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Abstract

By representing the absorption of a particle into the warped potential by the complex warped potential, it is shown that as a result of the warp, together with periodicity of the complex potential, the generations for fermions are born. We propose a semi-empirical mass formula for leptons, up quarks and down ones. It is shown that our mass formula, which is based on our warped model, may reproduce the overall aspects of approximate linear rises of the logarithmic experimental mass values versus the generation number. This gives the evidence that the physical space is warped. Further, it is also shown that our warped model is able to explain the origin of non-existence of ν_R and $\overline{\nu}_L$ (C-P violation) in nature.

1. Introduction

The warped model has been widely used to solve the hierarchy problems in the elementary particle physics. Many of such theories have been based on the gravitational ones.[1)~6)] We concerned with in this paper the hierarchy structure for the fermion mass and the C-P violation in the standard model in the framework of our warped model. We set up the five-dimensional warped space with a constant curvature and consider the physics in it.

In section 2, the basic equation needed for our later discussion is derived. In section 3, the semi-empirical mass formula is derived by using the complex warped potential. In section 4, the hierarchy structure for fermion mass is discussed. In section 5, the origin of C-P violation is given in our warped model.

2. Warped space

We set up our model space as follows. We here consider the simplest model space. Consider a five dimensional hyper surface with which an infinitesimal line element

$$ds^2 = e^{(-2\xi/a)}(dx^{02} - dx^{i2}) - d\xi^2, \tag{1}$$

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is equipped, where x^0 is time coordinate , x^i , $i=1,\ 2,\ 3,$ space ones and ξ is an extra coordinate newly introduced in our model space and 1/a is a constant curvature of our model space.

We identify the hyper surface with our model space, namely, our physical space. Hence, the fundamental metric tensor of our model space is

$$(g_{AB}) = \begin{vmatrix} e^{(-2\xi/a)} & & & & \\ & -e^{(-2\xi/a)} & & & \\ & & -e^{(-2\xi/a)} & & \\ & & & -e^{(-2\xi/a)} & \\ & & & & -1 \end{vmatrix}. \quad (2)$$

We define

$$q^0 = \mu dx^0/ds,\ q^i = \mu dx^i/ds,\ i=1,\ 2,\ 3,\ \text{and}\ q^{\xi} = \mu d\xi/ds, \quad (3)$$

where μ is a mass of a particle, hence we assume $\mu > 0$. They are the energy- momentum which forms a five-vector in our model. One obtains from Eq.(1) and Eq.(3)

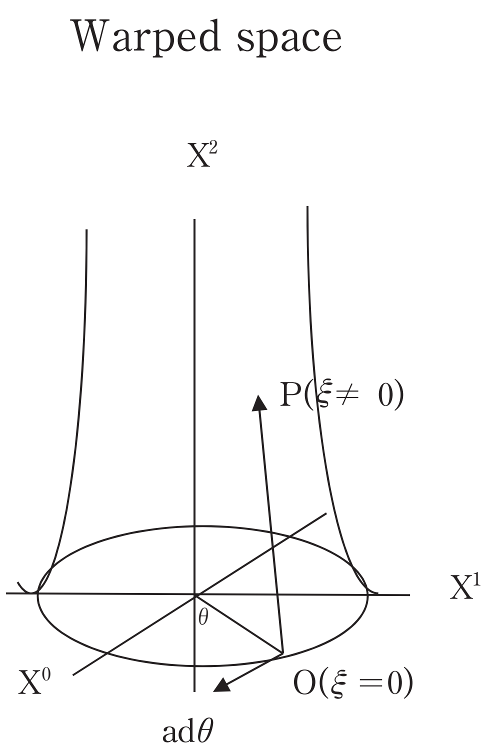

Fig.1. Warped space embedded, for illustrative purpose, in a three dimensional reduced Euclidean space. In the case of constant curvature $-(1/a^2)$, an infinitesimal line element in the space is

$$ds^2 = e^{(-2\xi/a)}(dX^{02} + dX^{12}) + d\xi^2,$$

where $(dX^0\ \ dX^1)$ are the components of ad θ in the $(X^0\ \ X^1)$ plane.[12)]

$$\mu^2 = e^{(-2\xi/a)}(q^{02} - q^{i2}) - q^{\xi 2}, \tag{4}$$

where q^0 is the energy, q^i , $i = 1,\ 2,\ 3$, three momentums and q^{ξ}, the momentum in ξ - direction of the particle (see Appendix A).

Since our model space is five dimensional, the charge-current density have five components which will be denoted by $(j^0 = \rho,\ j^i,\ j^{\xi})$ $i = 1,\ 2,\ 3$. When the coordinate system changes, the value of ρ changes, hence, it will be more convenient for our discussion to rewrite ρ in an invariant form irrespective of the coordinate system (recall proper time in special theory of relativity). Hence, we adopt "relativistic" invariant form

$$\| \rho \|^2 = g_{AB} j^A j^B,\ A,\ B = 0,\ 1,\ 2,\ 3,\ \xi, \tag{5}$$

as our charge density where (g_{AB}) is the metric tensor given by Eq.(2).

In Eq.(5), if one makes the coordinate transformation $(x^A) \to (x^{A'})$, one has

$$\| \rho \|^2 = g_{AB} j^A j^B = g_{A'B'} j^{A'} j^{B'}. \tag{6}$$

For an example, for up quarks, if (v^A) is a velocity of the particle and (q^A) , the corresponding momentum of the particle, since $\rho = 2e/3$, then one has

$$(j^A) = ((2e/3)\, v^A) = (2e/3)\,(q^A / \mu_u) \quad \text{and} \quad (j^{A'}) = ((2e/3)\, v^{A'}) = (2e/3)\,(q^{A'} / \mu_u), \tag{7}$$

where we put $\mu = \mu_u$, for up quarks. If one substitutes Eq.(7) in Eq.(6), one has

$${\mu_u}^2 = g_{AB}\, q^A\, q^B = g_{A'B'}\, q^{A'}\, q^{B'}, \tag{8}$$

which is rewritten from Eq.(2) as

$${\mu_u}^2 = e^{(-2\xi/a)}(q^{02} - q^{i2}) - q^{\xi 2} = e^{(-2\xi'/a)}(q^{0'2} - q^{i'2}) - q^{\xi' 2}. \tag{9}$$

Similarly one has the invariant forms,

for down quarks

$${\mu_d}^2 = e^{(-2\xi/a)}(q^{02} - q^{i2}) - q^{\xi 2}, \tag{10}$$

and for leptons

$${\mu_1}^2 = e^{(-2\xi/a)}(q^{02} - q^{i2}) - q^{\xi 2}. \tag{11}$$

3. Semi-empirical mass formula

In our following discussion, we take the simplified physical picture in the classical mechanics in order to make the underlying essential points in our physics clearer though mathematical strictness somewhat fails.

In the conventional 4-dim gravitational theory the fundamental metric tensor $(g_{\lambda\mu})$, $\lambda,\ \mu = 0, 1, 2, 3$, plays the equivalent roles as the gravitational potential in Newtonian mechanical theory. This can be proved by comparing the equation of the geodesic curve which is the equation of motion of a free particle in the gravitational theory, with the equation of the motion of the particle in the gravitational potential in Newtonian mechanical theory.[7)]

For above reason, we regard the metric tensor $e^{(2\xi/a)}$ given in Eq.(2) as the potential in our warped model space.

Our scenario is the following; A particle which has already acquired mass μ through the coupling to Higgs particle, propagates in the extra dimension and is absorbed into the potential $e^{(\xi/a)}$ (we do not discuss about the mechanism of the mass generation of the particle). We use the complex potential, i.e. the complex warp factor to represent the absorption of a particle into the potential. This is an analogy to the optical model in nuclear reaction theory.[8)] Hence we assume that whatever the matter is, distributes uniformly in the warped potential corresponding to an average single-particle potential in the nuclear theory. Furthermore, our warped potential is not a microscopic object and huge compared with one used in the nuclear theory so that the energy levels of the particle in the warped potential will be narrow and dense. Hence, one will need not necessarily to apply the quantum mechanics to discuss our problem. Moreover, since we are not dealing with high speed particle, it will be sufficient for our qualitative discussion here to use the elementary Newtonian mechanics.

In our case, the complex warped potential becomes the periodical one so that one may expect that the generation patterns in the elementary particle physics appear.

We assume that the curvature 1/a of our model space is a complex number.

$$1/a = \rho + i\sigma, \quad \rho,\ \sigma, \quad \text{real numbers.} \tag{12}$$

The complex potential is defined by

$$W = e^{(\xi/a)} = W_R + i W_I, \tag{13}$$

where $$W_R = e^{<\rho/\sigma>\Lambda}\cos\Lambda \quad \text{and} \quad W_I = e^{<\rho/\sigma>\Lambda}\sin\Lambda \quad \text{with } \Lambda = (\sigma\,\xi). \tag{14}$$

At rest frame, namely, $q^i = 0, \quad i = 1,\ 2,\ 3,$ and $q^\xi = 0$, the energy of the particle is, from Eq.(4)

$$q^0 = \mu W. \tag{15}$$

It should be noticed that q^0 is a complex number. A particle absorbed into the potential due to the attractive part, *i. e.* the negative parts of W_R, falls down toward lower energy region and acquires the equal amount of energy used to drop down. Hence the particle gains the maximal energy at the bottom of the valleys of W_R . These positions on ξ -axis are determined by the solutions of the following equations

$$\partial_\Lambda W_R = 0 \quad \text{and} \quad \partial^2{}_\Lambda W_R > 0, \tag{16}$$

from which one obtains

$$\Lambda = (2n-1)\pi + \delta, \quad n = 1,\ 2,\ 3,\ \cdots, \tag{17}$$

where $\sin\delta = \rho/\sqrt{\rho^2+\sigma^2}$.

If one substitutes Λ given by Eq.(17) in W_R given by Eq.(14), one obtains the minimal values of W_R,

$$W_R = (-)\Gamma e^{<\rho/\sigma>(2n-1)\pi}, \quad n = 1,\ 2,\ 3,\ \cdots, \tag{18}$$

where $\Gamma = e^{<\rho/\sigma>\delta}\cos\delta$.

Since the bottom of the valleys of W_R are the most stable places for particle to stay, we identify the energy of the particle situated at that place with the "physical mass" . Hence, the magnitudes of the physical mass are given by

$$m_n = |\mu W_R| = \mu\Gamma e^{<\rho/\sigma>(2n-1)\pi}, \quad n = 1,\ 2,\ 3,\ \cdots. \tag{19}$$

(see Appendix B)

Taking logarithm of both sides of Eq.(19), one obtains

$$\log_e m_n = \log_e(\mu\Gamma) + <\rho/\sigma>(2n-1)\pi, \quad n = 1,\ 2,\ 3,\ \cdots. \tag{20}$$

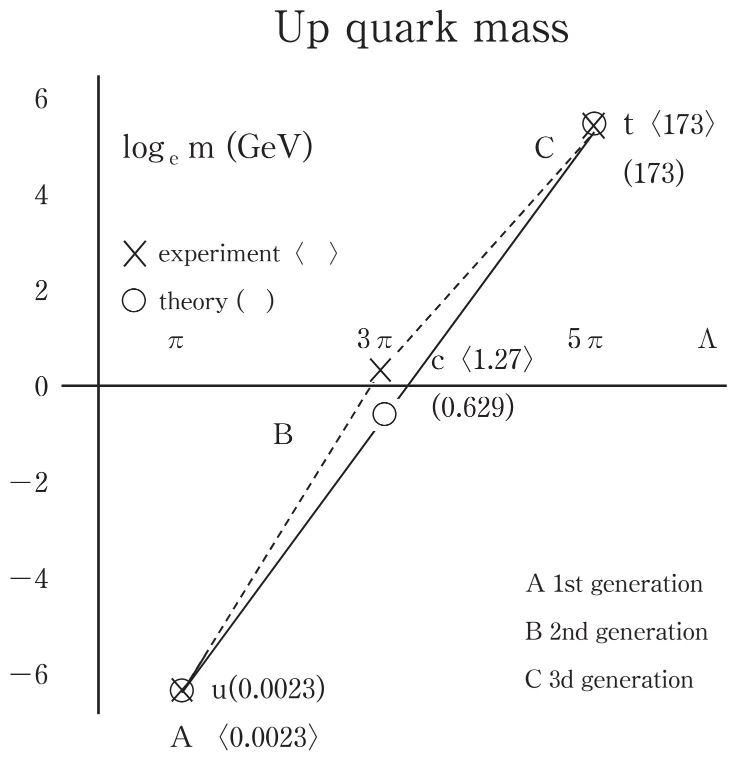

Fig.2. m_c is estimated from the experimental values of m_u and m_t by using the simplified mass formula $(T-1)$ in Table.

In the theory of 1)~ 6), the physical mass m^{phys} is derived from the mass eigen value m of Kaluza-Klein reduction equation, by rescaling due to the warped factor, i.e. the metric of the form $e^{(k(s-si))}$ where s is a location of an observer and s_i , a singularity in the extra dimension,. Thus one has

$$m^{phys} = e^{(k(s-si))}\, m,$$

where k is a curvature and s_i is an arbitrary parameter of the model.

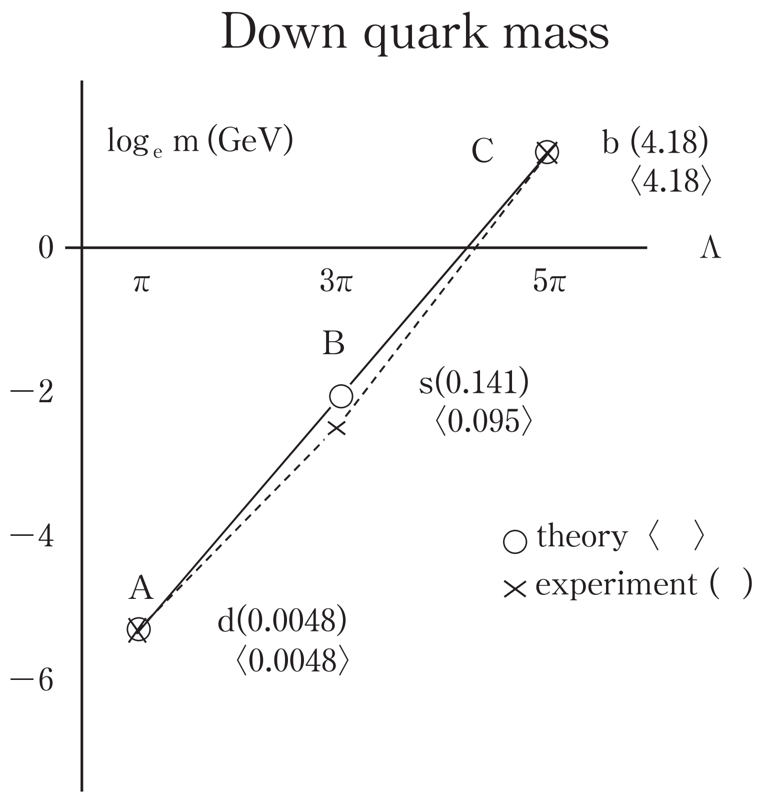

Fig.3. m_s is estimated from m_d and m_b by using the simplified mass formula $(T-1)$ in Table.

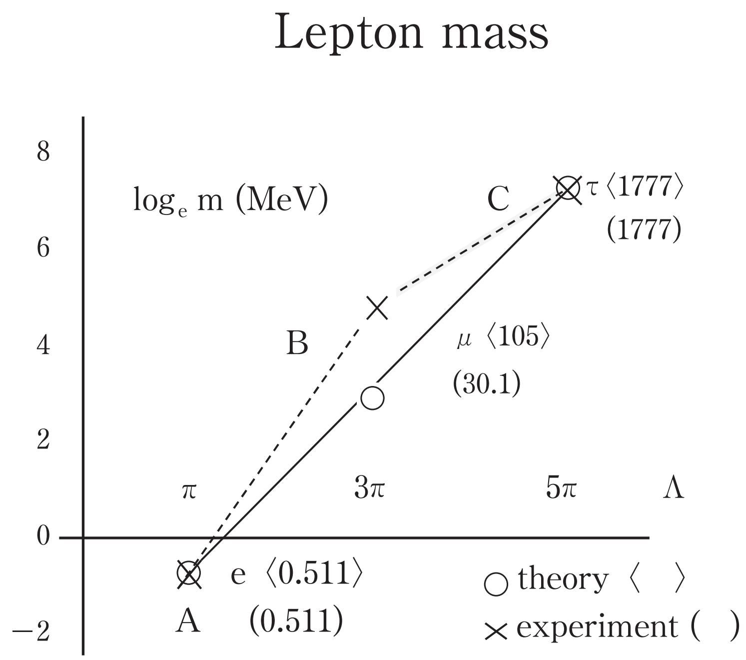

Fig.4. m_μ is estimated from m_e and m_τ by using the simplified mass formula $(T-1)$ in Table.

The numerical calculation of W_R, for example, for down quarks will be given in Fig.5, where the comparison of the predicted mass values with the experimental ones is made.

If one identifies the n with generation number and plots the logarithmic mass values against n , the logarithmic mass values must lie on a straight line.

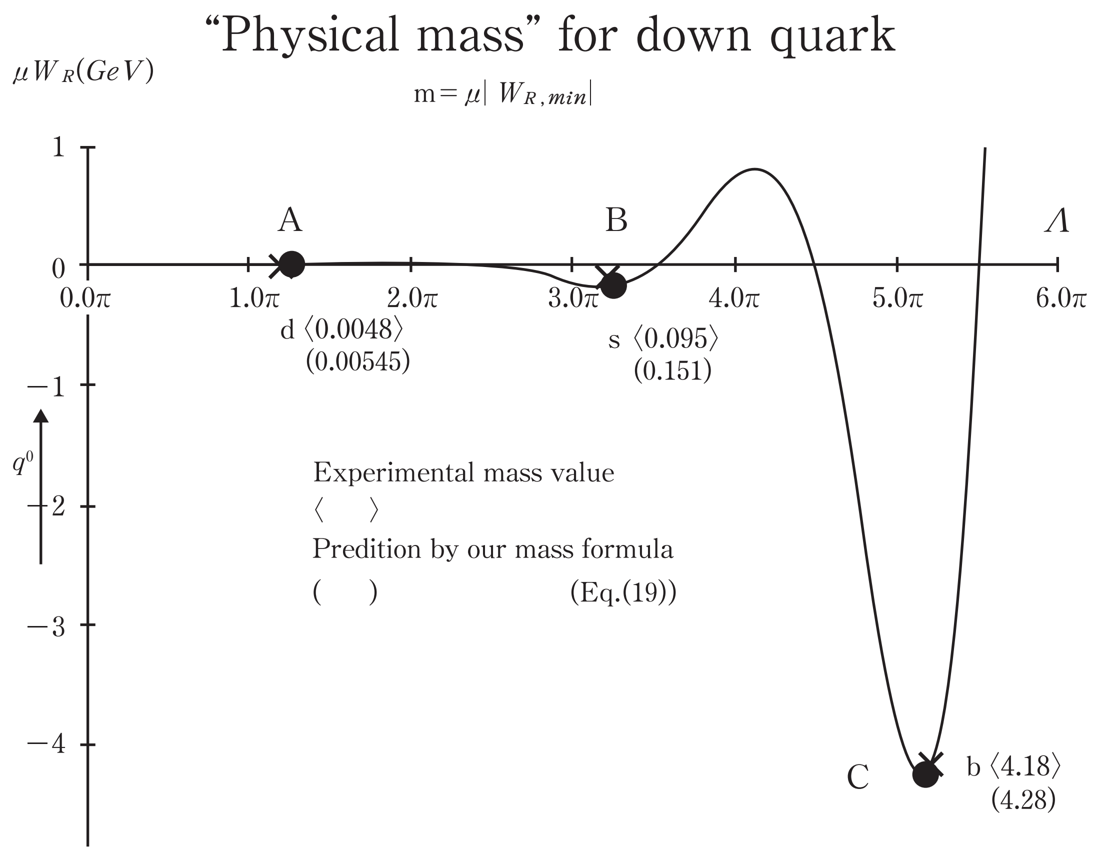

Fig.5. Warped potential $W_R = e^{<\rho/\sigma>\Lambda}\cos\Lambda$ with $\Lambda = (\sigma\xi)$ and the parameters used are $\mu = 0.0008838\, GeV$ and $<\rho/\sigma>= 0.5389$. The d, s, and b quarks are situated at bottoms (with negative sign) of the valleys of the Warped potential.

These are shown in Fig.2, Fig.3 and Fig.4 and the values of parameters used are listed in Table. In the case of the leptons $(e,\ \mu,\ \tau)$ except for the neutrinos, the deviation of the predicted value of μ (particle) from the experimental one is considerably large. The deviation of the experimental mass value of μ from the straight line of Eq.(20) implies the break of the invariance of Eq.(11) (or Eq.(6)); the energy difference between e and τ is about 1777 eV whereas the deviation of the experimental mass value of μ from our predicted one is about 75 eV , hence the ratio 75/1777 ~ 0.04 and from this result, one may say that the break of the invariance of Eq.(11) is about 0.04. Similarly, for the down quark, the break is about 0.01 and for up quark, about 0.001.

4. Hierarchy for the fermion masses

-----the interpretation of its structure in terms of our warped model-----

Our model has the space-time structure in which a charge density is unchanged under the coordinate transformation in our model space. In the space-time with such symmetry, law of physics are the same to all observers living in it and in particular, the form of fundamental equation must not be changed under the transformation which connects any two points in the model space.

For example, we here restrict to the case of the down quarks. In Fig.1, one denotes by $(q^0 q^i q^\xi)$, $i = 1,\ 2,\ 3$, the energy-momentum of a particle at $O(\xi = 0)$ and by $(q^{0'} q^{i'} q^{\xi'})$, $i = 1,\ 2,\ 3$, those at $P(\xi \neq 0)$. One assumes that $q^\xi = 0$ at $O(\xi = 0)$ and $q^{\xi'} = 0$ at $P(\xi \neq 0)$, that is, the particle is at rest at both points on the ξ coordinate axis . Under the above restriction, one can write from the invariance of Eq.(10)

$$q^{02}-q^{i2}=e^{(-2\xi/a)}(q^{0'2}-q^{i'2}), \tag{21}$$

which, with an appropriate choice of our coordinate axes, reduces to the rescaling

$$q^0=e^{(-\xi/a)}q^{0'}, \tag{22}$$

$$q^i=e^{(-\xi/a)}q^{i'}, \quad i=1,\ 2,\ 3. \tag{23}$$

The basic equation of a fermion at $O\,(\xi=0)$ is written as (see Eq.(32))

$$(\gamma^0 q^0-\gamma^i q^i-\mu_d)\,\phi\,(x)=0, \tag{24}$$

If one substitutes Eq.(22) and Eq.(23) in Eq.(24),one obtains the equation of the particle at $P\,(\xi\neq 0)$

$$(\gamma^0 q^{0'}-\gamma^i q^{i'}-\mu_d\,e^{(\xi/a)})\,\phi\,(x)=0. \tag{25}$$

The comparison of Eq.(24) with Eq.(25) shows that an observer at $P\,(\xi\neq 0)$ measures the mass μ_d at $O\,(\xi=0)$ as $\mu_d\,e^{(\xi/a)}$.

Fig.5 shows the phenomenological warped potential for down quarks, The mass of μ_d at $\xi=0$ is 0.0008838 GeV . An observer at A (1st generation) measures μ_d as $\mu_d\,e^{0.5324x\pi}=0.0054\,GeV$, an observer at B (2nd generation) measures μ_d as $\mu_d\,e^{0.5324x3\pi}=0.150\,GeV$ and an observer at C (3d generation) measures μ_d as $\mu_d\,e^{0.5324x5\pi}=4.28\,GeV$.

According to the standard model of electroweak theory, a fermion acquires its mass through Yukawa coupling to Higgs particle.[9)]

For down quarks, for example, the masses of the fermions are given by

$$m_h=g_h v \quad or \quad g_h=m_h/v, \quad h=d,\ s,\ b, \tag{26}$$

where g_h are the respective coupling constants of particle h to Higgs one and v is a vacuum expectation value of the Higgs field.

In our model, from Eq.(19), one has

$$m_d=ge^{<\rho/\sigma>\pi}\,v, \quad m_s=ge^{<\rho/\sigma>3\pi}\,v, \quad m_b=ge^{<\rho/\sigma>5\pi}\,v, \tag{27}$$

where $g=\mu_d\varGamma_d/v$ is a coupling constant.

These consequences seem to suggest a universality of coupling constant.

The particle μ is not present in the standard model. This is somewhat defective and so we give the somewhat modified scenario to remedy this. Here, we restrict to the case for the down quarks. New scenario is that, as for the masses $(m_d,\ m_s,\ m_b)$, firstly, the mass of the down quark m_d (instead of μ) is generated through Yukawa coupling to Higgs particle and next, the remaining two masses $(m_s,\ m_b)$ are enlarged from m_d due to the respective rescaling factors.

Various other possibilities to remedy this will be considered.

5. The origin of non-existence of a particle with positive helicity and an antiparticle with negative one

In the conventional theory, for the specific representation of γ -matrices

$$\gamma^0 = \begin{vmatrix} 0 & I \\ I & 0 \end{vmatrix}, \quad \gamma^i = \begin{vmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{vmatrix}, \quad i = 1,\ 2,\ 3, \tag{28}$$

where σ^i, $i = 1,\ 2,\ 3,$ are Pauli matrices and I, unit 2x2 matrix, the massless Dirac equation takes forms

$$(E - (\sigma p))\, u = 0, \tag{29}$$
$$(E + (\sigma p))\, v = 0, \tag{30}$$

where u represents a particle with positive helicity or an antiparticle with negative one and v represents a particle with negative helicity or an antiparticle with positive one.

Since Eq.(29) and Eq.(30) are independent of each other, if the equation of motion are invariant under space reflection, both solutions of $u \neq 0,\ v = 0$ and $u = 0,\ v \neq 0$ must exist. However, it is known from the phenomenology of neutrino physics that the invariance under space reflection breaks down. It is likely that nature needs only solution $u = 0,\ v \neq 0$ but not $u \neq 0$, $v = 0$.

We discuss this in our warped model. For that purpose, one has to derive Dirac equation in our model space. However, for simplicity, we assume that the curve is loose, i.e. 1/a $\ll 1$ so that the spin connection can be ignored and the covariant derivative may be replaced by the ordinary one. (see Appendix A and Appendix C). One writes Eq.(4) in the squared form of linear combination of q^0, q^i, $i = 1,\ 2,\ 3$ and q^ξ

$$\begin{aligned} \mu^2 &= e^{(-2\xi/a)}(q^{02} - q^{i2}) - q^{\xi 2} \\ &= (e^{(-\xi/a)}(\gamma^0 q^0 - \gamma^i q^i) - \gamma^\xi q^\xi)^2 . \end{aligned} \tag{31}$$

For simplicity, in derivation of Eq.(31), the approximation $[q^\xi,\ e^{(\xi/a)}] = 0$ is made.
Thus one obtains Dirac equation in our model space

$$(\gamma^0 q^0 - \gamma^i q^i - e^{(\xi/a)} \gamma^\xi q^\xi - e^{(\xi/a)} \mu)\, \varphi(x^0 x^i \xi) = 0, \tag{32}$$

where $\varphi(x^0 x^i \xi)$ is the wave function.

In Eq.(31), γ^ξ is newly introduced γ -matrix in our model space and γ^0, γ^i, $i = 1,\ 2,\ 3$ and γ^ξ must satisfy ant-commutation relations

$$\{\gamma^\alpha \gamma^\beta\} = 2\eta^{\alpha\beta}, \quad \alpha,\ \beta = 0,\ 1,\ 2,\ 3,\ \xi, \tag{33}$$

where $(\eta^{\alpha\beta})\alpha,\ \beta = 0,\ 1,\ 2,\ 3,\ \xi$ is metric tensor with signature $(+----)$ in five dimensional Minkowski space. In the following discussion, we use the explicit representations of γ -matrices given by Eq.(28), As for γ^ξ , one has to seek a 4x4 traceless matrix satisfying anti-commutation relation given by Eq.(33). It is well known that $\gamma^5 = -i\gamma^1\gamma^2\gamma^3\gamma^0$ is such a matrix.

However one cannot use it as our γ^{ξ} -matrix because in order that q^0 be energy γ^{ξ} must be anti-hermitian. Hence, one may put

$$\gamma^{\xi} = i\gamma^5, \tag{34}$$

and adopt it as our γ^{ξ} - matrix. Using the representations given by Eq.(28),from Eq.(34), one obtains the explicit representation of γ^{ξ} -matrix

$$\gamma^{\xi} = \begin{vmatrix} iI & 0 \\ 0 & -iI \end{vmatrix}. \tag{35}$$

If one uses the explicit representation of Eq.(28) and Eq.(35),one obtains from Eq.(32)

$$\begin{vmatrix} -e^{(\xi/a)}(\mu + iq^{\xi}) & q^0 + (\sigma q) \\ q^0 - (\sigma q) & -e^{(\xi/a)}(\mu - iq^{\xi}) \end{vmatrix} \varphi(x^0 x^i \xi) = 0. \tag{36}$$

After quantization , namely, making the following replacements in Eq.(36)

$$q^0 \to i\partial^0, \quad q^i \to -i\partial^i, \quad i = 1,\ 2,\ 3, \text{ and } q^{\xi} \to -i\partial^{\xi}, \tag{37}$$

if one puts

$$\varphi(x^0 x^i \xi) = \begin{vmatrix} u_A \\ u_B \end{vmatrix} \exp(-i(q^0 x^0 - q^i x^i)\phi(\xi)), \tag{38}$$

where u_A and u_B are the spin states of the particle, and substitutes Eq.(38) in the resultant equation, one obtains from Eq.(36)

$$(\mu + \partial^{\xi})\phi(\xi) u_A = e^{(-\xi/a)}(q^0 + (\sigma q))\phi(\xi) u_B, \tag{39}$$

$$(\mu - \partial^{\xi})\phi(\xi) u_B = e^{(-\xi/a)}(q^0 - (\sigma q))\phi(\xi) u_A, \tag{40}$$

We examine asymptotic behavior of $\phi(\xi)$ for large ξ . Since wave function must be $|\phi| \leqq 1$, and since the right hand side of Eq.(39) (and Eq.(40)) approaches to zero due to presence of $e^{(-\xi/a)}$ when ξ goes to infinity, as for Eq.(39), from $(\mu + \partial^{\xi})\phi(\xi) = 0$ in the left hand side of Eq.(39), one obtains $\phi(\xi) = Ae^{(-\mu\xi)}$ which remains finite for large ξ . On the other hand, as for Eq.(40), from $(\mu - \partial^{\xi})\phi(\xi) = 0$, one obtains $\phi(\xi) = Be^{(\mu\xi)}$ and hence ,in this case, the wave function $\phi(\xi)$ goes to infinity as ξ increases . This is not physically acceptable. Hence, Eq.(40) is excluded. To say more precisely, in the case of $\phi(\xi) = Be^{(\mu\xi)}$, the normalization of ϕ is given by

$$\int |\phi|^2 \sqrt{g}\, d\xi = B^2 \int e^{2(\mu - 2/a)\xi} d\xi, \tag{41}$$

where $\sqrt{g} = e^{(-4\xi/a)}$. Hence, if $\mu > 2/a$, the normalization of the wave function is impossible.

In conclusion, if a fermion has a mass, then the fermion obeying Eq.(40) which corresponds to Eq.(29), are not allowed to exist.

It should be noticed that the helicity for a massive particle is not the relativistic invariant concept which makes the above conclusion obscure.

In the case of $\underline{\gamma}^{\xi}=-\gamma^{\xi}$ satisfying the same anti-commutation relation (Eq.(33)) as γ^{ξ}, if one puts $\xi'=-\xi$, one has $e^{(-\xi/a)}\underline{\gamma}^{\xi}\partial_{\xi}=e^{(\xi'/a)}\gamma^{\xi'}\partial_{\xi'}$ and $\mu e^{(-\xi'/a)}=\mu e^{(\xi'/a)}$, thus, in addition to $\partial^{\xi}\to-\partial^{\xi'}$, the metric changes from $e^{(-\xi/a)}$ to $e^{(\xi'/a)}$ in Eq.(39) and in Eq.(40).

Recently It has been confirmed that a neutrino has small mass.[10] As a consequence a neutrino has only ν_L and $\overline{\nu}_R$ states.

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Appendix A

The equation for scalar field $\phi(x^0 x^{\xi})$

Action is

$$I=\int\sqrt{g}\;\;dv\;\;L, \tag{A-1}$$

where $dv=dx^0dxd\xi$, $\sqrt{g}=e^{(-4\xi/a)}$ and L is the Lagrangean density for scalar field

$$L=(1/2)(g_{AB}(\partial^A\phi)(\partial^B\phi)-\mu^2\phi^2). \tag{A-2}$$

The variation of I with respect to ϕ under the assumption $\delta\phi=0$ at the boundary in the partial differentiation, yields the equation for the scalar field

$$\partial^B(\sqrt{g}\,g_{AB}(\partial^A\phi))+\mu^2\phi=0. \tag{A-3}$$

For the explicit representation of Eq.(2), Eq.(A-3) becomes

$$(\partial^{02} - \partial^{i2} - e^{(2\xi/a)}(-\mu^2 + \nabla^{\xi 2}))\,\phi(x^0 x^i \xi) = 0, \tag{A-4}$$

where $$\nabla^{\xi 2} = (\partial^{\xi} - 4/a)\,\partial^{\xi}. \tag{A-5}$$

If one makes the separation of variables, $\phi(x^0 x\, \xi) = \phi(x^0)\,\phi(x)\,\phi(\xi)$ and puts it into Eq.(A-4), one obtains

$$(\partial^{02} + q^{02})\,\phi_{q^0}(x^0) = 0, \tag{A-6}$$

$$(\partial^{i2} + q^{i2})\,\phi_{q^i}(x^i) = 0, \quad i = 1,\ 2,\ 3, \tag{A-7}$$

and $$(\nabla^{\xi 2} - \mu^2 + e^{(-2\xi/a)}(q^{02} - q^{i2}))\,\phi(\xi) = 0. \tag{A-8}$$

If one makes the following approximation and subsequent replacement in Eq.(A-8)

$$\nabla^{\xi 2} = (\partial^{\xi} - 4/a)\,\partial^{\xi} \to \partial^{\xi 2} \text{ and } \varphi(\xi) = N\exp(-iq^{\xi}\,\xi), \tag{A-9}$$

one obtains Eq.(4).

However, Eq.(4) is not an approximate but correct formula.

Appendix B

Warped potential is defined by

$$W = e^{(\xi/a)}, \quad = W_R + iW_I, \quad \text{with} \quad 1/a = \rho + i\sigma, \quad \rho,\ \sigma \text{ real numbers} \tag{B-1}$$

where $$W_R = e^{<\rho/\sigma>\Lambda}\cos\Lambda \quad \text{and} \quad W_I = e^{<\rho/\sigma>\Lambda}\sin\Lambda \quad \text{with } \Lambda = (\sigma\,\xi). \tag{B-2}$$

The derivatives of W_R and W_I with respect to Λ are respectively given by

$$\partial_{\Lambda} W_R = -Ke^{<\rho/\sigma>X}\sin X \quad \text{and} \quad \partial_{\Lambda} W_I = Ke^{<\rho/\sigma>X}\cos X, \tag{B-3}$$

where $$X = \Lambda - \delta, \quad K = e^{<\rho/\sigma>\delta}/\cos\delta \quad \text{and} \quad \sin\delta = \rho\sqrt{\rho^2 + \sigma^2}. \tag{B-4}$$

One obtains from $\partial_{\Lambda} W_R = 0$

$$X = n\pi, \quad n = 0,\ 1,\ 2,\ 3,\ \cdots, \tag{B-5}$$

which gives

maximum values of $$W_R = \Gamma e^{<\rho/\sigma>2(n-1)\pi}, \quad n = 1,\ 2,\ 3,\ \cdots, \tag{B-6}$$

and minimum values of $$W_R = -\Gamma e^{<\rho/\sigma>2(n-1)\pi}, \quad n = 1,\ 2,\ 3,\ \cdots, \tag{B-7}$$

where $$\Gamma = e^{<\rho/\sigma>\delta}\cos\delta. \tag{B-8}$$

From these consequences, one obtains the physical mass

$$m_a = \mu_a \Gamma e^{<\rho/\sigma>a(2n-1)\pi}, \quad n = 1,\ 2,\ 3,\ \cdots. \tag{B-9}$$

a = lepton, up quark and down one.

Appendix C

The equation of fermion in our model space
Lagrangean is given by

$$L = \sqrt{g}\,(i\,\overline{\varphi}\,\Gamma^A D_A \varphi - \mu\,\overline{\varphi}\,\varphi), \quad A = 0,\ 1,\ 2,\ 3,\ \xi. \tag{C-1}$$

The variation of L with respect to $\overline{\phi}$ gives the equation

$$(i\Gamma^A D_A - \mu)\,\phi = 0, \tag{C-2}$$

where Γ^A is the γ -matrix for our curved space satisfying

$$\{\Gamma^A \Gamma^B\} = 2g^{AB}, \quad A,\ B = 0,\ 1,\ 2,\ 3,\ \xi, \tag{C-3}$$

where (g^{AB}) is the contravariant metric tensor of our model space and Γ^A is related to the flat γ^A -matrix

$$\Gamma^\mu = e^{(\xi/a)}\gamma^\mu, \quad \mu = 0,\ 1,\ 2,\ 3 \quad \text{and} \quad \Gamma^\xi = \gamma^\xi. \tag{C-4}$$

(γ^A) satisfy the anti-commutation relation

$$\{\gamma^A \gamma^B\} = 2\eta^{AB}, \tag{C-5}$$

where (η^{AB}) is the metric tensor for five dimensional Minkowsky space with signature $(+----)$.

The spin connection in our model space is given by [6),11)]

$$D_\mu = \partial_\mu + (1/4)\,e^{(2\xi/a)}\,\partial_\xi\, e^{(-2\xi/a)} = \partial_\mu - (1/2a), \quad \mu = 0,\ 1,\ 2,\ 3 \quad \text{and} \quad D_\xi = \partial_\xi. \tag{C-6}$$

Therefore, if one makes the approximation $1/a \cong 0$, then one has $D_\mu \cong \partial_\mu$ and $D_\xi = \partial_\xi$. Applying these results to Eq.(C-2), Eq.(39) and Eq.(40) may be obtained from Eq.(C-2) by using the explicit representation of Eq.(28) and Eq.(35).

Table

For simplicity, the simplified mass formula

$$m_a = \mu_a e^{<\rho/\sigma>a(2n-1)\pi}, \quad n = 1,\ 2,\ 3, \tag{T-1}$$

a = lepton, up quark and down quark.

is used to obtain the following parameters.

The formula $(T-1)$ is obtained if one puts $\delta = 0$ (shift parameter) and hence $\Gamma = 1$ in Eq.(20)(or Eq.(19)).

Experimental mass values

lepton	up quark	down quark
$m_e = 0.510\,(MeV)$	$m_\mu = 0.0023\,(GeV)$	$m_d = 0.0048\,(GeV)$
$m_\mu = 105.6$	$m_c = 1.275$	$m_s = 0.095$
$m_\tau = 1776$	$m_t = 173$	$m_b = 4.18$

(Particle Data Group, Phys. Rev.D58. (2012)).

Parameters

$$\mu_e = \sqrt{m_e m_\tau}\ \ e^{(-3\pi<\rho/\sigma>e)}, \qquad <\rho/\sigma>_e = (1/4\pi)\log(m_\tau/m_e).$$
$$\mu_u = \sqrt{m_u m_\tau}\ \ e^{(-3\pi<\rho/\sigma>u)}, \qquad <\rho/\sigma>_u = (1/4\pi)\log(m_t/m_u).$$
$$\mu_d = \sqrt{m_d m_b}\ \ e^{(-3\pi<\rho/\sigma>d)}, \qquad <\rho/\sigma>_d = (1/4\pi)\log(m_b/m_d).$$

Numerical values

$\mu_e = 0.06656\,(MeV)$, $\mu_u = 0.0001389\,(GeV)$, $\mu_d = 0.0008838\,(GeV)$,

$<\rho/\sigma>_e = 0.6491$, $<\rho/\sigma>_u = 0.8939$, $<\rho/\sigma>_d = 0.5389.(13)$.