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Wicksell's Indeterminacy, Disequilibrium Analysis, and Cumulative Process

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Tomoki HIRASE

《**Abstract**》

This paper is devoted to the examination of two widely accepted understandings of Wicksellian cumulative process, i.e. Wicksell's indeterminacy and disequilibrium analysis. It should be noted that these two understandings are generally based on Myrdal's or Patinkin's interpretations, not Wicksell's descriptions, because it is difficult to grasp Wicksell's thought from his vague, unclear, and confusing statements on his works. Thus, in this paper, these two understandings are examined based on Wicksell's descriptions. In particular we shall deal only with cumulative process on the assumption of the stationary state in Wicksell (1936) while the main target of other historians of economic thought has been to elucidate Wicksell's thought from all Wicksell's works. This is because the natural rate of interest remains fixed in the stationary state, making it easy to point out why the process is cumulative. Moreover, after the essential theoretical characteristics of trade, price expectations, and price determination on cumulative process have been discussed, we examine whether the two understandings are found in cumulative process. As a result, it was discovered that there is a large difference between these two understandings and Wicksellian cumulative process.

1. Introduction

This paper examines interpretations of studies on the cumulative process proposed by Knut Wicksell. Wicksell studied "fluctuations in the price level," a topic we share, from an original point of view described below. His theory is markedly characterized by: 1. the assumption of endogenous money supply (wherein a central bank is supposed to use not the quantity of money but the level of interest rate, i.e., the money rate of interest, as a control variable), 2. the idea that economic changes, particularly fluctuations in the price level, are not circular but cumulative, moving upward or downward (cumulative process), and 3. an emphasis on explicit optimizing entrepreneurs (micro-foundation).[1] With regard to point 1, Woodford (2003) highly regards Wicksell's argument linking fluctuations in the price level and an interest-rate policy, considering it a revolutionary turn in the history of economic thought.[2] The idea in point 2 exerted a great influence on subsequent research on inflation. For instance, Hicks (1982), Iwai (1981), and Morishima (1992) all refer to the cumulative process. Point 3 is conceivably of particular importance to us. It is certain that numerous studies have been conducted on Wicksell's cumulative process from the perspective of old Keynesian macroeconomics by now. Many commentators with an interest in Keynesian economics wanted to regard Wicksell's studies as pioneers of Keynesian revolution without examining the structure of Wicksell's theory in detail.[3] But it is a well-known fact that Lucas' critique in the mid-1970s swept away almost all old

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1) Hirase (2006) presents Wicksell's theory of cumulative process as a dynamic model based on optimization.

2) Woodford (2003), p.49.

3) What is called the Wicksell connection debate can be cited as a recent example.

Keynesian macroeconomics (which are not based on optimization) and macroeconomic models and relevant arguments have since been based on optimization. Therefore, a contemporary evaluation of Wicksell's work should not be one that has the Keynesian Revolution in mind as mentioned above – at least, for us who live in the period after the Lucas critique. In fact, many interesting issues for modern macroeconomics, which are not found even in the work of Keynes (1936) and his successors, are involved in Wicksell's work.[4] So, we think that it is a mission given to a contemporary historian of economic thought to elucidate the theoretical structure of the cumulative process.

It is not, however, easy to understand what Wicksell really wanted to say; for Wicksell's description is so vague, unclear, and confusing that we must give the interpretation of Wicksell's analysis very careful attention. Siven explained this as follows:

It is not always easy to find the most sensible interpretation of Wicksell's theory of money. Contrary to wicksell's theory of value and capital, his representation of the theory of money was merely verbal. This makes it difficult to evaluate the level of his discussion: In some places it seems to

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4) The original point of view presented by Wicksell, along with an awareness of problems with the quantity theory of money, a transmission mechanism of monetary policy, was developed by his successors – including Lindahl and Myrdal – in a variety of ways. A series of studies by Wicksell and his successors eventually came to be recognized as the Stockholm School. As the Stockholm School persisted in overly heterodoxical assumptions, however, there was never any interaction with neoclassical economists. This is in contrast with Keynes (1936), who found the same problems with the Stockholm School and managed to establish a theory going against the quantity theory of money as one field of economics in a manner likely to be acceptable to neoclassical economists by deliberately adopting such assumptions as exogenous money supply. For example, Ohlin (1978) recalls the time when he met Keynes in Belgium in 1935: "I made a brief outline of our reasoning with cumulative processes and employ variations." Keynes' comment was that this sounded very much like the kind of reasoning he himself had been working with a couple of years earlier. He believed, however, that he could convey the essential ideas in a simpler way [Ohlin (1978), p.147].

be pure theory, at other places empirical applications or pedagogical simplifications. His discussion is inconsistent. One can find different opinions at different places. It is thus a question of finding a sensible interpretation of what seems to be the theoretical hard core of his exposition.[5]

In fact, as described below, there are two different interpretations of the purpose of Wicksell's analysis, especially the cumulative process. As pointed out above, it is a well-known fact that many theoretical economists have recognized that the cumulative process is unstable and explosive. On the other hand, Patinkin insisted:

Thus Wicksell's "cumulative process" is not the unstable explosive process that almost all commentators have tried to make of it, but a stable equilibrating process whose function it is to achieve the long-run equality of the money and natural rate of interest.[6]

Though the issue of whether Patinkin's view is right or not is controversial, it is true that we can easily find contradictions in Wicksell's description of the cumulative process. So it is natural that, especially after the 1970's, Wicksell's original analysis of the cumulative process itself is seldom referred to, and many theoretical economists have regarded Wicksell's analysis as the stable equilibrium or disequilibrium analysis based on the clear interpretations of others, i.e. Myrdal (1939), Patinkin (1956) and so on, without reference to Wicksell's descriptions. This would be why a "modern" understanding of Wicksell's cumulative process has now been widely accepted. Clearly, however, their interpretations are wrong, because they try to discuss or formulate Wicksell's analysis on new

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5）Siven (1997), p.215.

6）Patinkin (1965), p.368

classical assumptions which are different from Wicksell's. For example, "Wicksell's indeterminacy" would be a more famous concept among the theorists interested in monetary economics than the "original" cumulative process now. Ever since Sargent and Wallace (1975) referred to a kind of nominal indeterminacy under the assumption of rational expectations by the term "Wicksell's indeterminacy," this concept has drawn the attention of theorists. For instance, in Blanchard and Fisher (1989), which has been a standard textbook on macroeconomics, Wicksell's indeterminacy is introduced as what Wicksell himself had studied, without reference to Wicksell's description. Wicksell's indeterminacy, however, means the indeterminacy of the price level arising under the assumption of rational expectations. So we can assert that Wicksell's original analysis does not have any relationship with Wicksell's indeterminacy. In addition, we can provide the other example discussed in this paper, which is related to the disequilibrium analysis. The misunderstanding here is that Wicksell's cumulative process should be regarded as the disequilibrium analysis. Iwai (1981) is a good example of that kind of misunderstanding. The cumulative process, however, is formulated on assumption of deviation, not the disequilibrium. It is easy to show that we have to distinguish the deviation from the disequilibrium, because the former does not need the assumption of disequilibrium in the market.[7] It is clear that such confusion could be conceivably caused by the failure of historians of economic thought to fulfill their responsibility to provide a correct interpretation of the theory of the cumulative process. So, in order to fulfill this responsibility, we are thus going to examine the cumulative process, bearing in mind its theoretical characteristics.

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7) This view is in accordance with that of Siven (1997).

Sections 2 and 3 present Wicksell's ambitious attempt and his analysis of the cumulative process, respectively.[8] Wicksell was strongly dissatisfied with the fact that the quantity theory of money, which is applicable only under limited conditions, was considered as if it were an omnipotent theory in economics at the time. He thus distanced himself from the quantity theory of money and made an attempt at theoretically analyzing the quantity of money and fluctuations in the price level from a new perspective of the relative level of the money rate of interest. Section 4 examines the meaning of the assumptions, a static expectation and a stationary state in the cumulative process. As a result, it can be pointed out that Wicksell's original theory is based on perfect competition – that is, flexible pricing – so, as pointed out above, interpretations such as Iwai's (1981) would be incorrect. Section 5 explains the concept of Wicksell's indeterminacy, established by Sargent and Wallace (1975), in detail. It should not be surprising that Sargent and Wallace (1975) misunderstood Wicksell's theory, because they did not refer to Wicksell's description at all.

2. On Wicksell's View of Problems

In this section, let us clarify what Wicksell, who produced the theory of the cumulative process, considered a problem.[9] Wicksell had a strong dissatisfaction with too much neglect of such a limitation of the quantity theory of money by its believers that it works only under the limited condition (of a constant velocity of circulation).[10] Wicksell's view was that;

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8) Unless otherwise indicated, we shall deal with the discussion in Wicksell (1936), where it is assumed that real sector is in a stationary state and the natural rate of interest remains fixed even during the cumulative process.

9) Major studies on Wicksell include Davidson (1899), Patinkin (1956), and Uhr (1962). Gårdlund (1996) can be cited as his bibliography.

[the Quantity Theory] assumes that everybody maintains, or at least strives to maintain, his balance at an average level that is constant (relatively to the extent of his business or of his payments).[11]

Of course, if one follows such a view, fluctuations in the price level have to be explained not by the quantity theory of money but by another theory. He thus paid attention to the work of the Banking School, who, like him, took a critical position against the quantity theory of money, which is based on the assumption of exogenous money supply. He thought that the theory they advocated based on the assumption of endogenous money supply was appropriate for analyzing fluctuations in the price level (Ibid, p.87).

Wicksell further proposed to limit the scope of analysis to a pure credit economy in order to perform an analysis under the assumption of an endogenous money supply without being bothered by physical restrictions such as a gold-standard system. This pure credit economy refers to an economy in which all payments are effected not by notes or coins but only by bookkeeping transfers, that is, only by transactions through credit currency (endogenous money supply) (*Ibid,* p.70). It is thus considered always possible for the policy authorities in the pure credit economy to meet all monetary demands. That is, for the policy authorities in the pure credit economy, it is the money rate of interest that is a control variable, and the quantity of money is an endogenous variable determined within an economy.

How, then, did Wicksell think the price level would be determined under the assumption of the pure credit economy? What came to his notice was the empirical fact that "[a] low rate of interest is by no means always

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10）The historical context is discussed by Boianovsky and Trautwein (2001).

11）Wicksell (1962), p.41

accompanied by high, or by rising, prices" and "[i]n fact the opposite is the general rule" *(Ibid,* p.88). On the basis of this empirical fact, Wicksell thought that fluctuations in the price level are caused by a deviation of the money rate of interest from a certain level. As regards this level, Wicksell notes,

> [a]t any moment and in every economic situation there is a certain level of the average rate of interest which is such that the general level of prices has no tendency to move either upwards or downwards. This we call the *normal* rate of interest.[12]

How, then, is this relative level of the money rate of interest defined? According to Wicksell, the relative magnitude of the money rate of interest is determined by the natural rate of interest – which has the three characteristics described below – and he calls the money rate of interest equal to this natural rate the normal rate of interest. Although Wicksell's explanation of the natural rate of interest is not necessarily clear, it is considered 1. to be equal to the marginal productivity of real capital; 2. to put supply and demand of savings in equilibrium, i.e., to stabilize the capital market; 3. to stabilize the price level.[13] For instance, suppose the relative level of the money rate of interest declines as a result of the rate reduction. In this case, investment will exceed savings, i.e., the total demand will exceed the total supply. Furthermore, due to the assumption of the pure credit economy, the supply of credit currency will increase, to the extent

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12) *Ibid,* p.120

13) It is well known that there has been much discussion regarding the definition of the natural rate of interest. Not only do its definitions in Wicksell (1962) and Wicksell (1978) differ, but Wicksell did not even clarify whether it is a nominal rate of return or real rate of return, to begin with. Nevertheless, we have to regard the natural rate of interest as a nominal rate of return insofar as we postulate the assumption of static expectations. This view also accords with that of Myrdal (1939).

that money demand exceeds savings. Since this increase in credit currency will result in increased expenditure by entrepreneurs, a rise in the price level is eventually expected to occur. (As will be clarified in the next section, this process is cumulative.) In this way, Wicksell started by breaking out of the quantity theoretic manner of thinking and managed to present a new perspective by comparing the money and natural rates of interest.

On the other hand, Wicksell was forced to deny the neoclassical theory itself, since he used the concept of the relative level of the money rate of interest; for, according to the neoclassical theory, monetary factors are not supposed to have any influence on real sectors. To begin with, even an increase in demand through divergence between the two interest rates is supposed to be impossible within the framework of identity version of Say's law. Therefore, Wicksell had to break out of the world of neoclassical theory itself in order to complete his theory, but it was not what Wicksell hoped – for his objective was a departure from the quantity-theoretic way of thinking and not the denial of neoclassical theory, i.e. the classical dichotomy.[14] Such persistence of Wicksell's in neoclassical is a point of major difference from his successors. Although the break from new-classical theory lay ahead of the new trial by Wicksell, he refused to accept it. For this reason, by postulating strong assumptions of a stationary state and one-sector model, Wicksell argued that monetary factors do not influence real sector "in effect"; that is, an increase in the quantity of money causes a rise in the price level only. Of course, it is clear that this is exactly what the quantity theory of money and neoclassical theory tells us.

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14) We shall disregard Wicksell's reference to changes in the production structure, since this reference is not about an essential factor in the process becoming cumulative, but only a note regarding the application of his theory to reality.

The addition of strong assumptions for the sake of staying within the framework of neoclassical thus greatly impaired the innovativeness of his theory, and Wicksell's ambitious attempt remained insufficient as a result.

3. What is a Cumulative Process?

This section takes a look at the cumulative fluctuations in the price level that occur when the money rate of interest deviates from the natural rate of interest – that is, the cumulative process. In particular, we would like to clarify the processes through which a cumulative process arises. Wicksell argues that when the money and natural rates of interest deviate from each other, the price level continues to fluctuate indefinitely (*Ibid*, p.95). For instance, suppose that the money rate of interest is set by the policy authorities at a lower level than the natural rate of interest. In this case, according to Wicksell, a cumulative rise in the price level occurs. First, since the deviation of the money rate from the natural rate of interest implies the emergence of a profit margin for entrepreneurs, they will try to extend their businesses. Concretely, rational entrepreneurs will try to expand the scale of their businesses until the money rate of interest at the new level coincides with their subjective rate of return. In addition, it should be noted that, since assumption of the pure credit economy is adopted here, credit will be supplied so as to meet all demand for credit that has emerged under the current level of the money rate of interest. Even if the entrepreneurs increase their expenditure for factors of production by using money thus procured, however, they cannot actually acquire new factors of production due to the assumption of stationary state. That is, even though they have increased the expenditure for factors of production, those entrepreneurs will not acquire new factors of production,

and a rise is caused instead in the prices of factors of production and consumption goods. Thus, the increase in their expenditure causes a rise in the prices of factors of production. Such a rise in the prices of factors of production implies an increase in income for workers and landlords, which is considered to result in their greater expenditure on consumption goods. Therefore, the prices of consumption goods that entrepreneurs produce will conceivably rise eventually. Incidentally, we can easily confirm that the rise in the price level described above occurs repeatedly, for a rise in the prices of entrepreneurs' consumption goods also means an increased rate of return of their businesses. The realized rate of return remains at the level different from the entrepreneurs' subjective rate of return, i.e., at the previous level equal to the natural rate of interest. This means not merely that entrepreneurs' expectations have been wrong, but also that their incentive for expanding the scale of business emerges once again. Wicksell describes this as follows:

> At the end of this period he will make a pleasant discovery that he can actually sell his goods at higher than the normal price.[15]

That is, the entrepreneurs' discovery of the emergence of a new profit margin between the money rate of interest and the natural rate of return conceivably generates an increase in new demand for money. They then start raising and spending money just as before. It is Wicksell's view that through the repetition of such a process, a rise in the price level becomes cumulative. Such a process is considered to continue until the money rate of interest is restored to the level of the natural rate of interest. Thus, to repeat, for the cumulative process to occur, it is necessary that the entrepreneurs' subjective rate of return does not coincide with the realized

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15) *Ibid.*, p.95

rate of return. This is made possible in Wicksell's theory owing to special assumptions, including those of static expectations and the stationary state.

What is important in the above cumulative process is that such fluctuations in the price level are, unlike the fluctuation in relative prices, not of the equilibrating nature. Wicksell compares the fluctuation and equilibrium of relative prices to the movement of a pendulum and fluctuations in the price level to the motion of a cylinder which has no tendency to be restored to its original position. In Wicksell's theory, since the rate of return and the natural rate of interest are constant due to the assumption of stationary state, the money rate of interest must be adjusted in order for the money and natural rates of interest to coincide. Unless these two rates of interest coincide, the process must be considered to repeat itself. Fluctuations of the price level in Wicksell's theory are, in this sense, of the cumulative nature.

4. Is the Cumulative Process a Disequilibrium Analysis?

We have thus looked at details of Wicksell's cumulative process above. Since many preceding studies on this cumulative process rely on the interpretations of the cumulative process in Myrdal (1939) or Patinkin (1965), this kind of inquiry can be said to be significant in itself. In order to understand a theory completely, however, one needs to know what assumptions it is based on. Thus, in an attempt to further develop our foregoing discussion, in this section we will investigate what assumptions make the cumulative process occur and last so long.[16]

First of all, let us consider the assumption concerning entrepreneurs'

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16) The arguments in this section are based on the model analysis in Hirase (2006).

expectation. Wicksell makes an assumption that entrepreneurs form static expectations about the prices of their products (consumption goods). Therefore, entrepreneurs calculate the *ex ante* rate of return for their investment, based on existing prices. Next, confirm that Wicksell's theory is fundamentally a one-sector model in which the production of only consumption goods takes place.[17] For this reason, we have to think that the expenditure by entrepreneurs does not imply the purchase of capital goods but the purchase of factors of production. As the assumption of stationary state mentioned in the previous section is postulated, however, even if entrepreneurs increase their expenditure for factors of production for the purpose of expanding the scale of their business, the increased expenditure causes a rise in the prices of factors of production. Furthermore, in Wicksell's theory, an emphasis is placed on the existence of a certain type of demand. That demand is derived from the expenditure for factors of production; namely, it is the expenditure for consumption goods by the owners of factors of production. Therefore, the change in relative prices caused by a rise in prices for factors of production is completely offset by the rise in the prices of consumption goods which is conceivably caused by such additional demand. In this way, the rate of return for business in Wicksell's theory is not affected by entrepreneurs' behaviors and remains at the level of the natural rate of interest. Hence, once the money rate of interest deviates from the natural rate of interest, unless the money rate of interest returns to the natural rate of interest, the deviation is never eliminated. Of course, it is presumably obvious from the discussions in the previous section that this deviation is the motive power of the cumulative

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17) The cumulative process is such a short-term phenomenon that makes impossible changes in a round-about production structure, i.e., changes in relative prices.

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process.

On the basis of the above arguments, let us examine one of the interpretations of the cumulative process, that the cumulative process is a disequilibrium analysis. This interpretation is widely accepted by the old Keynesians, who persist in the concept of *ex ante* disequilibrium, in which there is no equality between investment and saving, i.e., aggregate planned demand and aggregate planned supply. They emphasize only Wicksell's denial of identity of investment and saving and regard Wicksell as a pioneer of Keynesian revolution, without examining the structure of his theory in detail. In fact, we can find this interpretation in many literatures of the old Keynesians. In particular, Iwai (1981) calls his monopolistic competition market model, where trades are done even if supply and demand are not equal to each other, a modern reformulation of Wicksell's cumulative process. It has often been observed that *ex ante* expected demand function by the entrepreneurs would not coincide with the *ex post* demand function in the monopolistic competition market. Therefore, we think that trade with rationing would come into existence in this market if not for stationary state. Iwai (1981) presents the dynamics that the surprise of entrepreneurs, which is equal to the gap between *ex ante* and *ex post* demand function, gives birth to motivation for entrepreneurs to resolve the optimization problem of profit maximization once again. Iwai (1981) regards this disequilibrium analysis could be the same as the cumulative process in that the fluctuation occurs lastly by disequilibrium.

The modern reformulation by Iwai (1981) may capture the characteristics of the dynamics in the cumulative process because Wicksell's cumulative process is caused by deviation of the entrepreneurs' *ex ante* rate of return from the natural rate of interest. But we have to distinguish

disequilibrium from deviation, which does not need the assumption of disequilibrium in market – that is, Wicksell's cumulative process, based on deviation, is consistent with equilibrium analysis. Wicksell's cumulative process should be different from Iwai's monopolistic competition model in that the former is based on assumption of perfect competition and the latter on imperfect competition – monopolistic competition. In fact, we have discussed how the prices of products should be determined by the supply, assumed to be constant in Wicksell's analysis, and the demand of the entrepreneurs facing the optimization problem in the cumulative process above. It is certain that the deviation, which means entrepreneur's surprise, gives an entrepreneur the incentive to resolve his optimization problem based on a new price and the repeat of this process brings about the cumulative process, but the trade in this process is done on not disequilibrium but equilibrium. Thus, while the modern reformulation of the cumulative process in Iwai (1981) is disequilibrium analysis, it is not what Wicksell had in mind. In other words, these old Keynesian interpretations could be said to neglect Wicksell's ambivalent attitude toward neoclassical economics highlighted in the previous section, which regards the perfect competition market as a normative one. We should consider the cumulative process to be based on equilibrium, not disequilibrium, analysis.

5. What is Wicksell's Indeterminacy?

Ever since Sargent and Wallace (1975), mentioned above, referred to monetary indeterminacy by the term "Wicksell's indeterminacy," the original cumulative process discussed by Wicksell has come to be identified with Wicksell's indeterminacy, static monetary indeterminacy. Wicksell's indeterminacy, however, means the indeterminacy of the price level arising

under the assumption of rational expectations, so this must be distinguished from Wicksell's original cumulative process, which arises under the assumption of static expectations. That is – as will be clarified below – whereas the former represents a static problem, the latter deals with a dynamic problem.[18]

First of all, we describe Wicksell's indeterminacy, referring to McCallum's famous classification. According to McCallum (2001), fluctuations in the price level in contemporary macroeconomics can be classified into two kinds. McCallum proposes to classify those fluctuations in the price level by using two concepts: "multiple(-solution) paths" and "indeterminacy of the value of a nominal variable." The former, "multiple paths," concerns a problem that arises when a solution path in the real economy is not uniquely determined – in other words, when paths for multiple or innumerable solutions satisfy all the conditions imposed on a given model. McCallum attributes this to what are called self-fulfilling expectations. On the other hand, the latter, "indeterminacy of the value of a nominal variable," refers to a situation in which the value of a nominal variable cannot be fixed for some reason. In this case, it should be noted that no indeterminacy exists for the real economy. That is, what matters here, presupposing the quantity theory of money, is a situation in which the quantity of money supplied by the policy authorities is not uniquely determined: namely, whereas the former is a dynamic problem, the latter is a static problem.[19] According to McCallum, Patinkin (1949) (1961), Gurley

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18) This view is in accordance with those of Laidler (1984) and McCullum (1986).

19) Although McCallum's attitude clearly supports the latter studies as he himself is a supporter of the quantity theory of money, he avoids making a judgment as to which line of research is more realistic. McCallum, however, emphasizes that the result of analysis in one line of research gives no answer to other line of research. What McCallum has in mind here is the fact that fixing the interest rate is often argued to be capable of eliminating fluctuations in the

and Shaw (1960), Sargent and Wallace (1975), Sargent (1979), McCallum (1981) (1986), and Canzoneri et al. (1983) can be cited as examples of the analysis of nominal indeterminacy. On the other hand, the analysis of multiple paths includes Sargent and Wallace (1973), Black (1974), Block (1975), Flood and Garber (1980), Obstfeld and Rogoff (1983), Taylor (1977), Woodford (1990) (1995), and Sims (1994). It is clear that Wicksell's cumulative process is different from not only the latter but also the former in that there would be no indeterminacy problem in Wicksell's cumulative process; that is, monetary demand is uniquely determined in the process.

We have to think that the cumulative process and Wicksell's indeterminacy, the problem of indeterminacy of a nominal variable value, could be different in their nature as well for, whereas nominal indeterminacy assumes rational expectations, Wicksell's cumulative process assumes static expectations.[20] As has already been pointed out, Wicksell's theory has a structure in which nominal money demand is uniquely determined when real money demand is determined due to the assumption of static expectations of price level. That is, in his theory, since the price level is given by the assumption of static expectations, the problem of indeterminacy of a nominal value is avoided.[21] Let us discuss

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price level. It is McCallum's view that the former strand of studies, however, only argues for the indeterminacy of the price level under a given quantity of money and does not address fluctuations in the price level by changes in the quantity of money at all, which the latter strand of studies addresses. McCallum thus emphasizes that the former and the latter arguments should be completely separated.

20) This point is also indicated in McCallum (1986). Defining the Wicksellian cumulative process as continuous fluctuations in the price level that occur when the nominal rate of interest is kept constant, McCallum argues that the Wicksellian cumulative process is at odds with the assumption of rational expectations. McCallum, however, only asserts it intuitively and does not examine it theoretically as we do.

21) Note, however, that the above argument does not give any answer to the interesting problem of how a Wicksellian disequilibrium dynamic model should be treated under rational expectations.

this point in detail. As has already been pointed out in the previous section, it is clear that there are two kinds of trade in one period of the cumulative process. At first, at the beginning of the first period, entrepreneurs make a business plan and borrow the money to carry out that plan. At this point, it is assumed that these entrepreneurs make a plan presuming the prices of consumption goods produced by them are unchanged; that is, the expected prices are equal to the existing prices of consumption goods. We therefore could present the monetary demand function in the model below:

$$M_t = L(P_t^e) = L(P_{t-1}) \qquad\qquad (1)$$

where nominal money supply of the t-th period is M_t and money demand is $L(\cdot)$, we define P_t^e as an expected price of their products, consumption goods, and P_{t-1} as an existing price. The entrepreneurs in question form their expectations of the price of their products statically, so we could think that P_t^e should be equal to P_{t-1}. It is clear that we could treat P_{t-1} or P_t^e of (1) as given from the discussion above, so the left side could be determined uniquely. Thus, because it is assumed that entrepreneurs formulate their expectations of the price of their products statically, there is no nominal indeterminacy in Wicksell's cumulative process. In addition, in the cumulative process, the other trade would be done in the same period as (1). In that trade, money supply is passed on to the owners of the factors of production through the entrepreneur's expenditure for these factors. If we assume that there is no money supply hoarded, this second trade should be formulated as follows:

$$P_t Y_t = M_t \qquad\qquad (2)$$

The left side of (2), money supply, is determined in the previous trade as (1). And, because Wicksell limited the scope of his argument to stationary

state, we have to treat Y_t as given. So the price of their product – that is, price level in Wicksell's analysis – should be determined uniquely. If other conditions were equal we can think this process, consisting of two kinds of trade, would be repeated in the next period. Therefore, the trade in the t+1th period should be formulated as follows:

$$M_{t+1}=L(P_{t+1}^e)=L(P_t) \tag{3}$$

$$P_{t+1}Y_{t+1}=M_{t+1} \tag{4}$$

where P_{t+1}^e denotes the entrepreneurs' expected price of the product and P_{t+1} denotes the price of the product in the t+1th period. So, we can see that there is not nominal indeterminacy as Wicksell's indeterminacy in the original cumulative process for (1). Wicksell's indeterminacy should correspond to only (2) in our discussion. It is certain that we could pin down only real money supply , M_t/P_t by only (2), and in that case there are many combinations of M_t and P_t. This special case, where all trades in one period could be characterized by only (2), would be explained by Wicksell's indeterminacy. Wicksell's cumulative process, however, is characterized by not only (2) but also (1), and this means that M_t could be pinned down uniquely in this model. This is how nominal indeterminacy should be avoided in Wicksell's cumulative process. So it should be concluded that we could not call Wicksell's cumulative process, in which the combination of M_t and P_t is determined uniquely, Wicksell's indeterminacy.

6. Conclusion

We have made a detailed examination of Wicksell's studies on the cumulative process and two widely accepted understandings of these. First,

we considered the theory of the cumulative process proposed by Wicksell. Through this examination, we have not only clarified Wicksell's view of problems but also identified the assumptions his theory is based on. *Inter alia*, we pointed out that what we call the assumption of static expectations today plays an important role in that theory. Next, by investigating the assumptions in Wicksell's theory, we confirmed that Wicksell's dynamic analysis is not the same as Iwai's (1981) disequilibrium analysis. It is certain that Wicksell's cumulative process is characterized by the deviation of the entrepreneurs' *ex ante* rate of return from the natural rate of interest. All trades in this process, however, are done on equilibrium, not disequilibrium like the monopolistic competition market model in Iwai (1981). Furthermore, we confirmed that one widely accepted concept, Wicksell's indeterminacy, is misleading in that Wicksell's indeterminacy deals with only parts of the original cumulative process. Thus, it can be said that we have managed to reveal the original contribution made by Wicksell.

We have to recognize, however, the existence of work that remains to be done. That is, we are still left with the task of presenting the cumulative process as a theoretical model. This work is unavoidable if we wish to utilize the consideration that we have made so far for contemporary purposes in one way or another. Constructing models of his theory means translating his research into our language of contemporary economics, and it is seemingly only through this process that we can give a truly contemporary appraisal of a classic. We leave this to our future efforts.

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