A Synthesis of Heterodox Economic Approaches: From the Perspective of the Motion of Capital

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A Synthesis of Heterodox Economic Approaches: From the Perspective of the Motion of Capital*

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Introduction

The aim of this paper is to synthesize three heterodox economic approaches to the problem of how accumulation is determined. I will therefore look at the Marx-Morishima, Keynes-Robinson and Marris-Wood approaches, and provide a synthesized approach, from which the three approaches can be derived.

In section 1, I first lay out a basic model. It contains the common features shared by the three approaches. They all have three unknown variables (the rate of profit, growth, and real wage) and two equations (the Cambridge equation and the wage-profit frontier). In order to complete the model, it is necessary to add one more equation. In the heterodox tradition, this takes the form of one of the three above-mentioned approaches. Second, I outline the three approaches. In the Marx-Morishima approach, the wage-profit frontier is introduced with a constant conventional wage. Under the Keynes-Robinson approach, the model is closed by an investment function. The Marris-Wood approach, to which little attention has been given, insists that it should be closed by what Marris calls the "growth-profitability function." Thus, we have before us three different approaches in a somewhat disjointed state. A simple question is raised: which is correct?

In section 2, I lay out a synthesized approach in a constructive way, which takes into explicit account the fact that it takes time for capital to move through its circuit.[1] I introduce two different growth rates: the supply-side growth rate and the demand-side growth rate. I first discuss the supply-side growth rate which is determined by two equations. The first is the Cambridge equation, and the second describes how long it takes for capital to pass through the processes of production and circulation. I call this second equation the generalized wage-profit frontier, which relates the profit rate not only to the wage rate but also to the growth rate. This equation can be identified with the growth-profitability frontier, as well as the wage-profit frontier if we add the simplifying assumption that the capital turnover rate is unity. If the real wage is assumed to be a constant, those two equations in this model, which are named circuit conditions, determine the profit rate and growth rate. I call this growth rate the supply-side growth rate, and it represents the point at which capitalists will be content with what they are doing. It is, therefore, interpreted as Harrod's warranted rate of growth. That means it is not the actual rate of growth which one observes at a point of time. If circuit conditions determine the warranted rate of growth, then what determines the actual rate of growth? Second, the paper aims to derive a Marxian investment function

[1] This is an expanded model of Foley (1982) approach, inspired by Marx's circuit of capital.

which determines the demand-side growth rate, or rather, the actual growth rate. The Marxian investment function is derived from the continuous metamorphosis of capital through which the self-expansion of the capital-value takes place. We can interpret the self-expansion of the capital-value as being the objective, and the metamorphosis as the constraint. Therefore capitalists maximize the capital-value, which is interpreted as the present value of net profits, discounted at the interest rate, subject to the circular movement of capital, which can be translated as the circuit conditions. The first order condition indicates that the marginal rate of profit equals the promoter's profit per capital. This condition is nothing more than the Marxian investment function which determines the demand-side growth rate. Finally, I discuss the existence of the steady-state growth rate and address the problem of how the growth rate is determined.

1 The Basic Model and Three Different Approaches

1.1 The Basic Model

Based on Marglin (1984a,b) and Dutt (1990), we construct a basic model.

Production is characterized by one-commodity model. We assume that the output is produced by means of two factors of production: input and labor input. Technology is assumed to exhibit a fixed coefficient of input a and labor input l.

Price formation is

$$p = (1+\pi)pa + wl.$$

This equation states that the nominal price of the commodity p is the sum of the cost of the input pa, the profit πpa (in which π is the profit rate), and the cost of wage wl (in which w is the nominal wage rate).[2]

This equation should be expressed in real, not nominal, terms. Let us introduce ω as the real wage w/p. Dividing through by p, and after some manipulation, we get:

$$\pi = (1 - a - \omega l)/a. \tag{1}$$

We call equation (1) the "wage-profit frontier."

Let us turn now to the quantity side. It is assumed that the workers' wage should not be saved, and capitalists save a fraction, s of their profit. s is called the

[2] We assume that the wage is paid at the end of the period.

capitalist's propensity to save, and is assumed to be a constant. These assumptions imply that capitalist's savings equal their investment, so that

$$s(p - pa - wl)x(t) = pa\dot{x}(t),$$

where $x(t)$ is output level at time t, $\dot{x}(t) \equiv \mathrm{d}x(t)/\mathrm{d}t$ is the increment of output at time t. This equation says that capitalists' savings from the profit at time t must equal their investment.

Similarly, the scale of quantity is arbitrary and needs to be normalized. We set the volume of capital as a numéraire, which is $pax(t)$ in the basic model. Dividing by $pax(t)$, we get:

$$g = s\pi, \tag{2}$$

where $g \equiv \dot{x}(t)/x(t)$, which is the rate of growth under the stationary state. This is the so-called Cambridge equation.[3]

In the basic model, there are three unknown variables: π, ω, g, but there are only two equations: the wage-profit frontier (1) and Cambridge equation (2). One equation needs to be added in order to close the basic model. What kind of equation should be added?

There is not a single answer. We have, at least, three answers in the heterodox tradition: the Marx-Morishima Approach, Keynes-Robinson Approach and Marris-Wood Approach. We examine these approaches in the following subsections.

1.2 Three Approaches

1.2.1 Marx-Morishima Approach

In the Marx-Morishima approach, the real wage is conventionally determined, or in short, exogenously given.[4] This real wage is called the conventional wage, denoted by b. We get:

$$\omega = b. \tag{3}$$

In this approach, there are three unknown variables: π, ω, g, and three equations: the wage-profit frontier (1), Cambridge equation (2), and conventional wage (3). Assuming that $1 > a + bl$, this system has a unique solution.

[3] See Pasinetti (1974) for details.

[4] See Marx (1977), Morishima (1973), and also see Marglin (1984a,b).

1.2.2 Keynes-Robinson Approach

In the Keynes-Robinson approach, the system is closed the by long-run investment function:[5]

$$g = g(\pi), \qquad g'(\pi) > 0. \tag{4}$$

This investment function shows that the rate of growth is determined by the rate of profit. More precisely, this equation should be written as $g = g(\pi^e)$ where π^e is the expected rate of profit. This means that the higher the expected profit rate, the higher the growth rate. In order to close the model, static profit expectations are employed, $\pi^e = \pi$.

Therefore, three equations (1), (2), and (4) determine three unknown variables: π, ω, g. This equation system is completed.

1.2.3 Marris-Wood Approach

In the place of the investment function, the Marris-Wood approach is closed by the so-called growth-profitability function.[6]

$$\pi = \pi(g), \qquad \pi'(g) < 0, \quad \pi''(g) < 0. \tag{5}$$

This function states that there is a negative relationship between the profit rate and the growth rate. This is because, in order to achieve faster growth, the firm must increase its "development expenditure," which is, for example, the cost for R & D and/or advertising.

This system consists of three equations: (1), (2), and (5) and three variables: π, ω, and g.

1.3 What is the Relation between the Three Approaches?

We now have three approaches for completing the basic model. But these approaches are mutually incompatible. As Figure 1 illustrates, there are too many equations for the number of unknown variables. For example, the Marx-Morishima approach and Keynes-Robinson approach are incompatible, for the conventional wage and the investment function overdetermine the system.

[5] This equation is formulated by Robinson (1962). Also see Roemer (1981, chap. 9).

[6] See Marris (1967). And also see Marris (1971, chap. 1) and Wood (1975).

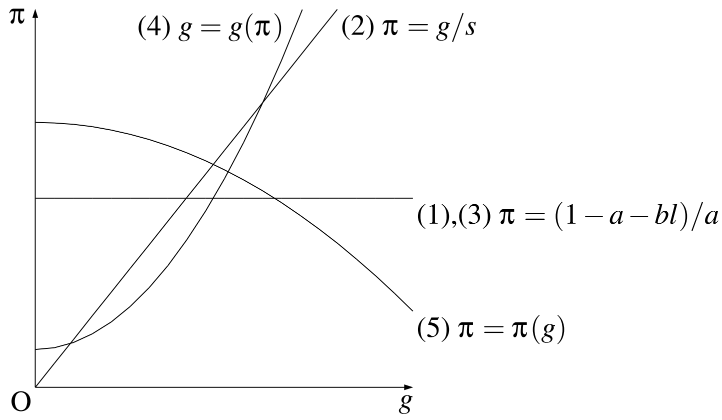

Figure 1: Incompatibility between Three Approaches

More importantly, each approach has its own entirely different closures. In $\pi \times g$ as Figure 1, the Marx-Morishima, Keynes-Robinson, and Marris-Wood approaches are closed by the horizontal, upward, and downward curve, respectively. There is thus an enormous difference between their approaches, and each claims to be most fundamental in the construction of the theory of the dynamics of capitalism.

In next section, we present a synthesized approach, from which the three approaches can be derived.

2 A Synthesized Approach

The circuit of capital, in *Capital*, volume II, chapter 1, provides the analytical tool for constructing a synthesized model. Marx represents the circular motion of capital in the following formula:

$$\mathrm{M} - \mathrm{C} \cdots \mathrm{P} \cdots \mathrm{C}' - \mathrm{M}'.$$

For Marx, the word capital does not have the same meaning as in modern economic literature. Capital undergoes a metamorphosis which transforms M, money-capital, successively into P, productive capital, C′, commodity-capital yet again and finally money-capital, with the cycle being completed.

But we would like to employ the following formula as the circuit of capital, slightly differently from Marx's formulation:

$$M \longrightarrow P \dashrightarrow C' \longrightarrow M'.$$

The difference is that C is omitted. The reason for this omission is that C in Marx's formulation does not represent any capital: money, productive, and commodity-capital, but represents the transaction of commodities. In other words, M, P, and C' are stock variables but C is exceptionally a flow variable.[7] In our formulation, we can distinguish between stock and flow variables: each of the nodes corresponds to the stocks, and the arrows between the nodes correspond to the flows. Our formulation leaves less room for misunderstanding than Marx's. We present our formula of the capital circuit with mathematical expressions in the next subsection.

2.1 The Determination of the Supply-Side Growth Rate

2.1.1 The Circuit of Capital

Figure 2 shows the entire picture of the capital circuit.

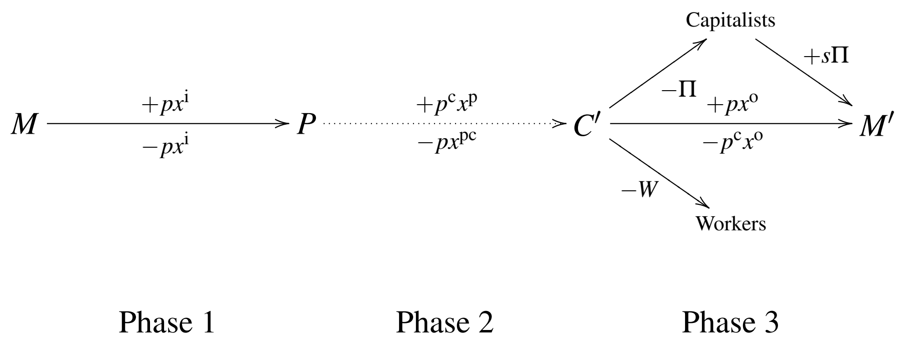

Figure 2: The formula of capital in diagrammatic form

[7]Indeed, C may be viewed consistently either from the stock or flow point of view, since there are inventories of raw materials awaiting production. But the inventories of raw materials should be viewed as a part of the productive capital P.

Phase 1 represents the transformation from money-capital M into productive capital P, or in short, the purchase phase. Purchasing commodities, say, raw materials, means increasing the amount of productive capital and decreasing the amount of money-capital. Let the inflow of commodities be denoted by x^{i}, so that the amount of productive capital is increased by $+px^{\mathrm{i}}$ in price terms, and money-capital is decreased by $-px^{\mathrm{i}}$. In Figure 2, the element above the arrow represents the amount of flow increasing the right-side stock, and the element below the arrow represents the amount of flow decreasing the left-side stock. Naturally, the sum of the two elements above and below the arrow must equal zero. We call this the "bookkeeping rule."

The phase 2, indicated by the dotted arrow, represents the transformation from productive capital P into commodity-capital C', or in other words, the production phase. In this process, the products are produced and the raw materials are productively consumed. Let x^{p} be the amount of production, and x^{pc} be the amount of productive consumption, so that $x^{\mathrm{pc}}/x^{\mathrm{p}} = a$ by definition is the Leontief coefficient. The products should be measured at cost price, which is denoted by p^{c}. P is decreased by px^{pc}, and C' is increased by $p^{\mathrm{c}}x^{\mathrm{p}}$. According to bookkeeping rule, it must hold that $p^{\mathrm{c}}x^{\mathrm{p}} = px^{\mathrm{pc}}$, so that we get $p^{\mathrm{c}} = pa$.

In phase 3, there are two different kinds of arrows. One is a horizontal arrow, and the others are diagonal arrows. The horizontal arrow represents the selling phase, and the diagonal arrows represent the phase of distribution. The sale of commodities is the transformation of commodity-capital C' into money-capital M'. It decreases C', the merchandise inventory and increases M', the fund reserve. Let x^{o} be the outflow of commodities. Then, the total amount of commodities' outflow is $-p^{\mathrm{c}}x^{\mathrm{o}}$, and the gross amount of cash inflow is $+px^{\mathrm{o}}$. The amount of these differences, $px^{\mathrm{o}} - p^{\mathrm{c}}x^{\mathrm{o}}$, is called income.

The income is distributed to workers as wages, W, and to capitalists as profits, Π. W can be expressed as wlx^{o}, where w is the nominal wage rate and l is the labor coefficient. From bookkeeping rule, we get $\Pi = px^{\mathrm{o}} - p^{\mathrm{c}}x^{\mathrm{o}} - W = (p - pa - wl)x^{\mathrm{o}}$. The some fraction of the profits is recommitted in the case of expanded reproduction. This fraction is the capitalists' propensity to save, or rather, to accumulate. We denote this retained fraction as s, and the amount of recommitted profit is $s\Pi$.[8] The circuit of capital will then be repeated.

In next subsection we derive relational expressions of the capital circuit from Figure 2.

[8]Therefore, $(1-s)\Pi$ represents the dividends in the case of a joint stock company.

2.1.2 Accumulation of Capital

First, we investigate how the capital stocks increase in the circuit. The stocks in the circuit, M, P and C,[9] are governed by the following rule:

$$\begin{aligned}
\dot{P}(t) &= px^{\mathrm{i}}(t) - pax^{\mathrm{p}}(t), \\
\dot{C}(t) &= pax^{\mathrm{p}}(t) - pax^{\mathrm{o}}(t), \\
\dot{M}(t) &= px^{\mathrm{o}}(t) - W(t) - \Pi(t) + s\Pi - px^{\mathrm{i}}(t),
\end{aligned} \tag{6}$$

where

$$W(t) = wlx^{\mathrm{o}}(t), \tag{7}$$

$$\Pi(t) = (p - pa - wl)x^{\mathrm{o}}(t). \tag{8}$$

These equations (6) follow from the bookkeeping rule. They represent the fact that each unit of capital is increased by the inflow and decreased by the outflow. Each unit of capital is accumulated when the inflow is greater than the outflow. If we define the total volume of capital as $K(t) \equiv P(t) + C(t) + M(t)$, then the increment of capital is denoted by $\dot{K}(t) = \dot{P}(t) + \dot{C}(t) + \dot{M}(t)$. We can easily get:

$$\dot{K}(t) = \dot{P}(t) + \dot{C}(t) + \dot{M}(t) = s(p - pa - wl)x^{\mathrm{o}}(t). \tag{9}$$

We concentrate on analyzing the stationary state, where the growth rate is $g = \dot{K}(t)/K(t)$ and the profit rate is $\pi = \Pi(t)/K(t)$, both being independent of time t.

The scale of output is arbitrary. Dividing (9) by the volume of capital $K(t)$, we get:

$$g = s(p - pa - wl)x^{\mathrm{o}}(t)/K(t) = s\pi.$$

The Cambridge equation (2) also holds in a synthesized model. But the interpretation differs from that of the basic model. The Cambridge equation in the basic model means that saving equals investment. On the other hand, it does not hold true in a synthesized model, because the increment of capital $\dot{K}(t)$ contains the increment of the money-capital $\dot{M}(t)$ which is not real investment. $s\Pi$ in this model represents the additional funds which increase the volume of capital and have not been expended yet. g in this model should be interpreted as the rate of supply-side growth, which is determined by the additional funds $s\Pi$.

[9] Henceforth in this paper we denote the commodity-capital as C, not C', for the symbol of the prime may be misunderstood as the derivation.

2.1.3 Turnover of Capital

In the previous subsection, we evaluated the increase of the total volume of capital, $\dot{K}(t)$. In this subsection, we evaluate the total volume of capital, $K(t)$. To that end, we consider the transfer process and formulate the ratio of capital turnover. The inflow and outflow are related by the convolution:

$$
\begin{aligned}
ax^{\mathrm{p}}(t) &= \int_{-\infty}^{t} x^{\mathrm{i}}(t')\alpha(t-t')\,\mathrm{d}t', \\
x^{\mathrm{o}}(t) &= \int_{-\infty}^{t} x^{\mathrm{p}}(t')\beta(t-t')\,\mathrm{d}t', \\
px^{\mathrm{i}}(t) &= \int_{-\infty}^{t} \big(px^{\mathrm{o}}(t') - W(t') - \Pi(t') + s\Pi(t')\big)\gamma(t-t')\,\mathrm{d}t'.
\end{aligned}
\tag{10}
$$

α represents a distributed lag in the process of production, interpreted as the proportion of commodity inflow at time t, that is consumed productively at time $t+t'$. β is a distributed lag in the process of sale, interpreted as the proportion of products at time t, that are sold at time $t+t'$. γ is also a distributed lag in the process of purchase, which is also interpreted as the proportion of money inflow obtained by selling at time t, and which is paid to get in a stock at time $t+t'$. α, β and γ are nonnegative and integrate to 1 over the positive half-line.

Under the stationary state, the initial conditions must satisfy the following equations.[10] Reasoning from the three equations (10),

$$
\begin{aligned}
ax^{\mathrm{p}} &= x^{\mathrm{i}}\alpha^*(g), \\
x^{\mathrm{o}} &= x^{\mathrm{p}}\beta^*(g), \\
px^{\mathrm{i}} &= \big(p-(1-s)(p-pa-wl)-wl\big)x^{\mathrm{o}}\gamma^*(g),
\end{aligned}
\tag{11}
$$

where

$$
\alpha^*(g) = \int_0^{\infty} \alpha(t)\exp(-gt)\,dt
$$

which is the Laplace transform of the lag function $\alpha(.)$ and similarly for $\beta^*(g)$ and $\gamma^*(g)$. The Laplace transform has specific properties as follows: $\alpha^*(0)=1$, $\mathrm{d}\alpha^*(g)/\mathrm{d}g<0$, $\lim_{g\to\infty}\alpha^*(g)=0$.

[10]We omit the initial time subscript such as $x(0)=x$.

From these equations, we can derive the stock variables P, C, and M. Noting that all stock variables grow at the rate of g, and substituting (11) to (6), we get:

$$\begin{aligned} P &= pax^{\mathrm{o}}\big(1-\alpha^*(g)\big)/g\alpha^*(g)\beta^*(g), \\ C &= pax^{\mathrm{o}}\big(1-\beta^*(g)\big)/g\beta^*(g), \\ M &= pax^{\mathrm{o}}\big(1-\gamma^*(g)\big)/g\alpha^*(g)\beta^*(g)\gamma^*(g). \end{aligned} \tag{12}$$

Summing up (12), we get

$$K = P + C + M = pax^{\mathrm{o}}/\tau(g), \tag{13}$$

where

$$\tau(g) \equiv g\alpha^*(g)\beta^*(g)\gamma^*(g)/\big(1-\alpha^*(g)\beta^*(g)\gamma^*(g)\big). \tag{14}$$

$\tau(g)$ represents the ratio of capital turnover, which is calculated by dividing the total cost pax^{o} with the total volume of capital K.

Substituting the total profit (8) and the total capital (13) to the definition of profit rate $\pi = \Pi/K$, we get:

$$\pi = (1-a-\omega l)\tau(g)/a. \tag{15}$$

This equation (15) states that the profit rate depends not only upon the wage rate but also upon the growth rate. We call this equation the "generalized wage-profit frontier." It resembles the wage-profit frontier in that it depends upon the real wage rate, and is slightly similar to the growth-profitability function in that it depends on the growth rate. How does it relate to the other approaches? We will answer that question in the next subsection.

2.1.4 The Generalized Wage-Profit Frontier and Circuit Conditions

The Marx-Morishima approach is a special case of a synthesized approach. It is derived from a generalized model by the addition of the assumption that the ratio of capital turnover is identically unity: if $\tau(g) \equiv 1$, then (15) is reduced to (1). More precisely, it is assumed that the period of circulation is instantaneous, i.e., $\beta^*(g) = \gamma^*(g) = 1$, and the period of production is unity. Under the Marx-Morishima approach, in other words, the elasticity of the profit rate with respect to the growth ratio is assumed to be zero.

There is little difference between a synthesized approach and the Marris-Wood approach, for the ratio of capital turnover in a synthesized approach depends upon g, and the profit rate is expressed as $\pi = \pi(g)$ as in the case of the Marris-Wood approach. There is a slight difference between the two approaches in the assumption for the derivatives of the profit rate. In the Marris-Wood approach, it is simply assumed that both $\pi'(g)$ and $\pi''(g)$ are negative. In a synthesized approach, on the other hand, these signs are generally indeterminate, for the sign of τ depends upon the shape of the distributed lags. Hereafter we assume, for the sake of simplicity, that they are negative. [11]

Applying (2), (15) and the assumption of the conventional wage (3), we obtain the unique growth rate without any investment function. From (2), (15), and (3), we derive:

$$1/\big(1+s(1-a-bl)/a\big) = \alpha^*(g)\beta^*(g)\gamma^*(g).$$

The value of the left hand side is the constant less than unity. The value of the right hand side has some specific features: it is unity when $g = 0$ and zero when $g \to \infty$. And yet it is a continuously decreasing function on g. Therefore it has a unique solution even if our model lacks any investment function. We define (2) and (15) as the "circuit conditions" and denote this "supply-side" growth rate as g^s. As illustrated by Figure 3, the circuit conditions with a constant wage determine the supply-side growth rate.

Why is the growth rate determined without any investment function? The answer is that this rate of growth is not the actual growth rate balancing saving and

[11] What is the significance of this assumption? In order to interpret this assumption, we define two concepts. One is the increment of capital, which is defined as $I \equiv gK$, and the other is the elasticity of the increment of capital with respect to the growth ratio, which is defined as $\eta \equiv gI'/I$.

First, the sign of the first derivative of the profit rate is negative if and only if we assume:

$$\eta - 1 > 0.$$

In other words, the elasticity is greater than unity. The meaning of this condition is clear. It indicates that the rate of change of the growth rate is less than the rate of change of the capital increment. This assumption, therefore, eliminates the possibility of increasing returns to scale.

Second, the sign of the second derivatives is negative if and only if we assume:

$$\frac{g\eta'}{\eta} > \eta - 1.$$

In other words, "the elasticity of the elasticity" is more than the elasticity minus unity. This formulation is mathematically clear, but is too complex to interpret.

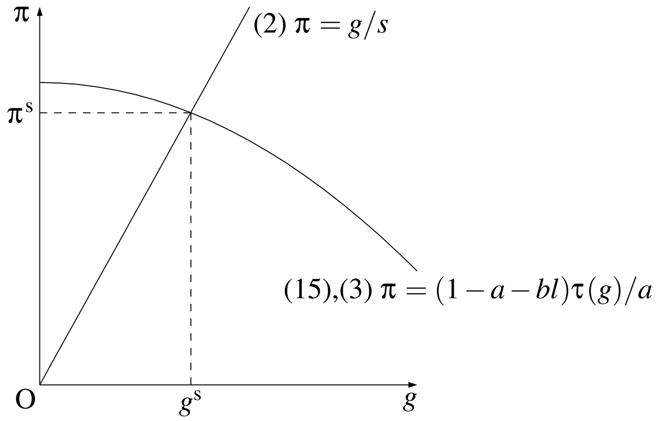

Figure 3: Circuit Conditions Determine the Supply-Side Growth Rate

investment. It is the rate of steady-state growth at which capitalists feel they have the right level of capital and do not wish to increase or decrease their capital increment. When capital is behaving as in this case, this rate of growth is interpreted as the "warranted rate of growth" *à la* Harrod.[12] It should be natural that this rate is not the actual rate of growth, or the demand-side growth rate.

Therefore, the problem here is that we have to research the investment function which will determine the actual rate of growth, and the consistent relationship between the warranted and actual rate of growth.

2.2 The Determination of Demand-Side Growth Rate

2.2.1 The Metamorphosis of Capital

All we have to do here is to investigate the Marxian investment function. For that purpose we should reconsider the logic of capital.

What is capital? Capital is an ongoing process oriented to the expansion of its own value, passing through the circuit of its metamorphoses: from money to productive to commodity capital, and back. We can focus on two moments in this movement. One is the expansion of the value, and the other the metamorphosis.

We can interpret the expansion of its value as the objective of the movement of capital. The expansion of value is generally identified with profit maximization.

[12]See Harrod (1939, 1948).

This is true in the case of a static model, but is not correct in the dynamic model we employ here. In a dynamic context, the aim of the movement of capital is to maximize the value of capital in the long run, as measured by the capitalization of the dividends, or in short, the discounted present value of the net profit, which equals the profit minus the capital increment.

On the other hand, what role does metamorphosis play in the movement of capital? In the circuit of capital, the value of capital cannot grow directly from M to M' without passing through the metamorphosis from M to P, P to C and C to M'. The movement of capital cannot fully eliminate such circular restrictions. The metamorphosis of capital is, therefore, interpreted as the constraining conditions for the expansion of its value. We have already formulated these constraints as "circuit conditions."

Now we can formulate the motion of capital as a constrained maximization problem: the objective of capital is to maximize the value of capital, and the constraints are the circuit conditions. Letting V be the value of capital and i be the interest rate, capitalists maximize the following objective function:

$$V = \int_0^\infty (1-s)\pi K(t)e^{-it}\,dt$$

subject to the circuit constraints: (2) and (15). Substituting (2) and (15) to the above objective function and after some manipulation, we get:

$$V = K + \int_0^\infty (\pi(g) - i)K(t)e^{-it}\,dt. \tag{16}$$

The value of capital, V, equals the sum of the volume of capital, K, plus the discounted value of the difference between the profit and the opportunity cost of capital. This discounted value is called the promoter's profit by Hilferding.[13] Therefore, the value of capital equals the sum of the volume of capital plus the promoter's profit.

V should be normalized. Let v be the value of capital per unit of the volume of capital, V/K. Dividing V by K and after some manipulation,[14] we get:

$$v \equiv \frac{V}{K} = 1 + \frac{\pi(g) - i}{i-g} = \frac{\pi(g) - g}{i-g}. \tag{17}$$

[13] See Hilferding (2006, chap. 9).

[14] We assume that $\lim_{t\to\infty} K(t)e^{-it} = 0$. In other words, the discounted value of the volume of capital at infinite horizon approaches zero. And we also assume $i > g$.

Capitalists try to maximize this rate, which is called the valuation ratio by Richard Kahn.[15]

2.2.2 Marxian Investment Function

Setting $v'(g) = 0$, we get the following first-order condition for capital-value maximization:

$$-\pi'(g) = \frac{\pi(g) - i}{i - g} = v - 1. \tag{18}$$

This equation states that the marginal profit rate with respect to the growth rate (MPG) equals the promoter's profit per unit of capital. MPG represents the marginal opportunity cost incurred by the increasing growth rate. The promoter's profit per unit of capital represents a kind of the marginal revenue that an additional growth will bring to capitalists. The growth rate is determined when this equation holds. Therefore we can interpret this equation as a Marxian investment function, which determines the rate of growth. We denote this "demand-side" growth rate as g^{d}.

We shall now look more carefully into how g^{d} is determined. It depends upon $v - 1$, and v depends upon g and i, noting that π depends upon g. Then g^{d} is a negative function of i: $i \to g^{\mathrm{d}}$. [16] If i is an exogenous parameter, then g^{d} is determined as a constant value. But we interpret the interest rate as an unknown variable in this model.

Following Wicksell,[17] we introduce two kinds of interests. One is the natural rate of interest, at which the market equilibrium of supply and demand in the real market is achieved. It is denoted by i^{n}. Second, the money rate of interest is the interest rate in the capital market. It is used to discount the net profit to the present value. It is the same as shown in (16) and (18). It is obvious that if $i < i^{\mathrm{n}}$ then $g^{\mathrm{s}} < g^{\mathrm{d}}$, and vice versa.[18] That is to say, an economy is overheating

[15] See Kahn (1972). This notion is the same as Tobin's q.

[16] Because we can easily get

$$\frac{\mathrm{d}g}{\mathrm{d}i} = \frac{\pi'(g) - 1}{-\pi''(g)(i - g)} < 0.$$

[17] See Wicksell (1898).

[18] Note that there is a certain type of "duality" between the natural-market rates of interest of Wicksell-type model and the warranted-actual rates of growth of the Harrod-type model. Hicks (1965) pointed to a similar correspondence between them.

when the natural rate of interest is higher than the market rate. We cannot discuss whether the scenario of Wicksell's famous "cumulative process" is correct or not, for our analysis concentrates on steady state growth. In other words, it is difficult to derive the (in)stability of equilibrium under capitalism from our model. But it is natural that we ask whether the equilibrium exists or not. Does the market-clearing equilibrium exist?

The answer is, of course, "Yes." Because if $i = i^{\mathrm{n}}$, then $g^{\mathrm{s}} = g^{\mathrm{d}} = g^*$ as illustrated by Figure 4. The synthesized model is constituted by four equations: (2), (3), (15), and (18), and four unknown variables: π, g, ω, and i. In this model, therefore, the four equations (Cambridge equation, generalized wage-profit frontier, conventional wage and Marxian investment function) can be deemed to determine our four variables (profit, growth, real wage and interest rate).

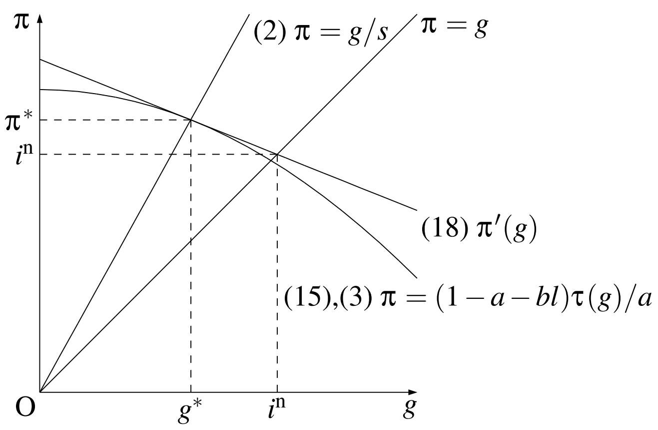

Figure 4: Growth Rate in a Synthesized Model

Concluding Remarks

This paper can provide a microfoundation for the growth model in the heterodox tradition. The wage-profit frontier is derived from the generalized wage-profit frontier if we assume that the rate of capital turnover is unity. The growth-profitability frontier is also derived from the generalized wage-profit frontier. The Keynes-Robinsonian investment function lacks a microfoundation, but the Marxian investment function possesses one. It is derived from the constrained maximization problem: maximizing the capital-value subject to circuit conditions. This is the most general approach to solve the problem of how to close the system.

A synthesized approach is fully compatible with other heterodox approaches. For example, the Marx-Morishima and Keynes-Robinson approaches are not compatible, but a synthesized approach is compatible with both them. It allows both the conventional wage and investment function. This is because it introduces a new unknown variable: the interest rate. Then, four variables are determined by the four equations: the generalized wage-profit frontier, Cambridge equation, conventional wage, and Marxian investment function.

But the introduction of interest can be seen as both a strength and weakness of the approach. Indeed it makes the approach generalized, but it is not general for the interest rate to be determined in the goods market. It implicitly accepts the loanable funds theory, and thus rejects the theory of liquidity preference. At any rate, this approach needs to incorporate the analysis of the financial market. [19]

The approach should also incorporate the labor market. In other words, we have to relax the assumption of the constant real wage. It should be endogenous variable, like the quantities of the supply and demand of labor-power. This means that the natural rate of growth must be introduced into the model.

This approach has another weakness. It concentrates analytical attention on the steady state growth. This eliminates the analytical domain of the instability of capitalism.

The future direction of this study will be one that encompasses these topics.

[19] All of monetary elements in this model, for example G and V, represent quantities of demands for money. In other words, they are not money endowments. Therefore, we have to formulate the structure of money supply. It needs to be clear on whether the model assumes commodity money like gold (in which case the accumulation of the money commodity is indeed part of the capital accumulation) or a pure credit money, in which case the expansion of credit, either through private or public channels, allows aggregate demand to expand. As far as this model concerned, this model is more compatible with the horizontalists' approach than verticalists' approach. Because this model assumes that the interest rate is given exogenous for capitalists. This assumption is similar to that of the horizontalists. See Moore (1988).

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