre and posttest scores; however, she achieved the highest score on the released SOL word problems posttest, scoring 87%. Although she did not use the protocol sheet during her completion of the SOL measure, her work on the SOL word problems posttest demonstrated a close adherence to the steps outlined in the bar model drawing protocol used throughout the lessons. This calls into question whether the protocol and bar model drawing instruction served as a conceptual or procedural support for Child Three. While SBI is designed to assist students to conceptualize word problems, it is possible that the steps in the protocol served as a procedural aid in arriving at the accurate answers for the word problems given. This would explain the disparity between the extremely modest gain in *KM—3* posttest scores and the success Child Three achieved on the SOL measure.

Behavioral Concerns. It should be noted that there were behavioral concerns that impacted the performance of some participants, although no behavioral disabilities were recorded in the participants' profiles. Child One and Child Two, in particular, demonstrated behaviors that negatively affected their mastery of some lessons/intervention sessions. At other times, participants would arrive agitated from situations that had occurred during that school day. For example, Child One and Child Three arrived at times in tears due to perceived injustices at the hands of teachers and/or other students. In addition, Child One would often become agitated if she somehow felt that she could not be successful at the current bar model drawing lesson/intervention session. On several occasions, she retreated under the table at which she was working. Child Two would often display acute distractibility. He was easily distracted by Child One's behavior and any sound outside of the work room. He also consistently drummed and tapped out "beats" on the table. Child One and Child Two verbally argued at times, and Child One was eventually suspended from school three weeks before the end of the school year for physically attacking other students during the school day. Although Child One had been suspended previously during the school year, it should be noted that she did not experience any suspensions during the time she was involved in this research. By the time of her suspension, she had fully completed all lessons and testing required for the completion of this study, except the social validity survey which was sent to her by the school secretary.

Most of participants' failures on intervention probes were products of their lack of desire or refusal to complete the given probe and did not appear to be from a lack of understanding of the concepts taught. During remediation, participants often demonstrated an understanding of the lesson that had been taught previously on the same content.

Child Five and Child Six, displayed no challenging or interfering behaviors. In addition to the small rewards mentioned in Chapter 3 that were provided for all participants at the successful conclusion of each lesson/intervention session, this dyad appeared to value the minutes spent waiting with the researcher for parents' arrival following the successful completion of a lesson/intervention session. During this time, the students were helped with their homework, or the two participants would ask permission to be allowed to record a "music video," provided there was battery left in the FlipCam video recorder.

Conclusions

Implications for practitioners' use of bar model drawing in the classroom and recommendations for next steps in the research of bar model drawing will be discussed in this section.

Implications for Action. Since math word problems are an important component of math instruction with which participants historically struggled, this research offers practical, long-term implications in the classroom. First, since empirical evidence supports explicit instruction in the use of cognitive strategies for participants who have difficulty with math word problem solving and because SBI implicitly includes cognitive strategies, emphasizing the connection between the two strategies which have historically been studied separately could increase the value of SBI for educators.

The application of a cognitive strategy at the outset of solving a math word problem, such as restructuring the question being asked into an answer statement and leaving a blank for the answer, supports the student in paraphrasing the problem and thinking about how the problem needs to be answered, structured, organized, and computed. It may train students to thoughtfully form their own procedural foundation for successfully solving the problem. In Virginia, this is particularly important since paraphrasing is a key English standard for third graders and a powerful comprehension strategy (Hagaman & Reid, 2008). This research suggests that paraphrasing of math word problems may deserve more attention in the classroom, and this process may have to be explicitly taught.

Lastly, the generalizability of all components of bar-model drawing in comparison to other forms of SBI could mean that, as is the case with its use in Singapore, young students could be trained to use the model to support their understanding of the earliest, most fundamental word problems, and then teachers could build on this same conceptual understanding of bar-model drawing each year to support gradually more complex word problem solving (Forsten, 2010). In this manner, students can build upon their prior knowledge of bar model drawing and math word problem solving to lay a foundation for higher level, more complex problems in later grades. For example, word problems involving ratios and percentages can be solved using bar models, so that students can base their new understanding on a solid conceptual foundation built while solving other types of word problems.

Recommendations for Further Research. Swanson, Lussier, and Orosco (2013) recently investigated whether students with MD who possess lower cognitive abilities (i.e., compensatory model) or relatively high cognitive skills (i.e., high cognitive skills model) benefit more from the use of CSI. Their research indicated that students with MD with relatively high cognitive skills benefited more from CSI than students with lower cognitive abilities. They also posed the question of whether or not some cognitive strategies were more helpful in supporting students during word problem solving activities than others. Their results determined that students with MD participating in their study benefited more from the cognitive strategy involving visual schematics instruction than combining that cognitive strategy with what the researchers termed the general heuristic strategy, which involved underlining the question sentence, circling relevant numbers, placing squares around key words, and crossing out irrelevant information. The authors asserted that the visual-schematic condition assisted students in mapping the numbers in the problems, thus improving their accuracy.

Further research should be conducted in which students' cognitive skills, including working memory, are measured prior to intervention to determine if bar model drawing produces a different effect depending on students' cognitive skill levels. This could help determine whether bar model drawing serves as a conceptual rather than procedural tool for even low-performing students. This would also have implications for Response to Intervention (RTI) models since it would better inform educators which students may benefit from different kinds of supports, resulting in more time-efficient interventions. Further research could determine if direct instruction in bar model drawing is most effective as a second-tier, small group, intervention or a more intensive third-tier level of remediation.

In conducting this study, the question was raised about the utility of requiring some third-grade students to formulate answer statements. As noted earlier, participants in this study struggled with this cognitive strategy, a form of paraphrasing. This observation warrants further investigation. As noted previously, the bar-model drawing protocol used for this research included paraphrasing (i.e., rewriting the question as an answer statement), visualizing (i.e., constructing a bar model), hypothesizing about problem solutions (i.e., manipulating the bar model), and checking work (i.e., writing the answer in the previously written answer statement and ensuring it makes sense). The cognitive strategies paraphrasing and checking could easily be separated from visualizing and hypothesizing. Future research needs to examine this question further.

Limitations

This study was conducted with only six participants in the third grade, so the generalizability of the results to students in other grade levels is limited. In addition, all participants were African-Americans, only one male, who attended a low-performing urban school in Virginia. Therefore, the results may not apply to students in other locations or from other ethnic backgrounds. Since the intervention was provided in small groups of two (i.e., dyads), the results may not be applicable to other types of school settings, such as inclusion classrooms or self-contained special education classrooms when instruction is given in larger groups.

Also, since the study was conducted during the spring, from March to June, classroom preparation for math SOL testing was a high priority in the setting in which the study was conducted. Some successful results attributed to the study, such as *KM—3* and interview posttest results, could possibly have been a result of classroom activities, producing internal validity threats in the form of history and maturation.

All intervention lessons were taught by the researcher. This could have affected the researcher's attention during the intervention sessions and it may have influenced the participants' performance more than the content of the intervention. In addition, researcher bias is a realistic threat to the validity of the study.

The use of non-standardized testing instruments for the screening, baseline, intervention, and maintenance probes, and pre and posttesting measures (except for the *KM—3*) is another limitation. The tests and probes were constructed by the researcher based on released Virginia SOL word problem questions. Data collected with non-standardized instruments can be prone to errors (Mitchell & Jolley, 2013). Also, the use of five baseline points across all groups and participants to avoid testing fatigue instead of increasing the number of baseline points across groups and participants who began intervention after the first group can be considered a limitation to the research.

In addition, the doctoral candidate responsible for determining inter-observer agreement and fidelity of the intervention throughout the study was in the same cohort as the researcher. It is conceivable that some bias could have occurred because of the friendship that existed between these individuals.

Finally, since explicit instruction of bar model drawing with cognitive strategy instruction imbedded was bundled into one intervention, it is not possible to determine the effectiveness of any of these components individually.

Conclusion

This research adds to the limited research that formally combines SBI and CSI. The research suggests that direct instruction of the bar model drawing which implicitly includes cognitive strategy instruction could extend the current SBI literature and serve as the next step in SBI research. However, this research also highlights the need for more research on the best use of bar model drawing as an intervention in regards to RTI tiers in educational settings. Additional research is needed to determine which of the cognitive strategies included in this research were most effective, and whether or not any of the cognitive strategies used were ineffective. Despite these limitations, the results of this study suggest that explicit teaching of bar model drawing as a form of SBI has the potential to enhance students' awareness of cognitive strategies through paraphrasing, visualizing, hypothesizing about problem solutions, and checking work, all of which are important steps in solving word problems. In addition, bar model drawing may lead to increased accuracy in solving math word problems for students with MD.

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APPENDICES

Appendix A. SBI Literature Review Chart

Authors	Study Design	Participants	Special Education Participants	Grade	Setting Duration	Word-Problem Types	Instruction	Outcomes
Griffin, Jitendra (2009)	Matched pairs randomly assigned: a. schema instruction b. General strategy instruction (GSI)	$N = 60$ a. $n = 30$ b. $n = 30$	LD (identified by school) a. $n = 3$ b. $n = 2$	3	Teaching in groups of 15: a. and b. 20 lessons; 100 minutes	Change, group, compare	a. Explicit schema instruction b. General strategy instruction	Pretest scores showed equivalency between groups. Students in treatment condition (m = 24) outperformed control group (m = 18) on Time Test 1, although no effect was found between other measures
Jitendra, DiPipi, & Perron-Jones (2002)	Single-subject multiple probe across students	$N = 4$	LD (identified by school)	8	Individual tutoring: a. 18 lessons; 35-40 minutes	Vary, multiplicative comparison	Explicit schema instruction	Improved from baseline to intervention: 1. 44% to 100%; 50% to 100%; 37% to 100%; 29% to 100%
Jitendra, Griffin, Deatline-Buchman, & Sczesniak (2007)	Pretest Posttest Pilot Study a. LD b. Low-achieving (LA)	$N = 38$	9 LD (identified by school)	3	Classroom teaching: 45 lessons; 30 minutes	Change, group, compare	Explicit schema instruction	Pretest scores showed equivalency between groups. Growth was comparable across groups.
Jitendra et al. (2007)	Matched pairs randomly assigned	$N = 88$ a. $n = 45$ b. $n = 43$	LD (identified by school) a. $n = 2$ b. $n = 2$	3	Teaching in groups of 15: a. and b. 41 lessons; 25 minutes	Change, group, compare	a. Explicit schema instruction b. General strategy instruction	Pretest scores showed equivalency between groups. Post-test showed students in treatment condition (m = 1,410) outperformed control group (m = 1,281)
Jitendra et al. (1998)	Pretest-Posttest random assignment a. schema instruction b. General strategy instruction (GSI)	$N = 34$ a. $n = 17$ b. $n = 17$	SPED (identified by school): a. 8 LD; 2 MR; 2 ED; 5 at-risk b. 9 LD; 3 MR; 1 ED; 4 at-risk	2, 3, 4, 5	Small-group tutoring (3-6 students): a. and b. 17-20 lessons; 40-45 minutes	Change, group, compare	a. Explicit schema instruction b. General strategy instruction	Pretest scores showed equivalency between groups. Post-test showed students in treatment condition increased in performance by 26%, while the control performance increase was 16%.
Jitendra & Hoff (1996)	Single-subject multiple probe across students	$N = 3$	LD (identified by school)	3, 4	Individual tutoring: a. 13-16 lessons; 40-45 minutes	Change, group, and compare	Explicit schema instruction	Improved from baseline to intervention: 1. 20 to 90%; 31 to 95%; 26 to 95%

SBI Literature Review Chart, cont.

Authors	Study Design	Participants	Special Education Participants	Grade	Setting Duration	Word-Problem Types	Instruction	Outcomes
Jitendra, Hoff, & Beck (1999)	Single subject multiple baseline across subjects and behaviors	$N = 4$ (and 21 normally achieving students for testing only)	LD (identified by school)	6, 7	Individual tutoring: a. number of lessons not specified; 45 minutes	Change, group, compare	Explicit schema instruction	Improved from baseline to intervention: 1. 55% to 87%; 39% to 78%; 27% to 79%; 24% to 75%
Jitendra et al. (2013)	Pretest-posttest comparison group with random assignment a. SBI b. Standards-based curriculum (SBC)	$N = 136$ a. $n = 72$ b. $n = 64$	136 at-risk for math difficulty	3	Small group instruction: a. and b. 60 days, 30 minutes	Change, group, compare, and two-step problems	a. Explicit schema instruction b. Standards-based curriculum instruction	Students with higher pretest scores who received SBI performed better and maintained performance compared with students with higher pretest scores who received SBC
Jitendra et al. (2009)	Pretest-posttest comparison group with random assignment a. SBI b. General strategy instruction (GSI)	$N = 148$ a. $n = 70$ b. $n = 78$	15 LD (identified by school)	7	Whole-group instruction: a. and b. 10 days, 40 minutes	Ratio, proportion	a. Explicit schema instruction b. General strategy instruction	Pretest showed similar scores for both groups. Post-test showed students in treatment condition (m = 15.32) outperformed control group (m = 14.48)
Van Garderen	Single subject multiple probe across participant	$N = 3$	LD (identified by school)	8	Individual tutoring: a. two to four times a week; 35 minutes	One and two step addition and subtraction	"Visualize" strategy, explicit schema instruction	Improved from baseline to intervention: 1. 29% to 77%; 44% to 83%; 40% to 77%
Xin (2008)	Single subject, multiple probe across participants	N = 4	1 LD, 1 MR; 2 at-risk of math failure	5	Teaching in pairs: 12 sessions; 30 minutes	Group, multiplicative compare	Explicit schema instruction	Improved from baseline to intervention: 58% to 92%; 67% to 100%; 58% to 83%; 50% to 92%
Xin, Jitendra, Deatline-Buchman (2005)	Pretest-posttest comparison group with random assignment a. SBI b. General strategy instruction (GSI)	$N = 22$ a. $n = 11$ b. $n = 11$	18 LD (identified by school); 3 at-risk for math failure; 1 ED	6, 7, 8	Small group instruction: a. 12 sessions; 60 minutes b. 12 sessions; 60 minutes	Multiplicative comparison, proportion	a. Explicit schema instruction b. General strategy instruction	From pre- to post test: a. 54.22% increase b. 17.59% increase

Appendix B. CSI Literature Review Chart

Authors	Study Design	Participants	Special Education Participants	Grade	Setting Duration	Word- Problem Types	Instruction	General Outcomes
Compton, Fuchs, Fuchs, Lambert, & Hamlett (2012)	Longitudinal ex post facto	N = 684	Identified by researchers	3rd through 5th grade	School setting, individually or in groups, depending on the measure	WJ-III Applied Problems	n/a	LD is academic specific, not generally found across content areas of reading and math
Fuchs, et al. (2005)	Pretest-posttest control group design. a. at-risk of LD control b. at-risk of LD tutored students (AR) c. students not at risk (NAR)	N = 564 a. n = 69 b. n = 70 c. n = 437	Identified by researchers	1	Small-group instruction	Change, combine, compare, and equalize relationships	Concrete-representational-abstract (CRA) sequence	Tutored at-risk students made gains in comparison to non-tutored at-risk students
Garrett, Mazzocco, & Baker (2006)	Longitudinal ex post facto a. Math learning disability (MLD) b. Typical achievement (non-MLD)	N = 196 a. n = 17 b. n = 179	Identified by researchers	2nd through 4th grade	School setting, one-on-one in sessions lasting no longer than 45 minutes	Applied questions	n/a	Metacognitive skills of off-line tasks involving problem solving is weaker in MLD group when compared to non-MLD group.
Hanich, Jordan, Kaplan, & Dick (2001)	Ex Post Facto a. math difficulties only (MD) b. math and reading difficulties (MD/RD) c. reading difficulties only (RD) d. normal achievement in math and reading (NA)	N = 210 students a. n = 53 students b. n = 52 students c. n = 50 students d. n = 55 students	At-risk (identified by researchers)	2	Testing delivered as 7 task assignments at school in one-on-one settings. Each session limited to 45-minutes.	Change, combine, compare, equalize	n/a	MD/RD group showed greatest disadvantage in solving problems. MD and MD/RD performed lower than NA students
Hutchinson (1993)	Single-subject modified multiple baseline design	N = 20	LD identification made by district	adolescents	Resource class setting	Relational, proportion, and two-variable, two equation problems	Cognitive and metacognitive strategy instruction	Metacognitive and cognitive strategy instruction can support students with LD in solving complex algebra problems
Krawec, Huang, Montague, Kressler, & de Alba (2013)	Pretest-posttest longitudinal control group design a. learning disability (LD) b. typically achieving (TA)	N = 161 a. n = 78 b. n = 83	LD identification made by district	7th through 9th, 8th through 10th	3 days' initial instruction, then 30 minutes once weekly over the course of the school year	Not stated	*Solve It!*	Students using *Solve It!* reported greater strategy use regardless of ability, with medium effect size
Montague & Applegate (1993)	Ex post facto a. learning disability (LD) b. Average achieving (AA) c. Gifted (G)	N = 90 a. n = 30 b. n = 30 c. n = 30	LD identification made by district	6-8	Two one-on-one 55-minute sessions in a school setting	Complex, multi-step problems	n/a	Students with LD showed a lack in ability to represent math word problems compared to peers in other two groups.

Authors	Study Design	Participants	Special Education Participants	Grade	Setting Duration	Word- Problem Types	Instruction	General Outcomes
Montague, Applegate, & Marquard (1993)	Pretest-posttest control group design. a. cognitive strategies b. metacognitive strategies c. both a and b	$N = 72$ a. $n = 25$ b. $n = 23$ c. $n = 24$ (and 24 normally achieving peers for pre- posttest comparison)	LD identification made by district	7th through 9th	School setting, group instruction (8 to 12 students)	One-, two-, and three-step story problems	Cognitive and metacognitive strategy instruction	All conditions showed increases in performance, and posttest scores rivaled their NA peers on posttests. Students in the c conditions performed best.
Montague, Enders, & Dietz (2011)	Cluster randomization a. intervention group, including LD, low achieving (LA), average achieving (AA) b. comparison group, including LD, LA, and AA	$N = 779$ students a. $n = 319$ students intervention group b. $n = 460$ students comparison group	LD identification made by district	8	Three days of intensive instruction, followed by weekly practice sessions over seven months	Four basic operations using whole numbers or decimals	*Solve It!*	Intervention showed significantly greater growth across students with LD, LA, and AA over those in comparison group.
Montague, Krawec, Enders, & Dietz (2014)	Randomized control trial a. Specific learning disability (SLD) b. Low-achieving students c. Average-achieving students	$N = 1{,}059$ a. $n = 86$ b. $n = 710$ c. $n = 263$	LD identification made by district	7	3 days' initial instruction, then once weekly over the course of the school year	Not stated	*Solve It!*	Students in the intervention group grew in their mastery of solving word problems across ability groups
Rosenzweig, Krawec, & Montague (2011)	Ex post facto a. learning disabilities (LD) b. Low achieving (LA) c. Average achieving (AA)	$N = 73$ a. $n = 14$ b. $n = 34$ c. $n = 25$	LD identification made by district	8	Students recorded and instructed to "think aloud" during problem solving. Individually tested in a school setting	Four basic operations using whole numbers or decimals	n/a	LD and LA groups showed increased nonproductive metacognitive verbalizations as word problem difficulty increased.

Appendix C. IRB Application and Approval Seal

January 6, 2014

Dear Parents,

We are conducting a study involving math word problem solving. To conduct this study we need the participation of children in grades three through seven who experience some measure of difficulty in math. The attached "Permission for Child's Participation" form describes the study and asks your permission for your child to participate.

Please carefully read the attached "Permission for Child's Participation" form. It provides important information for you and your child. If you have any questions pertaining to the attached form or to the research study, please feel free to contact Lisa Morin or Dr. Silvana Watson, Responsible Project Investigator, at the numbers below.

After reviewing the attached information, please return a signed copy of the "Permission for Child's Participation" form to your child's teacher if you are willing to allow your child to participate in the study. Keep the additional copy of the form for your records. Even when you give consent, your child will be able to participate only if he/she is willing to do so. **Participation is strictly voluntary and participation can be discontinued by you at any time.**

We thank you in advance for taking the time to consider your child's participation in this study.

Sincerely,

Silvana Watson, Ph.D.
Responsible Project Investigator
Associate Professor of Special Education
Old Dominion University
Child Study Center
Norfolk, VA 23529
Office # (757) 683-6364 / fax # (757) 683-5593

PERMISSION FOR CHILD'S PARTICIPATION DOCUMENT

The purposes of this form are to provide information that may affect decisions regarding your child's participation and to record the consent of those who are willing for their child to participate in this study.

<u>PROJECT TITLE:</u> **Using Schematic-Based and Cognitive Strategy Instruction to Improve Math Word Problem Solving for Students with Math Difficulties**

<u>RESEARCHERS</u>
Silvana Watson, PhD., Responsible Project Investigator, Associate Professor of Special Education, College of Education, Communication Disorders and Special Education department.
Lisa Morin, M.S., Doctoral Student, College of Education, Department of Communication Disorders and Special Education.

<u>DESCRIPTION OF RESEARCH STUDY</u>
Several studies have been conducted looking into the subject of visual representations in math that support students in math word problem solving. Few of these studies, however, have specifically looked at one type of visual representation, bar-model drawing, and whether or not it will assist students with disabilities or struggling students' learning achievement. If you decide to allow your child to participate in this study, your child will join a study involving research of use of bar-model drawing to teach math word problem solving to students with disabilities and struggling students. The researcher will provide your child with training in the form of tutoring each school day, for 25 to 40 minutes for each session, over the course of no more than 22 weeks. Total time required for this study will be no more than 15 hours cumulatively. The researcher will provide all resources needed for the study. If you say YES, sessions will be scheduled, although some follow up communication will be required to determine how well the student maintained any skills gained in the study. Approximately ten students will be participating in this study.

<u>EXCLUSIONARY CRITERIA</u>
In order for your child to participate in this study, your child must be considered as struggling in the content area of math, as shown by a score on three different math measures that will be given by the researcher. Students not struggling in math will not be allowed to participate.

<u>RISKS AND BENEFITS</u>
RISKS: If you decide to allow your child to participate in this study, then your child may face a risk of being identified. The researcher will try to reduce these risks by removing all linking identifiers. And, as with any research, there is some possibility that you or your child may be subject to risks that have not yet been identified. Stress and anxiety due to identification as a child struggling in math, or due to math word problem solving, could cause psychological harm to your child.
BENEFITS: There are no direct benefits to participation in this study. The potential direct benefit is that your child's comprehension of math word problems may improve. This, in turn, may benefit them by improving their learning and increasing academic achievement.

<u>COSTS AND PAYMENTS</u>
The researchers want your decision about participating in this study to be absolutely voluntary. You will receive no payment to help defray incidental expenses associated with participation, such as gas or travel expenses.
The researchers are unable to give you any payment for participating in this study.

<u>NEW INFORMATION</u>
You will be contacted if new information is discovered that would reasonably change your decision about your child's participating in this study.

<u>CONFIDENTIALITY</u>
The researchers will take reasonable precautions to keep private information, such as data from assessments, confidential. The researcher will remove identifiers from the information, store information in a locked filing cabinet prior to its processing. The results of this study may be used in reports, presentations, and publications; but the researcher will not identify your child by name. Pseudonyms will be used, and any geographic indicators will be omitted from reports. Of course, your records may be subpoenaed by court order or inspected by government bodies with oversight authority.

<u>WITHDRAWAL PRIVILEGE</u>

Your child's participation in this study is completely voluntary. It is all right to refuse your child's participation. Even if you agree now, you may withdraw your child from the study at any time. In addition, your child will be given a chance to withdraw at any time if he/she so chooses.

COMPENSATION FOR ILLNESS AND INJURY

Agreeing to your child's participation does not waive any of your legal rights. However, in the event of harm arising from this study, neither Old Dominion University nor the researchers are able to give you any money, insurance coverage, free medical care, or any other compensation. In the event that your child suffers harm as a result of participation in this research project, you may contact Dr. Silvana Watson, Responsible Project Investigator, at 757-683-6364, Dr. George Maihafer, Chair of the Institutional Review Board, at (757) 683-4520, or the Old Dominion University Office of Research at 757-683-3460.

VOLUNTARY CONSENT

By signing this form, you are saying 1) that you have read this form or have had it read to you, and 2) that you are satisfied you understand this form, the research study, and its risks and benefits. The researchers will be happy to answer any questions you have about the research. If you have any questions, please feel free to contact Dr. Silvana Watson, Responsible Project Investigator, at 757-683-6364 or swatson@odu.edu, or Lisa Morin at 757-683-6360 or lmorin@odu.edu.

If at any time you feel pressured to allow your child to participate, or if you have any questions about your rights or this form, please call Dr. George Maihafer, Chair of the Institutional Review Board Chair (757-683-4520) or the Old Dominion University Office of Research (757-683-3460).

Note: By signing below, you are telling the researchers YES, that you will allow your child to participate in this study. Please keep one copy of this form for your records.

Your child's name (please print): ______________________

Your child's birth date: ______________________

Your name (please print): ______________________

Relationship to child (please check one):
Parent: ____
Guardian: ____

Your Signature: ______________________

Date: ______________________

INVESTIGATOR'S STATEMENT: I certify that this form includes all information concerning the study relevant to the protection of the rights of the participants, including the nature and purpose of this research, benefits, risks, costs, and any experimental procedures.
I have described the rights and protections afforded to human research participants and have done nothing to pressure, coerce, or falsely entice the parent to allowing this child to participate. I am available to answer the parent's questions and have encouraged him/her to ask additional questions at any time during the course of the study.

Experimenter's Signature: ______________________**, Responsible Project Investigator**

Date: ______________________

Appendix D. Letter from Principal

[redacted]

Where Children Come First!

[redacted]

24 January 2014

Dear Parents,

We have a wonderful opportunity for our school and for your child!

Ms Lisa Monn, graduate student at Old Dominion University, has received permission to engage in research at SP Morton Elementary while satisfying the requirements for her Doctor of Philosophy in education. Ms Morin proposes to identify students for targeted math intervention and tutoring for approximately eight very specific lessons. Instruction would occur after school between approximately 3:30 and 4:15 p.m.

To identify the students who would most benefit from this targeted instruction, Ms Monn needs to administer two broad assessments to a pool of third grade students:

- The Woodcock-Johnson Applied Problem Sub-test
- The Key Math Assessment

Additionally, Ms Morin would assess students' mathematical abilities using math word problems taken directly from released Virginia Standards of Learning (SOL) Assessments.

We request your permission for your child to participate in the broad assessment and, if selected, to participate in the intensive after-school tutorial. Participating parents would need to provide transportation for their child at the conclusion of each tutoring session. A specific schedule will be shared with the parents of students selected for this intervention.

Please sign and return the bottom portion of this letter giving your child permission to participate in the broad assessment.

Sincerely,

[redacted]

I give my permission for my child, ______________________________, to participate in broad assessment to determine if (s)he would benefit from intensive after-school tutorial with Ms Lisa Monn. If selected, my child may participate in the after-school tutorial program, and I will provide transportation.

______________________________   _____/_____/________

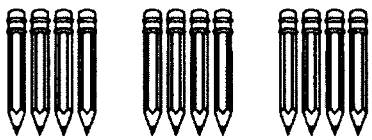
Parent Signature   Date

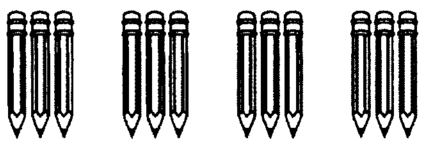
Appendix E. Sample SOL Word Problem Questions for Pre/PostTesting

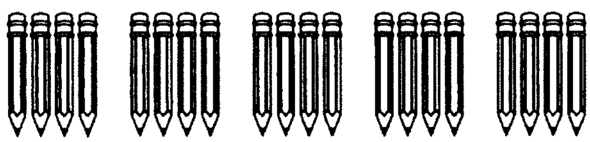
Name: ____________ **Date:** ________

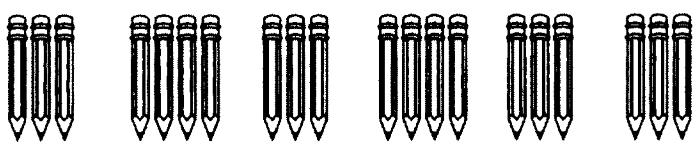
Posttest

1 **Rosa placed 20 pencils in groups of 4. Which of the following shows how Rosa placed the pencils?**

A 

B 

C 

D 

2. **Myra made 84 cupcakes for a bake sale. She put 3 chocolate candies on top of each cupcake. What was the total number of chocolate candies she used for the tops of the cupcakes?**

A 252
B 261
C 272
D 2,412

3.

Steve bought a package with 5 sheets of stickers in it. Each sheet had 32 stickers. What was the total number of stickers Steve bought?

F 37

G 160

H 180

J 1,510

4.

On Monday, 497 donuts were sold at a bakery. On Tuesday, 354 donuts were sold. What is the total number of donuts sold at the bakery on those two days?

A 43

B 143

C 741

D 851

5.

There were 12 puppies on a farm. If 8 of the puppies were brown and the rest were spotted, how many of the 12 puppies were spotted?

F 20

G 16

H 8

J 4

6.

Trey bought 4 rolls of film. Each roll could make 27 pictures. What was the total number of pictures that Trey could make with the 4 rolls he bought?

A 35

B 108

C 211

D 828

7.

An ice-cream shop used 1,287 gallons of vanilla ice cream and 956 gallons of chocolate ice cream last month. What was the total number of gallons of vanilla ice cream and chocolate ice cream sold last month?

F 331

G 1,133

H 2,243

J 10,847

8.

At a carnival, 817 tickets were sold on Monday. On Tuesday, 1,265 tickets were sold. What is the total number of tickets sold at the carnival on those two days?

F 2,082

G 1,652

H 1,072

J 448

9.

Tina bought 3 boxes of cookies. Each box had exactly 60 cookies in it. What is the total number of cookies Tina bought?

F 180

G 120

H 63

J 20

10.

Ming had 11 pencils in her pencil box. Each pencil was either yellow or red. If 8 pencils were yellow, how many red pencils were in Ming's pencil box?

F 19

G 9

H 4

J 3

11.

There are 4 tables. Chris put 6 plates on each table. What is the total number of plates Chris put on the tables?

F 2

G 10

H 18

J 24

12. Juan has 147 baseball cards and 259 football cards. How many more football cards than baseball cards does Juan have?

A 11

B 12

C 102

D 112

13.

Lorenzo and Shawn had a paper airplane contest. Lorenzo's airplane flew 20.25 feet. Shawn's airplane flew 16.50 feet. How many feet farther did Lorenzo's airplane fly than Shawn's airplane?

F 3.75

G 4.25

H 4.35

J 4.75

14.

Sam spent $3.29 for an ice cream sundae and $0.98 for a drink. What is the total amount Sam spent for the ice cream sundae and drink?

F $3.27

G $4.17

H $4.27

J $4.37

15.

Alyssa watched 3.5 hours of television last week. This week, she watched 4.7 hours of television. How many more hours did Alyssa watch television this week than last week?

A 0.2

B 0.8

C 1.2

D 8.2

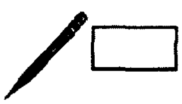
Appendix F. Bar Model Drawing Protocol

Step-by-Step Model Drawing

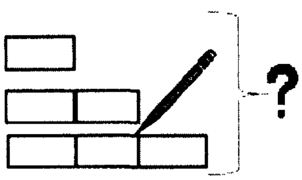
1. **Read** the entire problem aloud.

2. **Rewrite** the question in sentence form, leaving a space for the answer.

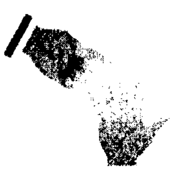
3. **Underline WHO** and/or **WHAT** is involved in the problem.



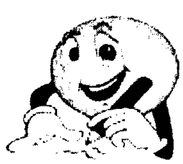
4. **Draw** the unit bar(s).



5. **Chunk** the problem and **adjust** the unit bars.



6. Correctly **compute** and solve the problem.



7. **Write** the answer in the sentence, and make sure the answer makes sense.

Appendix G. Social Validity Survey

Directions: Use the number lines below to show how much you agree or disagree with each of the statements below. Circle a number that best shows your opinion.

1. I liked learning how to draw models and solving word problems using model drawing.

1	2	3	4	5

No! I strongly disagree! ☹ I guess so... 😐 Yes! I strongly agree! ☺

2. Drawing models for word problems helps me solve the problems.

1	2	3	4	5

No! I strongly disagree! ☹ I guess so... 😐 Yes! I strongly agree! ☺

3. It was difficult for me to learn how to draw models for word problems.

1	2	3	4	5

No! I strongly disagree! ☹ I guess so... 😐 Yes! I strongly agree! ☺

4. I will draw models when I have to solve math problems in my classroom.

1	2	3	4	5

No! I strongly disagree! ☹ I guess so... 😐 Yes! I strongly agree! ☺

5. I think other kids my age should be taught how to draw models for word problems.

1	2	3	4	5

No! I strongly disagree! ☹ I guess so... 😐 Yes! I strongly agree! ☺

(Social validity measure adapted from Mahoney, 2012, p. 159.)

Appendix H. Sample Baseline Assessment

Name: ______________________________ Date: __________________

1. Olivia ate 3 cookies after lunch and 2 more cookies after dinner. How many did she eat all together?

2. Sam had 9 pencils, but he gave 2 away. How many did he have left?

3. Jeannette saw 4 snakes and 2 frogs while she was hiking. How many amphibians did she see all together?

4. Ari had 11 basketballs, while Nieco had 5 footballs. How many more balls did Ari have than Nieco?

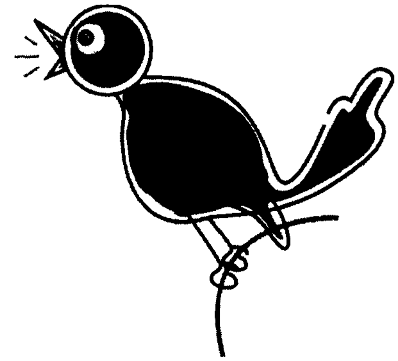
5. Sarah owned 53 fiction and 31 nonfiction books. How many books does Sarah have in all?

6. Thomas had 83 M&Ms, but he dropped 29. How many M&Ms does he have now?

7. Avery, Jackson, and Hayden each have 32 baseball cards. How many cards do they have in all?

8. Paula had 130 antique buttons in all. She had 65 metal buttons and the rest were glass. How many glass buttons did she have?

Appendix I. Sample Intervention Probe



Name: ______________________

Bar Model Drawing—Lesson Five Review

1. Daniel has 54 stamps in his collection. His uncle gives him 6 more. How many stamps does Daniel have in all?

2. There are 29 carnivores, 56 herbivores, and 24 omnivores in a large forest. How many consumers are in the forest?

3. Maola Milk Company produced 1,287 gallons of plain milk and 956 gallons of chocolate milk last week. What was the total number of gallons of milk sold last week?

4. Barry paid his bills. He paid $1,200 on rent and $481 on his car loan. How much money did he spend?

Appendix J. Sample Scripted Lesson

Lesson One Script

This is Lesson One. The first thing I would like you to do is to work the first two problems without me saying anything. They're very easy problems. So just do the first two problems as you would normally. Show your work. (Student completes first two problems of Lesson One.)*

So, these are very simple problems, but there's a way to do them with bar model drawing. And you have to learn with simple problems so that later on you'll be able to solve tougher problems with bar model drawing.

1. So we're going to follow these steps. [Teacher introduces the protocol sheet.]** The first step says read the entire problem. So go ahead with Number 3 and read the entire problem out loud. (Student reads out loud.)

2. Now the second thing is to "Rewrite the question in sentence form, leaving a space for the answer." So "Mya picked seven daisies, and then picked six more later on. How many did she pick all together?" So what does your answer need to be? Write it down at the bottom because you're going to need room. (Student responds and writes, "Mya picked _____ daisies.") Perfect. Your spelling doesn't matter.

3. Now, go back to your question and the protocol: "Determine who or what is in the problem." So, who's the *who*; what's the *what*? (Student responds, and teacher affirms, Mya and daisies are the *who* and *what*.) So now you're going to write the label for the unit bars. Yes, "Mya's daisies" is the label for the unit bars.

4. Now let's chunk the problem. What is the first chunk? "Mya picked seven daisies for her grandmother," yes, so right here you're going to put a little line that means "Stop." Very good. That's the first chunk. So now we can draw some bars to represent seven daisies. Your bars are representing Mya's daisies because that's what you're counting. And you're going to draw seven bars in a straight line. [Teacher demonstrates; student follows and draws seven bars.] The bars need to be about the same size. All your bars will look the same, no matter whether you're counting daisies or money. Okay, so now what's your next chunk? (Student responds, "Six daisies later on.") Yes, so on the same line, because we're still counting daisies, add six bars. On the same line, draw six bars.

5. Okay, now the protocol says, "Fill in the question mark." So where does a question mark need to go? (Student responds, "The question is how many daisies.")

So up here at the end of your line of bars, the question mark represents how many are there all together in that line of bars that you've drawn.

6. Okay, so now compute the problem. (Student counts bars.) That's exactly what you should do.

7. Then, "Write the answer in the sentence, making sure the answer makes sense." (Student writes "13" in her answer sentence.) Now reread the sentence and make sure your answer makes sense. (Student rereads the now completed sentence with the blank filled in: "Mya picked 13 daisies.")

Okay, very good, so you've done your first bar model drawing. So, now read the entire problem for Number Four. (Student reads.) Now rewrite the question in sentence form, leaving a space for the answer. (Student writes an answer statement, leaving a blank for the answer.) Now, who's the *who*, and what's the *what?* (Student responds, "Aleah and popsicles." Teacher affirms.) Yes, so what are you counting, actually? Are you counting *Aleahs* or *popsicles*? (Student responds, "Popsicles.") Yes, popsicles. So, what does your bar model label need to be? (Student responds, "Popsicles.") Yes, very good. Spelling doesn't matter (when student expresses concern about spelling *popsicles*). Okay, now chunk the problem. (Student chunks the problem by drawing a line and draws unit bars.) Now, where does your question mark go? Add the question mark, because when the problems get harder, the question mark will help you. (Student calculates, writes her answer in the blank in her answer statement, and reads the entire complete answer statement to make sure it makes sense.)

(Student proceeds to the next problem.) [Student is encouraged and reminded to adhere to the protocol, making sure each step is completed on each problem. Student is told there will be a test after she completes the problems in Lesson One correctly. Student is reminded of the order of work on the protocol. Teacher watches and makes verbal corrections as needed.]

This type of bar model is a discrete model, because for each one thing you're counting, you're drawing a corresponding one unit bar.

(Student takes the test for Lesson One independently.)

* Parentheses represent a response or action taken by the student.

** Brackets represent an action taken by the teacher.

Name: ________________________

Bar Model Drawing—Cumulative Mastery Check

1. It snowed for 5 days. Then it snowed for 2 more days. How many days of snow were there in all?

2. There were 9 horses on a farm. Four horses went out riding. How many horses were still in the pasture?

3. Five cats and 4 dogs live on Banks Street. How many pets live on Banks Street all together?

4. Beau has $8 and his brother has $10 to spend at the Dollar Store. How much more money does Beau's brother have than he does?

5. Holmes buys a big bag of M&Ms. He eats 87 M&Ms, and then 82 more later on. How many total M&Ms did he eat?

6. Oscar has a collection of stamps. He has 156 stamps from the United States and 224 from other countries. How many more stamps are there from other countries than the U.S.?

7. Carol purchases 5 new muffin pans for her bakery. Each muffin pan holds 10 muffins. How many muffins can she make at once?

8. There were 321 baseball fans in the stadium. 203 were Phillies fans. The rest were Mets fans. How many Mets fans were there?

Appendix L. Lesson Sequence Outline

Lesson Number	Bar Model Drawing Type	Operation(s)	Example
Lesson One	Discrete model with one variable	Addition	Olivia ate 3 cookies after lunch and 2 more cookies after dinner. How many did she eat all together?
Lesson Two	Discrete model with one variable	Subtraction	Sam had 9 pencils, but he gave 2 away. How many did he have left?
Lesson Three	Discrete model with more than one variable	Addition	Jeannette saw 4 snakes and 2 frogs while she was hiking. How many amphibians did she see all together?
Lesson Four	Discrete model with more than one variable	Subtraction	Ari had 11 basketballs, while Nieco had 5 footballs. How many more balls did Ari have than Nieco?
Lesson Five	Continuous model with one or more variables	Addition	Sarah had 53 fiction and 31 nonfiction books. How many books does Sarah have in all?
Lesson Six	Continuous model with one or more variables	Subtraction	Thomas had 83 M&Ms, but he dropped 29. How many M&Ms does he have now?
Lesson Seven	Discrete or continuous model with one or more variables	Multiplication	Avery, Jackson, and Hayden each have 32 baseball cards. How many cards do they have in all?
Lesson Eight	Part-Whole model with one or more variables	Addition and subtraction	Paula had 130 antique buttons. 65 were made of metal, and the rest were glass. How many glass buttons does she have?

Appendix M. Types of Math Word Problems Used in SBI Literature

Problem type	***Example***
Change	Three chickadees came to the bird feeder, and then some cardinals also landed on the bird feeder. Now there are 8 birds at the feeder. How many cardinals came?
Group	Khary had 5 video games; Tymele had 6. How many video games do they have if they put them all together?
Compare	There are 11 cats living on Virginia Ave. There are 8 dogs. How many more cats are there than dogs?
Multiplicative Compare	On Bald Mountain, one hiker counted 7 marmots. He counted 3 times as many picas. How many picas did he see on the mountain?
Vary	Sherita practices for the band concert for 3 hours, twice a day. How many hours does she spend practicing each day?
Part-Whole	There were 567 fans in the baseball stadium. 378 of the fans were Phillies fans. How many of the fans were routing for the visiting team, the Yankees?
Proportion	If a bat eats 1,250 bugs in 2 ½ hours, how many bugs can a bat eat in 6 hours?
Ratio	The ratio of water to lemon juice to sugar in a recipe is 5 cups : 2 cups : 1 cup. If Breanne has 6 cups of lemon juice, how much water and sugar will she need to add to make lemonade for a party?
Multistep addition and subtraction	Anthony goes shopping with his first paycheck from his first job. He buys a new pair of work boots for $45.00, some socks for $8.00, and a video game for $16.00. He gives the cashier a $100.00 bill. How much change will he receive?

Appendix N

Intervention Content Fidelity Checklist

Date: Time: Lesson:

Class time: Observer:

		Not Observed NA	Support Not Provided 0	Support provided 1
	Teacher ensures that students have supplies—protocol sheet, calculator, pencil(s). Students are reminded that the teacher will read any word problem to them upon request.			
	Evidence that student is given/completes the 4-question criterion probe			
	Teacher checks 4-question criterion probe to ensure that student received a 100%. If student did not, the former lesson will be reviewed instead of continuing on to the next lesson.			
	In the event that one student requires remediation of a lesson already taught, or another student finishes independent work early, the other student will be provided a math activity not related to bar model drawing or math word problems (e.g., hamburger fraction activities, Geoboard activities). Student requiring remediation will follow along while the teacher reviews all steps in checklist.			
	Teacher provides the appropriate lesson sheet and instructs the student(s) to complete the first two problems on the sheet on his/her/their own, showing the work.			
	Teacher discusses the work done on the first two questions completed independently by the student(s). She refers to the sequential steps in the protocol as she discusses the work completed: 1) Read the problem aloud. 2) Rewrite the question in sentence form leaving a space for the answer. 3) Underline *who* and *what* is involved in the problem. 4) Draw the unit bars. 5) Chunk the problem and adjust the unit bars. 6) Correctly compute and solve the problem. 7) Write the answer in the sentence and make sure the answer makes sense.			
Explicit Instruction Model	Unless independently completed work is exactly correct and sequentially aligned with the protocol, **teacher completes at least one question with the student(s) watching**, drawing close attention to solving the problem using the sequential steps of the protocol: 1) Read the problem aloud. 2) Rewrite the question in sentence form leaving a space for the answer. 3) Underline *who* and *what* is involved in the problem. 4) Draw the unit bars. 5) Chunk the problem and adjust the unit bars. 6) Correctly compute and solve the problem. 7) Write the answer in the sentence and make sure the answer makes sense.			

	Student(s) is/are instructed to complete at least two problems with the teacher offering immediate feedback as the student(s) work(s), following the sequential steps of the protocol: 1) Read the problem aloud. 2) Rewrite the question in sentence form leaving a space for the answer. 3) Underline *who* and *what* is involved in the problem. 4) Draw the unit bars. 5) Chunk the problem and adjust the unit bars. 6) Correctly compute and solve the problem. 7) Write the answer in the sentence and make sure the answer makes sense.			
	Student(s) is/are instructed to complete at least one problem on his/her/their own without any procedural feedback from the teacher until the problem is completed. (Teacher may provide general behavioral prompts such as "keep working"). The final product is assessed based on the sequential steps in the protocol: 1) Read the problem aloud. 2) Rewrite the question in sentence form leaving a space for the answer. 3) Underline *who* and *what* is involved in the problem. 4) Draw the unit bars. 5) Chunk the problem and adjust the unit bars. 6) Correctly compute and solve the problem. 7) Write the answer in the sentence and make sure the answer makes sense.			
	Teacher offers appropriate feedback throughout the lesson, including positive feedback.			

Appendix O. Measure of Use of Cognitive Strategies

Accurate Use of Cognitive Strategies Across Problems on Pre- and Post-Tests				
Problems	**Paraphrasing (rewrote the question as an answer statement)**	**Visualizing (constructing a bar model)**	**Hypothesizing about Problem Solutions (manipulating the bar model)**	**Checking Work (writing the answer in the previously written answer statement)**
Problem 1				
Problem 2				
Problem 3				
Problem 4				
Problem 5				
Problem 6				
Problem 7				
Problem 8				
Problem 9				
Problem 10				
Problem 11				
Problem 12				
Problem 13				
Problem 14				
Problem 15				

CURRICULUM VITAE

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EDUCATION:

2014 (projected)	Ph.D.	Old Dominion University, Norfolk, VA; Education
2005	M.S. Ed.	Old Dominion University, Norfolk, VA; Special Education
2004	B.S.	Old Dominion University, Norfolk, VA; Interdisciplinary Studies with a concentration in Early Childhood Education and a minor in Special Education

EXPERIENCE:

Academic Experience:

Graduate Teaching Assistant, Old Dominion University: Department of Communication Disorders and Special Education. Norfolk, VA. Spring/Summer 2012. Adjunct instructor for two semesters of *Students with Diverse Learning Needs in the General Education Classroom* (SPED 406). Spring semester, 33 students; summer semester, 25 students.

Classroom Experience:

Special Education Teacher, special project, Franklin High School. Franklin, VA. September 2013 to March 2014. Worked with students to complete the Virginia Substitute Evaluation Program (VSEP), an alternative assessment to high-stakes English testing required for graduation. Instruction focused on strategies for improving reading comprehension and vocabulary building.

General Education Teacher, Inclusive Classroom, Robertson Elementary School, Suffolk Public Schools. Suffolk, VA. September 2008-June 2011. Produced and implemented lesson plans designed for students preparing for high-stakes testing, employing differentiated teaching strategies to address each student's learning style and ability, within a Response-to-Intervention (RTI) model.

Special Education Teacher, Inclusive and Self-Contained Classrooms, J.P. King Middle School, Franklin City Public Schools. Franklin, VA. January 2005-June 2008. Created and executed lesson plans designed for students preparing for high-stakes testing, as well as alternative assessments. Developed and administered Virginia Grade Level Alternative (VGLA) assessment portfolios. Composed and oversaw the implementation of Individualized Education Plans (IEPs) that effectively addressed student needs and established goals to support and enhance student achievement. Participated as a team leader in the district-wide Reading First Initiative.

PUBLICATIONS:

Bobzien, J., Richels, C., Raver, S. A., Hester, P., Browning, E., & Morin, L. (2012). An observational study of social communication skills in eight preschoolers with and without hearing loss during cooperative play. *Early Childhood Education Journal*, 1-8. doi: 10.1007/s10643-012-0561-6

PROFESSIONAL PRESENTATIONS:

Watson, S., Morin, L. & Raymer, A. The importance of phonological interventions for older students. Division of International Special Education & Services, Council for Exceptional Children; Braga, Portugal, July 15, 2014.

Morin, L. & Agrawal, J. Evidenced-based strategies to teach mathematical problem solving to students with LD. Council for Exceptional Children; Philadelphia, PA, April 10, 2014.

Watson, S., & Morin, L. A synthesis of reading interventions for older students with dyslexia-type learning disability. Council on Learning Disabilities; Austin, TX, October 24, 2013.

Agrawal, J., & Morin, L. I can manipulate; I can draw. Look, I got it!—Using the concrete-representational-abstract sequence to support students in math. Council on Learning Disabilities; Austin, TX, October 25, 2013.

Morin, L., & Browning, E. Investigating a content-enhancement device that supports math word problem solving. Council for Exceptional Children; San Antonio, TX, April 6, 2013.

Morin, L., & Browning, E. Investigating content enhancement devices that support math word problem solving: A pilot study. Conference on Learning Disabilities; Austin, TX, October 10, 2012.

Watson, S., Gable, R., Cho, D., Reid, L., & Morin, L. The role of attention and working memory in the learning and teaching process. Council for Exceptional Children; Denver, CO, April 14, 2012.

Morin, L. Schematic representations for students solving math word problems. Virginia Council for Learning Disabilities; Harrisonburg, VA, March 24, 2012.

COURSES TAUGHT:

SPED 406, Students with Diverse Learning Needs in the General Education Classroom

AWARDS:

2010 Robertson Elementary School Teacher of the Year, Suffolk, VA

2007 J. P. King Middle School Teacher of the Year, Franklin, VA

2004 Outstanding Scholar Award, Old Dominion University, Norfolk, VA

CERTIFICATION AND LICENSURE:

K-12 Specific Learning Disabilities
K-12 Emotional Disabilities
PreK-6 Elementary Education
6-8 Middle Ed. English
6-8 Middle Ed. History/Social Studies
6-8 Middle Ed. Mathematics
6-8 Middle Ed. Science
Wilson Reading System Certification

PROFESSIONAL SERVICE:

Membership in Professional Organizations:

2011 – present Council for Exceptional Children (CEC)

2011 – present CEC Division on Learning Disabilities

2011 – present Council for Learning Disabilities (CLD)

2011 – present Virginia Council for Learning Disabilities

Service in Professional Organizations:

2012 – present CLD Technology Committee co-chair

2012 – present CLD Leadership Academy cohort leader

University Service:

2012 – 2013 Data Manager, Child Study Research Team, Old Dominion University, Norfolk, VA. Managed data from research conducted in cooperation with an oral preschool program, investigating best practices for promoting social and literacy development for preschool-aged students who are deaf.

2011 – 2012 Member, Child Study Research Team, Old Dominion University, Norfolk, VA. Participated in research design and data analysis to explore and expand best practices for promoting academic growth for preschool students who are deaf or hard of hearing.