Incentives for improving the public budget balance in local governments and resident migration

著者	Kobayashi Katsuya
出版者	Institute of Comparative Economic Studies, Hosei University
journal or publication title	Journal of International Economic Studies
volume	25
page range	63-80
year	2011-03
URL	http://hdl.handle.net/10114/7074

Journal of International Economic Studies (2011), No.25, 63–80

Incentives for improving the public budget balance in local governments and resident migration

Katsuya Kobayashi*

Abstract

This paper examines the incentives for local governments to improve their own fiscal balances in an environment of inter-regional resident migration, and shows that whether a local government chooses to make its fiscal balances efficient depends on the condition of population distribution. Since residents migrate between regions in the pursuit of higher utility, their utility will become equal at a migration equilibrium. We obtain the following results from this property of resident migration: a local government in a country with two regions will improve its own fiscal balance efficiently when the utility of residents in the other region increases with emigration from there. If the residents' utility in the other region decreases, the local government will have an incentive to deteriorate its own fiscal balance. However, residents' utility will not decrease under this deterioration.

Key words: Resident migration; Reducing public expenditures; Local governments

JEL classification numbers: H72, H11

1 Introduction

Do local governments always have incentives to improve their budgets on local public goods? The efficiency of local governments is as important as that of other sectors. [1] This is particularly important considering that the fiscal balance of Japanese local governments has deteriorated since the latter half of the 1990s. The central government has promoted decentralization to the local governments in order to ameliorate the balance of public finance. In concrete terms, it has transferred tax sources to local governments and reduced subsidies. The purpose is to make all sections of the public sector more efficient by transferring the authority related to resources from the central government to local governments, which have more information concerning their own regions.

Incidentally, it has been traditionally argued that if residents migrate freely under a decentralized public finance system, local governments have incentives to be efficient (Tiebout (1956)). However, many researches have clearly shown that resident migration does not always bring efficiency. [2] Even if it is assumed that local governments are benevolent and that there are no rent seeking activities by politicians and bureaucrats, resident migration does not guarantee Pareto efficient allocation. These studies have looked at the efficiency of the

* Faculty of Economics, Hosei University, E-mail katsuyak@hosei.ac.jp

[1] See Oates (1999) and Inman & Rubinfeld (1997). These papers survey researches on local governments.

[2] Refer to Itaba (2002), which surveys these researches in detail.

population distribution and of the provision of local public goods, but there are no studies looking at improving the public balances under resident migration. [3] Since resident migration does not always bring efficiency in allocation, some factors may exist that give local governments incentives to deteriorate their fiscal conditions under resident migration. The purpose of this paper is to elucidate the conditions under which local governments have incentives for improving their own public balances under resident migration. By showing these conditions, it will be possible to encourage the central government to arrange conditions that local governments will make efficient efforts to improve the public balance. This analysis is intended to contribute to the efficiency of Japanese local governments.

With regard to local public goods, in particular, if there are no externalities and local governments do not take account of resident migration, then they will provide local public good efficiently (Boadway & Flatters (1982)). However, the population distribution is not always Pareto efficient in this case. Even when local governments take account of migration, local public goods may not also be provided efficiently since local governments cannot directly control resident migration in a way that makes the population distribution efficient. In this case, local governments may distort the provision of local public goods in order to improve the distribution of the population. If so, the device of inter-regional transfers resolves this problem. Even if a local public good has externalities, this device makes the provision of local public goods Pareto efficient. Myers (1990), Wellisch (1994) and Caplan, Cornes & Silva (2000) show the effects of inter-regional transfers in cases where a local public good has no externalities and where it has externalities. [4] On the other hand, Mitsui & Sato (2001) shows that inter-regional transfers distributed by the central government without commitment bring about a concentration of residents into one region. As a result, the population distribution becomes inefficient. Thus, interregional transfers do not always bring about Pareto efficiency.

However, these researches lack the perspective of examining the efficiency of local governments. More precisely, they do not analyze whether each local government has an incentive to improve its own public expenditures under resident migration. The improvement of public expenditures means that local governments reduce the cost of local public goods and increase the revenues of public services such as water, gas and traffic services by increasing productivity. [5]

By improving their own public budget balances, local governments can attract immigration from other regions. The reason for this immigration is that the improvement of the public budget balance brings larger utility to residents in the improved region than those in other regions by allowing them to avoid increasing taxes and through reducing taxes. Since residents migrate to the region that provides the largest utility, considering continuous residents, all res-

---

[3] There are studies on the incentive for policy innovation in local governments. Rose-Ackerman (1980) and Strumpf (2002) show that under fiscal federalism, local governments become risk-averse and become free-riders that imitate new technologies which other governments develop. These studies are similar to this paper since local governments need new skills or know-how in order to reduce public expenditures without decreasing the level of public services. However, neither study looks at the environment of resident migration, but rather at the uncertainty of policy innovation.

[4] Myers analyzes the effect of inter-regional transfers managed by each local government in the case of local public goods without externalities. The result is that this transfer makes the population distribution Pareto efficient, so that there is no need for the central government. Wellisch analyzes the case of local public goods with externalities. In this case, the population distribution lead to Pareto efficiency, too. Caplan et al. analyze the effect of different timings, meaning that each local government is a leader and the central government a follower. In this case, the distribution becomes Pareto efficient. These studies show that inter-regional transfers are needed in order to achieve efficiency.

[5] These services often are in deficit. For example, many water services in Japan which are managed by local governments are in deficit. The average of the price in each region is 174 yen/m^3. On the other hand, the average cost is 178 yen/m^3 (data for 2004 from Ministry of Internal Affairs and Communications (2006)). This deficit, of 4 yen/m^3, is usually compensated with taxes. Thus, local governments should reduce their public expenditures by improving their revenue or reducing these costs.

idents obtain the same utility at a migration equilibrium that is defined as a state where there are no residents with incentives to migrate to other regions. Taking these factors into account, we can show in this paper that whether residents' utility is high or not at a migration equilibrium depends on the distribution of population. In a country with two regions, if a region where a local government improves its public balance is congested, meaning that it is an overpopulated region, the increase of residents' utility will be weakened through a worsening of this congestion. At the same time, if the other region is also congested, the utility will rise since the emigration from the other region will mitigate its congestion. As a result, we find that the improvement of the public budget balance will increase all residents' utility when both regions are congested. In this case, efforts to improve the public budget balance will be Pareto efficient at the migration equilibrium. On the other hand, when the other region is sparse, the improvement in the public budget balance in a congested region will cause residents to migrate from the sparse region into the congested region. Since the sparseness and congestion become worse in both regions, residents' utility decreases as a result of this improvement. In this case, the local government in the congested region does not make Pareto efficient efforts at a migration equilibrium. However, the local government in the sparse region always makes Pareto efficient efforts, since immigration mitigates the sparseness. In summary, it is shown that whether local governments make Pareto efficient efforts depends on the population conditions in other regions. Since local governments do not have any authority to improve the public balance in the other region, they will raise their own residents' utility by influencing migration by controlling their own public expenditures. As a result, the utility of the residents in their own region cannot be higher than that at the optimal population scale of the other region. Thus, no local government has an incentive to improve the public balance if the residents' utility in the other region decreases due to emigration from the region. While previous researches have not paid attention to the relation between the population distribution and the distortion of efforts to improve the public budget balance, the relation is demonstrated in this paper.

The outline of this paper is as follows. Section 2 presents the basic model structure and the main results. Section 3 describes the case in which local governments simultaneously decide to provide local public goods and make efforts to reduce public expenditures. The conclusion is given in Section 4. The proofs of the propositions and lemma are provided in the Appendix.

2 Model

A nation consists of two regions, named $i = 1, 2$, and a local government exists in each region. The population in region i is n_i, and the aggregate population is $\bar{n}$, thus $n_2 = \bar{n} - n_1$. We assume continuous residents, so that the weight of each resident is zero. In other words, the population is atom-less. Each resident has a homogeneous preference and is able to choose either region 1 or 2 to reside in. The utility function of a representative resident in region i is $u^i\ (x_i, y_i)$. x_i is the consumption of a private good and y_i is the consumption of a local public good which has no externalities to the other region and which is provided by the local government in region i. We assume that u^i is strictly quasi-concave, [6] and that y_i is a normal good. [7]

[6] The sufficient condition for strict quasi-concavity is $2u^i_y u^i_x u_{xy} - (u^i_x)^2 u_{yy} - (u^i_y)^2 u^i_{xx} > 0$. u^i_x, u^i_y and u^i_{xy} mean $\partial u^i / \partial x_i$, $\partial u^i / \partial y_i$ and $\partial^2 u^i / \partial x_i \partial y_i$, respectively.

[7] The sufficient condition for a normal good for y_i is $1/u^i_x\ (u^i_x u^i_{xy} - u^i_y u^i_{xx}) > 0$. We assume this condition.

Each resident is endowed with one unit of homogeneous labor which is supplied to firms in region i. Firms produce the private good and pay labor a wage equal to the marginal product. The collective production function for the private good in region i is assumed to be $f_i(n_i)$ [8] which is concave, $f_i' \geq 0$, $f_i'' \leq 0$ and $f_i(0) = 0$. The firms in region i are assumed to be owned by the residents of region i. Hence, the profits of the firms in region i are equally distributed to each resident of region i. We also assume there are no transfers between regions. Each local government collects a resident-based head tax in order to produce the public good. The marginal cost of the public good is c_i which is fixed for y_i. We consider that each local government makes efforts to improve the public budget balance, which is public revenue minus public expenditure. In this paper, improving this balance means that each local government decreases its net public expenditure. Thus, the constraint of resources in region i is $f_i(n_i) = n_i x_i + c_i y_i + F - a_i + d(a_i)$. a_i is efforts and $d(a_i)$ is the cost of efforts. One unit of effort leads to a reduction of one unit of public expenditure, but the effort cost is generated. F is the fixed cost of the local public good. We assume that $F - a_i + d(a_i) > 0$ for all a_i, and that $F = 0$ when $y_i = 0$. This means that F is larger than the surplus of the cost reduction effort. [9] We assume that $d(a_i)$ is strictly convex, $d(0) = 0$, $d'(a_i) > 0$, $d'(0) = 0$, $d'(+\infty) = +\infty$, and $d''(a_i) > 0$. In addition, we describe a_i satisfying $a_i - d(a_i) = 0$ as $\bar{a}$. We assume that local governments choose $a_i \in [0, \bar{a}]$. This assumption means that local governments do not make efforts that lead to deficits. We assume that each local government chooses the effort level a_i and the quantity of local public good y_i to maximize the utility of a representative resident in region i.

Pareto Efficiency Following Wellisch (1994), Pareto efficient allocation is defined as feasible allocations at which it is impossible to increase u^i without reducing u^j ($i \neq j$, $i, j = 1, 2$). However, we assume that neither government can compel residents to migrate from one region to another region. [10] Residents migrate to get higher utility. Since each resident is atom-less, if all residents in both region 1 and 2 have the same utility level, then they have no incentives to migrate to the other region. Thus, Pareto efficiency should lead to $u^i = u^j$. In addition, Pareto efficiency, which is defined as the impossibility of increasing u^i without reducing u^j under $u^i = u^j$, is characterized by a linear combination of u^1 and u^2 being maximized subject to the following (2) and (3).

$$\max_{x_1, x_2, y_1, y_2, n_1, a_1, a_2} \quad \delta u^1(x_1, y_1) + (1-\delta)u^2(x_2, y_2) \tag{1}$$

$$s.t. \;\; f_1(n_1) + f_2(\bar{n} - n_1) - n_1 x_1 - (\bar{n} - n_1)x_2 - c_1 y_1 - F + a_1 - d(a_1) - c_2 y_2 - F + a_2 - d(a_2) = 0 \tag{2}$$

$$u^1 = u^2 \tag{3}$$

$$0 \leq \delta \leq 1, x_1, x_2, y_1, y_2 \geq 0$$

(2) is the constraint of resources.

We use the Lagrange function, and define λ_1 as the Lagrange multiplier of the resources constraint (2) and define λ_2 as the multiplier of the migration equilibrium constraint (3). We assume an interior solution to x_i and y_i.

[8] We assume that productivity may be different between regions. Precisely, the product function depends on the land, $f(n_i, T_i) \equiv f_i(n_i)$. T_i means the land scale in region i. The larger T_i, the larger the productivity.

[9] If F is small, the effort surplus may be larger than the public cost in the region. More accurately, $c_i y_i + F - a_i + d(a_i) < 0$ for some a_i. In this case, the local government returns this surplus to residents. As a result, production of the private good increases in the region. This case is not intrinsically different from the following analysis. In addition, it may not be actually returned to private goods since the effort surplus is not always pecuniary revenue. In this paper, since it is not our purpose to clarify this, we assume that $F - a_i + d(a_i) > 0$ for all a_i.

[10] We also assume that even if there is a social planner like the benevolent central government, this planner cannot do that.

We achieve the following first-order conditions:

$$x_i \;:\; (\delta + \lambda_2)u_x^i - \lambda_1 n_i = 0, \tag{4}$$
$$y_i \;:\; (\delta + \lambda_2)u_y^i - \lambda_1 c_i = 0, \tag{5}$$
$$n_1 \;:\; f_1'(n_1) - x_1 = f_2'(\bar{n} - n_1) - x_2, \tag{6}$$
$$a_i \;:\; \lambda_1(1 - d'(a_i)) = 0, \quad i = 1, 2. \tag{7}$$

From (4) and (5), we obtain

$$(\delta + \lambda_2)\frac{u_x^i}{n_i}(n_i\frac{u_y^i}{u_x^i} - c_i) = 0. \tag{8}$$

λ_2 is positive as long as $u_1 = u_2$ at the optimal solution. We assume that there exists an interior solution of the migration equilibrium, $\lambda_2 > 0$. Thus, (8) means that Pareto efficiency for y_i is satisfied when the sum of the marginal rate of substitution for y_i equals the marginal cost of the local public good, which satisfies the Samuelson condition.

We denote the Pareto efficient a_i as a_i^*. Then, a_i^* is satisfied by

$$1 - d'(a_i^*) = 0. \tag{9}$$

since $\lambda_1 = (\delta + \lambda_2)u_x^i/n_i > 0$ from (4) and $\lambda_2 > 0$. This means that the socially optimal effort of each local government is to make the marginal effect of the improvement of the public budget balance equal to the marginal cost of efforts.

From (6), a distribution of population becomes socially efficient when the net marginal product in each region is equal. If (6) is not equal, more private goods can be produced in the nation by having the residents migrate from the region with the smaller net marginal product to the region with the larger one. We denote a Pareto efficient population in region i as n_i^*.

The purpose of this paper is to analyze the incentive for improving the balance of public revenues and expenditures. These incentives encourage efforts to reduce the costs and increase the revenues of public services. We can consider the following methods for cutting costs and increasing revenues: outsourcing and restructuring public services, reducing staff, introducing a system for evaluating public policies to avoid waste, computerization of office work, improving the revenue of or privatizing water, gas, traffic and childcare services, and so on. In practice, it is difficult for local governments to introduce these methods in the short term since they require the implementation of organizational and institutional changes. In addition, these methods involve costs. Thus, once local governments introduce such methods, it is difficult to withdraw them quickly since they usually require long-term contracts and changes in public institutions. For example, Ota city [11] made a contract for entrusting water service to a private firm. [12] The contract term was from April 1, 2002 to March 31, 2007. This contract contained a clause imposing a penalty if either of the parties canceled the contract. That is to say, Ota city was committed to this system of providing the water service in trust with the private firm. [13] From this viewpoint, improving the public budget balance requires a commitment. Thus, it is appropriate to set the behavior for improving the public balance on the first stage of the game in our model.

Following Mitsui & Sato, we assume that residents can freely migrate to the region in

[11] Ota city is in Gunma prefecture, Japan.

[12] Komiyama (2003) explains this entrusting contract in detail.

[13] Another example is the outsourcing of simple routines. Shizuoka prefecture in Japan established a center for general-affairs office work (Somu-jimu center) and introduced the outsourcing of general-affairs office work (*somu-jimu*) in 2002. Shizuoka prefecture has saved about 97 million yen every year. Wakasugi & Kobayashi (2006) explains this.

which they want to live, and that they cannot move between regions after they choose a region. This assumption is based on the following fact. Residents really always choose their occupations when they decide where to live. It is not easy for individuals to change jobs because of the time required to search for a new job. On the other hand, providing many local public goods is a daily task. For example, it includes police, fire fighting, library, health care services, and so on. From those features, the time structure in this model is based on the following three stages. First, local governments set their own effort level, a_i, which is how they will improve their public budget balance, ex-ante. Second, residents decide where to reside. Third, local governments provide local public goods, y_i. We will solve the game by backward induction, so that we will use a sub-game perfect equilibrium (SPE) in the following sections.

In Section3, we will consider not only the case where local governments take resident migration into account, but also where they do not take it into account.

Sub-game perfect equilibrium We will find a sub-game perfect equilibrium in this model. The model will be solved using backward induction.

Stage 3 At stage 3, local governments set y_i given n_i and a_i in order to maximize a representative resident in its own region. Specifically, our maximization problem is

$$\max_{y_i} u^i (x_i, y_i) \quad s.t.\ f_i(n_i) = n_i x_i + c_i y_i + F - a_i + d(a_i). \tag{10}$$

Then, the first order condition is

$$\frac{u_x^i}{n_i}(n_i \frac{u_y^i}{u_x^i} - c_i) = 0. \tag{11}$$

Therefore, y_i satisfies the Samuelson condition, namely that y_i is Pareto efficient. [14] y_i is set given n_i and a_i , so that y_i is a function of them. If n_i and a_i are not Pareto efficient, the quantity of y_i will not be equal to y_i of (8).

We denote the indirect utility of the resident in region i as

$$V_i (n_i, a_i) \equiv \begin{cases} u^i \left(\frac{f_i - c_i y_i - F + a_i - d}{n_i}, y_i \right) & \text{if } f_i - n_i x_i - c_i y_i - F + a_i - d \geq 0 \\ 0 & \text{if } f_i - n_i x_i - c_i y_i - F + a_i - d < 0. \end{cases} \tag{13}$$

From the fixed cost $F > 0$, the resources constraint in (10) may be in a breach in the neighborhood of $n_i = 0$. In this case, we assume $x_i = y_i = 0$ and $V_i = 0$.

Stage 2 At stage 2, residents choose either region 1 or 2 depending on which gives them a larger utility. If $V_i (n_i, a_i) > V_j (n_j, a_j)$ for given a_i and a_j, residents in region j emigrate from j to i. If the utilities in region 1 and 2 are equal, residents do not migrate. Since the migration equilibrium is defined as a state where each resident has no incentive for moving to the other region given others' choices, this is defined as the following migration equilibrium conditions, either 1 or 2.

---

[14] As the population in i increases under $f_i' - x_i \geq 0$, y_i becomes larger because y_i is a normal good. Specifically, we obtain

$$\frac{dy_i}{dn_i} = \frac{1/u_x^i\,(f_i' - x_i)(u_x^i u_{xy} - u_y^i u_{xx}^i) + u_y^i}{1/(u_x^i u_y^i)(2u_x^i u_y^i u_{xy}^i - (u_x^i)^2 u_{yy}^i - (u_y^i)^2 u_{xx}^i} > 0 \tag{12}$$

by differentiating (8) or (11) from the quasi-concavity of u_i and the normal good of y_i . However, dy_i/dn_i may be negative when $f_i' - x_i < 0$ This property affects the second order condition for n_i of the indirect utility function.

1. If $n_1, n_2 > 0$, then $V_1(n_1, a_1) = V_2(n_2, a_2)$.
2. If $n_i = 0$, then $V_i(0, a_i) \leq V_j(\bar{n}, a_j)$.

Statement 1 of the condition means that the migration equilibrium is an interior solution. Statement 2 of condition means that it is a corner solution. These conditions constitute a Nash equilibrium at stage 2. Since the weight of each resident is zero (atom-less), even if a resident who chooses region 1 migrates to region 2, he (she) cannot have an influence on the others and on the productivity in each region. Therefore, under these migration equilibrium conditions, each resident lacks any incentive for migration given the others' strategies. As a result, satisfying these conditions means a Nash equilibrium at this stage.

The migration equilibrium depends on the level of efforts, (a_1, a_2), which is determined at stage 1, so that the migration equilibrium becomes a function of (a_1, a_2). However, there may be multiple migration equilibria at (a_1, a_2). [15] In this case, each equilibrium is not always continuous at each value of (a_1, a_2). This discontinuity complicates the analysis in this paper. In order to prevent this complexity from centering the discussion, we denote one migration equilibrium as $n_1(a_1, a_2)$, as the case in which the migration equilibrium varies continuously with the change of (a_1, a_2), meaning that the residents' strategy is $n_1(a_1, a_2)$. [16] We consider the migration equilibrium in this limited class. In addition, the migration equilibrium $n_1(a_1, a_2)$ may disappear at some (a_1, a_2). This fact depends on the configuration of V_1 and V_2. In this case, the migration equilibrium becomes another equilibrium with the change of (a_1, a_2).

We obtain the effect of a unit resident migrating from the region j to i by differentiating (13): [17]

$$\frac{\partial V_i}{\partial n_i} = \frac{u_x^i}{n_i}[f_i'(n_i) - x_i] \tag{14}$$

$$\frac{\partial V_j}{\partial n_i} = \frac{u_x^j}{\bar{n} - n_i}[f_j'(\bar{n} - n_i) - x_j]$$

by using the envelope theorem on (11). Whether a flow of population into region i increases their utility or not depends on the sign of $[\,\cdot\,]$ in the above. V_i may have multiple peaks for n_i generally as Atkinson & Stiglitz (1980) suggested in their discussion about resident migration in Ch. 17. [18] We define the following condition.

Definition 1 $\forall n_i \in (\underline{n_i}, \bar{n}_i)$ *for some* $\underline{n_i}$ and $\bar{n}_i$ in $[0, \bar{n}]$,

1. *if* $f_i'(n_i) - x_i > 0$, *then region i is locally sparse,*
2. *if* $f_i'(n_i) - x_i = 0$, $f_i'(n_i - \delta) - x_i > 0$ *and* $f_i'(n_i + \delta) - x_i < 0$ *for all* $\delta > 0$ *such that* $(n_i - \delta, n_i + \delta) \subseteq (\underline{n_i}, \bar{n}_i)$, *then region i is locally optimal, and*
3. *if* $f_i'(n_i) - x_i < 0$, *then region i is locally congested.*

Statement 1 of the definition means that a flow of population into region i will bring increas-

---

[15] In this case, the migration equilibria will be not a function but a correspondence of (a_1, a_2).

[16] Mayers, Wellisch and Caplan et al. research models that assumed that each local government takes resident migration into account in the framework of the game. However, these studies do not consider the possibility of multiple migration equilibria and of the discontinuity of a migration equilibrium, so that they deal with the migration equilibrium as a function of some variables. This paper follows them.

[17] A unit resident does not mean one resident but one weight resident.

ing utility to the region from (14). Thus, the population in i is still sparse. Statements 2 and 3 are parallel logic. However, as we stated above, V_i may have multiple peaks for n_i, so that this definition is only local. If V_i has a single peak for n_i, this definition is global.

Next, we will consider the stability condition of a migration equilibrium. If a migration equilibrium is disturbed for some reason, it may diverge, for example to $n_i = 0$ or $n_i = \bar{n}$. To avoid this divergence, we look only at the case of a locally stable migration equilibrium. This stability means that when some residents with small positive weight migrate from region 1 to region 2 at a migration equilibrium, the utility of residents in region 2 becomes higher than that in region 1, and vice versa. As a result, the residents who migrated return. Hence, we define $\partial V_i / \partial n_i - \partial V_j / \partial n_i < 0$.[19] The stability condition is used in Atkinson & Stiglitz (1980), Boadway & Flatters, Wellisch, Caplan et al. and Mitsui & Sato.[20] In this model, the stability condition is

$$D \equiv \frac{u_x^1}{n_1}(f_1'(n_1) - x_1) + \frac{u_x^2}{\bar{n} - n_1}(f_2'(\bar{n} - n_1) - x_2) < 0. \tag{15}$$

A migration equilibrium under this stability condition is categorized into one of only the following three cases. Specifically,

Case 1 $f_i' - x_i < 0$ and $f_j' - x_i < 0$,

Case 2 $f_i' - x_i = 0$ and $f_j' - x_i < 0$, and

Case 3 $f_i' - x_i < 0$, $f_j' - x_i > 0$, and $\left|\frac{u_x^i}{n_i}(f_i' - x_i)\right| > \left|\frac{u_x^j}{n_j}(f_j' - x_j)\right|$, $i \neq j$, $i, j = 1, 2$.

In addition to this stability condition, we assume that V_1 and V_2 do not become tangent or overlap each other at any of the migration equilibria.

Stage 1 At stage 1, each local government chooses an effort level. First, we consider how residents migrate when a local government increases its effort level. In other words, how does the migration equilibrium vary when a local government slightly improves the public budget balance. To see this, we differentiate $V_1(n_1, a_1) = V_2(\bar{n} - n_1, a_2)$ by a_i. When the migration equilibrium $n_1(a_1, a_2)$ is continuous about a_i, we obtain

$$\frac{\partial n_i}{\partial a_i} = -\frac{1}{D}\frac{u_x^i}{n_i}(1 - d'(a_i)) \tag{16}$$

from (15). This means that increasing efforts to improve the public budget balance brings immigration when the effort is less than the Pareto efficient level, $1 - d'(a_i) > 0$. However, if a local government makes a major change in its effort, then the migration equilibrium may become discontinuous or move to the corner solutions, $n_i = \bar{n}$ or 0.

Now, if residents do not migrate when government i increases its effort, the change of the utility will be

$$\left.\frac{\partial V_i}{\partial a_i}\right|_{n_i \text{ is fixed.}} = \frac{u_x^i}{n_i}(1 - d'(a_i)). \tag{17}$$

---

[18] The second order condition of V_i for n_i is

$$\frac{\partial^2 V_i}{\partial n_i^2} = \left[\frac{V_i'(n_i)}{u_x}\frac{u_{xx}^i(f_i' - x_i)}{n_i} + \frac{u_x^i f_i''}{n_i} - \frac{V'(n_i)}{n_i}\right] + \left[\frac{V_i'(n_i)}{(u_x^i)^2}(u_x^i u_{xy}^i - u_y^i u_{xx}^i) + \frac{(u_y^i)^2}{u_x^i c_i}\right] y_i'(n_i)$$

The first bracketed expression on the right-hand side is negative if $f_i' - x_i > 0$. But the second bracketed expression is positive since $y_i'(n_i) > 0$ when $f_i' - x_i > 0$. Thus, we cannot identify the sign of the second order condition even if $f_i' - x_i > 0$. Of course, we cannot identify it in the case of $f_i' - x_i < 0$, either.

[19] Boadway & Flatters points out that both regions tend to have a unique stable migration equilibrium under a condition of overall overpopulation and that both tend to have an unstable migration equilibrium if there is under-population.

[20] Mitsui & Sato defines it more generally without using differential calculus.

This means that V_i is an increasing function of a_i till a_i^* and is a decreasing function of a_i beyond a_i^* when n_i is fixed. In other words, the indirect utility is maximized at the Pareto efficient effort a_i^* at each n_i, that is to say $V_i\,(n_i, a_i) \leq V_i\,(n_i, a_i^*)$ for all n_i and a_i.

Next, we consider each local government's decision regarding its own efforts, a_1 and a_2. We differentiate V_i by a_i and substitute (16) into it. We obtain

$$\frac{\partial V_i}{\partial a_i} = \frac{u_x^i}{n_i}(f_i'(n_i) - x_i)\frac{\partial n_i}{\partial a_i} + \frac{u_x^i}{n_i}(1 - d'(a_i)) = \frac{u_x^1 u_x^2}{Dn_1(\bar{n} - n_1)}(f_j'(n_j) - x_j)(1 - d'(a_i)) \tag{18}$$

If $\partial V_i / \partial a_i > 0$, then the local government i will get a larger payoff by slightly increasing its effort as long as the migration equilibrium varies continuously. If $\partial V_i / \partial a_i < 0$, then local government i will get a larger payoff by slightly decreasing its effort as well. We obtain the following proposition.

Proposition 1 *If the strategies chosen by residents become a migration equilibrium $n_1(a_1, a_2)$ which is a differentiable function such that $f_1' - x_1 < 0$ and $f_2' - x_2 < 0$ for all a_1 and a_2, then (a_1^*, a_2^*) becomes each local government's behavior in the strategy of a subgame perfect equilibrium.*

In the case of this proposition, two regions are congested at any (a_1, a_2). We can interpret this feature as the case where the effect of each local government's effort is relatively small and total population is relatively large since local governments have hardly any influence on the congested population. Then, (a_1, a_2) becomes Pareto efficient, but the population distribution may not be so since (6) does not always follow from (18). Therefore, an effort to improve the public budget balance does not have the effect of making the population distribution efficient. Of course, since Proposition 1 is a sufficient condition for the efficiency effort, the other conditions may hold, too.

Here, we assume that V_i has a single peak for n_i.[21] This assumption is plausible in practice. Hayashi (2002) measures a minimal efficient scale of population in Japanese local governments. According to Hayashi's analysis, each local government in Japan has a U-form with regard to the local public expenditure per capita, since the fixed costs and the congestion effects of the local public good, which are different in each region, bring a scale economy to each local government. This means that there exists a different intrinsic optimal population level for each local government, while Hayashi's analysis only shows optimal scales of the public expenditures per capita. Thus, it is thought that this assumption is plausible. We denote the optimal population level in region i as n_i^{**} when V_i has a single peak.

Assumption 1 *The following conditions about $V_i\,(n_i, a_i)$ for all a_i are assumed.*

1. *There exists a unique $n_i^{**} \in (0, \bar{n})$ that satisfies* $\left.\dfrac{\partial V_i}{\partial n_i}\right|_{n_i = n_i^{**}} = \dfrac{u_x^i}{n_i^{**}}[f_i'(n_i^{**}) - x_i] = 0$

2. $\forall n_i \in [0, n_i^{**})$, $\dfrac{\partial V_i}{\partial n_i} > 0$, *and* $\forall n_i \in (n_i^{**}, \bar{n})$, $\dfrac{\partial V_i}{\partial n_i} < 0$.[22]

Using this assumption, we obtain the following facts about each optimal population (n_i^{**}, n_j^{**}): $n_i^{**} < n_i(a_1, a_2)$ and $n_j^{**} < \bar{n} - n_i(a_1, a_2)$ in Case 1, $n_i^{**} = n_i(a_1, a_2)$ and $n_j^{**} < \bar{n} - n_i(a_1, a_2)$ in Case 2,

[21] The second order condition of V_i for n_i is not always negative even if u_i is strictly quasi-concave. Boadway & Flatters assumes that the graph of V_i for n_i is single peaked.

[22] We can consider the example case of a single peak. See Appendix.

and $n_i^{**} < n_i(a_1, a_2)$ and $n_j^{**} > \bar{n} - n_i(a_1, a_2)$ in Case 3. See Figure. With regard to the three cases under Assumption 1, the following lemma is obtained.

Lemma 1 *Under Assumption 1, there are no stable and interior multiple migration equilibria satisfying the combinations of Case 1 - Case 1, Case 1 - Case 2 and Case 2 - Case2.*

However, multiple migration equilibria satisfying Case 3 and another case may exist. As a result, the following proposition is obtained.

Proposition 2 *Under Assumption 1, local governments i and j choose the following effort level in a subgame perfect equilibrium.*

1. *If the migration equilibrium $n_1(a_1, a_2)$ satisfies $f_1' - x_1 < 0$ and $f_2' - x_2 < 0$* at (a_1^*, a_2^*), *then (a_1^*, a_2^*) is a Nash equilibrium at stage 1.*
2. *If the migration equilibrium $n_1(a_1, a_2)$ satisfies $f_i' - x_i = 0$, $f_j' - x_j < 0$ and $V_j(\bar{n}, a_j^*) \leq V_j(\bar{n} - n_i, a_j)$ at (a_i^*, a_j), then (a_i^*, a_j) is a Nash equilibrium at stage 1.*
3. *If the migration equilibrium $n_1(a_1, a_2)$ satisfies $f_i' - x_i < 0$, $f_j' - x_j > 0$, $\left|\frac{u_x^i}{n_i}(f_i' - x_i)\right| > \left|\frac{u_x^j}{n_j}(f_j' - x_j)\right|$ and is continuous for all (a_1, a_2), then $(0, a_j^*)$ and $(\bar{a}, a_j^*)$ is a Nash equilibrium at stage 1.*

a_j may not become a_j^*, and $n_i = n_1(a_1, a_2)$ if $i = 1$ and $n_i = \bar{n} - n_1(a_1, a_2)$ if $i = 2$.

Statements 1 and 2 in Proposition 2 state that (a_i^*, a_j^*) and (a_i^*, a_j) are the behaviors in SPE if the migration equilibrium is either Case 1 or Case 2 at (a_i^*, a_j^*) and (a_i^*, a_j), respectively. On the other hand, statement 3 means that the severer condition, meaning the existence of Case 3 for all (a_i, a_j), is required in order for it to be a strategy of SPE.

In statement 1, the migration equilibrium which satisfies $f_i' - x_i < 0$ and $f_j' - x_j < 0$ at (a_1^*, a_2^*) is interpreted as the case where the total population is large and where each region's potential gap is similar. The large total population in this paper means that $n_1^{**} + n_2^{**} < \bar{n}$ at (a_1^*, a_2^*), that is to say that each region cannot absorb the total population at the optimal population level. The potential gap means the difference of resident's utility at the optimal population level and at (a_1^*, a_2^*) in each region. In other words, each local government has similar technology and know-how to provide the public good, and the productivity of the private good is similar in each region. In this case, when the total population is large, both local governments choose the Pareto efficient effort level. The logic is the following. When local government *i* increases its effort to improve its public budget balance, then the residents' utility will increase in *i*. As a result, residents will migrate from region *j* to *i*. Since both regions are congested, this migration will dilute the increase of utility. However, the congestion in region *j* will be mitigated, and the utility of residents in *j* will be improved. As a result, at the new migration equilibrium, the utility of residents in both regions will increase. If local government *i* decreases its effort, the opposite will occur. Hence, the condition where both regions are congested encourages efforts by each local government to make efforts at efficiency.

In statement 2, the migration equilibrium which satisfies $f_i' - x_i = 0$ and $f_j' - x_j < 0$ at (a_i^*, a_j) is interpreted as the case where the potential gap is large and where the total population is large. In this case, the residents' utility at the optimal population level in region *j* is larger than it is in region *i*, when both local governments choose an efficient effort. Local government *j*, which has large potential, does not make efforts in spite of having room to improve its public budget balance, while local government i chooses an efficient effort. The reason is the following. If j increases its effort, residents in i will migrate from i to j. Then, j's congestion will worsen and i will become sparse. On the other hand, if j decreases its effort, residents in j will emigrate from j to i. And then, although j's congestion will be mitigated, i will become congested and this congestion will bring a larger deterioration than the mitigation. As a result, residents' utility will decrease. The condition $V_j(\bar{n}, a_j^*) \leq V_j(\bar{n} - n_i, a_j)$ at (a_i^*, a_j) means the utility level when the number of residents migrating to j at Pareto efficient effort is not very large. This case is where the total population is large and the effort effect is small. If $V_j(\bar{n}, a_j^*)$ is large, on the contrary, local government j may choose a Pareto efficient effort level and attract all residents. In statements 1 and 2, the total population is large since $n_1^{**} + n_2^{**} < \bar{n}$. Specifically, when the total population is large, the equilibrium tends to appear at either Case 1 or 2.

In statement 3, the total population level is small since $n_i > n_i^{**}$ and $\bar{n} - n_i < n_j^{**}$. In addition, if there is neither a stable nor unstable migration equilibrium, except for the corner equilibrium, the potential gap between regions is large since residents' utility at the optimal population level is region j is larger than it is in region i. In this case, local government i chooses either no effort or excessive effort, while local government j chooses the efficient effort. If i makes the efficient effort, residents will migrate from region j to region i. This migration will cause a worsening of the congestion for i and sparseness for j. This will invite a decline in the utility. Hence, j chooses the smallest or largest effort level at the equilibrium.

In conclusion, it is better for the total population to be large and for the difference of the potentiality in each region to be small in order to elicit Pareto efficiency from each local government. Incidentally, the population distribution at the equilibrium may not become Pareto efficient. The efficient population distribution is such that the net marginal product is equalized. (18) does not guarantee this efficiency.

3 The case with simultaneous decisions

In the previous section, we examined the behavior when neither local government could change at stage 2 or 3. However, we can also consider the case where y_i and a_i are decided simultaneously. This case is divided into the following two cases.

The case where migration is not taken into account First, we consider the case in which neither local government takes migration into account. In this case, each local government maximizes

$$\max_{a_i,\, y_i} u^i(x_i, y_i)$$
$$s.t.\ f_i(n_i) = n_i x_i + c_i y_i + F - a_i + d(a_i)$$

given n_i. The first order conditions are

$$y_i \;:\; \frac{u_x^i}{n_i}[n_i \frac{u_y^i}{u_x^i} - c_i] = 0 \tag{19}$$

$$a_i \;:\; \frac{u_x^i}{n_i}(1 - d'(a_i)) = 0. \tag{20}$$

The Samuelson condition and Pareto efficient effort level are fulfilled in (19) and (20) given n_i. However, we do not know how the population is distributed since neither local government considers the residents' migration at all in this case. In other words, Pareto efficiency of the population distribution is not at all guaranteed by (19) and (20). In addition, residents' utility in this case is not larger than that in the previous section. At first sight, it seems that the utility in the case where migration is not taken account of is larger than that in the case where it is taken account of since the effort is always Pareto efficient in the former case. However, when V_i is either statement 2 or 3 in Proposition 2, there exists some $a_i \neq a_i^*$ which makes V_i larger. This result applies in this case. Hence, the decision taking no account of migration is no more efficient than that taking account of migration at least under the single peak assumption.

The case where migration is taken into account Second, we consider the case where each local government takes migration into account. Then, each local government maximizes

$$\max_{a_i,\, y_i} u^i(x_i, y_i)$$
$$s.t.\ f_i(n_i) = n_i x_i + c_i y_i + F - a_i + d(a_i)$$

taking account of the migration equilibrium $n_1(a_1, a_2, y_1, y_2)$ which is a function of a_1, a_2, y_1 and y_2. [23] In the case where the resident migration is taken into account, we do not know whether this maximized problem fulfills the second order condition or not. Thus, we consider the following. We differentiate u_i for y_i and a_i. Then,

$$y_i \ :\ \frac{u_x^i}{n_i}(f_i'(n_i) - x_i)\,\frac{\partial n_i}{\partial y_i} - \frac{u_x^i}{n_i}\,c_i + u_y^i, \tag{21}$$

$$a_i \ :\ \frac{u_x^i}{n_i}(f_i'(n_i) - x_i)\,\frac{\partial n_i}{\partial a_i} + \frac{u_x^i}{n_i}\,(1 - d'(a_i)). \tag{22}$$

When y_i changes slightly under $u_1 = u_2$, the change of the migration equilibrium $n_1(a_1, a_2, y_1, y_2)$ will be

$$\frac{\partial n_i}{\partial y_i} = -\frac{1}{D}\frac{u_x^i}{n_i}(n_i\frac{u_y^i}{u_x^i} - c_i). \tag{23}$$

If there are fewer local public goods than the amount under Samuelson's rule, increasing y_i will bring immigration. We substitute (23) and (14) into (21) and (22), respectively. Then these equations are

$$y_i \ :\ \frac{u_x^1 u_x^2}{Dn_1(\bar{n} - n_1)}\,(f_j' - x_j)\,(n_i\frac{u_y^i}{u_x^i} - c_i), \tag{24}$$

$$a_i \ :\ \frac{u_x^1 u_x^2}{Dn_1(\bar{n} - n_1)}\,(f_j' - x_j)\,(1 - d'(a_i)). \tag{25}$$

(25) is as the same as (18). This means that Proposition 1 applies to this case about the effort level and local public good, [24] namely if $n_1(a_1, a_2, y_1, y_2)$ satisfies $f_1' - x_1 < 0$ and $f_2' - x_2 < 0$ for all a_1, a_2, y_1, y_2, then y_i and a_i will be Pareto efficient. In the other cases where $n_1(a_1, a_2, y_1, y_2)$ does not satisfy both $f_1' - x_1 < 0$ and $f_2' - x_2 < 0$, the effort and public goods may not be Pareto efficient.

---

[23] If $i = 1$, then $n_i = n_1\ (a_1, a_2, y_1, y_2)$ and if $i = 2$, then $n_i = \bar{n} - n_1\ (a_1, a_2, y_1, y_2)$. In this case, residents will migrate after they see (a_1, a_2, y_1, y_2), so that the migration equilibrium becomes a function of these variables.

[24] $f_i' - x_i$ is different from the previous section, since each local government maximizes for a_i and y_i given the other government's strategy. Then, the second order condition of u_i about n_i given a_i and y_i is

The population distribution does not always become Pareto efficient, that is (6). If the population distribution is Pareto efficient and stable, then a_i and y_i will become Pareto efficient, too, since $f_1' - x_1 = f_2' - x_2 < 0$. On the other hand, if local governments do not take migration into account when they provide the local public goods, the Samuelson condition will be satisfied at the migration equilibrium. At first sight, it seems more efficient for neither local government to take resident migration into account. However, the residents in the case where migration is taken into account may be better off than in the case where it is not taken into account, since the population distribution may be more inefficient and each local government will not adjust it in the former case.

4 Conclusion

This paper analyzed the incentives for local governments to improve their public balance under resident migration. The paper showed that in a country with two regions, whether a local government makes efforts or not depends on the population condition in the other region. Even if a local government intends to improve its public budget balance in order to improve the residents' utility in its own region, the utility will become identical to that of residents in the other region, since residents will migrate until their utility becomes equal. Then, if the outflow of population from one (congested) region improves the residents' utility, the local government in the other region will make efficient efforts. However, if the utility worsens due to the outflow of population from one (sparse) region, the local government in the other region will not make efforts. This local government then has the incentive to worsen its public balance in order to promote migration into its own region. Thus, local governments do not always make efforts efficiently under decentralization. In addition, the population distribution is not guaranteed to be Pareto efficient. However, if migration is taken into account in making this decision, it has the effect of making the population distribution efficient to a certain degree. Of course, if migration is not taken into account, this effect does not exist. Thus, even if there is no device such as an interregional transfer mechanism under the decentralized local public finance system, the effort of improving the public balance with migration taken into account has the effect of making the population distribution efficient to a certain degree.

Appendix

The example of a single peak population In this section, we consider the example of a single population peak. We consider the case where $u_i(x_i, y_i) = x_i y_i$, $f_i(n_i) = n_i^a (0 < \alpha < 1/2)$, $d(a_i) = a_i^2$, and $\bar{n} = 1$. Then, the constraint of resources in region i becomes $n_i^a = n_i x_i + c_i y_i + F - a_i + a_i^2$.

We calculate (11) in this example, and get

$$x_i = \frac{n_i^a + a_i - a_i^2 - F}{2n_i} \text{ and } y_i = \frac{n_i^a + a_i - a_i^2 - F}{2c_i}.$$

$$\frac{\partial^2 u^i}{\partial n_i^2} = \frac{f_i' - x_i}{n_i^2}\left(u_{xx}^i (f_i' - x_i) - 2u_x^i\right) + \frac{u_x^i}{n_i} f_i''.$$

If $f_i' - x_i \geq 0$, then $\partial^2 u^i / \partial n_i^2 < 0$. However, when $f_i' - x_i < 0$, this condition may become positive. Hence, the indirect utility may not fulfill the second order condition.

Of course, if $n_i^a + a_i - a_i^2 - F < 0$, then $x_i = y_i = 0$. Thus, the indirect utility function becomes

$$V_i = \begin{cases} \frac{n_i^a + a_i - a_i^2 - F}{4n_i c_i} & \text{if } n_i \geq (F + a_i^2 - a_i)^{1/a} \\ 0 & \text{if } n_i < (F + a_i^2 - a_i)^{1/a} \end{cases} \tag{26}$$

When n_i is small, $n_i^a + a_i - a_i^2 - F < 0$ in (26) because of the assumption of a large F. In this case, the utility becomes zero. [25]

Next, we prove that there is a case of a single peak for the population in this example. We differentiate V_i by n_i, and obtain

$$\frac{\partial V_i}{\partial n_i} = \frac{(n_i^a + a_i - a_i^2 - F)[(2\alpha - 1)n_i^a - a_i + a_i^2 + F]}{4n_i^2 c_i}.$$

For instance, when the parameters are $\alpha = 1/3$ and $F = 4/15$, V_i has only one peak at $n_i^{**} = 27(a_i^2 - a_i)^3 + 64/125$. The net benefit of the local government's effort is $0 \leq a_i - a_i^2 \leq 1/4$, so that the range of the population of the peak is $721/8000 \leq n_i^{**} \leq 64/125]$.

Proof of Proposition 1 *On (18), a_i which maximizes V_i satisfies $1 - d'(a_i^*) = 0$ since $f_1' - x_1 < 0$ and $f_2' - x_2 < 0$ for any a_1 and a_2 and $D < 0$. a_j does also. Hence, (a_1^*, a_2^*) is the strategy in a subgame perfect equilibrium.*

Q.E.D.

Proof of Lemma 1 *Supposing that there are two or more migration equilibria at (a_i, a_j) such that $a_i = a_j$, these satisfy one of three cases. We denote two of these migration equilibria as $(n_i, V_1 = V_2)$ and $(\tilde{n}_i, \tilde{V}_1 = \tilde{V}_2)$, respectively.* [26]

1. We suppose that one migration equilibrium n_i is Case 1 and that the other migration equilibrium $\tilde{n}_i$ is also Case 1. From Assumption 1, if $\tilde{n}_i > n_i$, $\tilde{V}_i < V_1 = V_2 < \tilde{V}_j$ and if $\tilde{n}_i < n_i$, $\tilde{V}_i > V_1 = V_2 > \tilde{V}_j$. However, these contradict $\tilde{V}_1 = \tilde{V}_2$ which is an interior condition.

2. We suppose that one migration equilibrium n_i is Case 1 and the other migration equilibrium $\tilde{n}_i$ is Case 2. Then, $\tilde{n}_i = n_i^* < n_i$ under Assumption 1 because of $f_i' - x_i < 0$ at n_i and $f_i' - x_i = 0$ at $\tilde{n}_i$. Therefore, $\tilde{V}_i > V_1 = V_2$. In addition, $\tilde{V}_j < V_1 = V_2$ since $\bar{n} - n_i^* > \bar{n} - n_i$. However, these contradict $\tilde{V}_1 = \tilde{V}_2$.

3. We suppose that one migration equilibrium n_i is Case 2 and the other migration equilibrium $\tilde{n}_i$ is also Case 2. Then, $n_i = n_i^* = \tilde{n}_i$ since $f_i' - x_i = 0$. From Assumption 1, there is only one n_i^*. Therefore, there are no multiple equilibria, and only Case 2 is satisfied.

Q.E.D.

Proof of Proposition 2 *The proof consists of three parts, 1, 2 and 3.*

1. Let us assume that the local government i deviates from a_i^* to $a_i \neq a_i^*$ given a_j^*. We denote the migration equilibrium as n_i^d [27] at this deviation (a_i, a_j^*).

1) First, we consider the case of a stable and interior migration equilibrium that varies continuously from a_i^* to a_i. (18) is 0 at a_i^* since $1 - d'(a_i^*) = 0$. For all $a_i > a_i^*$, V_i is a decreasing function under Assumption 1 since residents migrate from i to j if (16) < 0 and $f_j' - x_j < 0$ for all $a_i > a_i^*$. For all $a_i < a_i^*$, V_i is an increasing function because of the same logic. Then, V_i is

[25] See footnote 9.

[26] If $i = 1$, then $n_i = n_1(a_1, a_2)$, and if $i = 2$, then $n_i = \bar{n} - n_1(a_1, a_2)$.

[27] n_i^d means that $n_i^d = n_1^d(a_1, a_2)$ if $i = 1$ and $n_i^d = \bar{n} - n_1^d(a_1, a_2)$ if $i = 2$. $n_1^d(a_1, a_2)$ is the migration equilibrium when a local government deviates.

maximized at a_i^*.

2) Next, we consider the case where the migration equilibrium does not vary continuously from a_i^* *to* a_i. *This deviation brings the migration equilibria to either* $n_i = 0$, $n_i = \bar{n}$ *or Case 3 because of Lemma 1. Then, at* $n_i = 0$ *and* $n_i = \bar{n}$, $V_i(0, a_i) \leq V_i(0, a_i^*) < V_i(n_i, a_i^*)$ *and* $V_i(\bar{n}, a_i) < V_i(\bar{n}, a_i^*) < V_i(n_i, a_i^*)$ *because of (17) and Assumption 1. Thus, this deviation decreases i's payoff.*

In Case 3, region i becomes either $f_i' - x_i < 0$ *or* $f_i' - x_i > 0$ *at* n_i^d. *When* $f_i' - x_i < 0$ *at* n_i^d, $f_j' - x_j > 0$ *in region j. Then* $n_i < n_i^d$ *because of Assumption 1 and* $f_j' - x_j < 0$ *at* n_i. *Thus,* $V_i(n_i^d, a_i) < V_i(n_i^d, a_i^*) < V_i(n_i, a_i^*)$ *since* $f_i' - x_i < 0$ *in* $[n_i, n_i^d]$. *Hence, local government i does not have the incentive for this deviation.*

When $f_i' - x_i > 0$ *at* n_i^d, $f_j' - x_j < 0$ *in region j. Then,* $n_i^d < n_i$ *since if* $n_i^d \geq n_i$, $V_i(n_i, a_i^*) = V_j(\bar{n} - n_i, a_j^*) \leq V_j(\bar{n} - n_i^d, a_j^*) = V_i(n_i^d, a_i) < V_i(n_i^d, a_i^*)$ *from* $f_j' - x_j < 0$ *and (17). But this contradicts* $f_i' - x_i < 0$ *in* $[n_i, n_i^d]$ *at* a_i^* *under Assumption 1. Therefore,* $n_i^d < n_i$. *Then,* $V_i(n_i^d, a_i) = V_j(\bar{n} - n_i^d, a_j^*) < V_j(\bar{n} - n_i, a_j^*) = V_i(n_i, a_i^*)$ *since* $f_j' - x_j < 0$ *in* $[n_i^d, n_i]$ *and because of Assumption 1. Hence, local government i does not have the incentive for this deviation.*

2. 1) Let us assume local government i deviates from a_i^* *to* $a_i \neq a_i^*$ *given* a_j. *We denote the migration equilibrium of this deviation as* n_i^d. *When the migration equilibrium varies continuously from* a_i^* *to* a_i *given* a_j, V_i *is maximized at* a_i^*, *as is 1-1) in this proof.*

We consider the case where the migration equilibrium does not vary continuously from a_i^* *to* a_i. *This deviation brings the migration equilibria to either* $n_i = 0$, $n_i = \bar{n}$ *or Case 3 because of Lemma 1. Then, at* $n_i = 0$ *and* $n_i = \bar{n}$, $V_i(0, a_i) \leq V_i(0, a_i^*) < V_i(n_i, a_i^*)$ *and* $V_i(\bar{n}, a_i) < V_i(\bar{n}, a_i^*) < V_i(n_i, a_i^*)$ *because of (17) and Assumption 1. Thus, this deviation decreases i's payoff.*

In Case 3, region i becomes either $f_i' - x_i < 0$ *or* $f_i' - x_i > 0$ *at* n_i^d. *When* $f_i' - x_i < 0$ *at* n_i^d, $f_j' - x_j > 0$ *in region j. Then,* $n_i < n_i^d$ *because of Assumption 1 and* $f_j' - x_j < 0$ *at* n_i. *Then,* $V_i(n_i^d, a_i) < V_i(n_i^d, a_i^*) < V_i(n_i, a_i^*)$ *since* $f_i' - x_i < 0$ *in* $(n_i, n_i^d]$ *at* a_i^*. *Hence, local government i does not have the incentive for this deviation.*

When $f_i' - x_i > 0$ *at* n_i^d, $f_j' - x_j < 0$ *in region j. Then,* $n_i^d < n_i$ *since if* $n_i^d \geq n_i$, $V_i(n_i, a_i^*) = V_j(\bar{n} - n_i, a_j) \leq V_j(\bar{n} - n_i^d, a_j) = V_i(n_i^d, a_i) < V_i(n_i^d, a_i^*)$ *from* $f_j' - x_j < 0$ *and (17). But this contradicts* $f_i' - x_i$ *at* n_i *at* a_i^* *under Assumption 1. Therefore,* $n_i^d < n_i$. *Then* $V_i(n_i^d, a_i) = V_j(\bar{n} - n_i^d, a_j) < V_j(\bar{n} - n_i, a_j) = V_i(n_i, a_i^*)$ *since* $f_j' - x_j < 0$ *in* $[n_i^d, n_i]$ *and because of Assumption 1. Hence,* $V_i(n_i^d, a_i) < V_i(n_i, a_i^*)$, *namely local government i does not have the incentive for this deviation.*

2) Let us assume that local government j deviates from a_j *to* $\hat{a}_j \neq a_j$ *given* a_i^*. *First, we consider that the migration equilibrium moves to the other interior equilibrium from* a_j *to* $\hat{a}_j$. *The migration that causes this deviation decreases the utility of residents in region j since* V_i *is the optimal scale of population at* n_i. *Thus, j does not have the incentive for this deviation. Second, we consider the case where the migration equilibrium moves to the corner due to the change from* a_j *to* $\hat{a}_j$. *In this case the migration equilibrium becomes either* $n_i = 0$ *or* $n_i = \bar{n}$. *Then,* $V_j(0, a_j^*) \leq V_i(\bar{n}, a_i^*) < V_i(n_i, a_i^*) = V_j(\bar{n} - n_i, a_j)$ *because of the definition of the migration equilibrium and* $f_i' - x_i = 0$ *at* n_i, *and* $V_j(\bar{n}, a_j^*) \leq V_j(\bar{n} - n_i\ a_j)$, *which is the condition in this proposition. Hence,* $(a_i^*\ a_j)$ *is a Nash equilibrium.*

3. 1) V_i *is a decreasing function for* a_i *in* $[0, a_i^*]$ *since* $f_j' - x_j > 0$ *and* $1 - d'(a_i) > 0$ *in (18). Then, an optimal effort for j is* $a_i = 0$. *On the other hand* V_i *is an increasing function for* a_i *in* $[a_i^*, \bar{a}]$ *since* $f_j' - x_j > 0$ *and* $1 - d'(a_i) < 0$ *in (18). Thus,* a_i *which maximizes* V_i *is* $a_i = 0, \bar{a}$.

2) V_j *is an increasing function for* a_j *in* $[0, a_j^*]$ *since* $f_i' - x_i < 0$ *and* $1 - d'(a_j) > 0$ *in (18). On the other hand,* V_j *is a decreasing function for* a_j *in* $[a_j^*, \bar{a}]$ *since* $f_i' - x_i < 0$ *and* $1 - d'(a_j) < 0$ *in (18). Thus,* a_j *which maximizes* V_j *is* $a_j = a_j^*$.

Q.E.D.

Reference

Atkinson, A.B.& Stiglitz, J.E. (1980), *Lectures on Public Economics*, McGraw-Hill Education.

Boadway, R. & Flatters, F. (1982), "Efficiency and Equalization Payments in a Federal System of Government: a Synthesis and Extension of Recent Results", *Canadian Journal of Economics* 15-4, pp. 613-633.

Caplan, A.J., Cornes, R.C. & Silva, E.C.D. (2000), "Pure Public Goods and Income Redistribution in a Federation with Decentralized Leadership and Imperfect Labor Mobility", *Journal of Public Economics* 77, pp. 265-284.

Hartwick, J.M. (1980), "The Henry George Rule, Optimal Population, and Interregional Equity", *Canadian Journal of Economics* 13-4, pp. 695-700.

Mitsui, K. & Sato, M. (2001), "Ex Ante Free Mobility, Ex Post Immobility, and Time Consistency in a Federal System", *Journal of Public Economics* 82, pp. 445-460.

Myers, G.M. (1990), "Optimality, Free Mobility, and the Regional Authority in a Federation", *Journal of Public Economics* 43, pp. 107-121.

Inman, R. & Rubinfeld, D. (1997), "Rethinking of Federalism", *Journal of Economic Perspectives* 11, pp. 43-64.

Oates, W. (1999), "An Essay on Fiscal Federalism", *Journal of Economic Literature* 37, pp. 1120-1149.

Rose-Ackerman, S. (1980), "Risk Taking and Reelection: Does Federalism Promote Innovation?", *Journal of Legal Studies* 9, pp. 593-616.

Strumpf, K. (2002), "Does Government Decentralization Increase Policy Innovation?", *Journal of Public Economic Theory* 4-2, pp. 207-241.

Tiebout, C. (1956), "A Pure Theory of Local Expenditures", *Journal of Political Economics* 64 pp. 416-424.

Wellisch, D. (1994), "Interregional Spillovers in the Presence of Perfect and Imperfect Household Mobility", *Journal of Public Economics* 55, pp. 167-184.

Zeng, D. (2002), "Equilibrium Stability for a Migration Model", *Regional Science and Urban Economics* 31, pp. 123-138.

Hayashi, M. (2002), "Returns to Scale, Congestion and the Minimal Efficient Scales of the Local Public Services in Japan", *Financial Review* (Ministry of Finance Japan) 61 (Japanese).

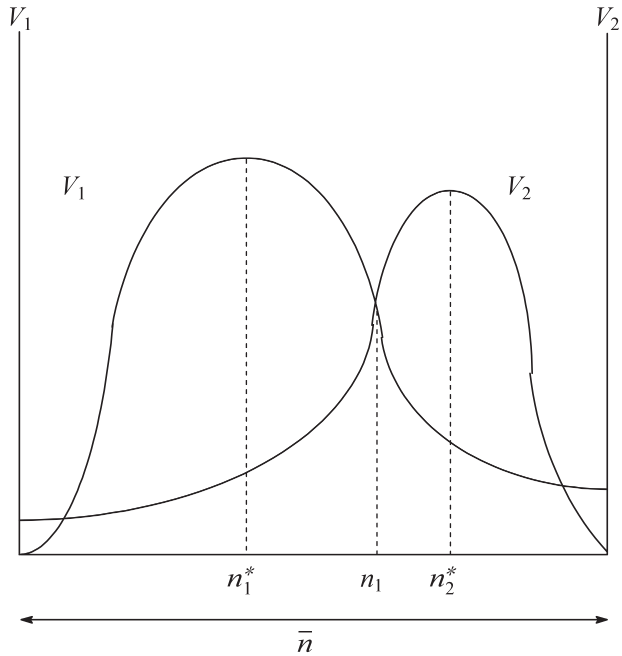
Itaba, Y. (2002), *Local Public Finance Systems for the Era of Decentralization*, Yuhikaku (Japanese).

Komiyama, Y. (2003), "Trust of Water Service to a Private Firm in Ota City", *Public Firm* 35-1 (Japanese).

Ministry of Internal Affairs and Communications (2006), *White Paper on Local Public Finance, 2006 edition*, National Printing Bureau (Japanese).

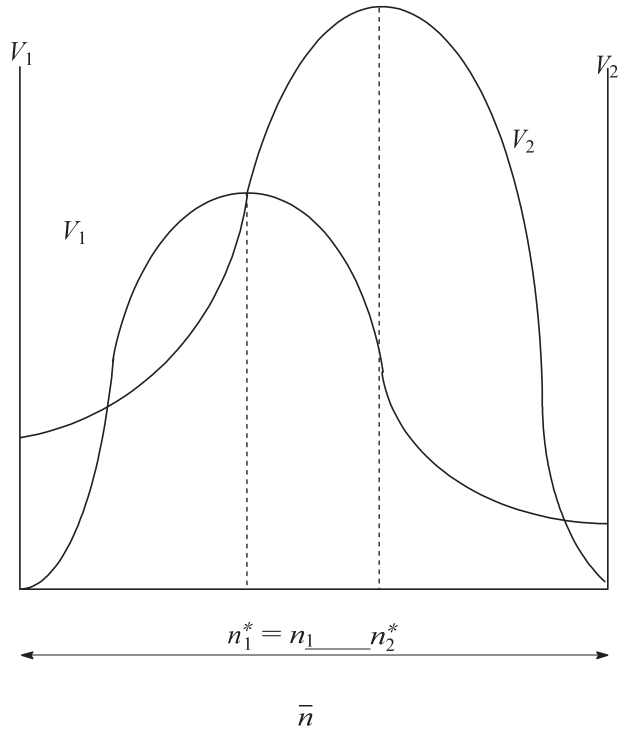
Wakasugi, T. & Kobayashi, H. (2006), "Enlarging Outsourcing to 80% of the GeneralAffairs Office Work Consigned", *Nikkei Glocal* 44 (Japanese).

Case 1

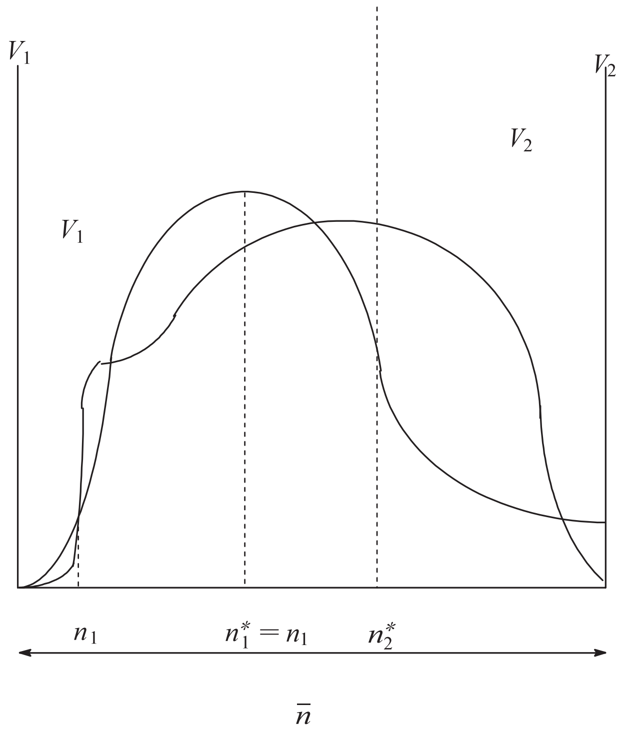


n_1 is the migration equilibrium at $(\overline{a}, \overline{a})$.

Case 2



Case 3



n_1 is the migration equilibrium of Case 3.