MODULAR MODELING FOR LARGE SCALE CANAL NETWORKS

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Abstract: Irrigation and drainage canals are some examples of water conveyance systems spread worldwide. These systems are characterized by transport phenomena and strong coupling having a relevant social and economic impact in society. The management of water networks is a complex task due to the large scale and uncertainty. This paper purposes a modular modeling framework based on canal and junction models to construct different water network topologies as irrigation and drainage networks. The framework presented is a valuable resource for studying the behavior of canal networks, first in a development stage and later for determining the benefits of introducing control algorithms to increase service level performance.

Keywords: Modeling, Large Scale Systems, Canal Networks, Unsteady Flow.

1. INTRODUCTION

Water is a vital resource for life on earth. Mankind way of life is based on water consumption: industry, agricultural and domestic activities. The use of such resource with efficiency is absolutely vital not to compromise future generations. For example agricultural has a great impact in water consumption and in respect to Portugal 81.8% of the available water is used for irrigation (Raposo, 1996). As water is not always available near the end consumers it is conveyed by a network of open canals. The objective of these facilities is to make water available to farmers while minimizing losses. The canal losses can be caused by bad network management due to oversupply which can cause spillage along the canal and outflows at the end of the networks system. Drainage networks like rivers and sewers must assure the necessary flow to prevent floods. Mathematical models that are able to simulate water networks behavior is an important supporting tool to improve water management.

Hydraulic simulation models are useful for studying flow routing in canal networks. The flow dynamics in canals is well described by the Saint-Venant equations (Akan, 2006), one dimensional nonlinear partial differential equations of hyperbolic type capable of describing the transport phenomenon. The Saint-Venant equations consist on mass and momentum conservation equations. Many hydraulic simulation models have been developed to study the flow behavior in canal networks based on numerical methods (Szymkiewicz, 2010) as finite difference (Akan

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and Yen, 1981) (Nguyen and Kawano, 1995) or finite elements.

This paper uses a linear model based on the linearization and discretization of the Saint-Venant equations for capturing the canal dynamics (Litrico and Fromion, 2009). The model has the interesting ability to accept either water depth of flow boundary conditions (Nabais *et al.*, 2011). The connecting elements can be either hydraulic structures, imposing flow boundary conditions, or reservoir type, imposing water depth boundary conditions. The modeling framework presented has the following features,

- a modular and flexible structure that keeps large scale systems tractable;
- it can represent different network topologies like irrigation and drainage systems;
- is a valuable resource for studying canal networks behavior and support control strategies;
- different problems of water management can be studied, namely the farmers request, the drainage after rainfall, tides impact or flood prediction.

In section 2 the structural elements present in canal networks are modeled, the canal and junctions. The canal networks used in this work are presented in section 3. Due to the strong coupling existing in canal networks the steady state algorithm for both networks are also presented. The framework accuracy is studied in section 4. First the convergence to different steady state configurations is analyzed then illustrative scenarios are tested for the irrigation and drainage networks. In section 5 final comments are drawn.

2. MODELING CANAL NETWORKS

Water conveyance networks are complex systems usually space distributed with a large dimension. Like other network systems they are composed by links and nodes. The link between nodes is accomplish by the water transportation element – the canal. The nodes establish the separation of different links and are represented by reservoir, gates or a combination of both.

2.1 *Canal Pool Models*

The flow dynamics in open canals is well described by the Saint-Venant equations (Akan, 2006), nonlinear partial differential equations of hyperbolic type capable of describing the transport phenomenon,

$$\frac{\partial Q(x,t)}{\partial x} + T(x,t)\frac{\partial Y(x,t)}{\partial t} = 0 \quad (1)$$

$$\frac{\partial Q(x,t)}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2(x,t)}{A(x,t)}\right) + \dots$$
$$\dots + g \cdot A(x,t) \cdot (S_f(x,t) - S_0(x)) = 0 \quad (2)$$

where $A(x,t)$ is the wetted cross section, $Q(x,t)$ is the water flow, $Y(x,t)$ is the water depth, $T(x,t)$ is the wetted cross section top width, $S_f(x,t)$ is the friction slope, $S_0(x)$ is the bed slope, x and t are the independent variables. One approach to this complex problem is to linearize (1) (2) around a nonuniform steady configuration defined by $(Q_0, Y_0(x))$. The Preissmman scheme is the numerical method used to discretize the linearized Saint-Venant equations into a finite linear model (Nabais and Botto, 2011).

To complete the model it is necessary to add boundary conditions. As the flow is considered subcritical one boundary condition for each end is introduced (Nabais *et al.*, 2011). In case of flow boundary condition, when the pool is connected to an hydraulic structure, as a gate or a pump, the model input is the flow. A similar approach is done for the water depth boundary condition, when the pool is connected to a reservoir or to another pool, the model input is the water depth. The pool linear model is given as,

$$\begin{aligned}\mathbf{X}(k+1) &= A\mathbf{X}(k) + B\mathbf{U}(k) + B_w\mathbf{W}(k,k-1)\\ \mathbf{Y}(k) &= C\mathbf{X}(k) + D_v\mathbf{V}(k)\end{aligned} \quad (3)$$

where k stands for time iteration, $\mathbf{X}(k)$ is the state-space vector, $\mathbf{Y}(k)$ is the output considered usually as water depths along the pool, $\mathbf{U}(k)$ is the model input that depends on the boundary conditions the pool is subjected to, $\mathbf{W}(k,k-1)$ is the state-space flow disturbance that accounts for lateral outflows/inflows along the pool, $\mathbf{V}(k)$ is the output disturbance considering noise measurements and A, B, B_ω, C and D_v are the state space matrices.

2.2 *Modeling Junctions*

Hydraulic conditions at a junction may be described by equations of mass and energy conservation. Assuming no change in storage volume within the junction the continuity equation at a junction formed by the parent canal i and the branches j and k can be written as,

$$Q_i = Q_j + Q_k \quad (4)$$

and when the flows in all branches joining at the junction are subcritical, the equation of energy conservation can be approximated by he kinematic compatibility condition (Akan and Yen, 1981),

$$Y_{id} = Y_{ju} = Y_{ku} \quad (5)$$

where Y_* stands for water depth and subscript u and d means upstream and downstream respectively. In irrigation canals subjected to control is common to find gates connecting canals. In this case the energy constraint (5) is replaced by the respective gate equation for computing the flow across the gate. This flow is the boundary condition for the connected pools.

3. CANAL NETWORKS

3.1 *Description*

For illustration purposes two different canal networks topologies (Adlul Islam and Sen, 2005) will be tested,

Drainage Network: composed by 14 pools and 14 nodes containing a loop, Fig. 1, with a total length of 29300m and a nominal flow of 70m^3/s;

Irrigation Network: composed by 41 pools and 42 nodes, Fig. 2, with a total length of 43500m and a nominal flow of 40m^3/s.

In Table 1–2 canal networks parameters are presented were L is the pool length, B is the cross section bed width, m is the cross section lateral slope $1 : m$, S_0 is the canal bed slope, n is the Manning roughness coefficient and N is the number of reaches by canal.

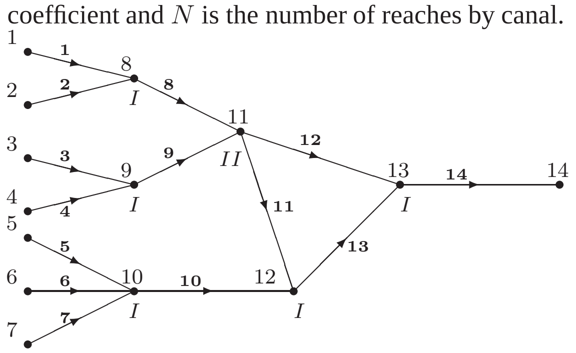


Fig. 1. Drainage network.

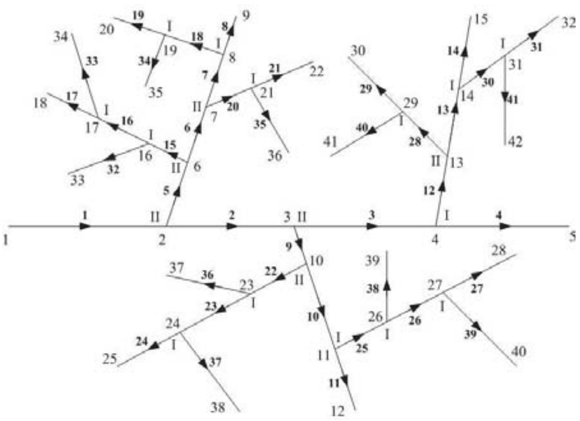


Fig. 2. Irrigation network (canal numbers are in bold).

3.2 *Steady State*

In a steady configuration, with no time derivatives, the Saint Venant equations becomes (Litrico and Fromion, 2009),

$$\begin{aligned} \frac{dQ(x)}{dx} &= 0 \\ \frac{dY(x)}{dx} &= \frac{S_0 - S_f(x)}{1 - F^2(x)} \end{aligned} \tag{6}$$

Table 1. Drainage network parameters.

Pool	L [m]	B [m]	m	S_0	n	N
1	1500	10	1	0.00027	0.022	20
2	1500	10	1	0.00027	0.022	20
3	3000	10	1	0.00047	0.025	40
4	3000	10	1	0.00047	0.025	40
5	2000	10	1	0.00030	0.022	25
6	2000	10	1	0.00030	0.022	25
7	2000	10	1	0.00030	0.022	25
8	1500	10	1	0.00027	0.022	18
9	1500	10	1	0.00027	0.022	18
10	2000	10	1	0.00030	0.022	22
11	1200	10	0	0.00033	0.022	14
12	3600	20	0	0.00025	0.022	38
13	2000	30	0	0.00025	0.022	21
14	2500	40	0	0.00016	0.022	25

Table 2. Irrigation network parameters.

Pool	L [m]	B [m]	m	S_0	n	N
1	2500	10.00	2.0	0.00013	0.015	22
2	2000	8.50	2.0	0.00015	0.016	20
3	1700	7.00	2.0	0.00016	0.017	18
4	1500	5.00	2.0	0.00017	0.018	16
5	1500	5.00	2.0	0.00020	0.020	16
6	1400	4.00	2.0	0.00021	0.020	16
7	1200	3.00	2.0	0.00022	0.020	15
8	1000	2.00	2.0	0.00024	0.022	13
9	1400	3.50	1.0	0.00025	0.022	15
10	1200	2.70	1.0	0.00022	0.022	15
11	1000	1.75	2.0	0.00024	0.022	15
12	1300	2.50	2.0	0.00022	0.022	16
13	1200	1.50	1.0	0.00025	0.022	15
14	1000	1.00	2.0	0.00022	0.022	17
15,18	1000	1.50	2.0	0.00024	0.022	13
16,21	1000	1.00	1.0	0.00025	0.022	13
17,26	1000	1.75	2.0	0.00024	0.022	15
19	900	0.90	0.9	0.00025	0.022	12
20,23	1100	1.50	2.0	0.00024	0.022	16
22	1200	1.75	2.0	0.00024	0.022	16
24	1000	1.00	1.0	0.00025	0.025	14
25	1200	2.00	2.0	0.00024	0.020	18
27	900	1.50	2.0	0.00024	0.022	14
28	900	1.50	1.0	0.00025	0.022	12
29	800	1.00	1.0	0.00025	0.022	11
30	800	1.25	2.0	0.00024	0.022	13
31	700	0.75	2.0	0.00024	0.022	12
32–41	700	0.50	1.0	0.00050	0.030	10

which corresponds to the gradually varied flow where F is the Froude number $F = \frac{V}{C}$ with $C = \sqrt{g\frac{A}{T}}$. The backwater $Y(x)$ can be obtained from (6) as long the nominal flow and downstream water level are given. Equation (6) is usually solved using numerical methods, e.g. Newton-Raphson method.

The network steady state configuration has to be determined from the known boundary conditions. Typically the upstream inflow and downstream water depths are known for the entire network. The complexity of this task depends on the network configuration. For a single canal the problem is solved straightforward from downstream to upstream intercalating the pool backwater computation (6) with the mass (4) and energy (5) conservation equations. For drainage networks the problem is similar as flow in each canal is known due to the network convergent nature. Iteration procedure is needed if some loop is present in the network. That is the case for the considered drainage network. The irrigation network steady state is the most challenging as the flow along the network is unknown and the solution is achieved through a complex iterative procedure.

The backwater computation in a single canal is classified into one of two categories,

Initial Value Problem (IVP): refers to the solution of (6) from the specified $(Q_0, Y_0(L))$;

Boundary Value Problem (BVP): refers to the solution of (6) from specified upstream and downstream water levels $(Y_0(0), Y_0(L))$. The shooting method can be used to overcome this problem. Using this method the BVP is solved as an IVP with iterations until the upstream water level is inside a predefined tolerance. The flow update Q^{k+1} is done using a simple extrapolation,

$$Q^{k+1} = Q^k + \frac{Y_0(0) - Y^k(0)}{Y^{k-1}(0) - Y^k(0)} \left(Q^{k-1} - Q^k\right) \tag{7}$$

The network nodes can be classified as type I or type II. Type I node requires the solution of a BVP after the determination of a IVP. Type II node means the BVP requires the solution of a group of canals. The Boundary Value Problem for a Group of Canals (BVPGC) starts with the solution of a IVP for a given canal that defines the upstream water depth. This value will be used to solve the BVP of the canal sharing the same upstream node. After the solution of the BVP the canal flow is determined and by continuity conservation the node inflow is computed. Then the IVP can be applied to the node upstream canal. The procedure continues until a node of type II is found, where typically the energy equation should be verified. The node classification is uniquely determined across the network and plays an important role in terms of computation efficiency. Accordingly to (B. J. Naidu and Narasimhan, 1997) the best path of marching should be determined before starting computations in particular the starting node. Parameters as the number of canals in the network, number of reaches into a canal is divided, the number of type II nodes and the number of loops in the algorithm for computing the solution affects the computational effort. For a given node the number of nodes of type I and II on the right and left side are counted. The solution should start from the side with the higher number of type II nodes, and in case of draw the side with more type I nodes should be chosen. The algorithm for computing the steady state is given by Algorithm 1 and 2 for the drainage and irrigation network, where E.E. means energy equation and C.E. means mass conservation equation. The boundary conditions are presented in Table 3–4. The drainage network steady state configuration is presented in Table 5. The irrigation network

Algorithm 1 Drainage network steady state

Solve C.E. for all nodes to set Q_i
IVP for canal [14]
repeat
    Assume Q_{11}
    Apply C.E. for node [11] and [12]
    IVP for canal [13]
    IVP for canal [11]
    IVP for canal [12]
until E.E. is verified at node 11
IVP for canal [8] and [9]
IVP for canal [10]
IVP for canal [1] and [2]
IVP for canal [3] and [4]
IVP for canal [5,6] and [7]

Algorithm 2 Irrigation network steady state

repeat
    Assume Q_{28}
    BVPGC for canals [27,39,26,38,25,11,10]
    repeat
        Assume Q_{25}
        BVPGC for canals [24,37,23,36,22]
    until E.E. is verified at node 10
    IVP for canal [9]
    repeat
        Assume Q_{32}
        BVPGC for canals [31,41,30,14,13]
        repeat
            Assume Q_{30}
            BVPGC for canals [29,40,28]
        until E.E. is verified at node 13
        BVPGC for canals [12,4,3]
    until E.E. is verified at node 3
    IVP for canal [2]
    repeat
        Assume Q_{20}
        BVPGC for canals [19,34,18,8,7]
        repeat
            Assume Q_{22}
            BVPGC for canals [21,35,20]
        until E.E. is verified at node 7
        IVP for canal [6]
        repeat
            Assume Q_{18}
            BVPGC for canals [17,33,15]
        until E.E. is verified at node 6
    until E.E. is verified at node 6
    IVP for canal [5]
until C.E. is verified at node 2
IVP for canal [5]

initial conditions can be consulted in (Adlul Islam and Sen, 2005).

4. SIMULATION RESULTS

Using elementary blocks for canals and junctions a simulator can be constructed for each network (Nabais

Table 3. Drainage network boundary conditions.

Node	Flow [m^3/s]	Node	Flow [m^3/s]	Level [m]
1	10.0	5	10.0	–
2	10.0	6	10.0	–
3	10.0	7	10.0	–
4	10.0	14	–	2.5

Table 4. Irrigation network boundary conditions.

Node	Flow [m^3/s]	Level [m]	Node	Level [m]
1	40.0	–	32	1.0749
5	–	0.9111	33	1.4777
9	–	1.6559	34	1.7107
12	–	0.9759	35	2.0070
15	–	0.9127	36	1.7769
18	–	1.8784	37	1.2190
20	–	1.6026	38	1.4745
22	–	1.6729	39	1.3719
25	–	1.3622	40	1.6091
28	–	1.4766	41	1.3310
30	–	1.1741	42	1.2535

Table 5. Drainage network steady state.

Pool	Flow [m^3/s]	Upstream [m]	Downstream [m]
1	10.0000	1.5870	1.8773
2	10.0000	1.5870	1.8773
3	10.0000	1.1393	1.8773
4	10.0000	1.1393	1.8773
5	10.0000	1.7360	2.2392
6	10.0000	1.7360	2.2392
7	10.0000	1.7360	2.2392
8	20.0000	1.8773	1.9525
9	20.0000	1.8773	1.9525
10	30.0000	2.2392	2.2713
11	10.1710	1.9525	2.2713
12	29.8290	1.9525	2.4849
13	40.1710	2.2713	2.4849
14	70.0000	2.4849	2.5000

et al., 2011). The simulator allows water depths and flows monitoring along network nodes and in each individual canal. In particular it is possible to analyze how perturbations propagate along the network. In a first stage the simulator accuracy for reaching a new steady state is analyzed for both networks. The comparison is made with the solution obtained by Algorithm 1 and 2 for new boundary conditions. In a second stage a physical scenario is tested for each network.

4.1 *Steady State Analysis*

The simulator accuracy is tested for both networks for different steady state configurations. Starting from the initial steady state a step flow is applied changing the upstream boundary condition. In the drainage network the boundary changed from 10m^3/s to $\begin{bmatrix} 12 & 14 & 16 \end{bmatrix}$m^3/s while for the irrigation network the boundary changed from 40m^3/s to $\begin{bmatrix} 45 & 50 & 55 \end{bmatrix}$m^3/s. The drainage network suffers a maximum flow deviation of 60% while for the irrigation network a 37.5% deviation is imposed. The simulator accuracy in converging to the new steady state configuration is evaluated for each canal in respect to the nominal flow, upstream water depth and downstream water depth. In Table 6–7 the simulator accuracy is presented using the following error criteria: Mean Absolute Error (MAE), Maximum Absolute Error (MXAE), Mean Absolute Relative Error (MARE) and Maximum Relative Error (MXRE).

The simulator ability to converge to the new final steady state is confirmed by the low Mean Absolute Relative Error. Only for the drainage network with a boundary flow deviation of 60% the mean relative error rises above 1% in water depths. Maximum nominal error values grow in respect to the boundary flow deviation and are bellow 70mm in water depth for all tested scenarios.

Table 6. Simulator accuracy for the drainage network.

12m^3/s	MAE	MXAE	MARE	MXRE
Flow	0.0054	0.0238	0.0002	0.0016
Upstream	0.0050	0.0082	0.0025	0.0042
Downstream	0.0045	0.0082	0.0020	0.0034
14m^3/s	MAE	MXAE	MARE	MXRE
Flow	0.0163	0.0747	0.0006	0.0050
Upstream	0.0190	0.0310	0.0087	0.0143
Downstream	0.0170	0.0310	0.0069	0.0116
16m^3/s	MAE	MXAE	MARE	MXRE
Flow	0.0323	0.1490	0.0010	0.0091
Upstream	0.0406	0.0656	0.0172	0.0277
Downstream	0.0362	0.0656	0.0136	0.0229

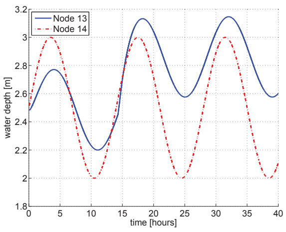
Table 7. Simulator accuracy for the irrigation network.

45m^3/s	MAE	MXAE	MARE	MXRE
Flow	0.0018	0.0059	0.0008	0.0028
Upstream	0.0011	0.0043	0.0007	0.0019
Downstream	0.0005	0.0036	0.0000	0.0018
50m^3/s	MAE	MXAE	MARE	MXRE
Flow	0.0063	0.0237	0.0019	0.0062
Upstream	0.0047	0.0168	0.0028	0.0069
Downstream	0.0021	0.0141	0.0013	0.0068
55m^3/s	MAE	MXAE	MARE	MXRE
Flow	0.0130	0.0558	0.0033	0.0095
Upstream	0.0105	0.0362	0.0060	0.0141
Downstream	0.0048	0.0304	0.0028	0.0140

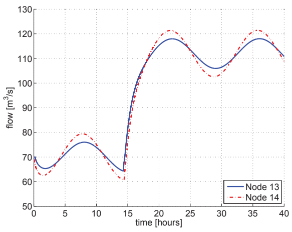
4.2 *Unsteady Flow Simulations*

Two different scenarios will be tested: the tides impact plus a rainfall scenario for the drainage network and an upstream flow disturbance propagation along the irrigation network.

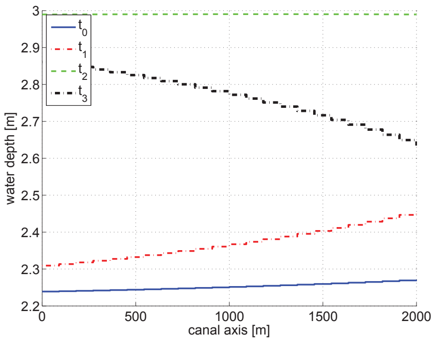
4.2.1. *Flood Prediction on Drainage Networks* For the drainage scenario two physical meaningful situations are tested. The ability to drain stormwater can be analyzed by raising the upstream boundary condition (60% on the network inflow for nodes 1 to 7) and the tides impact in the network can be imposed at the downstream boundary, modeled as a sine wave of amplitude 0.5m (relative deviation of 40%) and period 14 hours. In Fig. 3(a)–3(b) the variations of water depth and flow for nodes 13 and 14 are presented. The water depth along canal axis for pool 10 is presented in Fig. 3(c) for different situations: the steady state t_0, high tide t_1, high tide with rainfall t_2 and low tide with rainfall t_3.



(a) Stage hydrograph.



(b) Flow hydrograph.



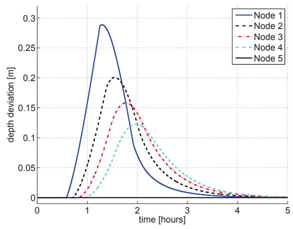
(c) Backwater for canal 10.

Fig. 3. Flood prediction under drainage network.

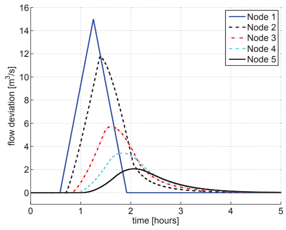
4.2.2. *Flow Disturbance on Irrigation Networks* The irrigation network upstream boundary condition is disturbed according to a triangle wave with a maximum flow deviation of 15m^3/s. The impact propagation can be studied looking into the flow and water depth variables along the network. In Fig. 4 the propagation effect is represented both in water depth and flow deviation for two shortest paths: from node 1 to node 5 (primary canals) and from node 2 to node 9 (secondary canals). It is clear the advection and dispersion effects on the disturbance propagation along the network.

5. CONCLUSIONS

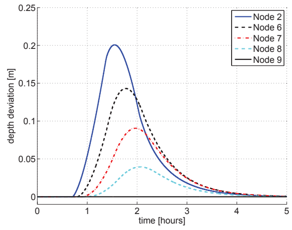
A modular modeling framework for channel network was presented. Different network configurations can be easily constructed by interconnecting the elementary components modeling canals and junctions. For illustrative purpose two different topologies, irrigation and drainage, were constructed without any hydraulic structure which is a challenge. The energy equation is used to impose water depth boundary conditions. The framework proved to be accurate in reaching new steady state configurations. For these case studies the response to boundary disturbance was analyzed corresponding to an increase in water demand by farmers or flood prediction due to the interaction between tides and rainfall drainage.



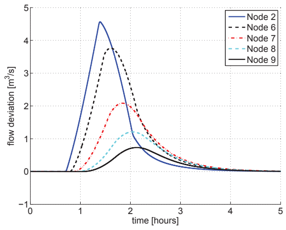
(a) Water depth hydrograph.



(b) Flow hydrograph.



(c) Water depth hydrograph.



(d) Flow hydrograph.

Fig. 4. Irrigation network hydrographs along the shortest path between nodes 1 to node 9 and for node 2 to node 9 for a maximum peak flow of 55m^3/s.

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