Numerical Method for Space Harmonic Waves in Polyphase Induction Motors

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Abstract

A numerical method is proposed to solve the problem of space harmonic waves in polyphase induction motors. Results are compared with those of other numerical methods.

1. Introduction

The accurate estimation of the torque produced by electric motors is important for the improvement of various mechanical equipments driven by them. However, magnetic saturation and space harmonic wave effects make the accurate computation of the torque a difficult problem. Of all types of electric motors, the polyphase induction motor is by far the most popular and the most widely used machine. Its space harmonic waves, however, produce abnormal torques and noise [1 to 5].

A numerical method not taking into account space harmonic wave is given in reference [6]. However, if this method is applied to the problem of space harmonic waves in a polyphase induction motor, a very small stepwidth must be selected to obtain accurate results, in spite of its time consuming nature. The reason is that the fundamental equations of polyphase induction motors taking into account the space harmonic waves consist of linear simultaneous differential equations with rapidly varying periodic coefficients. The purpose of this paper is to put forth one simple numerical method to solve these equations with periodic coefficients, quite effectively. The solution obtained by the method takes into account exactly the space harmonic waves and, as a result, describes transient and steady state characteristics of polyphase induction motors well. Furthermore, the method improves and generalizes the computational procedure described in reference [6].

The fundamental equations of polyphase induction motors considering space harmonic waves are introduced in terms of complex objects in section 2 of this paper, whose additional details are given in reference [5].

Section 3 describes the numerical method, where we at first approximate varying coefficients by keeping them constant over small time intervals and perform a direct integration of the fundamental differential equations over the above intervals to obtain analytical solutions.

Section 4 describes numerical results computed by the method of this paper. Comparison is made among various numerical schemes (see Appendix 2).

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2. Mathematical model

The linear simultaneous differential equations of polyphase induction motors are conveniently represented by matrix notations.

They involves the voltage vector $V[t]$ (t denotes the time), the impedance matrix $Z[\nu_m t]$ which is a function of the time t and of the mechanical angular velocity ν_m, and the current vector $I[t]$. The linear simultaneous differential equations with periodic coefficients are

$$V[t]=Z[\nu_m t]I[t]. \qquad (1)$$

The elements of the voltage vector $V[t]$ are the state voltages, whose amplitude are denoted by V_s and angular velocity by ν_s. Then, the voltage vector $V[t]$ (column vector of order 4) is

$$V[t]=\{V_s\exp(j\nu_s t),\ V_s^*\exp(-j\nu_s t),\ 0,\ 0\}, \qquad (2)$$

where the superscript * denotes the conjugate quantities and j is the imaginary unit ($j=\sqrt{-1}$).

The impedance matrix $Z[\nu_m t]$ (square matrix of order 4) consists of the resistance matrix R, the inductance matrix $L[\nu_m t]$ and the torque matrix $G[\nu_m t]$ which can be obtained by differentiating the matrix $L[\nu_m t]$ with respect to the time t and dividing the result by the mechanical angular velocity ν_m. This yields

$$Z[\nu_m t]=R+\nu_m G[\nu_m t]+L[\nu_m t](d/dt). \qquad (3)$$

The resistance matrix R (diagonal matrix of order 4) consists of the stator resistance R_s and the rotor resistance R_r as follows:

$$R=[R_s,\ R_s,\ R_r,\ R_r]. \qquad (4)$$

The inductance matrix $L[\nu_m t]$ is classified into four cases by the relations of numbers of rotor phases and of pole pairs p as described in reference [5]. The numerical method in this paper is applicable to all four cases. In order to illustrate the essential characteristics of the method, only a special case is treated as an example, where only the 19th space harmonic wave is taken into account. This example, at first, is practically useful because the 19th space harmonic wave has a big contribution to abnormal torque and then is theoretically very interesting as shown in Appendix 1. In this case, the elements of $L[\nu_m t]$ are the stator self-inductance L_s, the rotor self-inductance L_r, the mutual inductance of the fundamental wave M_1 and the mutual inductance of 19th space harmonic wave M_{19}. The inductance matrix $L[\nu_m t]$ (square matrix of order 4) is

$L[\nu_m t]=$

L_s	0	$M_1\exp(j\nu_m t)$	$M_{19}\exp(j19\,\nu_m t)$
0	L_s	$M_{19}\exp(-j19\,\nu_m t)$	$M_1\exp(-j\nu_m t)$
$M_1\exp(-j\nu_m t)$	$M_{19}\exp(j19\,\nu_m t)$	L_r	0
$M_{19}\exp(-j19\,\nu_m t)$	$M_1\exp(j\nu_m t)$	0	L_r

(5)

The torque matrix $G[\nu_m t]$ (square matrix of order 4) is given by

$$G[\nu_m t]=(\mathrm{d}/\mathrm{d}t)L[\nu_m t](1/\nu_m). \quad \cdots\cdots (6)$$

The current vector $I[t]$ (column vector of order 4) consists of the stator positive phase current i_{sp}, the stator negative phase current i_{sn}, the rotor positive phase current i_{rp} and the rotor negative phase current i_{rn}, viz.

$$I[t]=\{i_{sp}, i_{sn}, i_{rp}, i_{rn}\}. \quad \cdots\cdots (7)$$

The torque T is represented by the terms of the current vector $I[t]$, the torque matrix $G[\nu_m t]$ and the number of pole pairs p.

Namely, the torque is given as the real part of the following equation:

$$T=(p/2)I^{*t}[t]G[\nu_m t]I[t], \quad \cdots\cdots (8)$$

where superscript * and t denote the conjugate and the transpose of the current vector $I[t]$.

For mathematical convenience, the complex stator currents i_a, i_b, i_c are defined by the following equation whose real parts give the original coordinate stator currents (in three phase stator circuits).

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \exp(-j2\pi/3) & \exp(-j4\pi/3) \\ 1 & \exp(-j4\pi/3) & \exp(-j2\pi/3) \end{bmatrix} \cdot \begin{bmatrix} 0 \\ i_{sp} \\ i_{sn} \end{bmatrix}, \quad \cdots\cdots (9)$$

where the stator phases are represented by suffixes a, b and c.

3. Numerical method of solution

By mean of Eq. (3), Eq. (1) can be rewritten

$$(\mathrm{d}/\mathrm{d}t)I[t]=S[t]I[t]+U[t], \quad \cdots\cdots (10)$$

$S[t]$ is a square matrix of order 4 and $U[t]$ is a column vector of order 4. $S[t]$ and $U[t]$ are given by

$$S[t]=-L^{-1}[\nu_m t](R+\nu_m G[\nu_m t]), \quad \cdots\cdots (11)$$

$$U[t]=L^{-1}[\nu_m t]V[t]. \quad \cdots\cdots (12)$$

A formal solution of Eq. (10) can be obtained as

$$I[t]=P[t]I[0]+P[t]\int_0^t P^{-1}[\tau]U[\tau]\mathrm{d}\tau, \quad \cdots\cdots (13)$$

where the matrix $I[0]$ and $P[t]$ are respectively the initial values of current vector (cloumn vector of order 4) and the state transition matrix (square matrix of order 4). The state transition matrix $P[t]$ must satisfy the following equation

$$(d/dt)P[t]=S[t]P[t]. \quad \cdots\cdots (14)$$

The initial conditions of Eq. (14) are

$$P[0]=I_4, \quad \cdots\cdots (15)$$

where I_4 denotes the unit matrix of order 4.

The state transition matrix $P[t]$ is of paramount importance in the solution of Eq. (10). A staisfactory solution of Eq. (10) depends on finding efficient methods of determining the state transition matrix $P[t]$. Various numerical methods have been proposed to find $P[t]$ (see reference [7]).

As the principal purpose of this paper is to solve Eq. (10) by the most simple numerical method. It is assumed that the elements of the matrices $S[t]$ and $U[t]$ of Eq. (10) keep constant values in a small time interval h. Then, Eq. (10) can be solved as follows:

$$I[t+h]=\exp(S[t+Ah]h)I[t]-\{I_4-\exp(S[t+Ah]h)\}S^{-1}[t+Ah]U[t+Ah], \cdots (16)$$

where value for the parameter A in Eq. (16) will be given later.

A further approximation is applied to the matrix exponential function $\exp(S[t+Ah]h)$, namely

$$\exp(S[t+Ah]h)=\{I_4-(h/2)S[t+Ah]\}^{-1}\{I_4+(h/2)S[t+Ah]\}. \cdots\cdots (17)$$

The state transition matrix $P[t+h]$ is approximated by the central difference method as follows:

$$P[t+h]=\{I_4-(h/2)S[t+Ah]\}^{-1}\{I_4+(h/2)S[t+Ah]\}P[t]. \cdots\cdots (18)$$

To determine the value of the parameter A, Eq. (18) is rewritten by using a Taylor series expansions of the matrices $S[t+Ah]$ and $\{I_4-(h/2)S[t+Ah]\}^{-1}\{I_4+(h/2)S[t+Ah]\}$. These Taylor series expansions are

$$\begin{aligned}S[t+Ah]=S[t]+&Ah\,dS[t]/dt+(Ah)^2(1/2)d^2S[t]/dt^2+\\&+\cdots\cdots+(Ah)^k(1/k!)d^kS[t]/dt^k,\end{aligned} \quad \cdots\cdots (19)$$

and

$$\begin{aligned}\{I_4-(h/2)S[t+Ah]\}^{-1}\{I_4+(h/2)S[t+Ah]\}&=I_4+hS[t+Ah]+\\&+(h^2/2)S^2[t+Ah]+\cdots\cdots+(h^k/2^{k-1})S^k[t+Ah].\end{aligned} \quad \cdots\cdots (20)$$

Therefore, Eq. (18) can be rewritten as follows:

$$\begin{aligned}P[t+h]=\{I_4+hS[t]&+(h^2/2)(S^2[t]+2A\,dS[t]/dt)+(h^3/6)(1.5S^3[t]\\&+3A^2d^2S[t]/dt^2+3AS[t]dS[t]/dt+3A\,dS[t]/dt\,S[t])+\cdots\\&\cdots\cdots\}P[t].\end{aligned} \quad \cdots\cdots (21)$$

A rigorous Taylor series expansion of the state transition matrix $P[t+h]$ is described in referenc [7]. It is

$$\begin{aligned}P[t+h]=\{I_4+hS[t]&+(h^2/2)(S^2[t]+dS[t]/dt)+(h^3/6)(S^3[t]+\\&d^2S[t]/dt^2+S[t]dS[t]/dt+2dS[t]/dt\,S[t])+\cdots\cdots\}P[t].\end{aligned} \quad \cdots (22)$$

From a comparison of Eq. (21) with Eq. (22), because with this value $A=\frac{1}{2}$ the approximate state transition matrix Eq. (21) coincides with the rigorous expansion Eq. (22) up to the first three terms, the following value of the parameter A is shown as the most suitable,

$$A=1/2. \qquad (23)$$

To summarise this section, the formula for the linear simultaneous differential equations with periodic coefficients is given as

$$I[t+h]=\{1_4-(h/2)S[t+Ah]\}^{-1}\{(1_4+(h/2)S[t+Ah])I[t]+hU[t+Ah]\}. \qquad (24)$$

If the parameter A in Eq. (24) is 1/2, then the approximate state transition matrix Eq. (21) coincides with the rigorous expansion Eq. (22) in the first three terms. This method is called the "Improved central difference method". If the parameter A in Eq. (24) is 1, then this method is the same as described in reference [6] (the central difference method) and the approximate state transition matrix Eq. (21) matches the frist two terms of the rigorous expansion Eq. (22). In the case of linear simultaneous differential equations with constant coefficients, the formula denoted by Eq. (24)($A=1/2$) is reduced to the central difference method.

4. Numerical solution

The various parameters of a motor used in the numerical examples are listed in Table 1.

Table 1
Various constants of the calculated motor

Voltage	$V_s=\sqrt{2/3}\,200$	(V)
Currents	Initial currents are all zero.	
Angular velocities	$\nu_s=100\pi$	(rad/sec)
	$\nu_m=90\pi$	(rad/sec)
Resistances	$R_s=R_r=5$	(Ω)
Inductances	$L_s=L_r=0.31831$	(H)
	$M_1=0.30239$	(H)
	$M_{19}=0.30239/(19\times 19)$	(H)

When we consider only the 19th space harmonic wave, the fundamental equations can be reduced to a set of linear differential equations with constant coefficients, as shown in Appendix 1. To solve the coupled linear differential equations with constant coefficients, the Páde approximation method is known to be quite effective, although this method is not applicable to general problems of the polyphase induction motor including other space harmonic waves. Therefore, we obtain most accurate values for theoretical comparison by this method with very small stepwidth.

Eq. (1) is numerically solved respectively by the improved central difference method, the central difference method, the trapezoidal rule and the Páde approximation method (h=0.000001). A description of these methods is given in the Appendix 2.

The results of the numerical solution without considering the 19th space harmonic waves are shown in Fig. [1], fairly good results are obtained by any numerical method by using a small stepwidth (h=0.00001 or 0.0001). However, it is obvious that the improved central difference method is one of the most effective numerical methods.

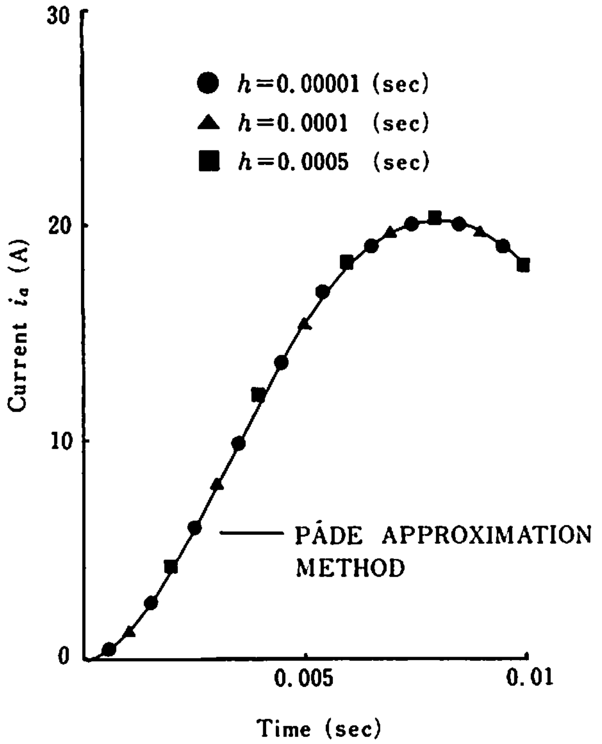

Fig. 1-a

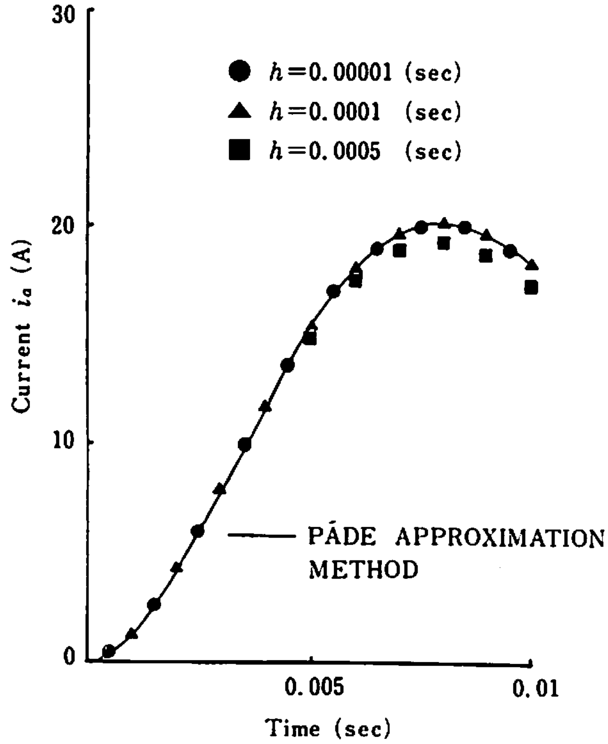

Fig. 1-b

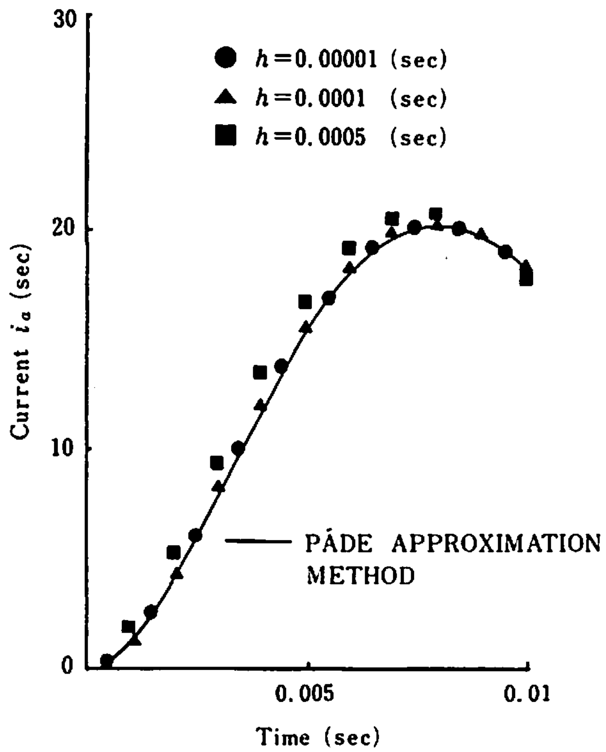

Fig. 1-c

Fig. 1. Comparison of the results not considering 19th space harmonic wave computed by various numerical methods:
(a) Improved central difference method.
(b) Trapezoidal rule.
(c) Central difference method.

When the 19th space harmonic wave is taken into account, the same conclusions are obtained by the results of Eq. (1) which are shown in Fig. [2].

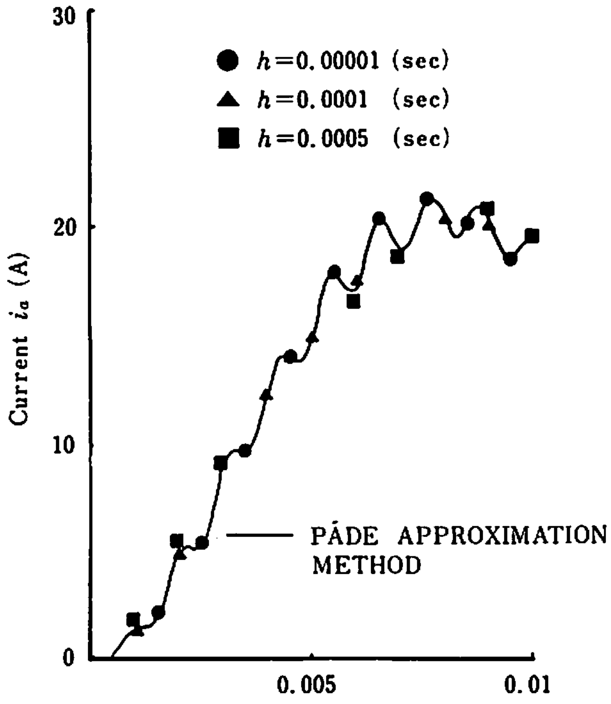

Fig. 2-a

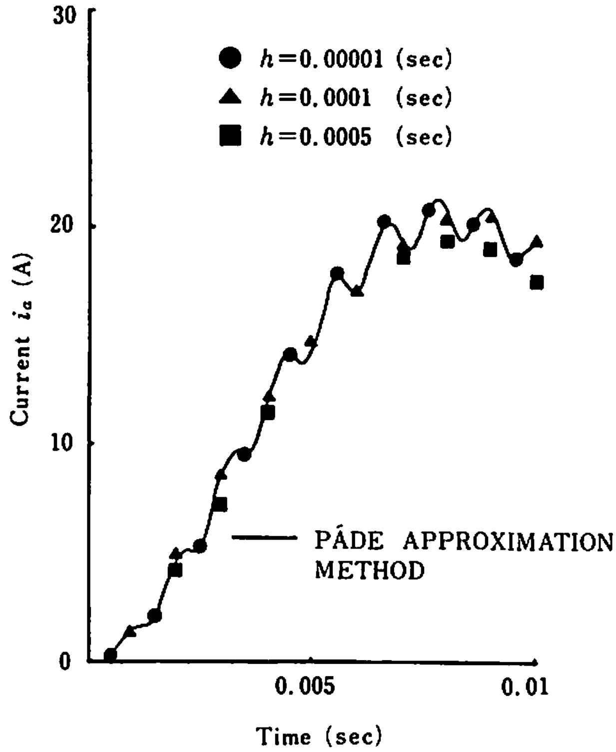

Fig. 2-b

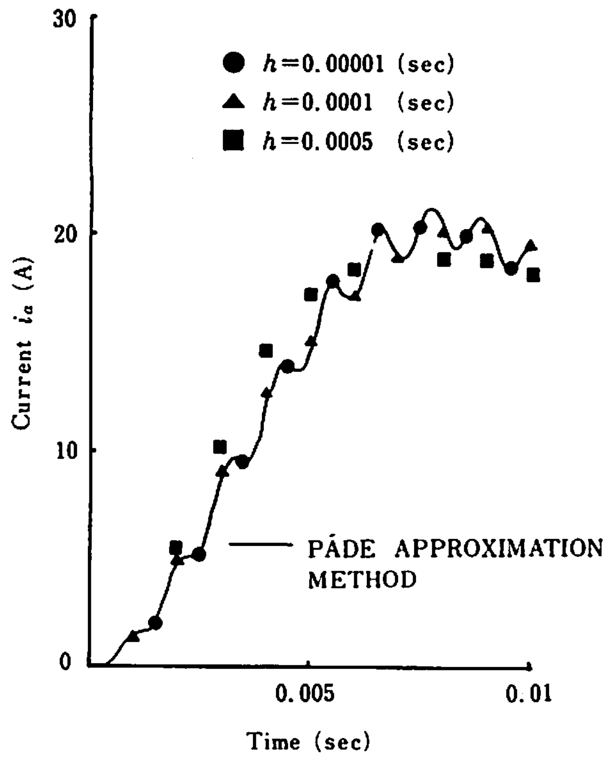

Fig. 2-c

Fig. 2. Comparison of the results taking into account the 19th space harmonic wave:
(a) Improved central difference method.
(b) Trapezoidal rule.
(c) Central difference method.

The state transition matrices of the rigorous Taylor series method Eq. (22), the improved central difference method, the trapezoidal rule and the central difference method are shown in Table 2. Table-2 shows that the state transition matrix of the central difference method is inferior in accuracy to the other methods. The third term in the state transition matrix of the improved central difference method is somewhat more accurate than the third term of the trapezoidal rule. Furthermore, the trapezoidal rule needs more terms such as $S[t]$ or $S[t+h]$ and $U[t]$ or $U[t+h]$. Therefore, the improved central difference method is superior to the others.

Table 2

Comparison of the approximate and the rigorous state transition matrix

Rigorous Taylor series expansion

$$P[t+h]=\{I_4+hS[t]+(h^2/2)\langle S^2[t]+\mathrm{d}S[t]/\mathrm{d}t\rangle+(h^3/6)\langle S^3[t]+\mathrm{d}^2S[t]/\mathrm{d}t^2 + S[t]\mathrm{d}S[t]/\mathrm{d}t+2(\mathrm{d}S[t]/\mathrm{d}t)S[t]\rangle+\cdots\}P[t]$$

Improved central difference method

$$P[t+h]=\{I_4+hS[t]+(h^2/2)\langle S^2[t]+\mathrm{d}S[t]/\mathrm{d}t\rangle+(h^3/6)\langle 1.5S^3[t] +0.75\mathrm{d}^2S[t]/\mathrm{d}t^2+1.5S[t]\mathrm{d}S[t]/\mathrm{d}t+1.5(\mathrm{d}S[t]/\mathrm{d}t)S[t]\rangle +\cdots\}P[t]$$

Trapezoidal rule

$$P[t+h]=\{I_4+hS[t]+(h^2/2)\langle S^2[t]+\mathrm{d}S[t]/\mathrm{d}t\rangle+(h^3/6)\langle 1.5S^3[t] +1.5\mathrm{d}^2S[t]/\mathrm{d}t^2+1.5S[t]\mathrm{d}S[t]/\mathrm{d}t+3(\mathrm{d}S[t]/\mathrm{d}t)S[t]\rangle +\cdots\}P[t]$$

Central difference method

$$P[t+h]=\{I_4+hS[t]+(h^2/2)\langle S^2[t]+2\mathrm{d}S[t]/\mathrm{d}t\rangle+(h^3/6)\langle 1.5S^3[t] +3\mathrm{d}^2S[t]/\mathrm{d}t^2+3S[t]\mathrm{d}S[t]/\mathrm{d}t+3(\mathrm{d}S[t]/\mathrm{d}t)S[t]\rangle +\cdots\}P[t]$$

Some examples of numerical solutions Eq. (1) and Eq. (8) computed by the improved central difference method are shown in Fig. [3].

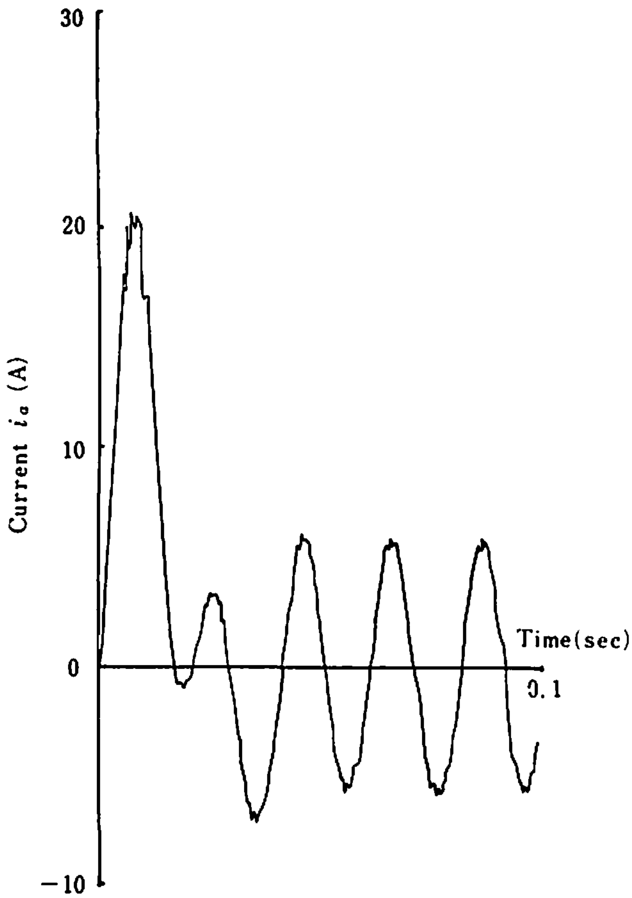

Fig. 3-a

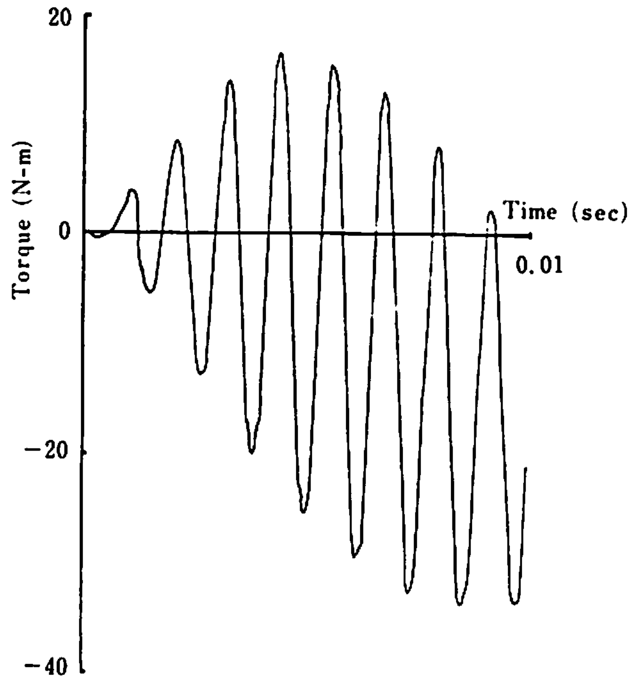

Fig. 3-b

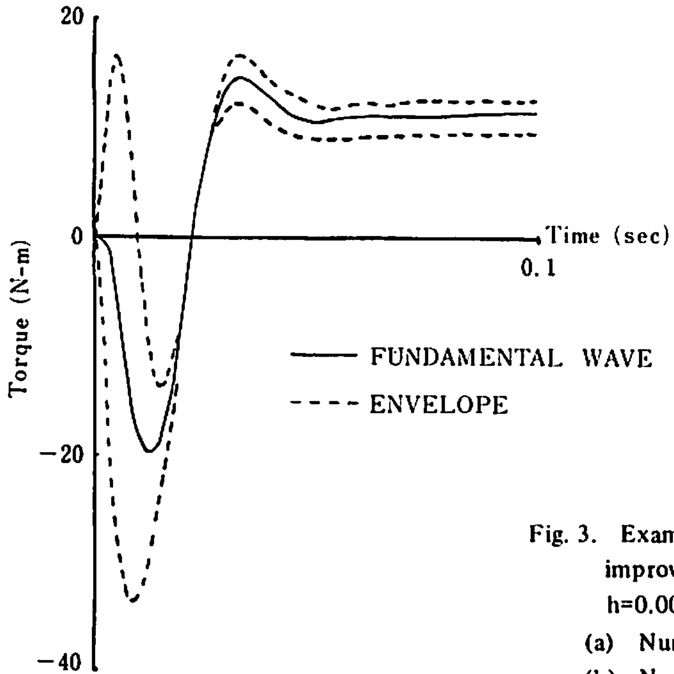

Fig. 3-c

Fig. 3. Examples of the numerical solutions computed by the improved central difference methods, using the stepwidth h=0.001.

(a) Numerical solution of the stator current

(b) Numerical solution of the torque.

(c) Numerical solution of the torque (the envelop in figure shows the region of the torque vibration due to the 19th space harmonic wave).

5. Conclusion

As shown above, this paper has proposed a simple and effective method applicable to the problem of space harmonic waves in polyphase induction motors. This method is superior in accuracy to the trapezoidal rule in spite of its simpler algorithmic form. A stepwidth can be chosen here by about 20 times as large as the one necessary in the method reported in reference [6]. It is an improvement on the method reported in reference [6].

For further study, the author plans to work out another numerical method for space harmonic waves in polyphase induction motors, taking into account the full system of mechanical equations.

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The practical computations given in this paper were carried out by using the computers FACOM 230-45S of Hosei University and HITAC 8800/8700 of Tokyo University.

Appendix 1. Linearized mathematical model

Eq. (1) in section 2 can be transformed into the linear simultaneous differential equations with constant coefficients by using the matrix $C[\nu_m t]$ (commutation matrix) for space harmonic waves. The commutation matrix $C[\nu_m t]$ (diagonal matrix of order 4) for the 19th space harmonic wave is

$$C[\nu_m t]=\ulcorner 1,\ \exp(-\mathrm{j}20\,\nu_m t),\ \exp(-\mathrm{j}\,\nu_m t),\ \exp(-\mathrm{j}19\,\nu_m t)\lrcorner. \cdots\cdots\cdots \text{(A-1)}$$

The linear simultaneous differential equations with constant coefficients are written as follows:

$$V^c[t]=Z^c I^c[t]. \cdots\cdots\cdots \text{(A-2)}$$

The transformed voltage vector (column vector of order 4) $V^c[t]=C^*[\nu_m t]\,V[t]$ is

$$V^c[t]=\{V_s\exp(\mathrm{j}\nu_s t),\ V_s^*\exp(-\mathrm{j}\nu_s t+\mathrm{j}20\nu_m t),\ 0,\ 0\}. \cdots\cdots\cdots \text{(A-3)}$$

The transformed impedance matrix $Z^c=C^*[\nu_m t]Z[\nu_m t]C[\nu_m t]$ (square matrix of order 4) consists of the resistance matrix R, the transformed inductance matrix L^c and the transformed torque matrix G^c, namely

$$Z^c=R+\nu_m G^c+L^c(\mathrm{d}/\mathrm{d}t), \cdots\cdots\cdots \text{(A-4)}$$

where the resistance matrix R is the same as Eq. (4), and the transformed inductance matrix L^c (square matrix of order 4) is

$$L^c = \begin{array}{|c|c|c|c|}\hline L_s & 0 & M_1 & M_{19} \\ \hline 0 & L_s & M_{19} & M_1 \\ \hline M_1 & M_{19} & L_r & 0 \\ \hline M_{19} & M_1 & 0 & L_r \\ \hline \end{array} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots (A\text{-}5)$$

The transformed torque matrix G^c (square matrix of order 4) is

$$G^c = \begin{array}{|c|c|c|c|}\hline 0 & 0 & 0 & 0 \\ \hline 0 & -j20L_s & -j20M_{19} & -j20M_1 \\ \hline -jM_1 & -jM_{19} & -jL_r & 0 \\ \hline -j19M_{19} & -j19M_1 & 0 & -j19L_r \\ \hline \end{array} . \quad \cdots\cdots\cdots (A\text{-}6)$$

As each current in the current vector $I[t]$ is an unknown quantity which must be computed, the transformed current vector $I^c[t]=C^*[\nu_m t]I[t]$ is represented by the new stator positive phase current i^c_{sp}, the new stator negative phase current i^c_{sn}, the new rotor positive phase current i^c_{rp} and the new rotor negative phase current i^c_{rn}. The transformed current vector $I^c[t]$ (column vector of order 4) is

$$I^c[t]=\{i^c_{sp},\ i^c_{sn},\ i^c_{rp},\ i^c_{rn}\}. \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (A\text{-}7)$$

The torque T is represented by the transformed current vector $I^c[t]$, the transformed torque matrix G^c and the number of pole pairs p. Namely, the torque T is given as the real part of the following equation:

$$T=pI^{ct*}[t]G^cI^c[t]. \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (A\text{-}8)$$

Of course, the real part of Eq. (A-8) equals the real part of Eq. (8).

The original coordinate stator currents (in three phase circuits) are derived as the real part of the following equation:

$$\begin{array}{|c|}\hline i_a \\ \hline i_b \\ \hline i_c \\ \hline \end{array} = \frac{1}{\sqrt{3}} \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & \exp(-j2\pi/3) & \exp(-j4\pi/3) \\ \hline 1 & \exp(-j4\pi/3) & \exp(-j2\pi/3) \\ \hline \end{array} \begin{array}{|c|}\hline 0 \\ \hline i^c_{sp} \\ \hline i^c_{sn}\exp(-j20\nu_m t) \\ \hline \end{array} \quad \cdots (A\text{-}9)$$

Appendix 2. Other numerical methods

1) Trapezoidal rule: Application of the trapezoidal integral formula to Eq.(10) in section 3 yields

$$I[t+h]-I[t]=(h/2)(S[t+h]I[t+h]+S[t]I[t]+U[t+h]+U[t]). \cdots\cdots (A\text{-}10)$$

Therefore, the following numerical solution of Eq. (10) can be obtained

$$\begin{aligned} I[t+h]=&\{I_4-(h/2)S[t+h]\}^{-1}\{I_4+(h/2)S[t]\}I[t] \\ &+\{I_4-(h/2)S[t+h]\}^{-1}(h/2)\{U[t+h]+U[t]\}. \end{aligned} \quad \cdots\cdots\cdots (A\text{-}11)$$

In this case, the state transition matrix $P[t+h]$ is approximated as follows:

$$\begin{aligned} P[t+h]=&\{I_4-(h/2)S[t+h]\}^{-1}\{I_4+(h/2)S[t]\}P[t] \\ \text{or}\quad =&\{I_4+hS[t]+(h^2/2)(S^2[t]+dS[t]/dt)+ \\ &+(h^3/6)(1.5S^3[t]+1.5d^2S[t]/dt^2+3(dS[t]/dt)S[t] \\ &+1.5S[t]dS[t]/dt)+\cdots\}P[t]. \end{aligned} \quad \cdots\cdots\cdots (A\text{-}12)$$

2) Páde approximation method: From Eq. (16) the numerical solution of Eq. (A-2) can be obtained as follows:

$$I^c[t+h]=\exp(S^c h)I^c[t]-\{I_4-\exp(S^c h)\}S^{c-1}U^c[t+0.5h] \quad \cdots\cdots \text{(A-13)}$$

where the parameter A in Eq. (16) is conveniently selected to be 1/2, and the matrices in Eq. (A-13) are

$$S^c=-L^{c-1}(R+\nu_m G^c), \quad \cdots\cdots \text{(A-14)}$$

$$U^c[t]=L^{c-1}V^c[t]. \quad \cdots\cdots \text{(A-15)}$$

The matrix exponential function $\exp(S^c h)$ in Eq. (A-13) is approximated by a Páde approximation [8], that is

$$\exp(S^c h)=\{12-6S^c h+(S^c h)^2\}^{-1}\{12+6S^c h+(S^c h)^2\}$$

$$\text{or} \quad =I_4+hS^c+(hS^c)^2/2+(hS^c)^3/6+(hS^c)^4/24 \quad \cdots\cdots \text{(A-16)}$$

$$+\cdots\cdots+.$$

As the righthand term of Eq. (A-16) is a relatively good approximation of the matrix exponential fucntion in Eq. (A-13), it is expected that the numerical solutions computed by the method of Eq. (A-13) using the Eq. (A-16) will yield fairly good results.

Notations

$V[t]$	$=\{V_s\exp(j\nu_s t),\ V_s^*\exp(-j\nu_s t), 0, 0\}$, voltage vector
$I[t]$	$=\{i_{sp},\ i_{sn},\ i_{rp},\ i_{rn}\}$, current vector
$Z[\nu_m t]$	$=R+\nu_m G[\nu_m t]+L[\nu_m t](d/dt)$, impedance matrix
R	$=\lceil R_s, R_s, R_r, R_r \rfloor$, resistance matrix
$L[\nu_m t]$	= inductance matrix
$G[\nu_m t]$	$=(d/dt)L[\nu_m t](1/\nu_m)$, torque matrix
$S[t]$	$=-L^{-1}[\nu_m t](R+\nu_m G[\nu_m t])$, coefficient matrix of the differential state equation
$U[t]$	$=L^{-1}[\nu_m t]V[t]$, input vector of the differential state equation
$P[t]$	= state transition matrix
$C[\nu_m t]$	= commutation matrix of the 19th space harmonic wave
$V^c[t]$	$=C^*[\nu_m t]V[t]$, transformed voltage vector
$I^c[t]$	$=\{i_{sp}^c,\ i_{sn}^c,\ i_{rp}^c,\ i_{rn}^c\}$, transformed current vector
Z^c	$=C^*[\nu_m t]Z[\nu_m t]C[\nu_m t]$(or$=R+\nu_m G^c+L^c d/dt$), transformed impedance matrix
L^c	= transformed inductance matrix
G^c	= transformed torque matrix
S^c	$=-L^{c-1}(R+\nu_m G^c)$, coefficient matrix of the linear differential state equation
$U^c[t]$	$=L^{c-1}V^c[t]$, input vector of the linear differential state equation
$I[0]$	= initial current vector
T	= torque (*N-m*)
t	= time (sec)
h	= stepwidth (sec)
ν_m	= mechanical angular velocity transformed into electrical angular velocity (rad/sec)
ν_s	= angular velocity of the impressed voltage source
V_s	= amplitude of the stator impressed voltage

p = number of pole pairs
j $=\sqrt{-1}$, imaginary unit
k = positive integer
I_4 = unit matrix of order 4
R_s, R_r refer to the stator and rotor resistances, respectively.
L_s, L_r refer to the stator and rotor self-inductances, respectively.
M_1, M_{19} refer to the fundamental and 19th space harmonic wave mutual inductances, respectively.

Superscripts *, t, -1 and c refer to the conjugate, transpose of matrix, inverse matrix and transformed matrix by commutation matrix $C[\nu_m t]$, respectively.
Subscripts a, b, c refer to the stator a-phase, b-phase, c-phase quantities; s r refer to the stator and rotor; and p, n refer to the positive and negative phase quantities, respectively.

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