A Hybrid Approach Coupling MLPG-R with QALE-FEM for Modelling Fully Nonlinear Water Waves

Shiqiang Yan, Qingwei Ma
School of Mathematics, Computer Science and Engineering, City, University of London
London, EC1V 0HB

ABSTRACT

The paper reports a progress on the development of a hybrid approach coupling the Meshless Local Petrov-Galerkin Method based on Rankine Solution (MLPG-R) and the Quasi Arbitrary Langangian-Eulerian Finite Element Method (QALE-FEM) for modelling nonlinear water waves. The former is to solve the one-phase incompressible Naiver-Stokes model using a fractional step method (projection method), whereas the latter is to solve the Fully Nonlinear Potential Theory (FNPT) using a time-marching procedure. They are fully coupled using a zonal approach. The hybrid approach takes the advantage of the QALE-FEM on modelling fully nonlinear water waves with relatively higher computational efficiency and that of the MLPG-R on its capacity on dealing with viscous effects and breaking waves. The model is validated by comparing the numerical prediction with the experimental data for a unidirectional focusing wave. A good agreement has been achieved.

KEY WORDS: Hybrid model; MLPG-R; QALE-FEM; Nonlinear water waves

INTRODUCTION

Wave-structure interaction has been a focus for the researches on offshore, coastal and ocean engineering for many years. For safety and survivability of the structures, extreme wave condition must be considered. Accurately modelling such extreme wave condition usually requires a large-scale (~ 10s km) and long-duration (e.g. 3-hour sea state) numerical simulation to capture the spatial-temporal propagation of the ocean wave. On the other hand, the response of the structure in extreme condition is considerably influenced by small-to micro-scale physics, such as the viscous/turbulent effect, hydro elasticity and so on. This implies that an effective numerical model shall be able to deal with both large-scale oceans wave and small-scale near-field physics simultaneously. The presence of the extreme waves invalids the routine wave diffraction analysis based on linear and second-order potential theory in frequency domain and a fully nonlinear analysis shall be considered using time domain analysis.

Advances have been made on the development of fully nonlinear potential theory (FNPT) on modelling highly nonlinear wave waves in large scale and for long duration, e.g. 3-hour sea state. Various numerical models based on the FNPT, e.g. the quasi-arbitrary Lagrange-Euler finite element method (QALE-FEM, Ma and Yan, 2006; Yan and Ma, 2010a) and Spectral Boundary Integral methods (e.g. Wang and Ma, 2015; Wang *et al*, 2016), have been developed and proven to be robust and highly efficient for modelling extreme water waves without breaking. The FNPT assumes that the flow is inviscid and irrotational, therefore, it cannot deal with breaking waves, slamming and other small-scale physics near structures.

Alternatively, NS models, which relying on the solutions to the Naiver-Stokes equation, Continuity equation and proper boundary conditions, have also been extensively developed and applied. These models may be solved by both the mesh based methods, e.g. the volume of fluid (VOF) and the meshless methods (particle methods), e.g. the smoothed particle hydrodynamics (SPH, Cummins and Rudman , 1999; Gotoh *et al*, 2004, 2014, 2016; Khayyer *et al*, 2008; Shao et al, 2006; Shao, 2012; Lind *et al*, 2012, 2016; Zheng *et al*, 2014), the moving particle semi-implicit method (MPS, e.g. Koshizuka and Oka, 1996; Khayyer and Gotoh, 2009, 2010, 2012 and Khayyer *et al*, 2008) and the Meshless Local Petrov-Galerkin method based on Rankine Solution (MLPG-R, Ma and Zhou, 2009; Zhou and Ma, 2010). The meshless methods have been recognized as promising approaches for modelling violent waves and their interaction with structures, attributing to their advantages that meshes are not required and numerical diffusion associated with the convection terms is eliminated through a fully Lagrangian formulation. Comparisons with the experimental data have suggested that the meshless methods can accurately capture the small-scale physics associated with breaking waves, slamming and hydro-elasticity. A recent review has been given by Gotoh and Khayye (2016) on current achievements and future perspectives for projection-based particle methods with applications in ocean engineering; and Ma et al (2016) on solving Poisson's equation in projection based particle methods. However, the NS models are relatively time-consuming. Consequently, the applications of the NS models often limit to a short time frame near the breaking wave impacts on structures. A large-domain (say 20x20 wavelengths in three dimension) or long-duration modelling of wave-structure interaction using the NS models are practically impossible.

In practice, the viscous effects in the problems concerned play a significant role only near the breaking waves and/or structures and may be ignored in other areas. This initiated the idea to develop hybrid approaches coupling FNPT and NS using a spatially hierarchical strategy (zonal approach), in which a relatively efficient FNPT model is applied to a major area without wave breaking and offshore structures; and a NS solver covers the remaining small area to resolve the detailed physics related to viscosity/turbulence and breaking wave impacts (e.g. Sriram, Ma and Schlurmann, 2014; Sueyoshi *et al*, 2007). An alternative strategy is to decompose the velocity/pressure in the NS model into two parts (model-decomposition strategy). For example, Edmund *et al* (2013) decomposed the velocity and pressure into irrotational and rotational parts, which are governed, respectively, by a Laplace problem for a velocity potential in the whole computational domain and a complementary NS model (the summation of two parts satisfies the NS model) to be solved in small subdomains near the structures.

This paper presents the development of a hybrid approach combining the QALE-FEM and the MLPG-R for modelling nonlinear water waves using a zonal approach. The basic idea is that in a small subdomain near the wave breaking or structure, the MLPG-R is used and the relatively robust QALE-FEM covers the rest of the domain; an overlap zone is introduced between two sub-domains which not only exchange the data between two solvers but also smooth the discontinuity of the pressure-velocity relation in two sub-domains. However, this paper only aims to focus on a preliminary investigation of the feasibility and accuracy of the hybrid model by using a case with non-breaking unidirectional focusing wave, in which an accurate reading of the experimental data is available. The subdomain covered by the MLPG-R is centered at the location, where the extreme focusing wave occurs. Good agreement has been achieved.

MATHEMATICAL MODELS

QALE-FEM

The QALE-FEM method is based on a fully nonlinear potential theory(FNPT) for water waves, in which the flow is assumed to be inviscid and irrotational. In the FNPT, the velocity is computed from a velocity potential (ϕ) by $\vec{u}=\nabla\phi$, and the pressure is calculated by using the Bernoulli's equation,

$$p/\rho=-\frac{\partial\phi}{\partial t}-\frac{|\nabla\phi|^2}{2}-gz \qquad (1a)$$

or

$$\frac{D\phi}{Dt}=-p/\rho+\frac{|\nabla\phi|^2}{2}-gz \qquad (1b)$$

in which z is the vertical coordinate whose origin is located at the mean free surface. The velocity potential satisfies the Laplace's equation,

$$\nabla^2\phi=0 \qquad (2)$$

On the free surface, the following boundary conditions in a Lagrangian form is used,

$$\frac{Dx}{Dt}=\frac{\partial\phi}{\partial x},\frac{Dy}{Dt}=\frac{\partial\phi}{\partial y},\frac{Dz}{Dt}=\frac{\partial\phi}{\partial z} \qquad (3a)$$

$$\frac{D\phi}{Dt}=-gz+\frac{|\nabla\phi|^2}{2} \qquad (3b)$$

where D/Dt is the material derivatives following the motion of the fluid particle. The latter (Eq. 3b) corresponds to p = 0 at the free surface. On a rigid boundary, e.g. the wavemaker,

$$\frac{\partial\phi}{\partial n}=\vec{U}_b\cdot\vec{n} \qquad (4)$$

where $\vec{U}_b$ and $\vec{n}$ are the velocity and the unit normal vector of the rigid boundaries, respectively.

In order to estimate the pressure using Eq. (1), $\partial\phi/\partial t$ needs to be accurately estimated. This may be obtained using the backward finite difference scheme, which may lead to a numerical instability (Yan and Ma, 2007). In the QALE-FEM, $\partial\phi/\partial t$ is evaluated by solving a similar boundary value problem, where the Laplace's equation of $\partial\phi/\partial t$ is used to govern the fluid motion. The boundary condition on the free surface is consistent with Eq. (3b), i.e.

$$\frac{\partial\phi}{\partial t}=-\frac{|\nabla\phi|^2}{2}-gz \qquad (5)$$

And the boundary condition on the rigid boundary is written as,

$$\frac{\partial}{\partial n}\frac{\partial\phi}{\partial t}=\left[\dot{U}_c+\dot{\Omega}\times r\right]\cdot n-(\Omega\times U_c)\cdot n-\frac{\partial}{\partial n}\left((U_c+\Omega\times r)\cdot\nabla\phi\right) \qquad (6)$$

where $\dot{U}_c$ and $\dot{\Omega}$ are the translational and rotational acceleration of the rigid body; U_c and Ω are the translational and rotational velocity of the rigid body.

The boundary value problems defined by Eqs (1-6) are solved by using a time-marching approach. If the position of and the velocity potential at the free surface, the motions of the rigid bodies are known at n[th] time step, the following procedure is used to find the variables at (n+1) [th] step,

1) Replace the boundary condition for the velocity potential on the free surface by a Dirichlet condition, $\phi^n=f_p^n$, where f_p^n is the potential values on the free surface and can be predicted by that at the previous step and a time integration of $D\phi/Dt$ over the time step;
2) Solve the boundary BVP of the velocity potential using the finite element method (FEM);
3) Calculate the fluid velocity on the free surface and body surfaces using gradient calculation scheme;
4) Update $\partial\phi/\partial t$ and $D\phi/Dt$ on the free surface using Eqs. (3) and (5);
5) Solve the BVP of $\partial\phi/\partial t$ using the FEM;
6) Update the position of the free surface and the velocity

potential at (n+1)th step using Eq. (3) and a time integration scheme, i.e.. $\vec{r}^{\,n+1} = \vec{r}^{\,n} + \left(3\vec{u}^{\,n} - \vec{u}^{\,n-1}\right)\Delta t / 2$ based on the Adams-Moulton method.

The details of the FEM formulations and the time integration scheme have been discussed in our previous publications, e.g. Ma et al. (2001). The main differences between the QALE-FEM method and the conventional FEM method is that the computational mesh is moving during the calculation by using a novel methodology based on the spring analogy method but purpose-developed for wave-structure interaction problems. The novel methodology for moving mesh is that interior nodes and boundary nodes are considered separately; the nodes on the free surface and on rigid boundaries are considered separately; nodes on the free surface are split into two groups: those on waterlines and those not on waterlines (inner free-surface nodes); and different methods are employed for moving different nodes. To move the interior nodes which do not lie on boundaries, a spring analogy method is used. In this method, nodes are considered to be connected by springs. The positions of nodes on the free surface are determined by physical boundary conditions, i.e., following the fluid particles at most time steps. The nodes moved in this way may become too close to or too far from each other. To prevent this from happening, these nodes are relocated at a certain frequency, e.g. every 40 time steps. When doing so, the nodes on the waterlines is re-distributed according to a principle for a self-adaptive mesh. One key issue for the QALE-FEM to be addressed is the technique for efficiently calculating the velocity at the free surface, as shown in Eq. (3). For these purpose, a three-point method for computing the velocity on the free surfaces and body surfaces suitable for unstructured/moving meshes, and a special technique for coping with wave overturning and impacting, are developed. These techniques ensure high robustness of the QALE-FEM. The details of those techniques will not be repeated here. Readers are referred to Ma and Yan (2006, 2009) and Yan and Ma (2007). Nevertheless, for the coupling with other numerical model using a zonal approach, which will be discussed in the following Section, the velocity and the pressure near the interface between the QALE-FEM domain and others shall be exchanged between two models. This means that near the interface, robust interpolation and gradient calculation schemes are demanded, e.g. to find the velocity using $\vec{u} = \nabla\phi$ or find the pressure using Eq. (1). For this purpose, a modified SFDI (simplified finite difference interpolation) is developed and applied. The details can be found in Xu *et al* (2015).

MLPG-R

In the MLPG-R model, the Navier-Stokes equation and Continuity equation in Lagrangian form are considered,

$$\frac{D\vec{u}}{Dt} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\vec{u} + \mathbf{g} \tag{7}$$

$$\nabla\cdot\vec{u} = 0 \tag{8}$$

where $\vec{u}$, ρ, p, ν, $\mathbf{g}$ and t denote the velocity, density, pressure, kinematic viscosity, gravitational force and physical time. The kinematic and dynamic conditions on the free surface are given by

$$\frac{D\vec{r}}{Dt} = \vec{u} \tag{9a}$$

$$p = 0 \tag{9b}$$

where $\vec{r}$ is the position vector. On the rigid boundary surface, the following conditions are satisfied,

$$\vec{u}\cdot\vec{n} = \vec{U}_b\cdot\vec{n} \tag{10a}$$

$$\frac{\partial p}{\partial n} = \rho\left(\mathbf{g} - \dot{\vec{U}}_b\right)\cdot\vec{n} \tag{10b}$$

in which $\dot{\vec{U}}_b$ is the acceleration of the rigid boundary. If one knows the velocity, pressure and the position of the particles at nth time step, then in the MLPG-R algorithm, the following fractional step method (projection method) is used to find the variables at the next time step with a time increment of Δt ,

1) Calculate the intermediate velocity ($\vec{u}^{\,*}$) and position ($\vec{r}^{\,*}$) of the particles using

$$\vec{u}^{\,*} = \vec{u}^{\,n} + \nu\nabla^2\vec{u}^{\,n}\Delta t \tag{11a}$$

$$\vec{r}^{\,*} = \vec{r}^{\,n} + \vec{u}^{\,*}\Delta t \tag{11b}$$

2) Evaluate the pressure pn+1 using the semi-implicit equation,

$$\nabla^2 P^{n+1} = \frac{\rho}{\Delta t}\nabla\cdot\vec{u}^{\,*} \tag{12}$$

3) Calculate the velocity at all the particles using the pressure gradient and the gravitational force,

$$\vec{u}^{\,n+1} = \vec{u}^{\,*} + \Delta t\left(-\frac{\nabla p^{n+1}}{\rho} + \mathbf{g}\right) \tag{13}$$

4) Update the position of the particles

$$\vec{r}^{\,n+1} = \vec{r}^{\,n} + \vec{u}^{\,n+1}\Delta t \tag{14}$$

The key task of the procedure is to solve Eq. (12) in order to evaluate the pressure, which is transferred in to a weak form by integrating it over a circular/spherical integration deomain Ω_1 centered at each particle,

$$\int_{\partial\Omega_I}\vec{n}\cdot\left(p^{n+1}\nabla\varphi\right)ds - p^{n+1} = \frac{\rho}{\Delta t}\int_{\Omega_1}\nabla\varphi\cdot\vec{u}^{\,*}d\Omega \tag{15}$$

where the test function φ is taken as the Rankine source solution. The remarkable feature of Eq. (15) is that this formula does not include any derivatives of p. This is of great benefit because estimating unknown functions is much easier than estimating their gradient. Special techniques have been developed for the integration and solutions to the weak formulation within the framework. It shall be noted that the particles in this approaches move in a Lagrangian way following the material velocity, typically resulting in a disordered(random) particle distribution, even though they may be distributed uniformly in the initial state. A recent test by Ma et al (2016) has confirmed that the MLPG-R has a quadric convergent rate for solving Poisson equations by using disordered/random particles. The accuracy may be further improved by considering particle regularization or shift schemes (Lind et al, 2012; Khayyer et al, ,2017), though they are not used the version of the MLPG-R adopted by this paper. The further details of the MLPG-R method can be found in Ma and Zhou (2009), Zhou and Ma (2010) and Sriram et al (2014).

Two-way (Strong) Coupling Using a Zonal Approach

As indicated above, a zonal approach is used to fully couple the QALE-FEM with the MLPG-R. There are two types of methods coupling the FNPT models with NS models using the zonal approach. One is the weak (one-way) coupling and the other is the strong (two-way) coupling. For the former, the FNPT models covers the entire computational domain and the NS models cover small area near the wave breaking or structures, where the viscous effects are significant; the FNPT models provide boundary conditions, e.g. the volume fraction, velocity and pressure, for the NS models; the NS models do not feedback any data to the FNPT model and usually are equipped with a damping zone near the it outer boundary to dissipate the reflected/disturbed waves (e.g., Yan and Ma, 2010b; Hildebrandt and Sriram, 2014). The weak coupling is simpler as it needs to only transfer information in one direction and is more suitable to deal with the problem where the reflections from the structures or the disturbing due to the wave breaking is insignificant near the boundary of the NS models. The two-way (strong) coupling exchanges solutions in the interface between two subdomains governed by the FNPT and NS models (e.g. Sriram, Ma and Schlurmann, 2014). This means that the reflected/disturbing waves from the NS domain is expected to be transferred to the FNPT model and the NS model does not require a damping zone to omit such waves. Nevertheless, inner iterations at each time step may be essential to ensure that boundary conditions at both subdomains are satisfied simultaneously. To secure the success of this approach, one critical question is how to build a coupling between the two models at the interfaces of the subdomains where the velocity-pressure relationships are discontinuous at two sides (Bernoulli or Euler equation in the FNPT subdomains and NS equation in the NS subdomains). Sriram, Ma and Schlurmann (2014) has reviewed and tested different approaches, including a fixed boundary interface, moving boundary interface and overlapping zone interface using various cases associated with highly nonlinear water waves. Our preliminary tests also concluded that a single interface (no matter is fixed or moving) cannot ensure a smooth transformation of the velocity-pressure relationships (or the viscous effects) and normally introduces suspicious waves near the interface. Such suspicious waves may be dissipated in the subdomain governed by the NS model due to the viscous effect but they retain/propagate in the FNPT subdomain unless an appreciated artificial damping is imposed in the FNPT formulation.

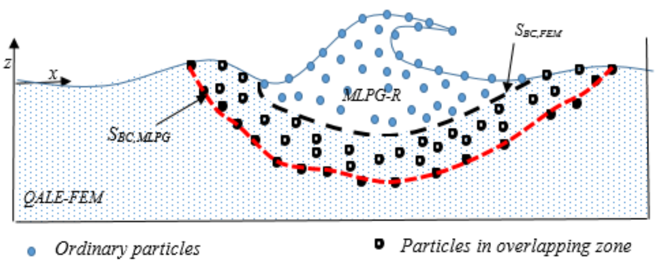


Figure 1. Schematic of the coupling configuration

In this work, the spatial domain is decomposed as sketched in Fig. 1, where an overlapping zone is introduced between the QALE-FEM subdomain (shadowed zone with an outer boundary denoted by S_{BC_FEM}) and the MLPG-R subdomain (discretized by particles with an outer boundary denoted by S_{BC_MLPG}). On the boundary of the MLPG-R subdomain (S_{BC_MLPG}), the velocity and pressure at discretized particles are fed by the solutions in the QALE-FEM using $\vec{u} = \nabla\phi$ and Eq. (1); one the boundary of the QALE-FEM subdomain (S_{BC_FEM}), the velocity potential and $\partial\phi/\partial t$ are specified by the solutions of the MLPG-R model using Eq. (1). In the overlapping zone, the solutions of the QALE-FEM and those of the MLPG-R are not consistent due to the fundamental difference, leading to, for example, a typical mismatch of the free surface position. To overcome this problem, the solutions of both the QALE-FEM and the MLPG-R in the overlapping zone need to be corrected by a weighted summation. In the overlapping zone, the velocity on the free surface, $\vec{u}_{sf}$, for both the QALE-FEM and the MLPG-R are corrected using

$$\vec{u}_{sf} = w\vec{u}_{sf}^{Q} + (1-w)\vec{u}_{sf}^{M} \tag{16}$$

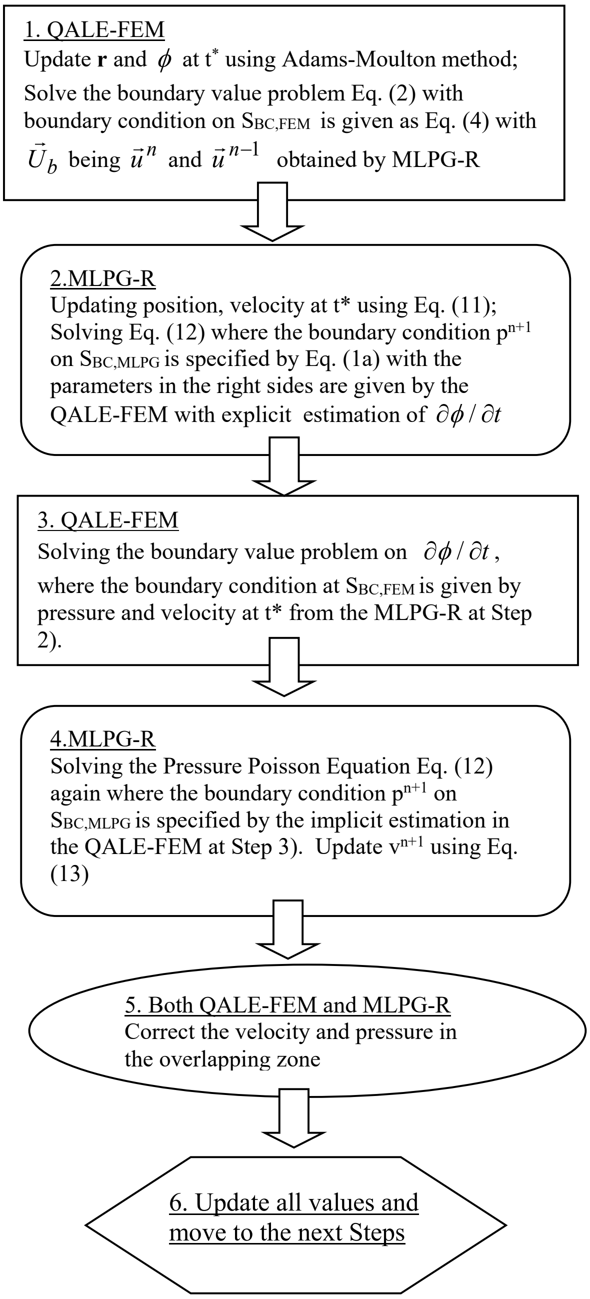


Figure 2. Flowchart of the coupling adopted in time domain

where w is a weighting function, which is zero on S_{BC_MLPG} and gradually changed to 1 on S_{BC_FEM} ; Subscripts 'Q ' and 'M' correspond to the QALE-FEM solution and the MLPG-R solution. The velocity and pressure at other MLPG-R particles in the overlapping zone are corrected using a similar way. Furthermore, the viscosity in the MLPG-R model is also changes from 0 on S_{BC_FEM} to the physical viscosity on S_{BC_MLPG}. One may notice that Eq. (16) is not valid if a breaking/overturning wave occurs in the overlapping zone. Indeed, modelling breaking waves exceeds the capacity of the FNPT and QALE-FEM. A criterion may need to be specified in the QALE-FEM to automatically determine the subdomain of MLPG-R based on the local wave steepness, ensuring no breaking waves occurs in the QALE-FEM subdomain (so does the overlapping zone).

One may also notice that the time integration scheme used in the QALE-FEM (i.e. the Adams-Moulton method) is different from that in the MLPG-R. These are taken into account in coupled procedure which is sketched in Fig. 2, assuming the solutions at n^{th} steps are known. The main differences between this procedure and the one used by Sriram, Ma and Schlurmann (2014) include (1) the boundary condition for solving the velocity potential at Step 1 is given as a Neumann condition in which the normal velocity on S_{BC_FEM} are fed by the MLPG-R using $3\vec{u}^n - \vec{u}^{n-1}$; (2) the boundary value problem for $\partial\phi/\partial t$ is solved in Step 3, which give an implicit estimation of the pressure on S_{BC_MLPG} in Step 4; and (3) the boundary S_{BC_FEM} is not a straight vertical plane but behaves similar to a free surface following the fluid motion. To ensure the boundary conditions of the MLPG_R and QALE-FEM satisfied simultaneously, Step (3) and Step (4) need to be repeated in an iterative matter. Nevertheless, one may agree that if the corresponding boundary values are predicted more accurately, the number of iteration will be reduced. This can be achieved by reducing the time step size and/or improve the accuracy of the time integration. For example, the Adams-Moulton method used in the QALE-FEM has shown its capacity on predicting the elevation and velocity potential at $(n+1)^{th}$ step using the corresponding data at previous two steps. We extend this idea to the configuration of the boundary condition of the QALE-FEM at S_{BC_FEM} , i.e. the fluid velocity and pressure at this boundary is predicted using the corresponding values at previous two steps by the MLPG-R solver. In such a way, the computed velocity and pressure at S_{BC_MLPG} ,which are required in Step (4), are well predicted. Therefore, the iteration may be practically avoided if the time step size is sufficiently small.

The time step size required by different models are different. Though both models adopt CFL condition, the QALE-FEM replaces the velocity in the CFL condition by the wave celerity (Yan *et al*, 2010a). However, the required mesh resolution is approximately 30-40 nodes per wave length. This leads to the time step size adopted by the QALE-FEM is approximately 1/32 ~ 1/200 significant wave period, depending on the steepness of the waves. Such time step is usually an order of magnitude larger that the MLPG-R time step, due to the fact that the particle resolution in the MLPG-R is significantly higher than the QALE-FEM. For simplicity, we use the same time step size for both models in the present work. By using such time step, the iteration of Steps (3-4) is practically unnecessary. Necessary convergence investigations are carried out for all cases. This will be discussed in the following section.

NUMERICAL RESULTS

The present hybrid model is validated by comparing the numerical results with experimental work. In the preliminary numerical investigation discussed in this paper, only non-breaking unidirectional focus wavs are considered, attributing to the fact that the free surface elevations are more easily to be measured with good accuracy. The experiments were performed using the 3D wave basin at the University of Plymouth. The wave basin is 35 m long and 15.5m wide. The water depth (d) used to perform the experiments is 2.93m. Flap wave paddles are installed to generate 3D waves. The focusing waves are generated by using flap wave paddles whose motion is specified by using a 2^{nd} order wavemaker theory using the spatial-temporal focusing mechanism. JONSWAP spectrum with a peak period of 1.456s is used. In order to explore the nonlinearity, different significant wave heights will be considered. For each wave component i, the phase, φ_i, is given by

$$\varphi_i = k_i x_f - \omega_i t_f \qquad (17)$$

where ω_i and k_i are the wave frequency and wave number, x_f and t_f are the theoretical focusing location and focusing time. The detailed can be found in Ma (2007) and Sriram Schlurmann and Schimmels (2013). For all focusing waves generated in the experiments, the focusing point is set at 13.886m from the wavemaker. Various wave gauges are used to record the wave elevations as sketched in Fig. 3. Though the experiments focus on both 3D bi-directional waves and 2D uni-directional waves, only the latter is considered in this paper. The domain decomposition in the hybrid model is shown in Fig. 4, in which two QALE-FEM subdomains are used at two ends of the wave basin and the MLPG-R domain covers small area where the wave focusing occurs. On the left side of the QALE-FEM domain 1, a 2^{nd} order piston wavemaker is used to generate the wave, whereas on the right side of the QALE-FEM domain 2, a self-adaptive wave absorber is used to absorb the incoming waves, representing the absorbing beach in the experiments.

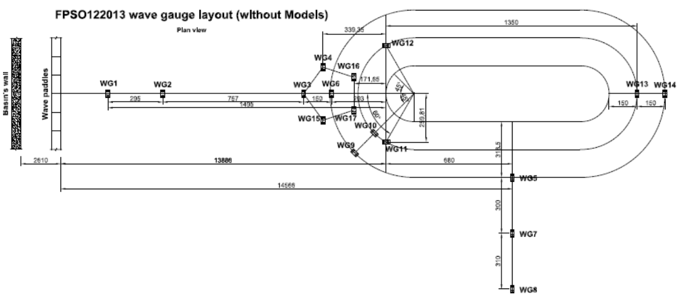


Figure 3: Locations of wave gauges

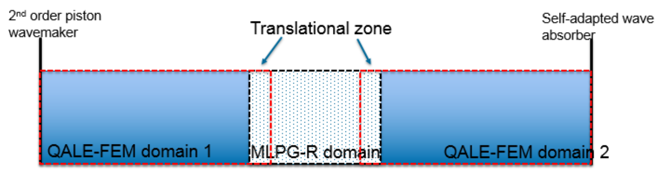


Figure 4: Domain decomposition of the hybrid method in the test case

In this preliminary test, convergence tests have been carried out individually for QALE-FEM and the MLPG-R by modelling the entire case (the computational domains of both models is the same as the experiment). The results suggest that the particle resolution (mean spacing) in the MLPG-R for achieving convergent results is ~ 0.01m; for the QALE-FEM, the mesh size on the free surface is 0.02m and the mesh size in vertical direction changes following an exponential function (approximately 16 layers of elements distributed from the free surface to the tank basin). The corresponding time steps leading to convergent results are 0.008s and 0.004s, for the QALE-FEM and

MLPG-R, respectively. To avoid defocusing our discussions on the hybrid modelling, the corresponding convergence tests are not presented here.

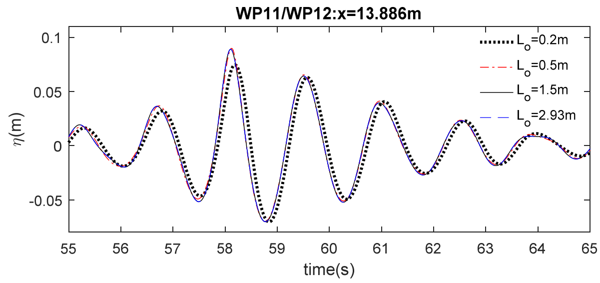


Figure 5: Comparison of the wave time histories recorded at the expected focusing point in the cases with different thickness of the overlapping zone(13.889m from the wave paddle; significant wave height 0.103m)

To reproduce the experiment in the numerical work, the QALE-FEM domain 1 covers area from the wave maker to 13m from the wavemaker and the QALE-FEM domain 2 covers from 15 to the end of the wave tank (i.e. 35m). Different thickness of the overlapping (translational) zone, ranging from 10 particle mean spacing to one water depths, L_o,, are considered first to test the sensitivity of the hybrid model to the thickness of the overlapping zone. Figs. 5 compares the wave time histories recorded at the expected focusing point, i.e. 13.889m from the wave paddle, which is located inside the MLPG-R domain. As observed that when $L_o \geq 0.5$m, all results are close and the relative different between the results with L_o = 0.5m and that with L_o = 2.93 is below 1%; whereas the corresponding results of L_o = 0.2m seems to be different from others. It is also observed that a further reduction of L_o (e.g. L_o = 0.1m) may lead to numerical instability.

The comparisons with the experimental data are also made for validation purpose. Two specific issues have been checked. One is the solution recorded in the MLPG-R domain but outside of the overlapping zone. The other is the solutions in the translational zone, in which the weighted summations are used to ensure a smooth transition of the viscos effects and the consistence of the solutions by two models. The thickness of the overlapping zone is chosen as 1.5m to ensure experimental data are available in the overlapping zone. Fig. 6 compares the wave elevation recorded at Gauge WP11/12 at x = 13.886m, which is close to the centre of the MLPG-R domain. For the purpose of comparison, the corresponding result obtained by the QALE-FEM only is also accompanied.

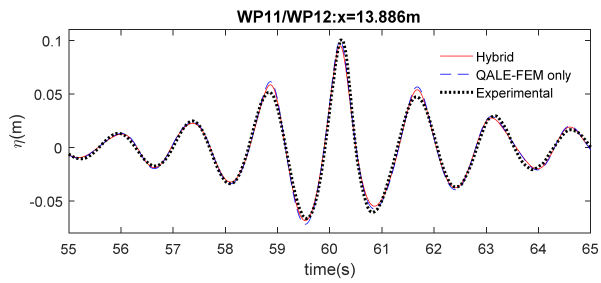


Figure 6: Comparison of the wave time histories recorded at the expected focusing point (13.889m from the wave paddle; significant wave height 0.103m;)

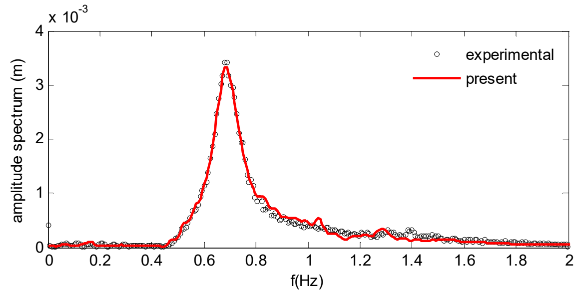


Figure 7: Comparison of the amplitude spectra at WG11(13.889m from the wave paddle; significant wave height 0.103m)

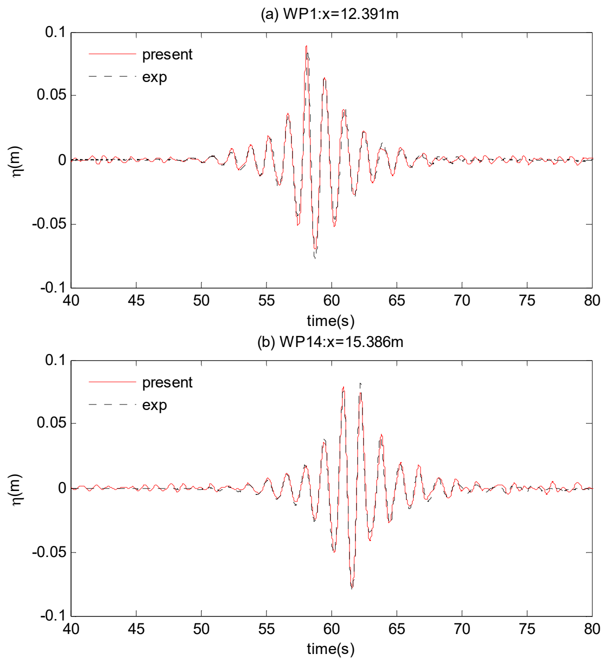
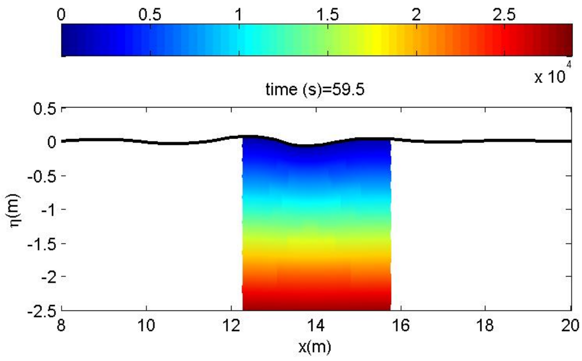


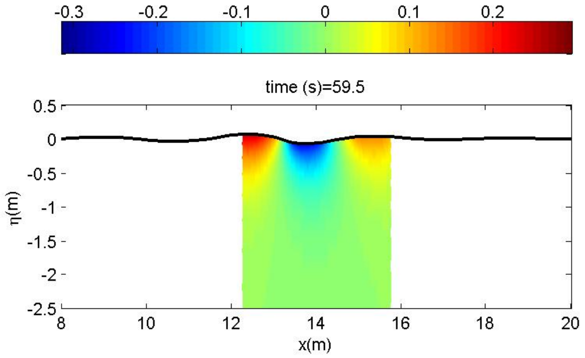
Figure 8: Comparison of the wave time histories recorded two positions in the overlapping zone (significant wave height 0.103m)

As demonstrated that the present numerical results by both the hybrid model and the QALE-FEM agree well with the experimental data, in particular near the time when the focusing occurs. The corresponding comparison of the wave spectrum is illustrated in Fig.7, which again shown a good agreement. Attentions are also drawn to the solutions in the overlapping zone. The wave time histories recorded by two gauges in the overlapping zones are also examined and the results are shown in Fig.8. Similar agreement to that shown in Fig.6 is observed. This implies that the present hybrid model combing the QALE-FEM with the MLPG-R has promising accuracy on modelling nonlinear water waves.

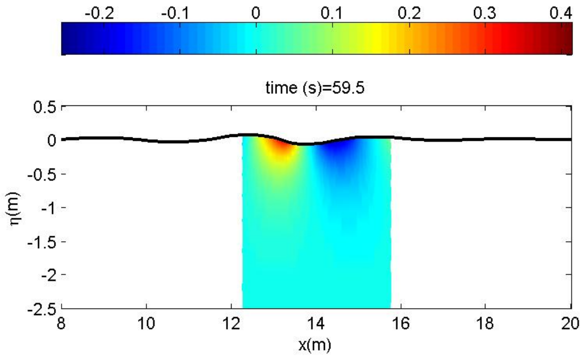
Attentions are also made to the spatial distribution of the free surface elevation, velocity and pressure near the focusing points at different time steps. Some results are shown in Fig. 9 and 10. The thick lines are the free surface consisting the solutions of the QALE-FEM and the MLPG-R in their subdomains. One may observe that the free surface in the entire domain is smooth and the free surface in the overlapping zone, i.e. at 11.5~13 and 15~16.5 in the QALE-FEM subdomain agree well with that in the MLPG-R domain. In the inner area of the MLPG-R domain, noises of the pressure and the velocity are not noticeable, as reflected by the smooth contours.



(a) Pressure



(b)horizontal velocity



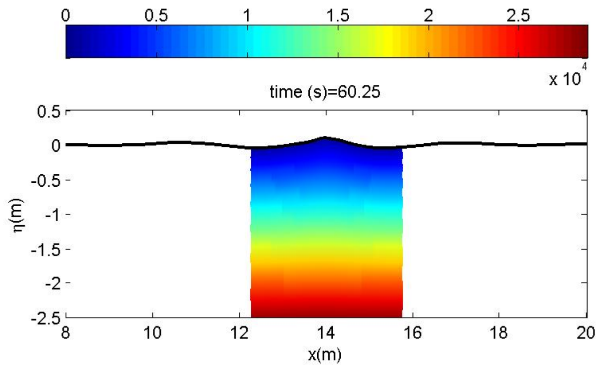
(c)vertical velocity

Figure 9: Pressure and velocity distribution near the focusing point at t = 59.5s (thick line: free surface; contours illustrate the solutions in the MLPG-R domain)

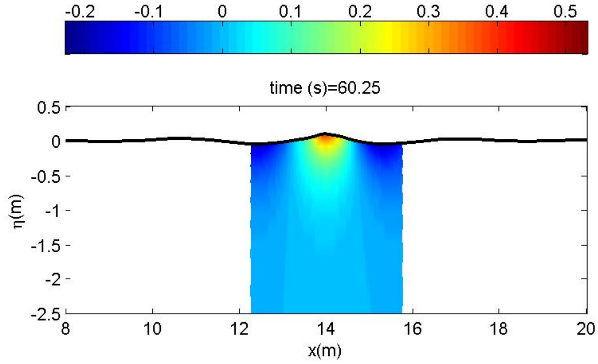
CONCLUSIONS

This paper reports our recent progress on developing a hybrid model coupling the fully nonlinear potential theory and the NS models. The QALE-FEM and the MLPG-R method are used to solve two models, respectively. A zonal approach with moving overlapping zones are used for a strong(two-way) coupling. The hybrid model is examined by the experiment on uni-directional focusing waves. Good agreements have been achieved.

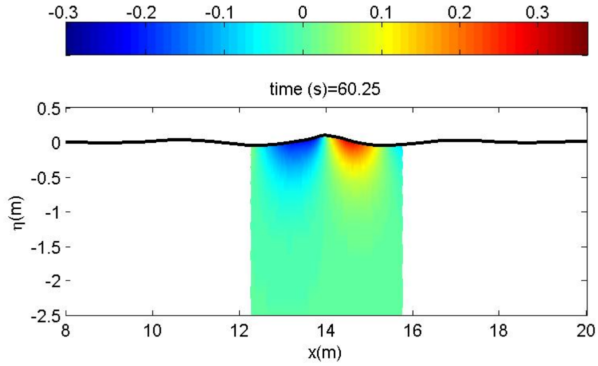
Although the ultimate objective of this work is to model breaking waves and/or their interaction with structures, only non-breaking unidirectional waves have been preliminarily tested in this paper. However, the results have shown a success of the key technique of the coupling, i.e. the overlapping zone technique. The relevant conclusion shall stand firmly even in the cases with violent breaking waves, since only non-breaking wave appears in the overlapping zone by carefully locating the subdomain of the MLPG-R. the hybrid model will be tested using the cases with breaking waves and/or structures in the next stage, which may be reported in the conference.



(a)pressure



(b) horizontal velocity



(c)vertical velocity

Figure 10: Pressure and velocity distribution near the focusing point at t = 60.25s (thick line: free surface; contours illustrate the solutions in the MLPG-R domain)

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