Mutually Consistent Expectations and the Nash Bargaining Problem

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Abstract

The Nash bargaining problem is considered as a game played by two expected-utility maximizers with prior probability distributions over the opponent's strategy set. A natural requirement of mutual consistency and interchangeability of pairs of priors is shown to be necessary and sufficient to single out the only distribution that generates the equivalence of the expected utility maximization and the Nash product maximization.

1. Introduction

The Nash bargaining solution, characterized as the utility-product maximization, can be derived in several ways: the well-established axiomatic approach due, originally to Nash[5], Bayesian negotiation models due to Harsanyi[2], and, more recent probabilistic models of Anbar and Kalai[1], or others. In contrast to the first two approaches, the last model due to Anbar and kalai is novel, but nevertheless interesting in that the Nash solution is related to the expected-utility maximization with a uniform prior probability distribution over the opponent's strategy choices.

They treated the bargaining game as a one-shot decision

Keywords: Nash product, mutual consistency, interchangeability, strong consistency

problem performed by two expected-utility maximizers who expect the opponent's behavior according to some probability distributions, and thereby characterized the form of distributions that generate optimal feasible payoff vectors. From a behavioral point of view, however, their result does not serve as a guide to a particular optimal decision. For example, as pointed out in Roth(7), it is not clear why players without knowledge about Nash's theory should come to draw the uniform distribution.

In this paper, we shall try to give a geme-theoretical ground on which one may explain the selection of the uniform prior distribution in such a bargaining situation. Two concepts will be needed: one is the mutual consistency, and the other is the interchangeability. The mutual consistency is essentially the condition of global optimality due to Anbar and Kalai(1). The interchangeability is a well known requirement of plausible non-cooperative equilibrium points. By bringing these two requirements together in an appropreate manner, we define what we call the strong consistency, which is the condition we need to single out precisely the uniform distribution.

The argument to this conclusion conceptually relies upon the mutually expected rationality postulate due to Harsanyi(2), which essentially states that a rational player must expect his opponent to follow the same rationality principles.

2. The Bargaining Game

We begin with the following bargaining situation: Two players have to bid independently their payoff demands. If the demands are compatible in the sense that they are mutually feasible, then they each receive precisely the amounts what they demanded. If incompatible, they receive predetermined conflict payoffs.

Let R denote the set of all feasible payoff vectors, and $c=$

(0, 0) the conflict payoff vector. We assume:

(a) R is a compact convex set in E^2, the 2-dimensional Euclidean space,

(b) $c=(0, 0)\varepsilon R$, and $(x, y)\varepsilon R$ for some $x>0$, $y>0$,

(c) $(1, 0)\varepsilon R$, $(0, 1)\varepsilon R$, and they are both Pareto optimal,

where the payoffs are given by the von Neumann-Morgenstern utilities, and normalized.

The game proceeds as follows: Player 1 and 2 independently choose as their strategies x in A and y in B, respectively, where the strategy set A of player 1 and that of player 2 are the same closed unit interval [0, 1]. If (x, y) is in R, then they receive (x, y). If otherwise, they receive $c=(0, 0)$. The **Nash solution** (x°, y°) of this game is given by the **Nash product maximization,** i. e.,

(d) $x^\circ y^\circ=\max\{xy|(x, y)\varepsilon R\}$, $(x^\circ, y^\circ)\varepsilon R$.

Let us consider, now, the bargaining game as one played by two expected payoff maximizers. Suppose, then, each player has an expectation about the opponent's behavior, in the form of a prior probability distribution over the strategy set of the opponent. So, let F and G be continuous probability distributions over B and A, respectively. By F, we express the expectation of player 1 over player 2's behavior, and by G, the expectation of player 2 over player 1's behavior.

Assume that the non-negative region of R can be represented by

$$\{(x, y)\varepsilon E^2_+|y\leqq\psi(x),\ x\,\varepsilon A\},$$

or equivalently by

$$\{(x, y)\varepsilon E^2_+|x\leqq\phi(y),\ y\,\varepsilon B\}.$$

Then, under the rule of the game, the expected-payoff to player 1 when he chooses $x\,\varepsilon\,A$ is given by

$$xF(\psi(x))+0(1-F(\psi(x)))=xF(\psi(x)).$$

Similarly, the player 2's expected-payoff when he chooses $y\,\varepsilon\,B$ is

$$yG(\phi(y))+0(1-G(\phi(y))=yG(\phi(y)).$$

Each player then tries to maximize his own expected-payoff. Thus, we may define the equilibrium state in this maximization as follows:

Definition 1. The payoff vector $(x^*, y^*) \varepsilon R$ is in **equilibrium** if (x^*, y^*) is Pareto optimal, and

$$x^* F(\psi(x^*)) = \max\{xF(\psi(x)) \mid x \varepsilon A\},$$
$$y^* G(\phi(y^*)) = \max\{yG(\phi(y)) \mid y \varepsilon B\}.$$

Note that any equilibrium payoff vector is a **Nash equilibrium** in the original bargaining game, since any strongly Pareto optimal payoff vector is a Nash equilibrium there. The following proposition, which is implied by Theorem 2 of Anbar and Kalai[(1)], is an immediate consequence of Definition 1.

Proposition 2. If F and G are both uniform, then the equilibrium payoff vector coincides with the Nash solution.

Proof. Since $F(y)=y$ and $G(x)=x$, the assertion is clear.

This proposition would be of little use in guiding the decisions to the Nash solution, unless there are certain grounds to make the prior distributions uniform. Is there any such behavioral principle? In the next section, we shall try to answer this question.

3. Mutually Consistent Distributions

The logical implication of the assumption that F and G are uniform is the equivalence of the expected-payoff maximization to the Nash product maximization, which is a stronger result than the local coincidence of the equilibrium payoff vector and the Nash solution. Noting this fact, we now state conditions that will lead to the uniform prior distributions.

Let R be the set of all feasible payoff vectors satisfying (a), (b) and (c). Let F be, as before, the prior distribution that player 1 has over the opponent's strategy set B. Further, let $\hat{G}$ be any probability distribution over the player 1's strategy set

A. We interpret $\hat{G}$ as the expectation that player 1 thinks player 2 has over A. Note that $\hat{G}$ is not a distribution over distributions but is simply a distribution over the strategy set A. $\hat{G}$ is the expression of the fact that just like player 1 regards the opponent's choice as a random variable, player 1's choice itself should also appear to the opponent as a random variable. Then, in player 1's mind, a hypothetical expected-payoff $\psi(x)\hat{G}(x)$ to player 2 can be associated to each choice $x \varepsilon A$ of strategy. It is therefore reasonable to require that F and $\hat{G}$ be such that the maximization of $xF(\psi(x))$ is compatible with that of $\psi(x)\hat{G}(x)$, since player 1 should also seek to reach an agreement. The same argument goes through for player 2 with distributions G and $\hat{F}$, where $\hat{F}$ is any distribution over B that player 2 thinks player 1 to assess.

To state this formally as a behavioral requirement we extend the above reciprocal consideration to the whole unit square $K=[0, 1]\times[0, 1]$ as follows.

Let Σ be the collection of all sets R satisfying (a), (b) and (c). Also, let $H=\{(x, y)\varepsilon K | x+y\geqq 1\}$. Then, to each point (x, y) in H, there corresponds at least one R in Σ for which (x, y) is Pareto optimal. The expected-payoff to player 1 associated with the point (x, y) is then given by $xF(y)$, and similarly, the hypothetical expected-payoff to player 2 is $y\hat{G}(x)$. Thus, to each point (x, y) in H, a pair $(xF(y), y\hat{G}(x))$ of the two expected-values can be associated. We require that in player 1's mind the order of desirability of the points (x, y) in terms of $y\hat{G}(x)$ should conform to the order in terms of $xF(y)$, i. e., his own criterion. Thus, for player 1 the following relation must hold:

(e) For all $x, x' \varepsilon[0, 1]$, $y, y' \varepsilon[0, 1]$ such that $(x, y), (x' y') \varepsilon H$;
$xF(y)\geqq x' F(y')$ if and only if $y\hat{G}(x)\geqq y' \hat{G}(x')$.

Similarly, for player 2,

(f) For all $x, x' \varepsilon[0, 1], y, y' \varepsilon[0, 1]$ such that $(x, y), (x', y') \varepsilon H$;
$yG(x)\geqq y' G(x')$ if and only if $x\hat{F}(y)\geqq x' \hat{F}(y')$.

Let (F, G) denote the pair of prior distributions of player 1 and player 2, respectively. We say the pair (F, G) is **mutually consistent** if (e) and (f) are satisfied with $\hat{G}=G$ and $\hat{F}=F$, respectively. If (F, G) is a mutually consistent pair, then (F, G) generates an equilibrium payoff vector for every R in Σ. Hence, in the terminology of Anbar and Kalai[1], a mutually consistent pair (F, G) is **globally optimal.**

The mutual consistency alone cannot be a guiding principle, however. For, there will be many mutually consistent pairs in general. In fact, if (F, G) is mutually consistent, then by definition, (G, F) also is. To reduce the number of mutually consistent pairs, we now introduce the notion of interchangeability. Two mutually consistent pairs (F, G) and (F', G') are said to be **interchangeable** if (F, G') and (F', G) are again mutually consistent pairs. Then we may define:

Definition 3. *A* pair (F, G) is **strongly consistent** if (F, G) and (G, F) are interchangeable.

The interchangeability used here is similar to that of equilibrium points in noncooperative games (see, Luce and Raiffa [Ref. (4), p. 106]). If (F, G) and (G, F) are mutually consistent interchangeable pairs, then (F, F) and (G, G) also are mutually consistent, so that for each player it will not affect the consistency whether the opponent's prior is F or G. As we will see below, the strong consistency is the solution that determines uniquely the form of F and G.

To show this, we finally assume:

(g) All F and G are increasing functions.

Lemma 4. *A* pair (F, G) is strongly consistent if and only if F and G are both uniform.

Proof. If F and G are uniform, then $F=G$ and $xF(y)=yG(x)=xy$. Hence, by (e) and (f), the pair (F, G) is strongly consistent.

Conversely, suppose (F, G) is a strongly consistent pair.

Then, (F, F) and (G, G) are mutually consistent. Consider the pair (F, F). Then, (e) must be true for $\hat{G}=F$. For any x' in $[0, 1]$, choose y in $[0, 1]$ so that they satisfy

(1) $x'=F(y)$.

This is always possible because F is a continuous, increasing function over $[0, 1]$ with $F(1)=1$. Then, letting $x=y'=1$, we have

$$xF(y)=F(y)=x'=x'F(y'),\ x+y>1 \text{ and } x'+y'>1.$$

Therefore, putting $\hat{G}=F$ in (e), we have $yF(x)=y'F(x')$, or

(2) $y=F(x')$.

Then, (1) and (2) together imply

$$x'=F(F(x')) \text{ for all } x' \varepsilon [0, 1].$$

Hence, the increasing function F is the inverse of F itself. So, this and continuity of F prove that F is uniform over $[0, 1]$.

As for G, by considering the pair (G, G) and condition (f), the proof is completely parallel. Q. E. D,

This lemma, combined with Proposition 2, implies that the Nash solution is attained via the expected-payoff maximization with strongly consistent prior distributions. As noted earlier, the Nash product maximization is equivalent to the expected-payoff maximization if the priors are uniform. Formally, the equivalence can be expressed by the following condition (h):

(h) For all $x, x' \varepsilon [0, 1], y, y' \varepsilon [0, 1]$ such that $(x, y), (x', y') \varepsilon H$;
$xF(y) \geqq x'F(y')$ if and only if $xy \geqq x'y'$,
and
$yG(x) \geqq y'G(x')$ if and only if $yx \geqq y'x'$.

Then, we may state:

Proposition 5. The expected-payoff maximization is equivalent to the Nash product maximization if and only if the pair (F, G) is strongly consistent.

Proof. It is clear that (h) is true if and only if F and G are uniform. Then, by the lemma, the assertion follows. Q. E. D.

4. Concluding Remarks

We have shown that the condition of strong consistency singles out the uniform distribution from all possible distributions over the opponent's behavior in the bargaining situation. The probabilistic model as considered here is not itself a game-theoretical one in that the players view the opponent's choices as governed by a random mechanism. What we have tried to show is, however, that some game-theoretical accounts can be given to the problem of how players draw expectations about the opponent's behavior. The mutual consistency is a natural prerequisite for this purpose. The key step is the application of the concept of interchangeability, which, in the context of non-cooperative theory, is counted as a desirable property that equilibrium points may possess. It is well known that the solutions to the classic two-person zero-sum games, or the Nash bargaining solution with variable threats[(6)] have this property. In our setting, this property gives a rationale to the expectation that the opponent will also have the same priors.

Finally, it may be interesting to observe that if (F, G) is a mutually consistent pair, then the equilibrium payoff vector (x^*, y^*) for R must be the point of R where the 'product of expected-payoff $xF(y)yG(x)$' is maximized in R. This apparent similarlity to the Nash product becomes real when F and G are uniform.

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