Automatic Balancing Mechanisms for Mixed Pension Systems under Different Investment Strategies

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Abstract

State pension systems are usually pay-as-you-go financed, i.e. current contributions cover pension expenditure. However, some countries combine funding and pay-as-you-go (PAYG) elements within the first pillar. The aim of this paper is twofold. First, using nonlinear optimisation based on Godínez-Olivares, Boado-Penas, and Haberman (2016), it seeks to assess the impact of a compulsory funded defined contribution (DC) pension scheme that complements the traditional defined benefit (DB) PAYG on the level of pension benefits. Future expected returns for both the funded part and the buffer fund of the PAYG are simulated through the non-overlapping block bootstrap technique. Second, in the case of a partial financial sustainability, we design different optimal strategies, that involve variables such as the contribution rate, age of retirement and indexation of pensions, to restore the long-term financial equilibrium of the system. We show that the adjustments needed to ensure sustainability for the mixed pension systems are less severe that the pure DB PAYG but the total replacement rate for the former is lower in most of the cases studied. When calculating the return that the individuals would receive, we prove that some cohorts are better off under a mixed pension system.

Keywords: Investment allocation, Optimisation, Pay-as-you-go, Public pensions, Risk, Simulation, Sustainability

JEL classification: E62, G17, H55, J11, J26

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1. Introduction

Pay-as-you-go (PAYG) pension schemes require a balance between income from contributions and pension spending as the revenue from current contributions is used directly to pay for current pensioners. The decline in fertility rates, the increase in longevity and the current forecasts for the ageing of the baby-boom generation all point to a substantial increase in the age dependency ratio, and this will raise serious concerns for the sustainability of PAYG pension systems. In particular, the life expectancy at birth is expected to increase by 7.8 for males and 6.6 years for females in 2010 comparing with 2070 (European Commission 2018). This is a worldwide problem, and consequently, many European countries (European Commission 2010, 2012) have already carried out some parametric reforms, or even structural reforms, of their pension systems.

In Europe, the common trend of the pension crisis is a wave of parametric adjustments including countries, among others, France, Greece, Hungary, Romania and Spain (see Whitehouse 2009a, 2009b) and OECD (2011, 2012, 2013). Among the major changes in pension reform is the introduction of what is known as Notional Defined Contribution pension schemes (NDCs), first developed about twenty years ago in countries such as Italy, Latvia, Poland and Sweden. NDCs aim at reproducing the logic of a funded defined contribution plan - (accumulated contributions are converted into a life pension annuity taking into account life expectancy according to standard actuarial practice) – albeit, under a pay-as-you-go framework.[5]

In Latin America, for most of their history, social protection systems have been based on a PAYG finance (Rofman, Apella and Vezza 2015). However, the pressures on their finances forced them to implement several reforms. Since the 1980's, most of the countries in the region made structural reforms replacing completely or partially their PAYG system with programs containing a fully funded component of individually capitalized accounts (Rofman, Apella and Vezza 2015; Titelman, Vera and Pérez-Caldentey 2009). The reforms began in Chile in 1981 and after 1993 expanded to various countries in the region. Chile exchanged its traditional system of defined benefit (DB) PAYG social protection for one of individually funded accounts to pay for retirement. As a result, a transfer of financial market and volatility risk from the state to the individual happened.[6]

The PAYG rate of return can be lower than the rate of return of funding schemes, especially in countries where the working population is not growing. In this case, the individual might consider that there is an implicit cost equivalent to the difference in return; see Robalino and Bodor (2009) and Valdés-Prieto (2005) for taxes implicit to PAYG schemes.

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[5] Pension contributions throughout the participant's working life are accumulated in a fictive or notional account where the return that contributions earn is calculated on the basis of a macroeconomic index that tries to reflect the financial health of the system. Pension accounts are called notional because there is no pot of pension fund money, just a series of individual claims on future public budget. Interested readers in NDCs can consult, for example, Lindbeck and Persson (2003), Williamson (2004), Börsch-Supan (2006), Holzmann and Palmer (2006), Vidal-Meliá, Domínguez-Fabián, and Devesa-Carpio (2006), Auerbach and Lee (2009), Vidal-Meliá, Boado-Penas, and Settergren (2010), Whitehouse (2010), Auerbach and Lee (2011), Chłón-Dominczak, Franco, and Palmer (2012), Holzmann, Palmer and Robalino (2012) and Alonso-García, Boado-Penas and Devolder (2018b). Other countries, including Egypt, China and Greece are seriously considering the introduction of NDCs (Holzmann, Palmer and Robalino (2012).

[6] At the same time with the aim of reducing political opposition and increasing interest in the new system, OECD (1998), contribution rates were set a low level enough to increase the net take-home pay.

However, the high-variability of the funding rate of return makes the choice between PAYG and funding less obvious and there might be advantages of mixing PAYG and funded schemes (De Menil, Murtin, and Sheshinski 2006 and Persson 2002). Also, PAYG is a useful social security financing technique which ensures income redistribution at both the inter and intra generational levels. On the other hand, Fajnzylber and Robalino (2012) state that the transition from a PAYG to a fully funded scheme has a high transition cost where current contributors pay twice: first to finance their own retirement capital and second to finance pension benefits of current retirees.

Currently, countries, such as Australia, Canada, Norway, Sweden, Latvia and Poland, amongst others, combine funded and PAYG elements within a compulsory basic pension system.[7] In particular, Sweden allocated 86.5% of the pension contributions to PAYG whereas Latvia and Poland allocate 70% and 62.6% respectively. The remaining part accrues funded pension rights and earns the market rate of return. These mixed systems have been advocated, particularly by the World Bank, as a practical way to reconcile the higher financial market returns compared with GDP growth with the costs of a scheme with a greater funded element.

With this in mind, the aim of this research is to assess both sustainability and adequacy for different types of pension systems. In the case of partial financial sustainability, we design different optimal strategies – automatic balancing mechanisms-, that involve variables such as the contribution rate, age of retirement and indexation of pensions, to restore the long-term financial equilibrium of the system. These mechanisms negatively affect the adequacy of the system in an ageing world. To counteract this, different types of mixed pension systems - with different exposure to equities- are proposed. Finally, we analyse the implications of the mixed systems on the return that the individual would receive.

We acknowledge that there are 2 limitations to our research methodology. The first limitation concerns the exogenous nature of the funded defined contribution (DC) pension system. Our main objective in this paper is to analyse whether mixing the classical DB PAYG and funding systems could lead to higher or lower replacement rates for pensioners and the consequences for the sustainability of the pension system. Thus, we assume the existence of an exogenous funded DC pension system in addition to the current DB PAYG scheme where the DC is superposed on the optimal path of the automatic balancing mechanism (ABM) of the DB scheme. We show how the inclusion of a DC scheme can affect the replacement rate, and other key financial measures, using a fixed contribution rate to the DC scheme. In this approach, we are working with real-world data which involves taking the real forecast demographic structure of the European population, the real salary structure, real initial age of retirement, contribution rate and indexation of pensions over 75 years. If we were to include of both the DC and DB elements in a single mathematical problem, the multi-stage stochastic optimization would become computationally very challenging. We are leaving this extension for future consideration.

The second limitation concerns our use of a complicated stochastic model for the financial risks facing the fund but no consideration of demographic uncertainty. We have taken this route on grounds of simplicity while recognising that the introduction of the

[7] See Chłón-Dominczak, Franco, and Palmer (2012) and Könberg, Palmer and Sunden (2006). In Australia, OECD (2017), the funded element comes from the superannuation guarantee that is not a part of the first pillar. However, it consists of a mandatory employer contribution for workers earning more than AUD 450 a month.

demographic uncertainty would be very valuable to compute, for example, in terms of the optimal path of the key variables. In further research, we will introduce demographic and other risks and a multi-stage stochastic optimization problem.

Following this introduction, the next section of the paper describes the methodology to calculate the optimal path of the decision variables to restore the financial sustainability together with the replacement rates for mixed pension systems. The procedure to follow to calculate future expected returns is also described in detail. Section 3 presents the data for a representative example together with the results and some sensitivity analysis under different investment strategies. Section 4 concludes and makes suggestions for further research.

2. Methodology to calculate the optimal path of the variables and replacement rates for mixed pension systems

This section describes first the optimisation model developed by Godínez-Olivares, Boado-Penas, and Haberman (2016) to identify the optimal paths of the key variables – contribution rate, retirement age and indexation of pensions – that guarantee not only the short run equilibrium (liquidity of the system) but also the long-run financial sustainability of the system. Sustainability is measured as the difference between the net present value of the future income from contributions and the expenditure on pensions in the long run. Second, the non-overlapping block bootstrap technique, that is used to simulate future expected returns for both buffer fund and accumulated capital of the funding scheme, is also described in this section. Future expected returns have a direct impact on pension levels the participants are expected to receive under the mixed pension system.

The objective of the optimal paths of the decision variables, also be named automatic balancing mechanisms (ABMs), is to re-establish the financial equilibrium of PAYG pension systems. ABMs are defined, Vidal-Meliá, Boado-Penas, and Settergren (2009), as a set of pre-determined measures to be applied immediately as required according to an indicator that reflects the financial health of the system.[8] The design of our ABM is asymmetric in the sense that adjustments in the decision variables only occur with negative deviations from the financial health indicator. In our setting, we include the amount of the buffer fund, F_n, because the amount of the fund, or even the return that it generates, might have a positive impact on a pension system facing unexpected changes in demographic, economic or financial conditions.

As per Godínez-Olivares, Boado-Penas, and Haberman (2016), the objective function to be minimised, subject to some constraints, is as follows:

$$min_{c_n, x_n, \lambda_n} \sum_{n=0}^{N} \frac{F_n(c_n, g_n, x_n^{(r)}, \lambda_n, J_n)}{(1+\delta)^n} \qquad (1)$$

[8] For more details on ABMs see D'Addio and Whitehouse (2012), Godínez-Olivares, Boado-Penas, and Pantelous (2016), Godínez-Olivares, Boado-Penas, and Haberman (2016) and Alonso-García, Boado-Penas, and Devolder (2018a), amongst others.

$$s.t. = \begin{cases} c_{min} \le c_n \le c_{max}; x_{min}^{(r)} \le x_n^{(r)} \le x_{max}^{(r)}; \\ \lambda_{min} \le \lambda_n \le \lambda_{max}; \\ c_{1\Delta} \le \frac{c_{n+1}}{c_n} \le c_{2\Delta}; x_{1\Delta}^{(r)} \le \frac{x_{n+1}^{(r)}}{x_n^{(r)}} \le x_{2\Delta}^{(r)}; \\ \lambda_{1\Delta} \le \frac{\lambda_{n+1}}{\lambda_n} \le \lambda_{2\Delta}; \\ F_n \ge 0 \end{cases}$$

where, the accumulation of the buffer fund over time, F_n, can be expressed as $F_n = (1+J_n)F_{n-1} + c_n W_n(g_n, x_n^{(r)}) - B_n(g_n, x_n^{(r)}, \lambda_n)$, and J_n is the return of both the buffer fund of the PAYG and the accrued benefits under the funding part[9] during year n; c_n is the contribution rate during year n; $W_n(g_n, x_n^{(r)})$ is the total contribution base paid at n that depends on the growth of salaries g_n and the retirement age at n, $x_n^{(r)}$; B_n is the total expenditure on pensions at n that depends on g_n, $x_n^{(r)}$, and the indexation of pensions at n, λ_n; $\delta > 0$ is the discount rate; N is the final year of our analysis.

$c_{min}, x_{min}^{(r)}, \lambda_{min} \in \Re$ and $c_{max}, x_{max}^{(r)}, \lambda_{max} \in \Re$ are lower and upper bounds of the decision variables respectively. These bounds are set in order to avoid possible unrealistic changes in the key variables of the pension system.

Smooth constraints are also necessary to prevent jumps in the paths of the contribution rate, age of retirement and indexation of pensions. Mathematically, smooth constraints are set as: $c_{1\Delta} \le \frac{c_{n+1}}{c_n} \le c_{2\Delta}; x_{1\Delta}^{(r)} \le \frac{x_{n+1}^{(r)}}{x_n^{(r)}} \le x_{2\Delta}^{(r)}; \lambda_{1\Delta} \le \frac{\lambda_{n+1}}{\lambda_n} \le \lambda_{2\Delta}$

where $c_{1\Delta}, c_{2\Delta}, x_{1\Delta}^{(r)}, x_{2\Delta}^{(r)}, \lambda_{1\Delta}, \lambda_{2\Delta} \in \Re$.

A liquidity restriction is also set as $F_n \ge 0$, for all n, to ensure liquidity in the system. The liquidity indicator that takes into account the accumulated value of the buffer fund can be defined as follows:

$$Lf_n = \frac{(1+J_n)F_{n-1} + c_n W_n\left(g_n, x_n^{(r)}\right) - B_n\left(g_n, x_n^{(r)}, \lambda_n\right)}{B_n\left(g_n, x_n^{(r)}, \lambda_n\right)} \tag{2}$$

Under a DB pension scheme, the initial pension is set as a percentage of a worker's pre-retirement income.

As in Godínez-Olivares, Boado-Penas, and Haberman (2016), the generalised reduced gradient algorithm is used to solve this non-linear discrete optimisation problem and find

[9] In practice, the investment strategy of the buffer fund and the accumulated contributions of the funding part could be different. However, as shown in the numerical application, we use the same return for both components as our results are not sensitive to the return associated to the buffer fund.

the decision variables – contribution rate, retirement age and indexation of pensions – over time.

Simulation of future returns[10]

Simulation of the returns is used for both the evolution of the buffer fund corresponding to the PAYG and the accumulated capital under the funding part of the mixed pension system.

We consider a general setup where the investment strategy combines both a risk-free asset and a diversified risky portfolio. This framework is consistent with the financial theory of Markowitz (1952), where the efficient allocation of a representative investor combines a given weight of risk-free asset plus the efficient risky portfolio, usually a stock index; see, among many others, Sharpe (1964) and Athanasoulis and Shiller (2000).

The simulation exercise is performed through the non-overlapping block bootstrap technique, one of the most widely used methodologies for time series simulation[11]; see, for instance, Künsch (1989) and Kreiss and Lahiri (2012).

The block bootstrap simulates the future behaviour of an asset by randomly concatenating blocks of its own historical returns. The procedure employed here for an individual stock[12] is summarized in four steps.[13]

Suppose that $\{S_t\}_{t\in N}$ is a time series of risky stock monthly prices and $\{S_1, S_2, \dots, S_T\}$ are observed.

i. Firstly, we compute the historical monthly returns $r_t = (S_t - S_{t-1})/\, S_{t-1}$, defining $\boldsymbol{r^*_T} \equiv (r_1, r_2, \dots, r_T)$ as the time series of monthly returns. Then, returns series are demeaned by subtracting their respective sample means from each observation, so $\boldsymbol{r_T} \equiv \boldsymbol{r^*_T} - \frac{1}{T}\sum(r_1 + \cdots + r_T)$. We assume that $\boldsymbol{r_T}$ are stationary weakly dependent time series, a common assumption for financial returns.
ii. Second, we add to each time series the long-term expected return obtained from the forecasts of a panel of analysts. This action allows us to adjust our simulations to the market consensus about future expected returns.[14] In particular, the long-term expected return for the European stock market is 5.72%, and 0.90% for the European sovereign bond market.
iii. Third, let k be an integer satisfying $1 \le k \le T$, define the overlapping blocks $B_1, B_2, \dots, B_N$ of length k contained in $\boldsymbol{r_T}$ as

$$B_1 = (r_1, \dots, r_k)$$
$$B_2 = (r_2, \dots, r_{k+1})$$
$$\dots$$

[10] We simulate returns instead of prices to construct the future evolution of the investment portfolio. This methodology simplifies the simulation procedure because it avoids a regular re-balancing of the portfolio to maintain the weights.

[11] Other simulation techniques such as continuous-time processes and discrete-time heterokedastic volatility models were also studied. However, given the long-time horizon of our simulation, block bootstrap was the preferred technique as it allows to carry out statistical inference without imposing structural assumptions on the underlying data-generating random process (Kreiss and Lahiri 2012).

[12] We show the details for the simulation procedure for an individual stock, since its translation to portfolio prices is analogous.

[13] A detailed exposition of the methodology is provided in Kreiss and Lahiri (2012) and references therein.

[14] The panel of analysts comprises estimations from BlackRock, Vanguard, JPMorgan or Franklin Templeton, among others. We have considered the average of the expected return forecasts as input data in our simulations.

$$B_N = (r_N, \dots, r_T)$$

where $N = T - k + 1$.

To generate the simulated samples, we select b blocks at random with replacement from the collection $\{B_1, B_2, \dots, B_N\}$. Every block is randomly selected from the realization of a uniformly distributed random variable, which sets the date where the block starts. Random variables are simulated using the same seed to keep the comparison among the different portfolio choice scenarios. The k-size of the blocks is fixed to 12, so each block contains the information in one-year.

iv. Finally, the realization of an individual simulated path of the asset is constructed by serially concatenating b blocks, yielding to a simulated path with $b \cdot k$ observations.

The deterministic optimisation framework uses several percentiles of the returns as a pre-defined fixed parameter, otherwise stochastic optimisation would be needed. As we are working with a real-world problem (European population, salaries, contribution rate, retirement age and indexation of pensions for 75 years) it would be computationally intense to include a time-dependent multi-stage optimisation. Therefore, each realisation path should be viewed as a one-scenario of a possible realisation of the time-dependent stochastic process. One possible approach is to use Monte Carlo techniques to generate scenarios by conditional sampling. However, as our model includes several smooth constraints to prevent jumps, the results of the contribution rate, retirement age and indexation of pensions should not vary dramatically since the parameters are bounded.

Under a mixed pension system, due to the funding DC nature, the amount of the pension also depends on the future returns according to an investment strategy. The replacement rate, defined as the amount of the initial pension divided by pre-retirement earnings, is computed in the following section. The replacement rate measures how effectively a pension system provides a retirement income to replace earnings, which is the main source of income before retirement.

3. Numerical application for a mixed pension system

This section illustrates the methodology presented in Section 2 by means of a representative example over the next 75 years. First, the main data and assumptions are presented, secondly the results are discussed and finally this section provides a sensitivity analysis which mainly depends on the percentage allocated to the DC funded part. In the main data, we discuss the demographic and economic data, and other assumptions related to the mixed pension system together with its DB PAYG and DC funded parts are follow:

Demographic and economic data

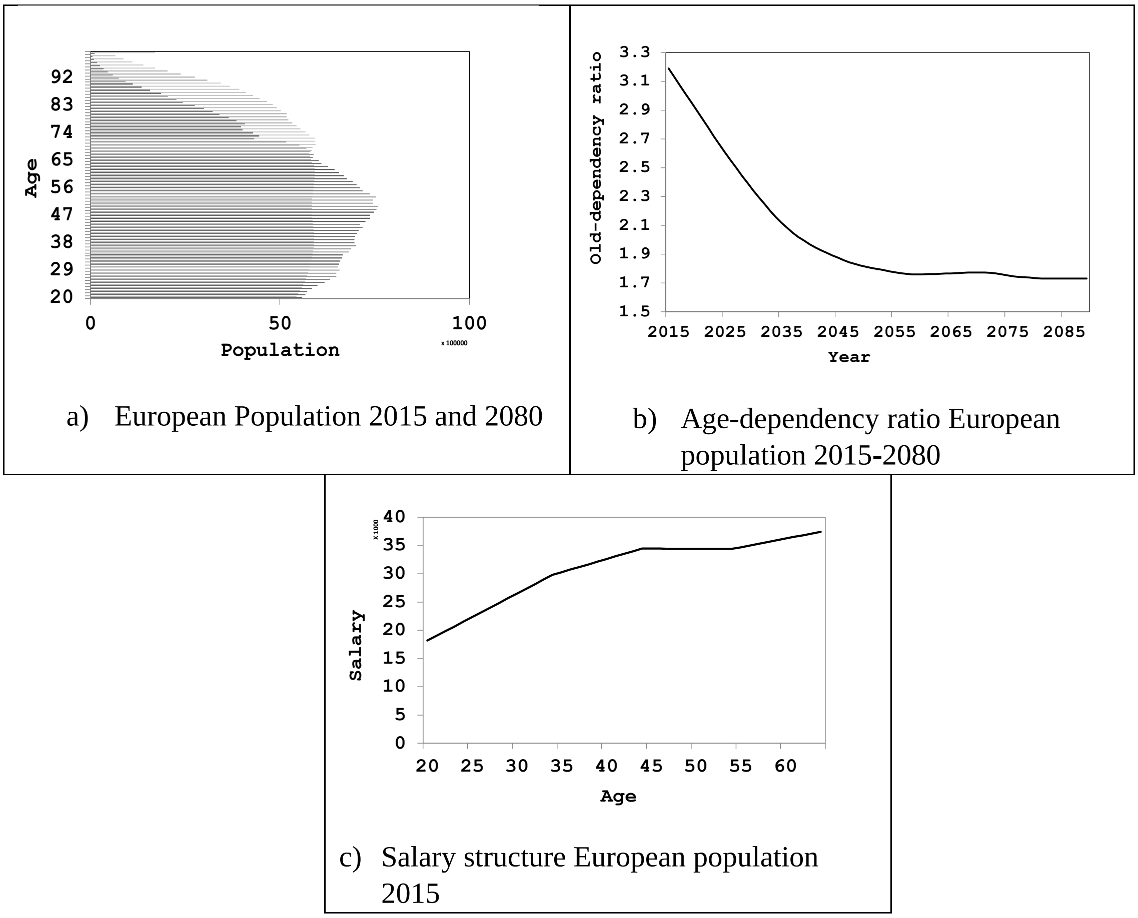
- We use the forecast demographic structure of the European population from 2015 to 2080 obtained from Eurostat.[15] It can be seen that the peak of population in 2015 (Figure 1(a), dark grey pyramid) is at ages 48-55, corresponding to the demographic boom in the 1960s and early 1970s. In 2080, we can notice the appearance of a more rectangular shaped pyramid (Figure 1(a), light grey

[15] Data obtained from Eurostat Database, http://ec.europa.eu/eurostat (baseline projection). Accessed 1 April 2018. Projections are only provided for the period 2015-2080. From 2080, population is supposed to remain constant and equal to the population in 2080.

pyramid) as a result of the aging process. Also, the age-dependency ratio,[16] decreases over time, with 3.2 contributor financing one pensioner in 2015 to 1.7 in 2080 as show in Figure 1(b).

- European salary structure is used in 2015 as shown in Figure 1(c). The salaries are assumed to increase at an annual constant rate of 2.5%.[17] No unemployment is considered in our analysis.

Figure 1: European population structure in 2015 (dark grey) and 2080 (light grey), evolution of the age dependency ratio, and salary structure in 2015



a) European Population 2015 and 2080

b) Age-dependency ratio European population 2015-2080

c) Salary structure European population 2015

Own source based on Eurostat

Choice of parameters and assumptions related to the mixed pension system

- The mixed pension system aims to provide a total replacement rate of 60% of participants' final salary. Creighton (2014) states that a level of 60% of a worker's pre-retirement income is in line with the average replacement rate which measures the pension as a percentage of a worker's pre-retirement income.
- For a replacement rate of 60%, under a pure DB PAYG pension scheme, the initial balanced contribution rate that makes income from contribution equal to pension expenditure in 2015 is 18.14%. We assume that 2%[18] is invested to the DC

[16] This ratio measures the number of elderly people relative to those of working age. It is calculated as the number of contributors divided by the number of pensioners.

[17] Average growth of the hourly labour costs in Europe from 2004 to 2015. Data obtained from Eurostat Database, http://ec.europa.eu/eurostat, accessed 1 March 2016.

[18] Since Sweden is the most quoted example of NDC scheme we took as a baseline scenario an allocation to the DC funded scheme similar to the one is used by the Swedish Pension Agency.

scheme while the remaining, i.e. 16.14%, is contributed to the DB PAYG part. Consequently, 2% invested into the DC part represents an allocation of 11% in the total contribution rate.

Choice of parameters and assumptions related to the DB PAYG part

- Initial pension for the DB pension part is set at 53% of a worker's pre-retirement income when the contribution rate to the DC scheme is 2%. The DC part would represent about 7% of the replacement rate (i.e. 2%*60% divided by 18.14%).[19]
- Under the DB PAYG scheme, the initial values for the contribution rate, age of normal retirement and indexation of pensions are 16.14%, 65 and 3% respectively, as shown in Table 1.
- Under the DB PAYG scheme, when the three variables are projected simultaneously, the lower bounds for the contribution rate and retirement age coincide with the initial values while the lower bound for the indexation of pension would be 0%.[20] The upper bounds are 20%, 72 and 2% respectively.
- Under the DB PAYG scheme, for smooth changes, we assume that the change in the contribution rate varies between 0% and 3%, the age of normal retirement between 0 and 3 months and the indexation of pensions between -1% and 0%. According to Godínez-Olivares, Boado-Penas, and Haberman (2016), these values are in line with the most important reforms in the 34 OECD member countries between January 2009 and September 2013.
- The buffer fund that might result from the PAYG part is invested 50% in bonds and 50% in equity.

Table 1: Initial values and bounds for the contribution rate, age of retirement and indexation of pension for the DB PAYG part

	Initial value	Lower bound	Upper bound	Change bound
Contribution rate (%)	16.14	16.14	20.00	0.00 – 3.00
Age of retirement (years)	65.00	65.00	72.00	0.00 – 0.25
Indexation of pensions (%)	3.00	0.00	2.00	-1.00 – 0.00

Own source

Choice of parameters and assumptions related to the DC funding part

- Contribution rate to the DC part is set at 2%.
- It is assumed that the DC scheme is financially sustainable in the sense that there are no risks regarding the assumptions of the mortality rates and discount rates.[21] Therefore the DC part is not affected by the automatic balancing mechanism but has an impact on the amount of the initial pension the individual receives at retirement age. Computationally, the inclusion of a DC funding part makes our analysis much more difficult as our study needs to be performed by cohort.

[19] This is an approximation as in practice the link between the evolution of the balanced contribution rates and replacement rates under a DB scheme is not linear.

[20] We set the indexation lower bound at 0% so that the pension payments do not decrease in nominal terms. However, pensions might decrease in real terms.

[21] In practice, funding schemes are subject to several risks such as longevity or interest rate risks. However, such risks are beyond the scope of this paper. In this case, this scheme can be seen as actuarially fair so that the expected present value of lifetime contributions is equal to the expected present value of benefits.

- Contributions to the DC scheme are invested 50% in bonds and 50% in equity.[22]
- With the aim of simulating returns for the DC part and buffer fund of the PAYG, we use monthly time series from 31/07/2006 to 29/12/2017.[23] The risk-free asset is proxied by a portfolio of investment grade Euro sovereign bonds.[24] The risky portfolio is captured by the index EuroStoxx 600, a composite index that accounts for the large, mid and small capitalization companies across 17 countries of the European region, and which could be considered as the proxy of the European stock market portfolio. Data has been downloaded from Datastream. The simulated paths using the block-bootstrap are built until December 2090, i.e. 75-year time horizon.[25] The forecasts are constructed by simulating 200,000 paths.
- Retirement age for the DC part is set to 65.
- Cohort projected mortality tables for a representative European country[26] are used to convert the accumulated capital of the DC part into a lifetime pension.
- Discount rate is set at 2%.
- At the retirement age, the accumulated capital is converted into a life annuity according to standard actuarial practice.

Results

We first show the results of the expected future returns that affect both the evolution of the buffer fund of the PAYG and the accumulated capital of the DC scheme. Second, we calculate the automatic balancing mechanisms that make the PAYG part financially sustainable in the next 75 years and show the total replacement rates of the mixed pension systems when the investment to the DC part is equal to 2% of the salary.

Results of simulated returns

- As shown in Figure 2 and Table 2, one euro invested in a 100% sovereign bond portfolio in 2015 results in 2.3 euros – on average - in 2090. In contrast, one euro invested in a 100% stock portfolio leads to 88 euros in 2090.[27] These results are

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[22] In countries such as Sweden and Norway the investment in equity is 60 and 40% respectively. For more information on buffer fund asset allocation please see Yermo (2008).

[23] There are some reasons to support our sample choice. First, we tried to be conservative in our forecasts by selecting a historical period which comprises the recent financial crisis 2008-2012. This introduces an additional degree of uncertainty to the asset portfolios. Second, there is a consensus about how the recent financial crisis resulted in a structural change in the financial system; see, for instance, the vast approval of regulation like the Dodd-Frank Act 2010 in the US, and MiFIDII in Europe. Under this perspective, the usage of long-term data could result in an unrealistic source of information for generating future scenarios.

[24] Time series of real bond portfolio funds are employed. In this case, bonds are captured by Vanguard Euro Government Bond Index Fund (ISIN IE0007472115), a pool of different Euro government bonds with average maturity 8.90 years and duration 7.34.

[25] As the base year is 2015 and years 2016 and 2017 are already realised, a total of 73 blocks are concatenated for year simulation.

[26] We use projected Spanish mortality rates over the period 2016-2065 for males obtained from the National Institute of Statistics. Mortality rates are assumed constant after 2065. Spain is one of the most ageing countries in Europe with only Sweden and Norway having a slightly higher life expectancy. We use the mortality tables for males whose mortality is slightly higher so that the rates are more representative of the European population.

[27] These results are in line with the empirical evidence on the high average market risk premium -average market return over the risk-free rate. The average return of equities on excess of long-term bond returns was 6.2% during the past 110 years; see Dimson, Marsh and Staunton (2011).

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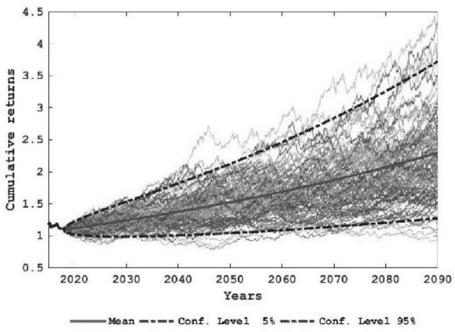
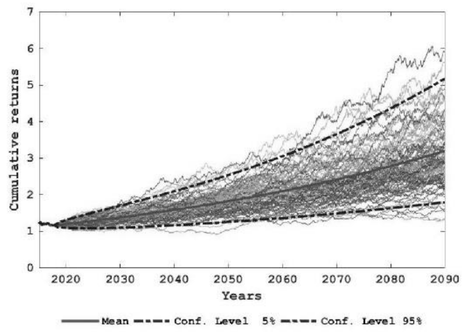
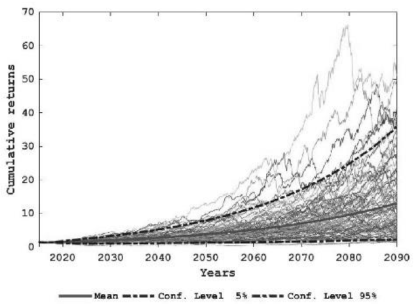
Figure 2: Cumulative returns for different allocations of investment



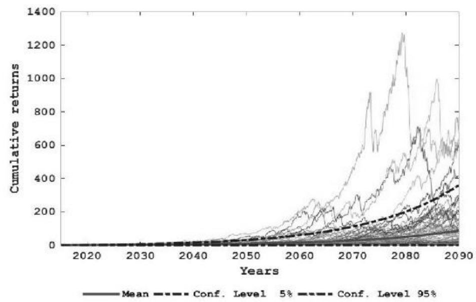
a) 100% in European sovereign bonds



b) 90% (10%) in European bonds (stocks)



c) 50% (50%) in European bonds (stocks)



d) 100% investment in European stocks

Own source

A detailed insight about the results is provided in Table 2, which provides in columns 2 to 12 some descriptive statistics (mean, standard deviation and some percentiles) of the simulated paths. To illustrate the median annual return, Column 13 computes the implied internal rate of return (IRR) of the investment while Column 14 (benchmark) shows the expected annual return according to the panel of analysts. We see that the volatility of the portfolio increases as the stock weights does: from 0.77 for the 100% bond portfolio, to 258.69 for the 100% stock portfolio. The lesson is clear: a higher exposure to risk is rewarded with a higher return. Second, the representative investor could be exposed to losses in her portfolio when investing her entire wealth in stocks. These losses could happen in 2.5 out of 100 of the scenarios for this portfolio. Finally, the implied IRR is lower than benchmark for those portfolios with higher weights in stocks. This result corroborates how our simulation procedure considers adverse scenarios (recessions, stock market crashes, etc.) for the stock market that could result in a lower performance of the stocks in the portfolio.

Table 2: Cumulative returns for different allocations of the portfolio

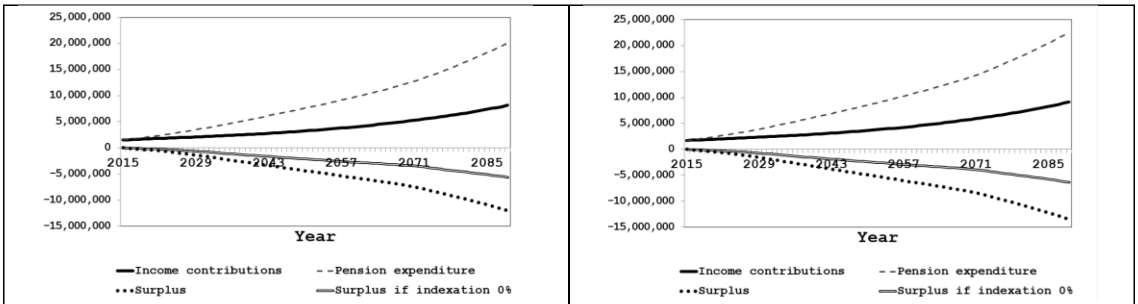
Portfolio (Equity)	Mean	Std	Percentiles 2.5	5	10	25	50	75	90	95	97.5	Implied IRR from median	Benchmark
0.00%	2.30	0.77	1.15	1.27	1.44	1.75	2.18	2.72	3.31	3.73	4.12	1.04%	0.90%
10.00%	3.20	1.06	1.62	1.79	2.01	2.44	3.03	3.77	4.60	5.17	5.73	1.49%	1.38%
50.00%	13.05	12.25	1.72	2.29	3.15	5.35	9.49	16.54	26.81	35.73	45.63	3.05%	3.31%
100.00%	88.78	258.69	0.73	1.32	2.55	7.57	24.41	75.55	199.79	357.87	583.46	4.35%	5.72%

Own source

Results of automatic balancing mechanisms for the DB PAYG scheme to get a replacement rate of 53%

If no automatic balancing mechanisms were implemented the system would accumulate a continuous deficit as shown in Figure 3. It can also be observed that the difference between the income from contributions and pension expenditure is increasing over time as a result of the demographic change. If the indexation of pension was to be set at 0% the amount of the deficit would be much lower at the end of the year of period of study. As expected, the amount of the deficit for a system that offers a replacement rate of 60%, Figure 3b, is higher than for a rate of 53%.

Figure 3: Income from contributions, pension expenditure and surplus (considering indexation of pensions 3% and 0%) for the 60% RR DB-PAYG and 53% RR DB-PAYG (in millions)



a) DB PAYG that provides a RR of 53%

b) DB PAYG that provides a RR of 60%

Own source

Figures 4 shows, for the PAYG part, the optimal of the contribution rate, retirement age and indexation of the pensions, respectively, to restore the financial sustainability of the system when the three decision variables are simultaneously modified. Unless stated otherwise, we always refer to the median of the distribution of investment outcomes.

It can be seen that, in order to get a replacement rate of 53%, the contribution rate and the retirement age need to increase to 18.39% and 71.62 respectively by the end of the time horizon while the indexation of pensions stabilises at 0% at the end of the period of analysis. Although the increase in the age of retirement is smoother during the first 35 years under the 90th percentile, we do not observe significant differences if returns on the buffer fund do not follow the median. Also, different investment strategies do not provide significant differences in the results.[28] This is mainly due to the fact that the amount of a

[28] Detailed results are not included in the paper as these are very similar to the ones in Figure 4.

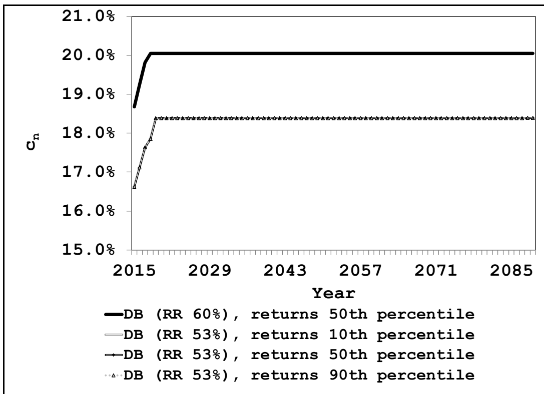
buffer fund under a PAYG scheme is very low as there is no accumulation of money and its main objective is to absorb unexpected events that might affect the liquidity of pension systems.

It is not surprising that the parametric reforms needed to make the PAYG scheme sustainable are harder if the replacement rate is 60%. In particular, the contribution rate and retirement age stabilise at 20% and 72 respectively. At the same time, the indexation of pension stabilizes at 0% at the end of the period of analysis.

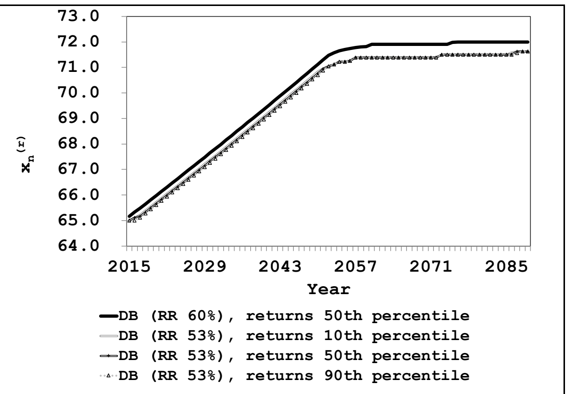
The trend of the liquidity ratio, Figure 4(d), presents a similar pattern for all scenarios studied and it presents an accumulation period for the first 15 years as it anticipates changes in the population structure, i.e. future increases in the age dependency ratio.

The balancing mechanism could also be designed around having only one modified variable rather than having three variables that are modified simultaneously. In particular, for the replacement rate of 53%, if the contribution rate were the only decision variable, it would need to increase to 39.67% by 2090, whereas if the age of retirement were the only variable, it would increase to 81.48. If the indexation of pensions were set as the only decision variable, the benefits would need to decrease at an annual rate of 5.08%.

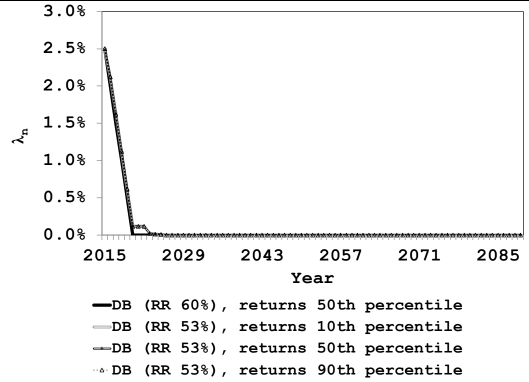
Figure 4: Results of ABM for PAYG scheme when the three variables are projected simultaneously with European population structure, 50% invested in equities and RR of 53%



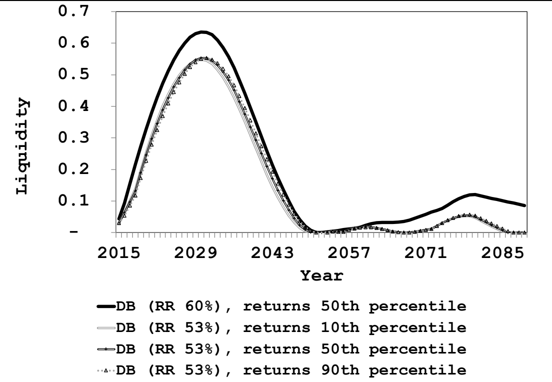
a) Contribution rate



b) Retirement age



c) Indexation of pensions



d) Liquidity ratio

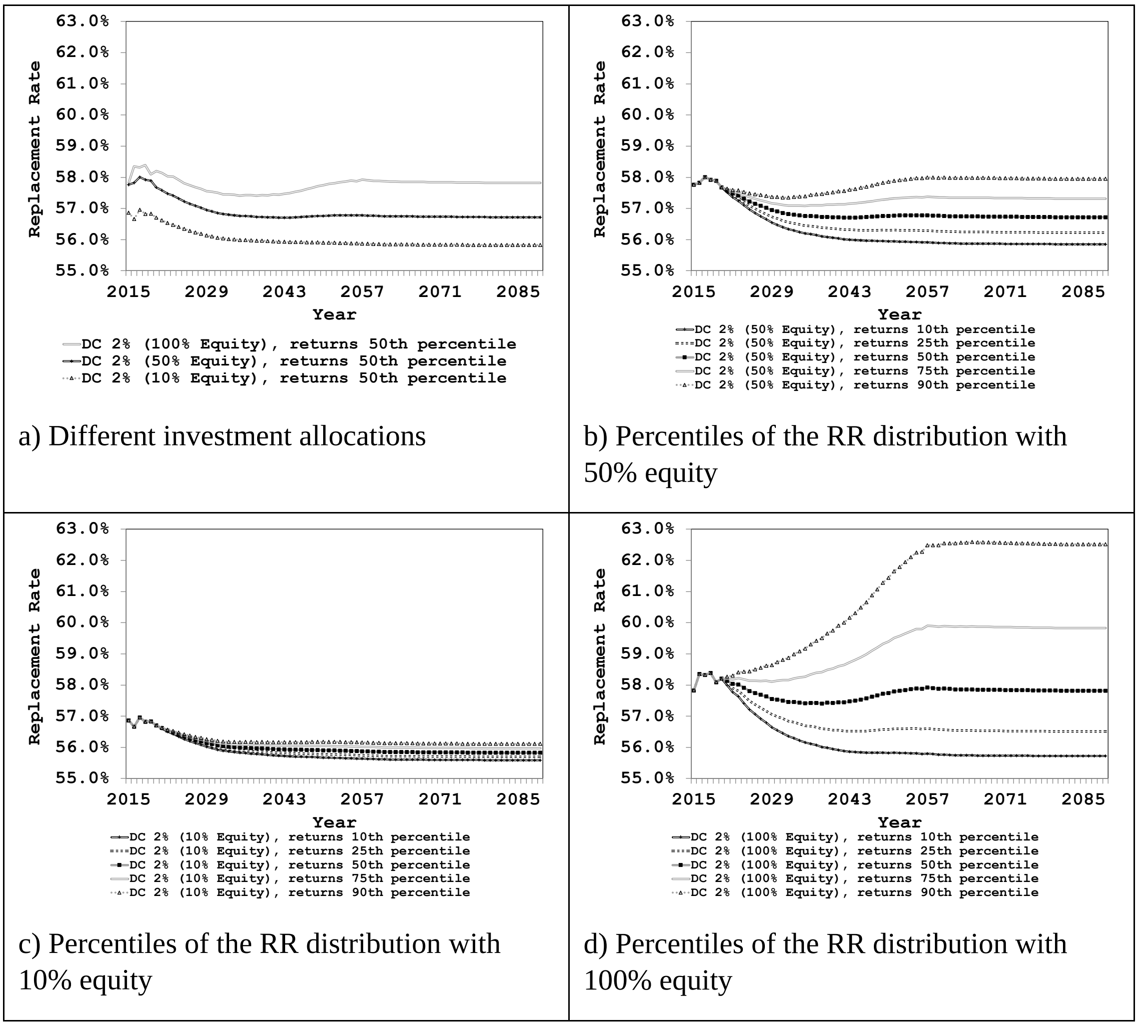
Own source

Results of total replacement rates for the mixed pension system

After investing 2% into a DC pension scheme, Figure 5(a) shows that the values of total replacement rate vary from 56.7% and 58%. We observe that the replacement rate increases to 58.4% if the investment is only made in equities. Replacement rates for this mixed pension system are lower than for a pure DB PAYG system that offers 60% of final salary; however, adjustments needed for this pure DB scheme are harsher, i.e. a retirement age of 72 years and a contribution rate of 20% - which correspond to the upper bounds imposed by the non-linear optimisation model - instead of 71.62 and 18.39% when the RR promised by the PAYG scheme of the mixed pension system is equal to 53%.

Results show a quite high volatility specially when investment is fully made in equities as shown in Figure 5(d). In particular, the RR reaches a value of 60% at the 75th percentile and 62.6% at the 90th percentile of the simulated distribution.

Figure 5: Total replacement rates for the mixed pension system with a 2% allocated to the DC part under different investment strategies



a) Different investment allocations

b) Percentiles of the RR distribution with 50% equity

c) Percentiles of the RR distribution with 10% equity

d) Percentiles of the RR distribution with 100% equity

Own source

Sensitivity of the results

In this subsection, we discuss the adjustments needed for the PAYG part depending on the percentage allocated to the DC part. Two other cases regarding contributions invested to the DB PAYG and DC funding schemes are studied where the initial total contribution rate still remains 18.14%. We want to assess whether a combination of DB PAYG and DC funded can provide a total replacement rate of 60%.

Specifically, we analyse a contribution rate to the DC scheme of 5% or 10% of the individual's salary. When the contribution rate is 5%, we set the initial pension of the PAYG scheme at 43%[29] and consequently the initial balanced contribution rate is 16.14%. In this case we assess whether the DC part provides the extra 7% of replacement rate. Similarly, if the contribution rate for the DC part is 10%, the replacement rate for the PAYG, the target replacement rate for the DC funding and its balanced contribution rate are 27%, 33% and 8.14% respectively. Table 3 shows the different combinations between the DB PAYG and DC funding that we will analyse in terms of replacement rates.

Table 3: Different combinations of DB PAYG part and DC funding for mixed pension systems.

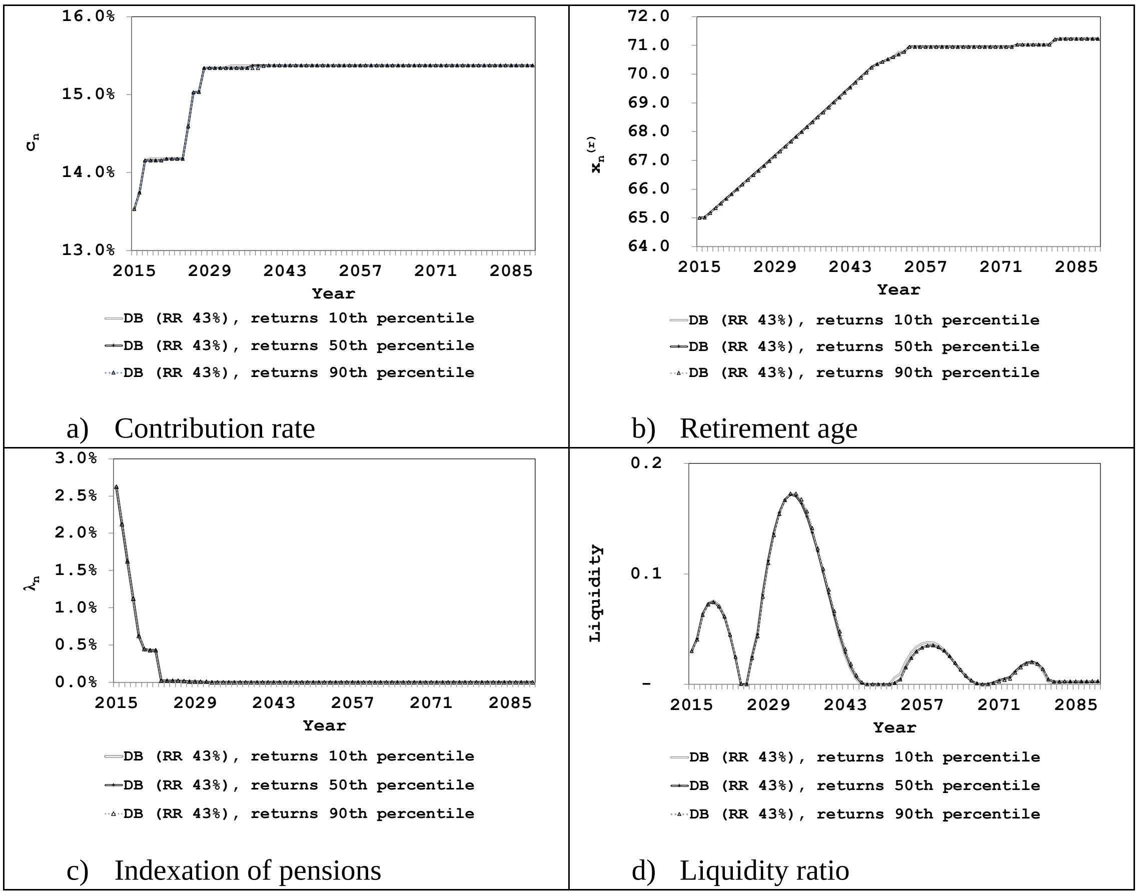
Mixed pension system	RR in the DB scheme	Target RR in the DC scheme
16.14% DB PAYG and 2% funded DC (baseline scenario)	53%	7%
13.14% DB PAYG and 5% funded DC	43%	17%
8.14% DB PAYG and 10% funded DC	27%	33%
18.14% DB PAYG	60%	0%

Own source

Under the replacement rate of 43% for the PAYG part, Figure 6 shows that the contribution rate needs to increase to 15.37% while the retirement age reaches a value of 71.23 at the end of the time horizon. At the same time, the indexation of pension stabilises at an annual rate of 0%. When the replacement rate of the DB part is 27%, Figure 7 shows that the contribution rate and retirement age need to increase to 12.01% and 70 respectively while the pensions are revalued at a 1% at the end of the period analysed. As in the previous case when the contribution rate is set at 2%, we also do not observe differences when analysing different percentiles (relative to the median) under either of the 2 different allocations to the DC part.

[29] When the contribution rate is 5%, we aim to assess whether the replacement rate of the DC part reaches 17% (i.e. 5%*60% divided by the balanced contribution rate when the replacement rate is set to 60%, i.e. 18.14%). Therefore, the replacement rate associated to the DB part is set to 43%. Similarly, when the contribution rate to the DC part is 10% the replacement rate associated to the DB part is 27%.

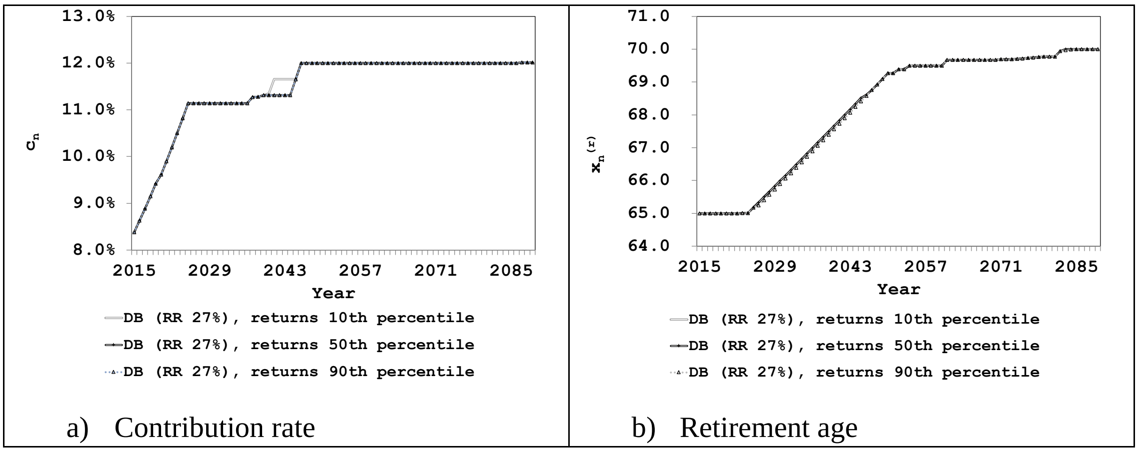
Figure 6: Results of ABM for PAYG scheme when the three variables are projected simultaneously with European population structure, 50% invested in equities and RR of 43%



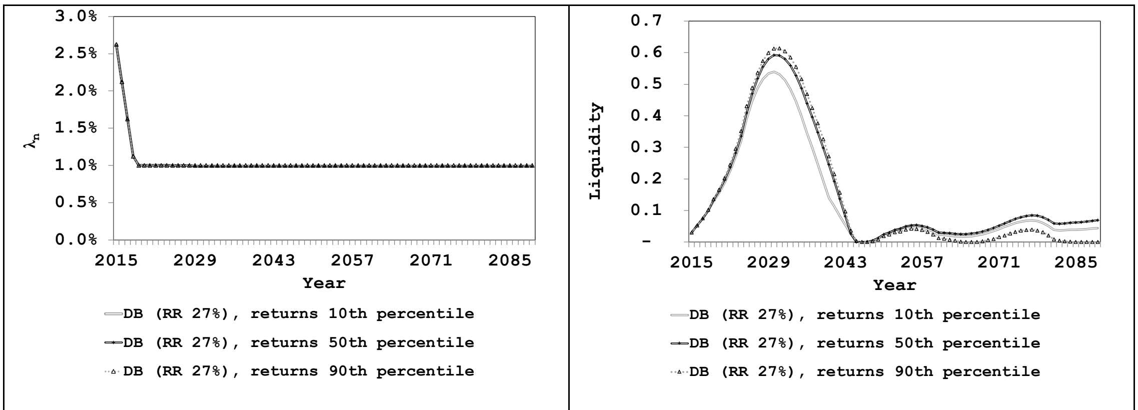
a) Contribution rate
b) Retirement age
c) Indexation of pensions
d) Liquidity ratio

Own source

Figure 7: Results of ABM for PAYG scheme when the three variables are projected simultaneously with European population structure, 50% invested in equities and RR of 27%



a) Contribution rate
b) Retirement age

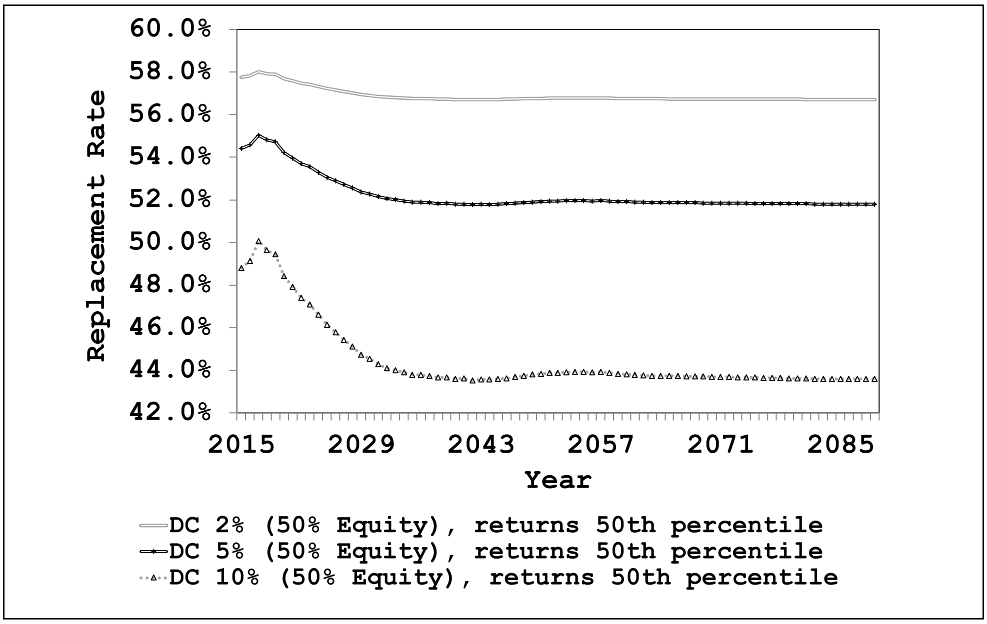


c) Indexation of pensions d) Liquidity ratio

Own source

As shown in Figure 8 the total replacement rate for the mixed pension system is lower when the percentage invested in the DC scheme increases. When the allocation to the DC part is equal to 10% of salaries and investment is made in 50% equities, the total replacement rate of the mixed pension system reaches a value of 43.6% at the end of the time horizon.

Figure 8: Total replacement rates for the mixed pension system with 50% invested in equities: different allocation to the DC part



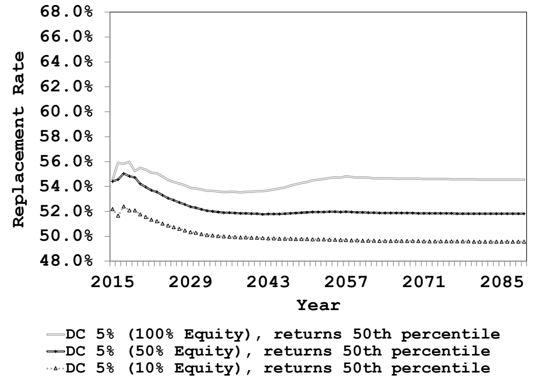
Own source

A greater investment in equities leads to an increase in the value of the total replacement rate as shown in Figure 9 and Figure 10. In particular, when investment to the DC is set at 5% (10%) and fully invested in equities the total replacement rate takes values between 53.2% and 56% (47% and 52%) at the 50th percentile.

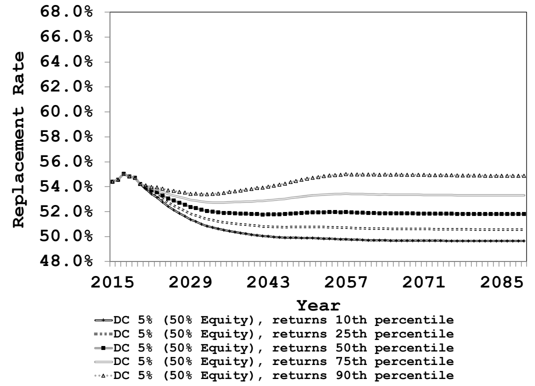
We note that the results are very volatile when the investment is fully made in equities. Specifically, when the allocation of the DC part is 5% the total replacement rate would reach a value of 66.3% at the 90th percentile as shown in Figure 9(d). If the allocation to the DC part is 10%, Figure 10(d), the total replacement rate could reach a value of 72.6% as shown at the 90th percentile.

At the 50th percentile, neither of the different percentages allocated to the DC or the different investment strategies analysed would lead to a total replacement rate equal to or higher than 60%. However, under a pure PAYG scheme that promises a replacement rate of 60% harsher adjustments to the key parameters are needed as explained above in Figure 6 and 7.

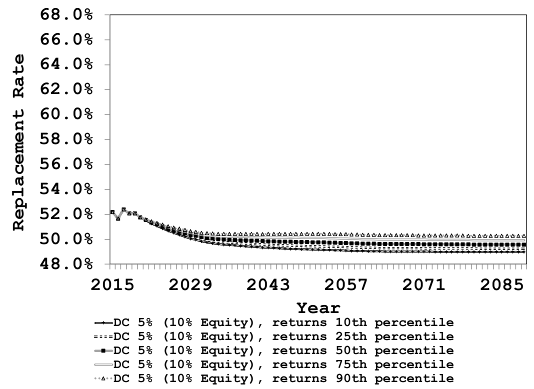
Figure 9: Total replacement rates for the mixed pension system with a 5% allocated to the DC part under different investment strategies



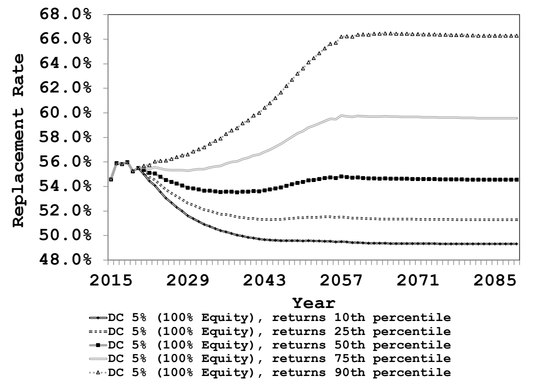
a) Different investment allocations



b) Percentiles of the RR distribution with 50% equity



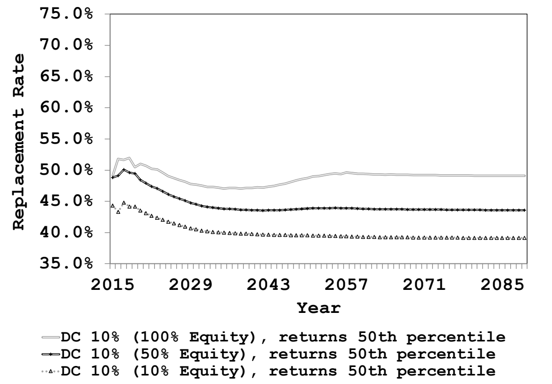
c) Percentiles of the RR distribution with 10% equity



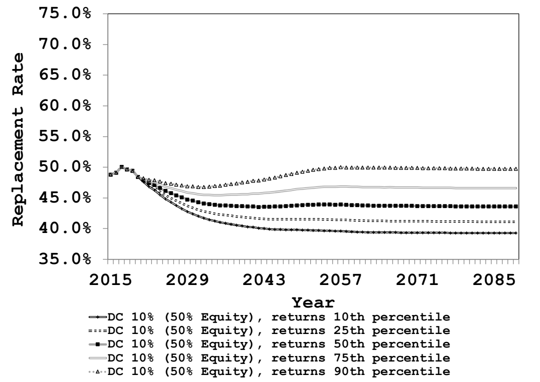
d) Percentiles of the RR distribution with 100% equity

Own source

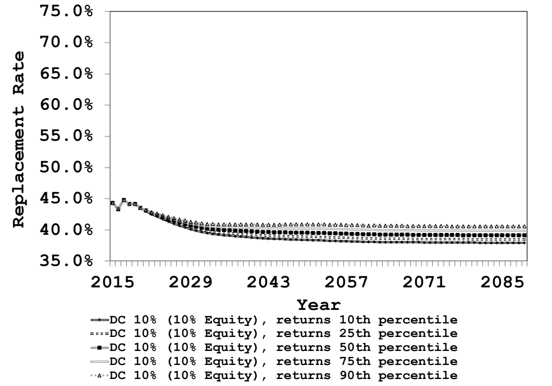
Figure 10: Total replacement rates for the mixed pension system with a 10% allocated to the DC part under different investment strategies



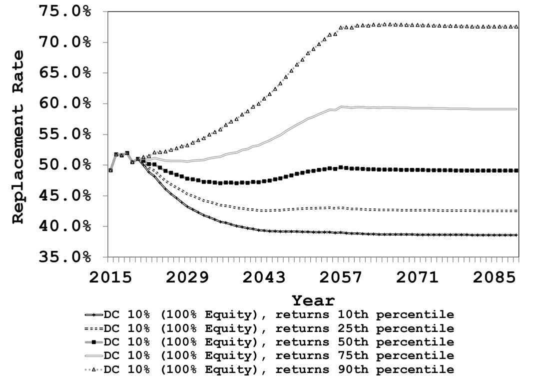
a) Different investment allocations



b) Percentiles of the RR distribution with 50% equity



c) Percentiles of the RR distribution with 10% equity



d) Percentiles of the RR distribution with 100% equity

Own source

Table 4 shows the relationship between the total contribution rate paid by the individuals and the total replacement rate received under different types of pension systems – pure PAYG with a RR of 60% versus different types of mixed pension systems depending on the allocation to the DC part. To make our results comparable we consider the optimal path of the contribution rate for the PAYG part as if it was the only decision variable with the retirement age and indexation of pensions set at 65 and 3% respectively. In all cases a higher total replacement rate at the end of the period analysed is associated to a higher value in the contribution rate. For instance, we can see that in 2090 the total RR obtained by the participants would be 43.60% if the contribution rate to the DC part is 10%. However, the level of the total contribution rate that makes the mixed pension system sustainable would be 30% - opposed to a level of 55.38% in the case of a pure PAYG system that promises a RR of 60%.

Table 4: Total contribution rates versus total replacement rates under different types of pension systems

	Total contribution rate				Total replacement rate			
Year	**Pure PAYG (RR=60)**	**Mixed with 2%**	**Mixed with 5%**	**Mixed with 10%**	**Pure PAYG (RR=60)**	**Mixed with 2%**	**Mixed with 5%**	**Mixed with 10%**
2020	28.86%	21.11%	20.47%	19.75%	60%	57.89%	54.21%	49.45%
2030	31.49%	32.66%	27.34%	23.20%	60%	56.95%	52.28%	44.74%
2040	32.51%	35.64%	32.16%	27.00%	60%	56.73%	51.80%	43.66%
2050	36.08%	35.87%	34.18%	28.80%	60%	56.76%	51.92%	43.80%
2060	43.82%	37.67%	35.97%	29.50%	60%	56.76%	51.91%	43.83%
2070	43.82%	40.98%	36.72%	29.50%	60%	56.74%	51.85%	43.70%
2080	44.68%	41.67%	37.14%	30.00%	60%	56.72%	51.82%	43.63%
2090	55.38%	41.76%	37.46%	30.00%	60%	56.72%	51.81%	43.60%

Note: Results are shown for investment in 50% equity at the 50th percentile.
Own source

Nevertheless, to avoid any biased analysis, it is necessary to calculate an indicator, the internal rate of return (IRR), that takes account of how the amount of both total contributions and pensions evolves with time for a cohort of individuals. In this setting the IRR for a contributor can be defined as the value of the parameter, in the law of compound interest, which in actuarial terms makes contributions equal to benefits.

When computing the IRR for the pensioners who retire in 2090 we can see that it is higher under a mixed pension system than for a pure PAYG with a RR of 60%. Specifically, its value when the allocation to the DC part is 10% (5%) is 2.76% (2.67%) compared to a 2.55% for the pure PAYG. For the rest of cohorts of pensioners, the results are not conclusive.[30]

4. Conclusions

Mixed pension systems have been advocated, particularly by the World Bank, as a practical way to reconcile the higher financial market returns compared with GDP growth with the costs of a scheme with a greater funded element.

This paper uses a nonlinear optimisation methodology based on Godínez-Olivares, Boado-Penas and Haberman (2016) and assesses the impact of a compulsory funded pension scheme that complements the defined benefit PAYG.

We explore the case of a pure PAYG scheme versus a mixed pension system considering different allocations to the funded scheme and investment strategies. The block-bootstrapping technique is used to calculate simulated returns. We show that the annual return is much higher if the investment is 100% in equities, i.e. 4.35% compared to 1.04% if the capital is only invested in bonds.

[30] The optimal adjustments are calculated for a close time horizon of 75 years and are not equitable in the sense that all cohorts receive the same IRR.

For the PAYG part, we observe that there are not substantial differences in the optimal paths of the decision variables - age of retirement, contribution rate and indexation of pensions, whatever the investment strategy to be followed by the buffer fund is. The main reason for this is the fact that, under a PAYG scheme, there is no accumulation of money with the aim of the fund being to absorb unexpected events that affect the financial equilibrium of the pension systems in the short run.

We show that the adjustments needed for the mixed pension system are less severe than the pure PAYG because the latter promises a higher replacement rate than the PAYG part associated to the mixed system. However, the total replacement rate for the mixed pension system is lower than the pure PAYG in most of the cases studied. Only if the investment is exclusively made to equities and at the 90th percentile, would the mixed pension system outperform the PAYG in terms of replacement rate. At the 75th percentile the mixed pension systems studies would reach a RR of 59-60%. There is an obvious trade-off between financial sustainability of the system and adequacy. However, the adjustments needed to ensure sustainability can affect the returns that the individuals would actually receive from the pension system.

When calculating the internal rate of return, we prove that individuals who retire in 2090 would benefit from mixed pension systems, whatever the allocation to the DC part is. The main policy recommendation deriving from all the above would be the possibility to reform the state pension system considering a mixed (DB PAYG + funding) system. As demonstrated in the paper, there might be some room for governments to benefit from a mixed pension system with respect to the classical PAYG financing not only in terms of financial sustainability of the system but also in terms of adequacy, i.e. replacement rates received by the individuals.

Finally, based on the model presented, at least two important directions for future research can be identified. First, it would be interesting to design an equitable automatic balancing mechanism for all cohort of individuals with a periodic recalibration of the optimal paths of the variables and study their consequences on both sustainability and adequacy considering the risk aversion of the participants. Another avenue for future work, following Alonso-García and Devolder (2016), would be to study the optimal mix between PAYG and DC funding scheme over time together with the automatic mechanisms that would ensure financial sustainability under a multi-stage stochastic optimisation framework. A third direction would be to include stochasticity for other risks inherent to pension systems, in particular, the demographic risk.

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